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DERIVATION OF CONTINUUM TRAFFIC MODEL FOR WEAVING SECTIONS ON FREEWAYS

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This paper presents a new continuum model describing the dynamics of multiclass traffic flow on multilane freeways including weaving sections. In this paper, we consider a specific freeway weaving type, which is formed when an on ramp is near to an off ramp and these two ramps are joined by an auxiliary lane. Traffic interactions in this weaving zone are very complex due to the involvement of weaving flows and non-weaving flows in the so-called mandatory lane-changing process. To handle this complexity, it is essential to have a good understanding of the (microscopic) driving behavior within the weaving zones. These behaviors are modeled based on a gap-acceptance model. The methodology to obtain a weaving continuum traffic model is thus twofold. On the one hand, we develop a (macroscopic) model to determine the mandatory lane-changing probability based on a renewal process. On the other hand, we implement the lane-changing model into a current gas-kinetic traffic flow model for heterogeneous traffic flow on multilane roadways. From this, corresponding macroscopic model is obtained based on the method of moments.

KEYWORDS: Gas-kinetic model, macroscopic model, gap-acceptance model, weavings

1. INTRODUCTION AND BACKGROUND

According to the Highway Capacity Manual (HCM) 2000 (HCM, 2000), weaving is defined as the crossing of several traffic streams moving on the freeway in the same direction. Due to the complexity of traffic interactions in a weaving area, the capacity of freeways is often reduced significantly and, consequently, the traffic operation is deteriorated. This illustrates the importance of research, theory and modeling for the traffic operation at weaving sections of freeways.

In the seventies, a weaving area operation study was initiated in a U.S.' Highway Research Program for reviewing the capacity of freeways. In this study, an analytical method was developed by MacShane and Roess (1970) to model the lane-changing processes at the weaving sections. Recently many researchers have been working on capacity analysis of weaving sections and its dependence on the length of the weaving area. Makigami and Iizuka (1993) developed a method of evaluating the weaving traffic operating conditions and finding a systematic way to determine weaving section length based on multiple merging probability theory. In this research, the weaving traffic considered only merging phenomena. Lertworawanich and Elefteriadou (2003) proposed a method for estimating the capacity at weaving sections based on a gap-acceptance theory. A similar approach is also chosen in this research because it allows to model at a high level of detail the lane-changing processes in weaving sections, which give rise to a significant impact on the traffic operation conditions in these zones. Almost all of these models are developed at microscopic level and, therefore, require much effort on calibration and validation. To contribute to this research stream, in this paper we propose a continuum (macroscopic) traffic model to describe the traffic operations at weaving sections. The main benefit of applying macroscopic traffic models over microscopic models is the relatively small number of parameters simplifying model calibration, while it still provides desirable results.

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Recently, research on macroscopic models of multilane traffic and multiclass traffic (MLMC) on freeways has become suitable for application in areas such as prediction and optimizing traffic control (model based). These strategies may allow network operators to use the existing infrastructure more efficiently. Specific examples of control measures that can only be modeled in a class-specific framework are dynamic truck overtaking prohibitions, uninterrupted passage for buses by ramp metering at on ramps, and dynamic lane allocation control. The significant contributions to these research topics are the works by Helbing (1997), Shvetsov and Helbing (1999), Hoogendoorn (1999), Hoogendoorn and Bovy (1999), etc. Most of these models are developed from gas-kinetic model for mixed traffic on multilane roadways. These types of model describe the evolution of the phase-space density (PSD) of vehicles on a freeway in which the left hand side (LHS) of the partial differential equation describes the continuous dynamics of the PSD due to the motions of traffic flow while the right hand side (RHS) describes the discontinuous changes of this function due to the events such as lane-changing, deceleration, etc. Extending this approach, an analytical model capturing the main characteristics of traffic operations on motorways such as congestion near on ramps has been derived mathematically from microscopic driving behaviors using renewal theory by Ngoduy et al. (2004a). This model explicitly takes into account the dynamics of traffic at on and off ramps by modeling the so-called mandatory lane-changing process within these zones. However, this model still distinguishes the interactions of merging and diverging traffic, which is an important part of traffic operations at a weaving section.

In this paper, a specific weaving section type (Figure 1) is considered in detail. In this type of weaving (defined as type B according to HCM 2000 (HCM, 2000)), an on-ramp is connected with an off-ramp by an auxiliary lane. Let 0 denote the lane index of auxiliary lane, and 1, 2 denote the lane index of the shoulder lane and the median lane, respectively. Traffic operation within the considered weaving section includes:

(a) Merging (b) Diverging

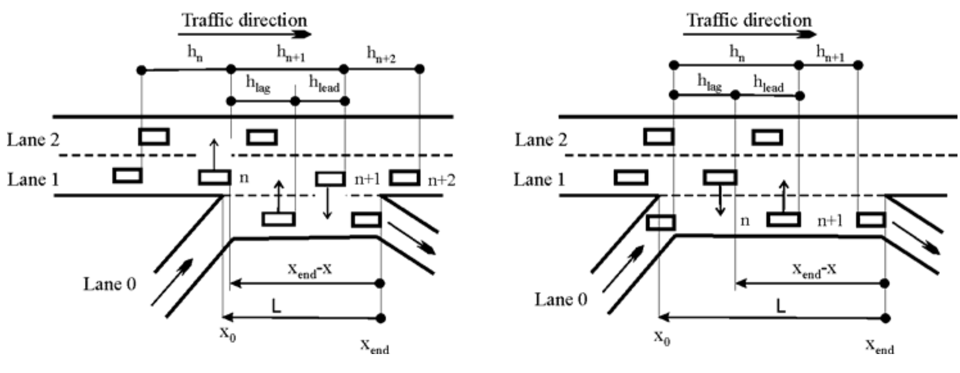

FIGURE 1: Lane changing at weaving section

- Merging traffic from the auxiliary lane to the shoulder lane with fraction $\alpha_{0,1}^{u}$ (defined as weaving flow).

- Diverging traffic from the shoulder lane to the auxiliary lane with fraction $\alpha^{u}_{1,0}$ (defined as weaving flow).
- Through traffic on the shoulder lane with fraction $\alpha^{u}_{1,1}$ (defined as non-weaving flow).
- Through traffic on the auxiliary lane (that is traffic that continues from the on-ramp to the off-ramp) with fraction $\alpha^{u}_{0,0}$ (defined as non-weaving flow).
- Note $\alpha^{u}_{i,j}$ equals the ratio between the flow rate that intends to change from lane *I* to lane *j* and the total flow rate in lane *i*.

Obviously the merging and diverging probability depend on all of these traffic streams. The calculation method for merging and diverging probability is based on gap-acceptance model and renewal theory, described in the ensuing of this paper. The main contribution of this paper to the state-of-the-art in continuum traffic flow theory is therefore the modeling of discontinuities of traffic flow at a weaving section on multilane freeways. It introduces the mandatory lane-changing process into the RHS of the current MLMC continuum models based on microscopic driving behaviors within the weaving zone. Based on the details of this mandatory lane-changing process, a relevant control measure at on/off ramps (ramp metering), or design of infrastructure (length of auxiliary lane) can be applied in order to obtain a better operation on freeways.

2. MODELING APPROACH

This section describes the approach to model the lane-changing processes in a gas-kinetic traffic flow model. Before going into details of the modeling approach, let us start with the most important assumptions for developing the gas-kinetic equations for interrupted traffic stream.

2.1 Behavioral assumptions

These assumptions are concerned with the drivers' behavior in traffic stream when making lane-changes:

1. The speed of lane-changing vehicles is not influenced by the lane-changing maneuver. That is, the lane-changing vehicles do not reach the target lanes instantaneously with a higher/lower speed.
2. The speed of the following and the leading vehicle in the target lanes is unaffected by the lane-changing maneuver (that is, there is no acceleration/deceleration during a lane-change).
3. Only one lane-change can be performed during each time interval (time gap available for a lane-change). That is, lane changes are not executed instantaneously.
4. When a fast vehicle catches up with a slow vehicle, the faster vehicle always intends to change to the adjacent lane in order to maintain its desired speed (used for the derivation of immediate lane-changing rate).
5. Lane selection model: when a driver intends to change lanes, first the target lane needs to be selected. The probabilities that a lane is selected as the target lane depend on a number of factors, such as the speed of the concerned vehicle, the user-class, traffic conditions in the current lane and the target lane. A lane selection model can be of either a discrete type (Ben-Akiva and Lerman, 1995) or a logit type (Ahmed,

1999). The result of the lane selection model is the probability to select either the right lane, the left lane or to remain in the current lane. If either the right lane or the left lane is selected, the vehicle will seek for an acceptable gap in the selected lane. The traffic regulations can also be in-cooperated in this model. For example, under European traffic regulations, the first option is always to move to the left when a slower vehicle needs to be passed.

