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Dispositions and the Principle of Least Action

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1. Introduction

My aim is to argue for the incompatibility of one of the central principles of physics, namely the *principle of least action* (PLA), with the increasingly popular view that the world is, ultimately, merely something like a conglomerate of objects and irreducible dispositions. First, I argue that the essentialist implications many suppose this view has are not compatible with the PLA. Second, I argue that, irrespective of whether this view has any essentialist implications, it is not compatible with the kind of explanation that the PLA affords.

2. Dispositionalism

Dispositional properties are, unlike categorical properties, here supposed to be properties that are not wholly manifest in the present. *Dispositionalism* is the view that the world is ultimately a conglomerate of objects and dispositions.[1,2] On this view, there is some level of true description of objects in the world that ascribes to them dispositions that are fundamental, that is to say that are not explicable in categorical terms. Moreover, change is said to consist in the manifestation of these

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[1] Supporters of dispositionalism include, among others, R. Harré and E. H. Madden (1975), J. Fetzer (1977), S. Shoemaker (1984), Cartwright (1989), S. Mumford (1998) and B. Ellis (2001).

[2] Some dispositionalists would also postulate processes along with their dispositions. Nothing will turn upon this here.

dispositions in response to appropriate prompting. Dispositions are thus supposed to be the ultimate ontological units that explain events.[3]

One of the advantages that dispositionalism is said to have over its rivals is its ability to account for the metaphysical presuppositions of science. Many non-dispositionalist accounts of laws of nature identify such laws with species of general truths.[4] However, Cartwright has argued that nature rarely manifests instances of fundamental general laws found in science and thus that laws construed as general statements are, for the most part, merely idealizations and abstractions that should not be regarded as true (1983). Consider, for example, Galileo's law of fall: whenever an object is released near the earth's surface, it falls with uniform acceleration. If we interpret this as a general truth, it comes out false. If, by contrast, we interpret it as describing objects' dispositions, it may yet be true. For even where an object does not (due to interfering factors) fall with uniform acceleration, it may be disposed to do so. It may not simply because dispositions typically act in conjunction in complex ways.

However, I want to argue that a dispositionalist ontology is not able to account for the metaphysical presuppositions of science. Specifically, I want to argue that dispositionalism is not compatible with the ontological presuppositions of the PLA.[5]

3. The Principle of Least Action

For each physical system, there is a quantity, the system's *action*, that depends on the way the system evolves. The PLA is the principle that a system's actual evolution is (and hence its equations of motion are) such that the system has a quantity of action

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[3] Dispositionalism does not, by itself, preclude supposing that there are fundamental categorical properties and relations. For example, the dispositionalist may think of spatio-temporal relations in categorical terms.

[4] See, for example, P. F. Ramsey (1978) and D. Lewis (1983).

[5] A. Chalmers also argues that dispositionalism is not able to account for the metaphysical presuppositions of science, though he does so on different grounds (1999).

that is an extremum relative to other quantities of action it might have had. Given the PLA, and once the action of a system has been formulated, we can deduce its equations of motion.

For example, in the case of a classical mechanical system of particles, the action between two moments in time, t_1 and t_2, is given by:

$$(1)\, S = \int_{t1}^{t2} L(q,\dot{q})dt$$

L is the system's *Lagrangian* and is, in such a system, a function of the coordinates, q, and the velocities, $\dot{q}$, of the particles in the system. L is also a function of all the other intrinsic properties ascribed to the objects in the system by classical mechanics, that is to say of properties such as charge and mass. However, unlike q and $\dot{q}$, these intrinsic properties are not supposed to be time dependent. In closed systems, that is to say systems that do not interact with others, L simply equals $T – V$, where T is the system's kinetic energy and V its potential energy.

The equations of motion of a system of particles between coordinates q_i at t_1 and coordinates q_f at t_2 can be derived from (1) by formulating L for the system and then applying the PLA, that is to say by formulating L for the system and then requiring that the system's action take on an extremum. Mathematically, this requirement is met when the variation of the action integral, δS, equals zero.

Thus, the PLA tells us that, from among all the possible paths of the particle or particles of a given closed classical mechanical system, the actual path or paths will be such that the integral over time of the difference between the system's kinetic and potential energy is an extremum. Indeed, for short enough paths, the extremum in

question will be a minimum and the evolution of the particle or particles between two points will be the 'laziest' possible (e.g., in the case of a single particle, its path will be a straight line).