6. Gap acceptance: in this paper, 'gap' stands for the space gap, that is, the distance between the front ends of two successive vehicles in the same lane. A vehicle is able to change to the target lane when both the lead-gap and the lag-gap are accepted. The lead-gap is accepted if, after the lane change has been carried out, the space between the lane-changing vehicle and the leader in the target lane is larger than a certain threshold distance (critical lead gap). The lag-gap is accepted if, after the lane-change has been carried out, the space between the lane-changing vehicle and the follower in the target lane is larger than critical lag gap (Tanner, 1962).
7. The critical gaps are considered single values, which are a linear function of speed and the reaction time of the follower (there is no different driving styles within a vehicle class and driver does not change his behavior with local traffic condition).
8. Drivers are willing to accept smaller gaps when approaching the end of the current lane. That is, the remaining distance to the end of the current lane influences the gap-acceptance behavior (used for the derivation of mandatory lane-changing probabilities).
9. The follower in the target lane may create gaps for the lane-changing vehicles by changing lanes to the left if there is an opportunity to do so (used for the derivation of mandatory lane-changing probabilities).

2.2 Statistical requirements

These assumptions are based on empirical data concerning the distribution of mean speed and distance gaps. These assumptions are useful for a simple mathematical derivation since it is almost impossible to derive a MLMC macroscopic model with assumptions that cover all realistic phenomena. Our purposes are to derive a macroscopic model that, on the one hand, is able to explain critical traffic phenomena theoretically (see Ngoduy et al., 2006) and, on the other hand, is able to reproduce well real-life observations without requiring complicated calibration and validation process (see Ngoduy et al., 2004a; Ngoduy, 2005).

10. The individual vehicle speeds conform to a Gaussian distribution. This requirement is used to determine the interaction lane-changing rate in the original models of Hoogendoorn (1999), Shvetsov and Helbing (1999).
11. The gap distribution is known. For analytical purposes, we use the exponential distribution of gaps. However, the derived model can be solved numerically with other types of distributions, such as, the M3 distribution type (Cowan, 1975).

2.3 Gap-acceptance model

In this section, we describe how a gap-acceptance model functions in lane-changing processes. The gap-acceptance model expresses the drivers' assessment whether or not gaps on the target lanes are acceptable for a lane-change (see Tanner, 1962). Let lead-gap and lag-gap denote the gap between the subject vehicle and its leader and between the subject vehicle and its follower in the target lane, respectively. In the case of merging

(Figure 1a), the decision to make a lane change from the acceleration lane to the shoulder lane is based on both the lag-gap and the lead-gap. When these gaps suffice, the subject vehicle can merge to the main. In the case of diverging (Figure 1b), the subject vehicle can exit from the main carriageway if the lead-gap between the subject vehicle and its leader, and the lag-gap between the subject vehicle and its follower in the off-ramp lane is sufficient. Let d_{lag} and d_{lead} denote the critical lag-gap and the critical lead-gap of vehicle, respectively, for lane-changing manoeuvres. By applying the safe-distance model (Jepsen, 1998) we obtain the following results: $d_{\text{lead}} = d_{\min} + l + \mu(x)Tv$ and $d_{\text{lag}} = d_{\min} + l + Tw$; where v and w are the speed of the subject vehicle and of the following vehicle in the target lane, respectively (Note that the second order term is neglected in the development of our model due to its small contribution, which is approximately $0.022v^2$ (see Hoogendoorn and Bovy, 1999). The ramp factor $\mu(x) \in [0,1]$ ($\mu(x_0) = \mu_{\max}$, $\mu(x_{\text{end}}) = \mu_{\min}$ and $\mu(x) = 1$ when $x < x_0$ and $x > x_{\text{end}}$) considers the willingness to accept smaller gap of the considered lane-changing drivers when approaching the end of the ramp.

Let $A(v|x, t)$ be the event that a vehicle driving with speed v finds sufficient gaps on the target lane at location x and time instant t. According to Ahmed (1999), in the context of lane-changing processes, there are desire to change lanes (for example, fast vehicles interact with slow ones and desire to change lane) and opportunities to do so. Since at the weavings, all weaving vehicles are forced to change lane (either enter or exit the main lane), we only consider the later, that is, to determine the probability to merge/diverge. According to gap acceptance model, the merging/diverging probability $\pi_{i,j}$ (i=0, j=1 when traffic enters to the freeway, while i=1, j=0 when traffic exits from the freeway) is equal to the probability that event A occurs, which is:

$$\pi_{i,j} = P(A) = P\left(h_{\text{lag}} \geq d_{\text{lag}}(w)\right)P\left(h_{\text{lead}} \geq d_{\text{lead}}(v)\right). \tag{1}$$

Equation (1) neglects the correlations between the critical lead-gap and the lag-gap for the sake of simple derivation. Indeed, these correlations can be captured by adding a specific error terms but this work will be done in the future. To determine $P(A)$, we need to calculate the probability distribution function of the lead-gap and the lag-gap using a so-called renewal process, described in the following section.

2.4 Renewal process in traffic flow

The concept of renewal processes can be found in statistical literatures, for example in Cox (1962). Let us consider a stream of vehicles which constitutes a renewal process N. Let h_n denote the total gap between two successive vehicles located at x_n and x_{n-1}. Let us assume that (h_1, h_2,…, h_n) are independently identically distributed random variables with p.d.f. $f(h)$. Suppose that we interrupt the renewal process N by inserting a vehicle at some specified location x between x_n and x_{n-1} (see Figure 2). Let h_{lag} and h_{lead} denote the distance between x and the next vehicle upstream of x and between x and the next vehicle downstream of x. That is, $h_{\text{lag}} = x - x_{n-1}$ and $h_{\text{lead}} = x_n - x$. In the terminology of the renewal process, the lead-gap h_{lead} acts as excess life, the lag-gap h_{lag} acts as current life and the total gap $h = h_{\text{lead}} + h_{\text{lag}}$ acts as total life.

Distribution of the lead-gap or excess life is calculated as:

$$f_{\text{lead}}(h) = \frac{1 - F(h)}{\bar{h}} \tag{2}$$

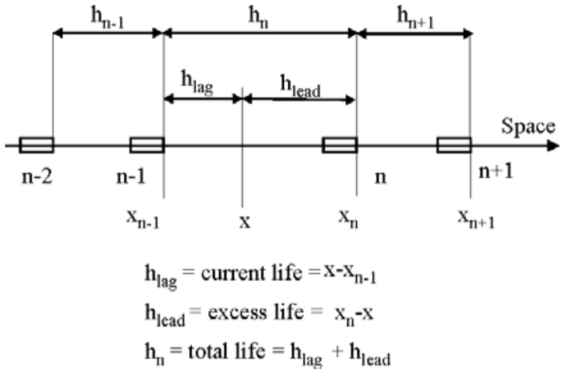

FIGURE 2: Excess, current, and total life at location *x*

where $\bar{h}$ is the total average distance gap, which is calculated by the following formula (Hoogendoorn et al., 2002):

$$\bar{h} = \frac{1-\left(d_0+l+TV\right)r}{r} = \frac{1}{\gamma r}\text{, with } \gamma = \frac{1}{1\text{-}\left(d_0+l+TV\right)r}. \tag{3}$$

The factor γ in equation (3) accounts for the space requirement (length of vehicle) in dense traffic, $r=r(x,t)$ and $V=V(x,t)$ denote the traffic density and mean speed, respectively. Since in this paper, we assume that the distance gap is exponentially distributed, equation (2) becomes:

$$f_{\text{lead}}\left(h\right) = \gamma r e^{-\gamma r h} \tag{4}$$

Distribution of the lag-gap or current life is calculated based on the total clear gap between the next vehicle in front of *x* and the next one behind *x*. That is, the lane-change of the follower on the target lane to create more gaps for the subject vehicle is taken into account. For example, if vehicle n-1 changes lanes, the total clear gap becomes $h_{n-1} + h_n$. If both vehicle n-1 and vehicle n change lanes, the total clear gap becomes $h_{n-1} + h_n + h_{n+1}$. In general, let $H = h_1 + h_2 + \ldots + h_n$ denote the total clear gap. The p.d.f. of the lag-gap becomes:

$$f_{\text{lag}}\left(h\right) = \frac{1\text{-}F\left(H\right)}{\bar{H}} \tag{5}$$

To calculate the c.d.f. of the total clear gap $F(H)$ in equation (5) , the so-called Laplace transform theorem is used. If the gap is exponentially distributed, that is, $f\left(h\right) = \gamma r e^{-\gamma r h}$ and the c.d.f. $F\left(h\right) = 1 - e^{-\gamma r h}$, the corresponding Laplace transform of $f(h)$ will be $L\left\{f\left(h\right);s\right\} = \gamma r / \left(\gamma r + s\right)$. Hence, the Laplace transform of $f\left(H_n = \sum h_n\right)$ becomes $L\left\{f\left(H_n\right);s\right\} = \left(\gamma r\right)^n / \left(\gamma r + s\right)^n$. It turns out that this is Laplace transform of the function:

$$f\left(H_n\right) = \frac{\gamma r\left(\gamma r h\right)^{n-1} e^{-\gamma r h}}{\Gamma\left(n\right)} \text{ and } F\left(H_n\right) = -\frac{\left(\gamma r h\right)^{n-1} e^{-\gamma r h}}{\Gamma\left(n\right)} + F\left(H_{n-1}\right) \tag{6}$$

where Γ denotes the gamma function, defined by $\Gamma\left(n\right) = \int x^{n-1} e^{-x} dx$.