Typically, physicists formulate an action integral for physical systems of certain types, say for the systems of classical mechanics or of General Relativity. They then derive equations of motion from this integral. They do not typically employ the PLA in the general form given here, that is to say in a form that makes a claim about all types of physical systems. I have nevertheless done so since the arguments to be developed below apply irrespective of the particulars of this or that formulation of the PLA. Moreover, since these arguments do apply across the board, they can easily be modified so as to appeal only to this or that restricted version of the PLA.

Traditionally, the PLA has been thought to have philosophical significance because it seems to suggest that physical systems are governed by final causes.[6] As in the case of a classical mechanical system, the quantity of action of a physical system depends on the whole of the system's evolution. Thus, if, as the PLA states, the equations of motion of an actual physical system depend upon its action, the evolution of such a system seems to be determined at the outset by its later states. Here, however, I assume that the supposition of the PLA does not involve the supposition of final causes, and proceed to examine its compatibility with dispositionalism.

4. The PLA and the Essentialist Implications of Dispositionalism

Dispositional essentialism about fundamental objects is the view that the dispositions of objects that have no internal structure are essential to such objects' being the kinds of objects they are. On this view, the dispositions of an object such as an electron are

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[6] See M. Stölzner (2003), and W. Yourgrau and S. Mandelstam (1968: ch. 14) for historically oriented discussions of the PLA.

essential to its being an electron. Now, dispositionalism is sometimes thought to imply dispositional essentialism about fundamental objects. Mumford, for example, argues that if, as the dispositionalist maintains, an electron is an electron solely in virtue of its dispositions to behave, anything that does not exhibit this kind of disposition to behaviour wouldn't be an electron (1998: 234). Ellis endorses the same argument (1999: 27). In this section I argue that, if Mumford and Ellis are correct, the PLA is not compatible with dispositionalism.

Consider a classical mechanical system that is comprised solely of a free point particle, a, with mass m and, at t_1, a velocity $\vec{v}_1$ and location $\vec{x}_1$. The action integral for this system is:

$$(2) \quad S = \frac{1}{2} \int_{t1}^{t2} mv^2 \, dt$$

The PLA tells us that among the equations of motion that might have described a's motion after t_1 those that will actually do so are such that S will be an extremum relative to quantities it might have been. But, the variation in S, and the corresponding variation in equations of motion that is thereby assumed to be possible, are variations that supposedly occur without changes to the physical system at t_1. In particular, it is assumed that different equations of motion could have described the evolution of a from t_1 onwards even given $\vec{v}_1$, m and $\vec{x}_1$. Thus, given the PLA, the equations of motion that describe the envisaged physical system are not made true merely by the intrinsic properties of the particle that comprises it along with the initial conditions in which it is found. However, if both dispositionalism and dispositional essentialism about fundamental objects are supposed, we must suppose that the equations of

motion that describe the envisaged system are made true by a's intrinsic properties. For if, as this combined view implies, the dispositions of a are essential to its being the kind of object it is, a's intrinsic properties fully determine which dispositions it possesses. But then, since the dispositions of the objects within a system supposedly determine how the system will evolve and hence which equations of motion describe it, the fundamental objects that comprise a physical system, along with their intrinsic properties, will supposedly determine how the system will evolve and which equations of motion describe it.[7]

I have assumed that if the dispositions of a are essential to its being the kind of object it is, a's intrinsic properties fully determine which dispositions it possesses. This follows from an assumption that dispositional essentialists make, namely that the intrinsic properties of a fundamental object fully determine which kind of object it is. To be sure, the dispositional essentialist might suggest instead that which kind of object a fundamental object is, and hence which dispositions it possesses, also depends on its history. Doing so, however, would not affect my argument. The variation in a's equations of motion that the PLA presupposes is possible at t_1 is presupposed to be possible irrespective of a's history prior to t_1. Thus, given the PLA, even a's intrinsic properties along with its history do not suffice to fix which equations of motion correctly describe its motion.

A different version of dispositional essentialism about fumdamental objects, one that takes its cue from Shoemaker's dispositional essentialism (1984), makes no use of the notion of kinds. On this version, dispositional essentialism about fundamental objects is the view that dispositions of objects are essential to such

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[7] Classical mechanics also allows the ascription of an acceleration to the envisaged particle, even though this quantity does not feature in the action integral. But since this additional property does not feature in the action integral, it is not relevant to determining the equations of motion that describe the system.

objects' possessing the intrinsic properties they possess. Dispositonalism combined with dispositional essentialism would then immediately imply, contrary to the PLA, that to specify the intrinsic properties of *a*, along with attending circumstances, is to specify its dispositions and hence its equations of motion.

To return to my main argument, its point is a general one. The PLA presupposes that the action of any given physical system could have taken various values, and thus that any such system could have been correctly described by different equations of motion, even once the objects that comprise it, along with their intrinsic properties and initial distribution, are determined.[8] However, given both dispositionalism and dispositional essentialism about fundamental objects, the equations of motion of a physical system must be made true solely by the objects that comprise it along with their intrinsic properties and corresponding dispositions.

Let me clarify the role dispositional essentialism about fundamental objects has in the above argument. If the dispositionalist rejects dispositional essentialism about fundamental objects, he allows that merely specifying that the objects of a physical system are of a certain kind, and hence that they have certain intrinsic properties, does not imply that they have certain dispositions. He could then accept, in accord with the PLA, that the kinds, and hence intrinsic properties, of the objects within a physical system do not suffice to make true the equations of motion that describe their motion. If, however, the dispositionalist assumes dispositional essentialism about fundamental objects, he must accept that it is not possible for an object to possess the intrinsic properties it possesses without possessing certain corresponding dispositions. Thus, he must accept, contrary to the PLA, that the

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[8] The term object is here used in a broad enough sense so as to include not only particles but also fields, strings and the like.

intrinsic properties of the objects that comprise a physical system do suffice to make true the equations of motion that correctly describe it.

I conclude that the combination of dispositionalism and dispositional essentialism about fundamental objects is not compatible with the PLA. Thus, if dispositionalism implies dispositional essentialism about fundamental objects, dipsositionalism is not compatible with the PLA. If dispositionalism does not imply dispositional essentialism about fundamental objects, dispositionalism and the PLA may yet be compatible. Whether this is the case depends on whether or not, given the PLA, dispositions can be fundamental. We have seen that descriptions of the intrinsic properties and initial circumstances of the objects of a physical system, taken together with the PLA and an appropriate action integral, imply that the objects in question have certain equations of motion and thus, for the dispositionalist, certain dispositions. This opens the door to the suggestion that, if the PLA is true, dispositions are not fundamental in that they supervene on the intrinsic properties of objects taken together with whatever makes the PLA true. In the following section, I argue that this is indeed the case.[9]

5. The Explanatory Force of the PLA

If the PLA is true, there is an explanation of why certain equations of motion rather than others describe the motion of particles in any given system. But, such explanations are not compatible with dispositionalism.

Let me clarify somewhat which explanations the PLA affords before explaining why such explanations are not compatible with dispositionalism. The

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[9] Proponents of dispositional essentialism about fundamental objects might think that, even if their view is not compatible with the PLA, it at least agrees with the PLA in implying that, given a system's starting point, there is only one way in which it might evolve. However, the truth of the PLA means only that given a system's starting point *and* the facts that make the PLA true, there is only one way in which the system might evolve.

requirement that a physical system's quantity of action be an extremum relative to the quantities of action it might have possessed, along with a formulation of that system's action integral, allows the selection of actual equations of motion from counterfactual equations of motion. Thus, the relational property of being a physical system that has a quantity of action that is an extremum can be appealed to in explaining why certain equations of motion describe the objects that comprise it.

Further, on the dispositionalist view, the equations of motion of a physical system describe the dispositions of the objects within it. On this assumption, if noting that a physical system has an action that is an extremum explains why it has the equations of motion it has, doing so thus explains why such a system has the dispositions it has.

Why are the deductions that the PLA affords explanatory? I can only offer a tentative response to this question. To begin with, these deductions appeal to the fact that certain quantities are extrema. But that some quantity is an extremum seems to imply that, if it is actual, its actuality is not an accident. Moreover, that something is not an accident enables appealing to it in explanations. The deductions that the PLA affords seem to gain further explanatory strength from their unifying force. Such deductions appeal to a single scalar quantity in order to deduce how the objects within a physical system will evolve, no matter how complex the system and its evolution.