Note that equation (6) is indeed an Erlang distribution which is well-documented in any statistical literatures, however for a general gap distribution other than exponential type the renewal theory and Laplace transform should be applied.

2. MODEL DEVELOPMENT

This section describes the derivation of high order multilane and multiclass continuum model for traffic dynamics on freeways with weaving from a gas-kinetic model. The model describes in detailed how drivers merge and/or diverge within this section. The developed model is able to capture the following situations:

- Drivers merge or diverge along a considerable stretch of freeway (auxiliary lane).
- Drivers are willing to accept smaller gap when reaching the end of auxiliary lane.
- Drivers on the shoulder lane are cooperative by changing to the median lane in order to give way for merging vehicles.

3.1 Generalized gas-kinetic model of weaving sections

This section establishes the gas-kinetic equation for weaving traffic flow. Since the traffic from on-ramp does not directly influence traffic flow on the left lanes of the shoulder lane (i=2), the generalized MLMC gas-kinetic model of Hoogendoorn (1999), Shvetsov and Helbing (1999) is applied for those lanes. This model holds also for those lanes in the diverging case when traffic desiring to exit switch to the shoulder lane before the beginning of the weaving area. The only difference in our model is that the immediate lane-changing (ILC) probability of vehicle class u from lane I to either adjacent lane j is determined by gap acceptance model. For traffic operation on the shoulder lane and on the auxiliary lane we need to determine the MLC rate. According to Shvetsov and Helbing (1999), the MLC rate is proportional to the traffic flow entering or exiting the freeway and inversely proportional to the length of ramps. Let $\omega_{i,u}^{\pm}\left(x,v,t\right)$ denote the incoming and outgoing flow rate to and from the freeway, respectively, at location x and time instant t. They are determined as follows:

$$\begin{aligned}
&\omega_{1,u}^{+}\left(x,v,t\right)=\delta\left(x\right)\frac{\alpha_{0,1}^{u}\rho_{0}^{u}\left(x,v,t\right)v}{L}\pi_{0,1}^{u};\omega_{1,u}^{-}\left(x,v,t\right)=-\delta\left(x\right)\frac{\alpha_{1,0}^{u}\rho_{1}^{u}\left(x,v,t\right)v}{L}\pi_{1,0}^{u},\\
&\omega_{0,u}^{-}\left(x,v,t\right)=-\delta\left(x\right)\frac{\alpha_{0,1}^{u}\rho_{0}^{u}\left(x,v,t\right)v}{L}\pi_{0,1}^{u};\omega_{0,u}^{+}\left(x,v,t\right)=\delta\left(x\right)\frac{\alpha_{1,0}^{u}\rho_{1}^{u}\left(x,v,t\right)v}{L}\pi_{1,0}^{u},
\end{aligned}\tag{7}$$

where $\delta(x)$=0 if x is outside of the weaving area, $\delta(x)$=1 otherwise. L denotes the weaving length.

The mandatory lane-changing rate $\omega_{i,u}^{\pm}\left(x,v,t\right)$ determined in equation (7) contributes to the discontinuous events of the dynamics of traffic flow in weavings. The generalized gas-kinetic equation describing the dynamics of the lane and class specific phase-space density $\rho_{i}^{u}\left(x,v,t\right)$ reads:

$$\partial_t \rho_i^u + \underbrace{v\partial_x \rho_i^u}_{\text{convection}} + \underbrace{\partial_v \left(\rho_i^u \frac{V_{i,u}^{\max} - v}{\tau_i^u} \right)}_{\text{acceleration}} = \underbrace{\left(\partial_t \rho_i^u\right)_{\text{int}}}_{\text{interaction}} + \underbrace{\left(\partial_t \rho_i^u\right)_{\text{lc}}^{\text{imm}}}_{\text{immediate lane-change}} + \underbrace{\left(\partial_t \rho_i^u\right)_{\text{lc}}^{\text{spon}}}_{\text{spontaneous lane-change}} + \underbrace{\left(\partial_t \rho_i^u\right)_{\text{lc}}^{\text{mand}}}_{\text{mandatory lane-change}} . \tag{8}$$

Equation (8) describes the changes of the phase-space density (PSD) within a small distance [x,x+dx] and time step [t,t+dt] due to the following processes:

- Convection, reflecting the changes of PSD due to the motion of vehicles along the road.
- Acceleration, describing the changes of PSD due to the tendency of vehicles to accelerate to the desired speed (free speed).
- Interaction, reflecting the interaction between fast vehicles and slow vehicles. That is, when the fast vehicles catch up with slow ones, they have to slow down in order to avoid the collision.
- Immediate lane-changing, reflecting the lane-changes due to the interaction between fast vehicles and slow vehicles. That is, when the fast vehicles catch up with slow ones, they might change their lane.
- Spontaneous lane-changing, accounting for the lane-changes due to the preference of drivers on a particular lane, which reflect the traffic regulations, for example the fast vehicles drive on the left lane while the slow ones drive on the right lane.
- Mandatory lane-changing, accounting for traffic entering and exiting the freeway (act as sink/source). The mandatory lane-changing rate is determined for the shoulder lane and the auxiliary lane within a weaving section as:

$$\left(\partial_t \rho_i^u\right)_{\text{lc}}^{\text{mand}} = \omega_{i,u}^{+}\left(x,v,t\right) + \omega_{i,u}^{-}\left(x,v,t\right) . \tag{9}$$

Equation (8) serves as an immediate step for the derivation of the macroscopic continuum model for traffic stream with on- and off-ramps, which is shown in the Section 3.4.

3.2. Immediate lane-changing probability

In this section, we refine the immediate lane-changing models of Hoogendoorn (1999), Hoogendoorn and Bovy (1999), and Shvetsov and Helbing (1999) by including driver behavior. The application of the gap-acceptance model to the immediate lane-changing process is shown in Figure 3. In this situation a vehicle approaches a slower vehicle and intends to change lanes to either adjacent lane if available. The subject vehicle then seeks for a sufficient gap in its selected target lane and changes lanes immediately when an appropriate gap is found. If the preferred target lane (that is, the right lane in compliance with European driving rules) is failed, the subject vehicle will consider the other lane for a lane-change (that is, the left lane). If all of these choices are failed, the subject vehicle will stay in the current lane. In the context of the immediate lane-changing process (Figure 4), let us consider a vehicle of class u (in lane 2) changing lanes to the left. The subject vehicle can change lanes if there are sufficient gaps between it and the leader and the follower, irrespective of vehicle class (so denoted by class s, $s \in U$). Obviously, the lead-gap depends among the other things on the speed of the subject vehicle (denoted by v) while the lag-gap depends on the speed of the follower (denoted by w). Let $A_{i,j}^{u,s}$ be the event that a vehicle of class u driving in lane i finds sufficient gaps between vehicles irrespective of class (denoted by s) in lane j. The average probability that $A_{i,j}^{u,s}$ occurs, according to the gap-acceptance model, is:

$$P\left(A_{i,j}^{u,s}\right)=\left[1-\left\langle F_{\text{lead}}^{j}\left(d_{\text{lead}}^{i,u}\left(v\right)\right)\right\rangle_{v}\right]\left[1-\left\langle F_{\text{lag}}^{j}\left(d_{\text{lag}}^{j,s}\left(w\right)\right)\right\rangle_{w}\right]. \tag{10}$$

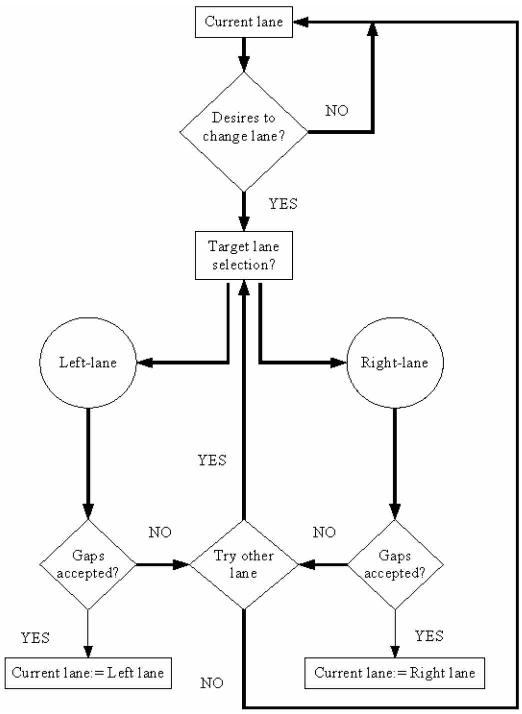

FIGURE 3: Structure of immediate lane-changing decision of a driver

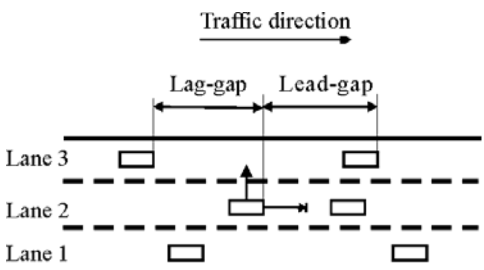

FIGURE 4: Immediate lane-changing on a multilane roadway

From equations (2) and (5) we obtain the c.d.f. of the lead-gap and the lag-gap for a lane-change as:

$$F_{\text{lead}}^{j}\left(d_{\text{lead}}^{i,u}(v)\right)=\int_{0}^{d_{\text{lead}}^{i,u}(v)} f_{\text{lead}}^{j}(h)\,\mathrm{d}h=\gamma_j r_j\int_{0}^{d_{\text{lead}}^{i,u}(v)}\left[1-F_j(h)\right]\mathrm{d}h,$$
$$F_{\text{lag}}^{j}\left(d_{\text{lag}}^{j,s}(w)\right)=\int_{0}^{d_{\text{lag}}^{j,s}(w)} f_{\text{lag}}^{j}(h)\,\mathrm{d}h=\gamma_j r_j\int_{0}^{d_{\text{lead}}^{j,s}(w)}\left[1-F_j(h)\right]\mathrm{d}h. \tag{11}$$

Substituting equation (11) into equation (10) results in:

$$P\left(A_{i,j}^{u,s}\right)=\left(\gamma_j r_j\right)^2\left\langle\int_{d_{\text{lead}}^{i,u}(v)}^{\infty}\left[1-F_j(h)\right]\mathrm{d}h\right\rangle_v\left\langle\int_{d_{\text{lead}}^{j,s}(w)}^{\infty}\left[1-F_j(h)\right]\mathrm{d}h\right\rangle_w . \tag{12}$$

Assumption of the exponential distribution of gaps results in:

$$\begin{aligned}P\left(A_{i,j}^{u,s}\right)&=\left\langle\exp\left(-\gamma_j r_j d_{\text{lead}}^{i,u}(v)\right)\right\rangle_v\left\langle\exp\left(-\gamma_j r_j d_{\text{lag}}^{j,s}(w)\right)\right\rangle_w\\&=\left\langle\exp\left[-\gamma_j r_j\left(d_{\min}+l_s+T_i^u v\right)\right]\right\rangle_v\left\langle\exp\left[-\gamma_j r_j\left(d_{\min}+l_u+T_j^s w\right)\right]\right\rangle_w\\&=\exp\left[-\gamma_j r_j\left(d_{\min}+l_s\right)\right]\left\langle\exp-\left(\gamma_j r_j T_i^u v\right)\right\rangle_v\exp\left[-\gamma_j r_j\left(d_{\min}+l_u\right)\right]\left\langle\exp-\left(\gamma_j r_j T_j^s w\right)\right\rangle_w .\end{aligned} \tag{13}$$

By substituting Taylor expansion $e^x=\sum_0^\infty x^n/n!$ to equation (13) we get:

$$\begin{aligned}P\left(A_{i,j}^{u,s}\right)&=\exp\left[-\gamma_j r_j\left(d_{\min}+l_s\right)\right]\exp\left[-\gamma_j r_j\left(d_{\min}+l_u\right)\right]\\&\times\sum_{n=0}^{\infty}\frac{(-1)^n\left(\gamma_j r_j T_i^u\right)^n}{n!}\left\langle v^n\right\rangle_v\sum_{n=0}^{\infty}\frac{(-1)^n\left(\gamma_j r_j T_j^s\right)^n}{m!}\left\langle w^n\right\rangle_w .\end{aligned} \tag{14}$$

Let $M_k=\left\langle(v-V)^k\right\rangle$ with $k\geq 2$, the assumption of Gaussian speed distribution leads to $M_{2z+1}=0$ (z=1,2,...). Using the definition of mean values for speed V and speed variance Θ as $\langle v\rangle=V$ and $\left\langle(v\text{-}V)^2\right\rangle=\Theta$, expansion of M_k results in:

$$\begin{aligned}M_2&=\left\langle(v-V)^2\right\rangle=\Theta,\\M_3&=\left\langle(v-V)^3\right\rangle=\left\langle v^3\right\rangle-V^3=0,\\&\ldots\\M_k&=\left\langle(v-V)^k\right\rangle=\left\langle v^k\right\rangle-V^k-0.5k(k-1)V^{k-2}\Theta=0.\end{aligned} \tag{15}$$

Hence:

$$\left\langle v^k\right\rangle=V^k+0.5k(k-1)V^{k-2}\Theta . \tag{16}$$

By substituting expression (16) into equation (14), after a straightforward algebraic calculation (see Ngoduy, 2005) we end up with:

$$P\left(A_{i,j}^{u,s}\right)=\exp\left[-\gamma_j r_j\left(d_{\min}+l_s+T_i^u V_i^u\right)\right]\exp\left[-\gamma_j r_j\left(d_{\min}+l_u+T_j^s V_j^s\right)\right]$$
$$\times\left[1+0.5\Theta_i^u\left(\gamma_j r_j T_i^u\right)^2\right]\left[1+0.5\Theta_j^s\left(\gamma_j r_j T_j^s\right)^2\right]. \tag{17}$$

The probability that any vehicle in lane j interacting with the lane-changing vehicle from lane i belongs to class s is calculated as: r_j^s / r_j, hence the probability that a vehicle of class u can change from lane i to lane j irrespective of vehicle class running in lane j becomes:

$$p_{i,j}^u=\sum_{s\in U}\frac{r_j^s}{r_j}\left\{\exp\left[-\gamma_j r_j\left(d_{\min}+l_s+T_i^u V_i^u\right)\right]\exp\left[-\gamma_j r_j\left(d_{\min}+l_u+T_j^s V_j^s\right)\right]\right.$$
$$\left.\times\left[1+0.5\Theta_i^u\left(\gamma_j r_j T_i^u\right)^2\right]\left[1+0.5\Theta_j^s\left(\gamma_j r_j T_j^s\right)^2\right]\right\}. \tag{18}$$

From equation (18), we can see that the immediate lane-changing probability depends on the current traffic condition in each lane such as the density, the speeds and speed variances. The influence of model parameters on traffic flow stability will be discussed in the following section.

3.3. Merging/diverging probability

According to Ahmed (1999), the so-called mandatory lane-changes exhibit different behavior compared to the immediate lane-changes presented in the previous section. Mandatory lane-changes often occur at bottlenecks and cause a major impact on road capacity due to the increase of traffic demand, which leads to traffic breakdown. In the remainder of this section, traffic dynamics at a weaving zone are discussed. The structure of a mandatory lane-changing process is shown in Figure 5, in which vehicles are forced to change to a fixed target lane.