Why is the type of explanation afforded by the PLA incompatible with dispositionalism? The dispositionalist assumes that the dispositions of objects alone are the ultimate explanatory ground for events. However, if we accept the PLA, we accept that there is an additional ground, and hence explanation, for why the objects that make up physical systems possess the dispositions they possess.

Insofar as dispositionalism might afford explanations for why objects have the dispositions they have, these explanations can ultimately only appeal to dispositions. As a result, dispositionalist explanations for why objects have the dispositions they have are historical. Events at one point in time are explained in terms of their being the manifestations of dispositions possessed by objects at earlier points in time and, perhaps, also in terms of their being the manifestations of dispositions possessed by objects that they are simultaneous with.

A dispositionalist might, to be sure, suppose that laws of nature explain why objects possess the dispositions they possess. But in order to maintain the assumption that dispositions alone are the ultimate explanatory ground for events, she will also have to assume that such explanations can be reformulated in historical terms.

However, since the PLA allows us to derive the equations of motion of a physical system by comparing the various quantities of action that the system might have had rather than by appealing to the system's actual history, it does not offer an historical explanation for why the actual equations of motion are actual. Indeed, not only does the PLA not offer an historical explanation for why certain equations of motion obtain, it does not even presuppose that the systems to which it applies have a history.

It might be objected that the PLA can, in the restricted mathematical formulations it has actually been given, itself be derived. For example, the restricted version of the PLA that is used in classical particle mechanics (PLAC) can be derived from the Lagrange formulation of the equations of motion of such systems. Thus, it might be thought, we can offer an explanation for the obtaining of the various restricted formulations of the PLA, and such explanations might be compatible with dispositionalism.

There may well be ways of explaining why the PLA, or some restricted version of it, obtains. However, the mere fact that the PLA, or some restricted version of it, can be deduced from other equations does not show that there is such an explanation. After all, not every deduction is an explanation. Thus, the mere existence of a deduction of some restricted formulation of the PLA does not undermine my argument.

Indeed, the fact that the PLAC can be derived from the corresponding Lagrange equations illustrates the fact that deduction and explanation are not the same. The PLAC allows the deduction of these equations and vice versa. Yet, as the fact that physicists typically use the PLAC to deduce corresponding equations of motion and not vice versa illustrates, the explanation only proceeds from the PLAC to the equations of motion.

In any case, even if it were to be shown that a deduction of the PLA, or of some restricted version of the PLA, is explanatory, this would not, in itself, assist dispositionalism. Rather, in light of the argument of this section, we would have reason to suppose that yet other principles, those from which the relevant formulation of the PLA is deduced, are not compatible with dispositionalism.

I conclude that the explanations afforded by the PLA are not compatible with dispositionalism. I think it is also fair to conclude that, if the PLA is accepted, it should also be accepted that the dispositions of physical objects are not ontologically fundamental. The view that dispositions are ontologically fundamental consists largely in the rejection of the view that an object's possessing some disposition supervenes on its possessing some intrinsic categorical property that, given the appropriate stimulus, has a nomic connection with a response (where both the nomic connection, the stimulus and the response are described in purely categorical terms). It

is, in part, the rejection of this view that allows the dispositionalist to maintain that dispositions, rather than categorical properties together with laws of nature, are the ultimate ontological grounds that explain events. But, it seems, the PLA requires that dispositions do supervene on non-dispositional properties taken together with something like a law of nature, namely whatever makes the PLA true. Given the PLA, the fact that the objects that make up a certain physical system possess the dispositions they possess will supervene on the intrinsic properties of the objects in question together with whatever makes the PLA true. The converse option, namely that the obtaining of the PLA supervenes on the dispositions that the objects of the physical system in question happen to possess, does not seem to be compatible with the fact that the PLA explains why these objects possess the dispositions they possess and not vice versa.

5. Conclusion

The PLA is not compatible with dispositionalism, irrespective of whether dispositionalism entails dispositional essentialism about fundamental objects. This is not to say that we ought to reject dispositionalism. Whether we ought to do so depends, among other things, on our view of the PLA itself. We might, for example, try to maintain dispositionalism by combining an instrumentalist view of the PLA with a realist view of equations of motion. I thus leave open the question of whether dispositionalism remains viable in light of the arguments of this paper.[10]

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