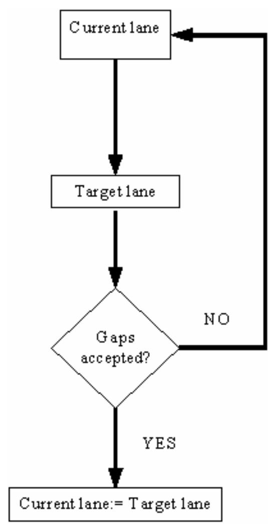

FIGURE 5: Structure of mandatory lane-changing decision of a driver

For causes of clarity, we assume that there is only one auxiliary lane (but the derivation for a multilane weave ramp is straightforward by treating possible lane-changes between auxiliary lanes as immediate lane-changes). Let us denote the lane index in decreasing order from the median lane I to the shoulder lane 1 and index 0 for the auxiliary lane. To determine the mandatory lane-changing probability at location x and time instant t, we need to take into account the lane-changing rate from the shoulder lane to the left lane in order to give way to merging vehicles (see Figure1a). Let $\Omega_{1,2}^{u}$ be the fraction of the vehicle class u in the shoulder lane (lane 1) which is willing to change to the adjacent lane (lane 2).

Let us consider a vehicle class u on the auxiliary lane trying to merge to the shoulder lane as shown in Figure 1a. The decision to make a lane change is based on the gap-acceptance model as described earlier. While the lead-gap p.d.f. is determined by equation (2), the lag-gap p.d.f. is determined by equation (5) in four situations as follows:

3.3.1. Case 1

In this case, both the follower and the leader are non-weaving vehicles, irrespective of vehicle class (denoted by s). The probability that Case 1 occurs is $\left(\alpha_{1,1}^{s}\right)^{2}$. There are two sub-cases described as below:

- Case 1.1: If the follower is willing to move to the median lane with probability $\Omega_{1,2}^{s}$ in order to give way to the merging vehicle and is able to do so, the space available for merging is $(h_{n+1}+h_n)$. Hence, in this case, the c.d.f. of the lag-gap on the shoulder lane for merging is:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=\int_{0}^{d_{\text{lag}}^{1,s}} f_{\text{lag}}^{1}\left(h\right)\mathrm{d}h=0.5\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}h\right]. \tag{19}$$

- Case 1.2: If the follower is not moving to the median lane, the space available for merging remains h_n. Hence, in this case, the c.d.f. of the lag-gap on the shoulder lane for merging is:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=\int_{0}^{d_{\text{lag}}^{1,s}} f_{\text{lag}}^{1}\left(h\right)\mathrm{d}h=\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(h\right)\mathrm{d}h\right]. \tag{20}$$

By summing these two sub-cases we obtain the lag-gap c.d.f. for Case 1 as:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=0.5\Omega_{1,2}^{s}p_{1,2}^{s}\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}h\right]+\left(1-\Omega_{1,2}^{s}p_{1,2}^{s}\right)\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(h\right)\mathrm{d}h\right]. \tag{21}$$

3.3.2. Case 2

In this case, the follower is a weaving vehicle and running just before the nose of the on-ramp while leader is a non-weaving one. The probability that Case 2 occurs is $\left(\alpha_{1,1}^{s}\alpha_{1,0}^{s}\right)$. Since both vehicles in the shoulder lane do not change their lane at the moment of interaction, the space available for merging remains h_n. Hence, the lag-gap c.d.f. on the shoulder lane available for merging is determined as equation (20).

3.3.3. Case 3

In this case, the follower is a non-weaving vehicle while the leader is a weaving one. The probability that Case 3 occurs is $\left(\alpha_{1,1}^{s}\alpha_{1,0}^{s}\right)$. There are four sub-cases described as below:

- Case 3.1: If the follower is willing to move to the left in order to give way to merging vehicle and is able to do so and the leader is able to move to the auxiliary lane immediately when it finds sufficient lead-gap in the auxiliary lane, the space available for merging is (h_{n+1}+ h_n+$h_{n\text{-}1}$). Hence, in this case, the c.d.f. of the lag-gap in the shoulder lane for merging is:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=\int_{0}^{d_{\text{lag}}^{1,s}} f_{\text{lag}}^{1}\left(h\right)\mathrm{d}\,h=\frac{1}{3}\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(3h\right)\mathrm{d}\,h\right]. \tag{22}$$

- Case 3.2: If the follower is willing to move to the left in order to give way to merging vehicle and is able to do so, and the leader is unable to move to the auxiliary lane, the space available for merging is (h_{n+1}+h_n). The lag-gap c.d.f. for this sub-case is as the same as equation (19).
- Case 3.3: If the follower is unable to move to the left while the leader is able to move to the auxiliary lane, the space available for merging is (h_n+$h_{n\text{-}1}$). The lag-gap c.d.f. is calculated for this sub-case as the same as equation (19).
- Case 3.4: If follower is unable to move to the left and the leader is unable to move to the auxiliary lane, the space available for merging now remains only h_n. The lag-gap c.d.f. is calculated for this sub-case as the same as equation (20).

By summing these four sub-cases we obtain the lag-gap c.d.f. for Case 3 as:

$$\begin{aligned}F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=&\frac{1}{3}\Omega_{1,2}^{s}p_{1,2}^{s}\lambda_{1,0}^{s}\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(3h\right)\mathrm{d}\,h\right]\\&+0.5\Omega_{1,2}^{s}p_{1,2}^{s}\left(1-\lambda_{1,0}^{s}\right)\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}\,h\right]\\&+0.5\left(1-\Omega_{1,2}^{s}p_{1,2}^{s}\right)\lambda_{1,0}^{s}\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}\,h\right]\\&+\left(1-\Omega_{1,2}^{s}p_{1,2}^{s}\right)\left(1-\lambda_{1,0}^{s}\right)\gamma_1 r_1\int_{0}^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(h\right)\mathrm{d}\,h\right].\end{aligned} \tag{1}$$

In equation (23), $\lambda_{1,0}^{s}$ denotes the probability that a vehicle irrespective of class in the shoulder lane finds sufficient lead-gap in the auxiliary lane determined as:

$$\lambda_{i,j}^{u}=\sum_{s\in U}e^{-\gamma_i r_j\left(d_{\min}+l_s+T_i^u V_i^u\right)}\left[1+0.5\Theta_i^u\left(\gamma_j r_j T_i^u\right)^2\right]. \tag{24}$$

3.3.4. Case 4

In this case the follower and the leader are both weaving vehicle, the follower is running just before the nose of the on-ramp. The probability that Case 4 occurs is $\left(\alpha_{1,0}^{s}\right)^{2}$. There are two sub-cases described as below:

- Case 4.1: If the leader is able to move to the auxiliary lane, the space available for merging is ($h_n + h_{n-1}$). The lag-gap c.d.f. is calculated for this sub-case as the same as equation (19).
- Case 4.2: If the leader is unable to move to the auxiliary lane, the space available for merging is h_n, the lag-gap c.d.f. is calculated for this sub-case as the same as equation (20).

By summing these two sub-cases, the lag-gap c.d.f. in Case 4 is calculated as:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=0.5\lambda_{1,0}^{s}\gamma_1 r_1\int_0^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}h\right]+\left(1-\lambda_{1,0}^{s}\right)\gamma_1 r_1\int_0^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(h\right)\mathrm{d}h\right] \tag{25}$$

From equations (21), (23), and (25), we obtain the lag-gap c.d.f. for merging event as:

$$\begin{aligned}F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)&=X_1^{s}\gamma_1 r_1\int_0^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(h\right)\mathrm{d}h\right]+Y_1^{s}\gamma_1 r_1\int_0^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(2h\right)\mathrm{d}h\right]\\&\quad+Z_1^{s}\gamma_1 r_1\int_0^{d_{\text{lag}}^{1,s}}\left[1-F_1\left(3h\right)\mathrm{d}h\right],\end{aligned} \tag{26}$$

where

$$\begin{aligned}X_1^{u}&=\left(1-\alpha_{1,1}^{u}\Omega_{1,2}^{u}p_{1,2}^{u}\right)\left(1-\alpha_{1,0}^{u}\lambda_{1,0}^{u}\right),\\Y_1^{u}&=0.5\Big[\left(\alpha_{1,1}^{u}\right)^{2}\Omega_{1,2}^{u}p_{1,2}^{u}+\alpha_{1,1}^{u}\alpha_{1,0}^{u}\Omega_{1,2}^{u}p_{1,2}^{u}\left(1-\lambda_{1,0}^{u}\right)\\&\qquad+\alpha_{1,1}^{u}\alpha_{1,0}^{u}\left(1-\Omega_{1,2}^{u}p_{1,2}^{u}\right)\lambda_{1,0}^{u}+\left(\alpha_{1,0}^{u}\right)^{2}\lambda_{1,0}^{u}\Big],\\Z_1^{u}&=\frac{1}{3}\alpha_{1,1}^{u}\alpha_{1,0}^{u}\Omega_{1,2}^{u}p_{1,2}^{u}\lambda_{1,0}^{u}.\end{aligned} \tag{27}$$

When the gap is exponentially distributed, applying equation (6) for n = 1, 2 and 3 gives:

$$F_{\text{lag}}^{1}\left(d_{\text{lag}}^{1,s}\right)=1-e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}-\tilde{X}_1^{s}d_{\text{lag}}^{1,s}e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}-\tilde{Y}_1^{s}\left(d_{\text{lag}}^{1,s}\right)^{2}e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}, \tag{28}$$

where

$$\begin{aligned}\tilde{X}_1^{u}&=0.5\left[\Omega_{1,2}^{u}p_{1,2}^{u}\left(1-\alpha_{1,0}^{u}\right)\left(1-\alpha_{1,0}^{u}\lambda_{1,0}^{u}\right)+\alpha_{1,0}^{u}\lambda_{1,0}^{u}\right]\gamma_1 r_1,\\\tilde{Y}_1^{u}&=\frac{1}{12}\alpha_{1,0}^{u}\left(1-\alpha_{1,0}^{u}\right)\Omega_{1,2}^{u}p_{1,2}^{u}\lambda_{1,0}^{u}\left(\gamma_1 r_1\right)^{2},\end{aligned} \tag{29}$$

and the merging probability is obtained as follows:

$$\pi_{0,1}^{u}=\sum_{s\in U}\frac{r_1^s}{r_1}\left\langle e^{-\gamma_1 r_1 d_{\text{lead}}^{0,u}}\right\rangle_v \left\langle e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}+\tilde{X}_1^s d_{\text{lag}}^{1,s} e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}+\tilde{Y}_1^s \left(d_{\text{lag}}^{1,s}\right)^2 e^{-\gamma_1 r_1 d_{\text{lag}}^{1,s}}\right\rangle_w . \tag{30}$$

The algebraic calculation of equation (30) is rather similar to the one mentioned in Section 3.2 (for more details see Ngoduy, 2005):

$$\begin{aligned}\pi_{0,1}^{u}=\sum_{s\in U}\frac{r_1^s}{r_1}e^{-\gamma_1 r_1\left(d_{\min}+l_s+\mu T_0^u V_0^u\right)}&\left[1+0.5\Theta_0^u\left(\mu\gamma_1 r_1 T_0^u\right)^2\right]e^{-\gamma_1 r_1\left(d_{\min}+l_u+T_1^s V_1^s\right)}\\ \times\Bigg\{&\left[1+\tilde{X}_1^s\left(d_{\min}+l_u+T_1^s V_1^s\right)+\tilde{Y}_1^s\left(d_{\min}+l_u+T_1^s V_1^s\right)^2\right]\left[1+0.5\Theta_1^s\left(\gamma_1 r_1 T_1^s\right)^2\right]\\ &-\left[\tilde{X}_1^s\gamma_1 r_1+2\tilde{Y}_1^s\left(d_{\min}+l_u+T_1^s V_1^s\right)-\tilde{Y}_1^s\right]\Theta_1^s\left(T_1^s\right)^2\Bigg\}.\end{aligned} \tag{31}$$

Similar approach is applied to derive the probability that a vehicle can go off the freeway:

$$\begin{aligned}\pi_{1,0}^{u}=\sum_{s\in U}\frac{r_0^s}{r_0}e^{-\gamma_0 r_0\left(d_{\min}+l_s+\mu T_1^u V_1^u\right)}&\left[1+0.5\Theta_1^u\left(\mu\gamma_0 r_0 T_1^u\right)^2\right]e^{-\gamma_0 r_0\left(d_{\min}+l_u+T_0^s V_0^s\right)}\\ \times\Bigg\{&\left[1+\tilde{Z}_0^s\left(d_{\min}+l_u+T_0^s V_0^s\right)\right]\left[1+0.5\Theta_0^s\left(\gamma_0 r_0 T_0^s\right)^2\right]-\tilde{Z}_0^s\gamma_0 r_0\Theta_0^s\left(T_0^s\right)^2\Bigg\},\end{aligned} \tag{32}$$

where

$$\tilde{Z}_0^u=0.5\alpha_{0,1}^u\lambda_{0,1}^u\gamma_0 r_0 . \tag{33}$$

From equations (31) and (32), it is clear that given local traffic condition, the reaction time plays a very important role in the lane-changing behavior. The higher the values of reaction time the lower the possibility to make a lane-change. That means the higher fraction of trucks in the shoulder lane and the auxiliary lane results in the more congestion of the traffic operations within weaving section. Furthermore, taking a closer look at the different driving style within a vehicle class, an aggressive driver often accepts smaller gaps than a timid driver. Consequently the former often causes more disruption for traffic flow than the latter. Another factor that contributes considerably to the lane-changing behavior within weaving section is the ramp factor μ, which is decreasing with the remaining distance to the end of the auxiliary lane. The lower the value of μ leads to the higher probability to change lanes and, consequently results in the more unstable traffic flow in the target lane. That explains why approaching the end of auxiliary lane, drivers are willing to accept smaller gaps to change lanes and disturb traffic in the target lane more considerably.

3.4. Generalized macroscopic continuum model of weaving sections

This section will establish a new macroscopic model for traffic flow dynamics at weaving section. The derivation bases on the gas-kinetic model developed above using the so-called method of moments, which is described in the ensuing of this section. That is:

$$\int_0^\infty \{\text{gas-kinetic model}\}\,\mathrm{d}v \to \{\text{macroscopic model}\}\,. \tag{34}$$

Compared to the gas-kinetic model, the macroscopic model is more suitable for theoretical and numerical analysis of traffic phenomena, real-time application in traffic control. The derivation of continuum traffic flow models from gas-kinetic traffic models using the method of moments has been applied by many researchers such as Leutzbach (1988), Helbing (1997), Hoogendoorn (1999), Hoogendoorn and Bovy (1999), etc. By applying the method of moments we can obtain the equations for the dynamics of density, mean speed or flow rate. Let us briefly describe the working of the method of moments as following. We multiply both sides of equation (8) with v^k (k = 0, 1, 2, …), then integrate them over the range of speed v. That is:

$$\begin{aligned}&\partial_t \int_0^\infty \rho_i^u v^k\,\mathrm{d}v + \partial_x \int_0^\infty \rho_i^u v^{k+1}\,\mathrm{d}v + \partial_v \int_0^\infty \left(\rho_i^u \frac{v^k V_{i,u}^{\max} - v^{k+1}}{\tau_i^u}\right)\mathrm{d}v \\ &= \int_0^\infty \left[\left(\partial_t \rho_i^u v^k\right)_{\text{int}} + \left(\partial_t \rho_i^u v^k\right)_{\text{lc}}^{\text{imm}} + \left(\partial_t \rho_i^u v^k\right)_{\text{lc}}^{\text{spon}} + \omega_{i,u}^{+} v^k + \omega_{i,u}^{-} v^k\right]\mathrm{d}v.\end{aligned} \tag{35}$$

Let $m_{i,k}^u(x,t) = \int_0^\infty \rho_i^u(x,v,t) v^k\,\mathrm{d}v = r_i^u \left\langle v^k \right\rangle_i^u$ and $m_{i,k}^{u,0}(x,t) = r_i^u V_i^{u,\max} \left\langle v^{k-1} \right\rangle_i^u$, the LHS of equation (35) becomes:

$$\begin{aligned}&\partial_t \int_0^\infty \rho_i^u v^k\,\mathrm{d}v + \partial_x \int_0^\infty \rho_i^u v^{k+1}\,\mathrm{d}v + \partial_v \int_0^\infty \left(\rho_i^u \frac{v^k V_{i,u}^{\max} - v^{k+1}}{\tau_i^u}\right)\mathrm{d}v \\ &= \partial_t m_{i,k}^u + \partial_x m_{i,k+1}^u + k\frac{m_{i,k}^u - m_{i,k}^{u,0}}{\tau_i^u}.\end{aligned} \tag{36}$$

For the RHS of equation (35), all the first three terms have been described in literature, for example in Shvetsov and Helbing (1999). The last term is determined below:

$$\int_0^\infty \omega_{i,u}^{\pm}(x,v,t) v^k\,\mathrm{d}v = \tilde{\omega}_{i,u}^{\pm}(x,t)\left\langle v^k \right\rangle_{i,u}^{\pm} \tag{37}$$

where

$$\tilde{\omega}_{i,u}^{\pm}(x,t) = \int_0^\infty \omega_{i,u}^{\pm}(x,v,t)\,\mathrm{d}v \quad \text{and} \quad \left\langle v^k \right\rangle_{i,u}^{\pm} = \int_0^\infty \frac{\omega_{i,u}^{\pm}(x,v,t)}{\tilde{\omega}_{i,u}^{\pm}(x,t)} v^k\,\mathrm{d}v\,. \tag{38}$$

From the obtained results, we come up with macroscopic continuum model when we set k = 0 for first order model, k = 1 for second order model, or even higher order model if k = 2. In this paper, we show equations for the dynamics of density (k = 0) and flow rate (k = 1). By definition, the macroscopic traffic variables for MLMC traffic flow are:

$$
\begin{aligned}
&r_i^u(x,t)=\int_0^\infty \rho_i^u(x,v,t)\,\mathrm{d}v,\\
&V_i^u(x,t)=\frac{1}{r_i^u(x,t)}\int_0^\infty \rho_i^u(x,v,t)\,v\,\mathrm{d}v,\\
&\Theta_i^u(x,t)=\frac{1}{r_i^u(x,t)}\int_0^\infty \rho_i^u(x,v,t)\left(v-V_i^u\right)^2\mathrm{d}v,\\
&V_{i,u}^{\pm}(x,t)=\langle v\rangle_{i,u}^{\pm},
\end{aligned}
\tag{39}
$$

where $V_{i,u}^{\pm}(x,t)$ denotes the mean speed of vehicles class u entering to lane i (with plus sign) or exiting from lane i (with minus sign) at location x and time instant t. By substituting expressions (39) into the RHS of equation (36) for $k = 0$ and $k = 1$, we obtain the corresponding expressions as follows:

$$
\partial_t r_i^u+\partial_x q_i^u=\underbrace{\sum_{s\in U}\sum_{j=i\pm1}\left(p_{j,i}^u\Psi_j^{u,s}-p_{i,j}^u\Psi_i^{u,s}\right)}_{\text{immediate lane-change}}+\underbrace{\sum_{j=i\pm1}\left(\Delta_{j,i}^u r_j^u-\Delta_{i,j}^u r_i^u\right)}_{\text{spontaneous lane-change}}+\underbrace{\tilde{\omega}_{i,u}^{+}+\tilde{\omega}_{i,u}^{-}}_{\text{mandatory lane-change}},
\tag{40}
$$

$$
\begin{aligned}
\partial_t q_i^u+\partial_x r_i^u\left[\left(V_i^u\right)^2+\Theta_i^u\right]=&\underbrace{\frac{r_i^u V_i^{u,\max}-q_i^u}{\tau_i^u}}_{\text{relaxation}}-\underbrace{\left[1-\alpha_{i,i-1}^u\pi_{i,i-1}^u-\left(1-\alpha_{i,i-1}^u\right)p_{i,i+1}^u\right]\sum_{s\in U}\Pi_i^{u,s}}_{\text{braking}}\\
&+\underbrace{\sum_{s\in U}\sum_{j=i\pm1}\left(p_{j,i}^u\Phi_j^{u,s}-p_{i,j}^u\Phi_i^{u,s}\right)}_{\text{immediate lane-change}}+\underbrace{\sum_{j=i\pm1}\left(\Delta_{j,i}^u q_j^u-\Delta_{i,j}^u q_i^u\right)}_{\text{spontaneous lane-change}}+\underbrace{\tilde{\omega}_{i,u}^{+}V_{i,u}^{+}+\tilde{\omega}_{i,u}^{-}V_{i,u}^{-}}_{\text{mandatory lane-change}}.
\end{aligned}
\tag{41}
$$

In equation (41), $\alpha_{i,i-1}^u\pi_{i,i-1}^u$ denotes the fraction of weaving vehicles that exit the shoulder lane while $\left(1-\alpha_{i,i-1}^u\right)p_{i,i+1}^u$ denotes the fraction of non-weaving vehicles that change lanes to the left due to the interaction process. Accordingly, $\left[1-\alpha_{i,i-1}^u\pi_{i,i-1}^u-\left(1-\alpha_{i,i-1}^u\right)p_{i,i+1}^u\right]$ becomes the fraction of vehicles that remain in the shoulder lane, which is multiplied with the interaction rate $\Pi_i^{u,s}$ to cause the changes of lane and class specific traffic flow rate q_i^u due to braking process. Note in weaving sections, the equilibrium relations mentioned in Hoogendoorn (1999) are not applied.

To close system (40) and (41), we need to determine the speed variance Θ_i^u by assuming that the speed variance is a function of mean speed V_i^u and density r_i^u as follow:

$$
\Theta_i^u=\eta\left(r_i^u\right)\left(V_i^u\right)^2,
\tag{42}
$$

where $\eta\left(r_i^u\right)$ is a step-like function of density, which conforms to the following expression:

$$\eta\left(r_i^u\right) = \eta_i^{u,0} + \delta\eta_i^u \left[1 + \exp\left(\frac{r_i^{u,cr} - r_i^u}{\delta r_i^u}\right)\right]^{-1}. \tag{43}$$

The step-like function parameters $\eta_i^{u,0}$, $\delta\eta_i^u$, δr_i^u, and critical density $r_i^{u,cr}$ are estimated from empirical observations (see the typical values for these parameters estimated from the Dutch free way A9 in Ngoduy, 2005).

Set of equations (40) and (41) are of hyperbolic type and can be solved using numerical methods presented in Ngoduy et al. (2004b). All factor functions describing the changes of traffic variables due to braking, ILC and SLC processes such as: $\Pi_i^{u,s}$, $\Psi_i^{u,s}$, $\Phi_i^{u,s}$, and $\Delta_{i,j}^u$ are given in Shvetsov and Helbing (1999).

4. NUMERICAL STUDY

To give an insight into the model performance, let us simulate the model developed in Section 3.4 with a two lane freeway 3km in length and having a weaving from KM2.0 to KM2.5 in 3 hours (Figure 6a). Traffic composition consists of car (90%) and truck (10%). Traffic demand to the main lanes and to the weaving section is given in Table 1. Traffic fractions are 0.5 in all movements. The numerical scheme presented in Ngoduy et al. (2004b) in which the road is divided into cells 100m in length, the simulation time step is 1 second, is applied. Open boundary condition is used for this simulation. The parameters used for the simulation are given in Table 2. Note that for the sake of simplicity, we assume that the parameters are the same for lane but different for truck and car.

TABLE 1: Traffic demand to the main lanes and to the weaving section

Time (sec)	Unit	0	1800	3600	7200	10800
Main lanes	Veh/h/lane	1800	1800	1800	1800	1800
Weaving	Veh/h	800	800	1800	1800	800

TABLE 2: Traffic demand to the main lanes and to the weaving section

Parameters	Notation/Unit	Truck	Car
Ramp factor	μ_{max}	0.9	0.9
	μ_{min}	0.2	0.2
Free speed	$V_i^{u,\max}$ (km/h)	85	120
Jam density	$r_i^{u,\max}$ (veh/km)	120	160
Reaction time	T_i^u (sec)	2.2	1.6
Relaxation time	τ_i^u (sec)	35	18
Speed variance coefficients	$\eta_i^{u,0}$	0.008	0.01
	$\delta\eta_i^u$	0.05	0.08
	δr_i^u (veh/km)	3	6
	$r_i^{u,cr}$ (veh/km)	28	38

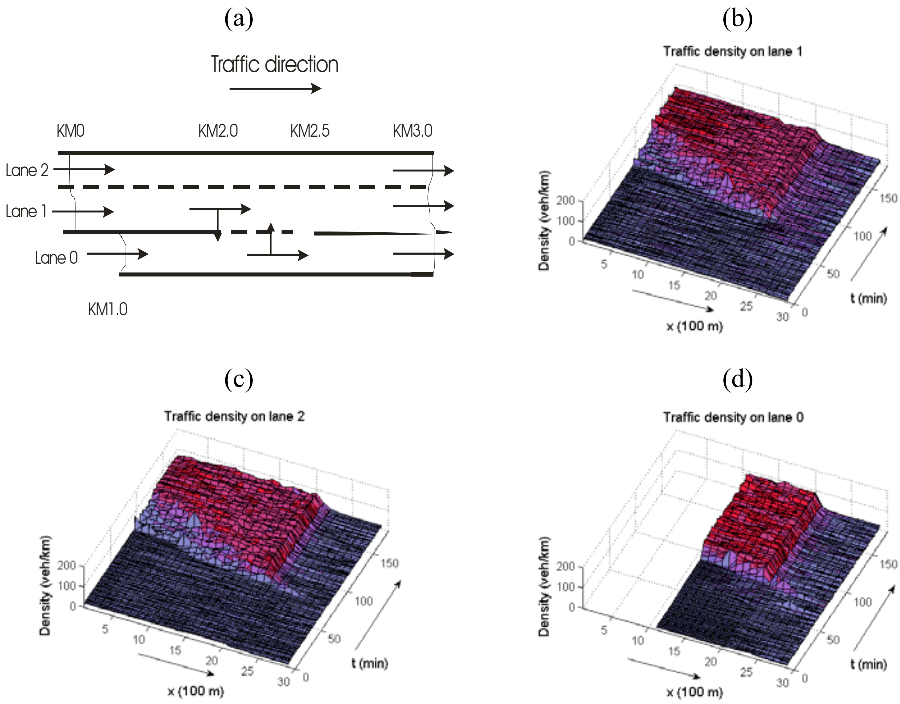

FIGURE 6: Density in the right lane (b), in the left lane (c) and in the auxiliary lane (d)

Figures 6b and 6c illustrate the dynamics of traffic densities in main lanes. It is clear that behind the bottleneck (e.g. KM2.0), a growing region of congested traffic forms immediately due to the peak of demand to weaving section (t = 3600-7200 sec), which makes the flow in the shoulder lane (lane 1) exceed the capacity. Congestion in the main lanes blocks the vehicles desiring to merge which leads to the congestion in the auxiliary lane although the peak flow is less than the capacity of this lane (Figure 6d). We find that the structure of congested traffic flow reproduced by the model is well-consistent with observations in microscopic models (Helbing and Tilch, 1998).

By varying the traffic demand, we construct a contour diagram of the weaving lengths with respect to the combination of different traffic demands for a given merging probability (Figure 7). Here we assume that the traffic operation is good when the merging probability is 85%. This is an important property of the developed model since it can be used to set up a nomogram to support the design guidelines in HCM 2000 for weaving geometry.

5. CONCLUSIONS

In this paper, we have developed a gas-kinetic traffic flow model for mixed traffic at weaving sections on freeways and derived the corresponding macroscopic model using the so-called method of moments. In this model the lane changing maneuvers have been modeled using renewal theory. The calculated lane-changing probabilities depend on a lot of factors such as density, speed, speed variance, weaving flow fraction and vehicle compositions on the target lane. Moreover, the model has also taken into account the give-way effect and the willingness to accept smaller gaps of subject drivers when approaching the end of weaving area, which have been often neglected in previous macroscopic traffic models.

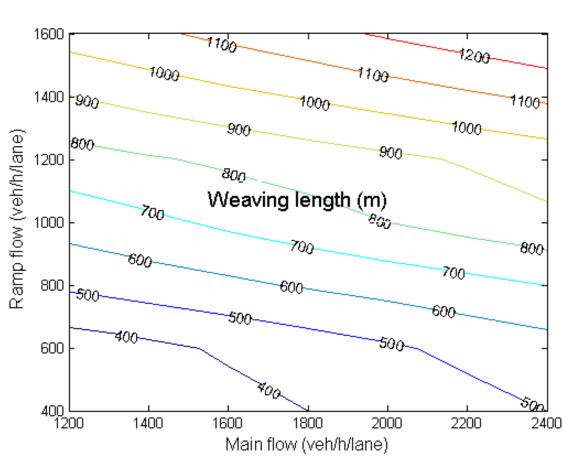

FIGURE 7: Contour diagram for necessary weaving lengths to keep main traffic flow stable

We argue that the proposed model is more generic than other continuum models since it is based on microscopic principle in terms of modeling the inflow from on-ramp or out-flow to off-ramp but requires fewer parameters than microscopic models, which results in the simplification of the calibration and validation process. Numerical simulation results show that the developed model is well-consistent with microscopic models in reproducing the formation of traffic congestion within weaving sections. Furthermore, the developed model is useful in supporting the geometry design manual for weaving sections by means of constructing counter diagram of necessary weaving lengths based on probabilistic method.

Our current work is to investigate the working of the proposed model in case the behavior of drivers differs within a vehicle class (e.g. timid or aggressive driver) or changes due to the current traffic conditions, for example the reaction time increases when the local traffic becomes unstable. This work requires an application of the distribution of critical gaps within a vehicle class instead of a single value as assumed in this paper. This research direction may result in a model to explain better the wide scattering of the flow-density relation, which is often observed at weaving sections.

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APPENDIX. NOTATION

Shorthand

p.d.f.	:	probability density function
c.d.f.	:	cumulative density function
PSD	:	phase-space density
ILC	:	immediate lane-change
SLC	:	spontaneous lane-change
MLC	:	mandatory lane-change
RHS	:	right hand side
LHS	:	left hand side
MLMC	:	multi-lane multi-class
exp	:	exponential function

Definition of mean operator

Let $\langle \psi(x) \rangle$ denote the mean operator of any function ψ(*x*). It is determined by the following expression: $\langle \psi(x) \rangle_x = \int_{-\infty}^{+\infty} \psi(y) f(y) \, \mathrm{d} y$, where *f*(*x*) denotes the probability density distribution of variable *x*.

Independent variables

x	:	location x (m)
x_{end}	:	location of the end of weaving section (m)
x_0	:	location of the beginning of weaving section (m)
v	:	speed v (m/s)
t	:	time instant t (s)
L	:	weaving length (m)
I	:	freeway lane index (i=0,1,2,…,I)
u	:	vehicle class index (u=1,2,…,U)

Mesoscopic variables

$\rho_i^u(x,v,t)$	:	lane and class specific phase-space density at location x and time instant t
$f_i^u(x,v,t)$	:	lane and class specific gap probability density function
$F_i^u(x,v,t)$	:	lane and class specific cumulative gap probability density function
$f_{\text{lead}}^{i,u}(x,v,t)$	:	lane and class specific lead gap probability density function
$F_{\text{lead}}^{i,u}(x,v,t)$	:	lane and class specific cumulative lead gap probability density function
$f_{\text{lag}}^{i,u}(x,v,t)$	:	lane and class specific lag gap probability density function

$F_{\text{lag}}^{i,u}(x,v,t)$: lane and class specific cumulative lag gap probability density function

Macroscopic variables

$r_i^u(x,t)$: lane and class specific density at location x and time instant t (vehicle/m)

$V_i^u(x,t)$: lane and class specific mean speed at location x and time instant t (m/s)

$q_i^u(x,t)$: lane and class specific flow rate at location x and time instant t (vehicle/s)

$\Theta_i^u(x,t)$: lane and class specific speed variance at location x and time instant t (m^2/s^2)

$V_i^{u,e}(x,t)$: lane and class specific equilibrium speed at location x and time instant t (m/s)

$\alpha_{i,j}^u(x,t)$: lane and class specific traffic flow fraction at a weaving section.

Lane-changing variables

$\pi_{i,j}^u(x,t)$: class specific average probability to find sufficient gaps in the target lanes j for a mandatory lane-change from lane I at location x and time instant t

$p_{i,j}^u(x,t)$: class specific average probability to find sufficient gaps in the target lanes j for an immediate lane-change from lane I at location x and time instant t

$V_{i,u}^{\pm}(x,t)$: class specific mean speed of traffic merging to (plus sign) or diverging from (minus sign) lane I at location x and time instant t (m/s)

$\omega_{i,u}^{\pm}(x,t)$: class specific average rate of merging traffic to (plus sign) or diverging traffic from(minus sign) lane I at location x and time instant t (vehicle/m^2).

$\Pi_i^{u,s}(x,t)$: lane and class specific interaction rate (vehicle/(m.s))

$\Psi_i^{u,s}(x,t)$: lane and class specific changes of density due to interaction between vehicles (vehicle/(m.s))

$\Phi_i^{u,s}(x,t)$: lane and class specific changes of flow rate due to interaction between vehicles (vehicle/s^2)

$\Delta_i^u(x,t)$: lane and class specific spontaneous lane-changing rate (1/s)

Model parameters

$V_i^{u,\max}$	: lane and class specific free speed at location x and time instant t (m/s)
τ_i^u	: lane and class specific time needed to relax to the equilibrium situation (s)
l_u	: class specific average length of a vehicle (m)
$d_u^{\min}$	: class specific minimal safe distance (m).
T_i^u	: lane and class specific reaction time (s)
D_i^u	: lane and class specific gross distance headway (m)
$d_{\text{lead}}^{i,u}$	: lane and class specific lead distance gap (m)
$d_{\text{lag}}^{i,u}$	: lane and class specific lag distance gap (m)
$\overline{h}_i$	: lane specific expected unoccupied space (m)
$r_i^{u,\max}$	: lane and class specific jam density (vehicle/m)
$r_i^{u,cr}$	: lane and class specific critical density (vehicle/m)
$\eta_i^{u,0}$, $\delta\eta_i^u$, δr_i^u	: step-like function parameters
$\mu(x)$	: ramp factor, which is a function of location x