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THE MATHEMATICAL REPRESENTATION OF A MODEL OF THE RELATIONSHIPS BETWEEN TRANSPORT AND LAND USE

BY ROGER L. MACKETT

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ABSTRACT

MACKETT, R.L. (1979) The mathematical representation of a model of the relationships between transport and land use. Leeds: University of Leeds, Inst. Transp. Stud., WP 123

This paper contains the mathematical description of a model that represents the relationships between transport and the spatial distribution of population, housing, jobs, employment, shopping and land. This paper complements the verbal description of the model elsewhere (Mackett, 1979b), and is intended for the reader interested in the detailed workings of the model. In the model, transport costs are measured in generalised costs units for the public and private modes. The demolition and building of housing are calculated within the model, taking any exogenous information into account. Economic activity is divided into twelve sectors which are allocated to three groups according to the degree of locational response to changes in transport costs. In the model a distinction is made between infrastructure, for example, houses and jobs, and the allocation of people to them. The allocation procedure ensures that those who have not moved retain the same residential or employment location over time. The distinction between infrastructure and activities means that inequalities in their provision, for example, vacant houses and jobs, are included. Land availability is taken into account in the location of activities. The equation system is solved using iterative techniques. After convergence a wide range of information is obtained, including the spatial distribution of population, housing and jobs, accessibility indicators, modal splits and time and money expenditure on travel.

1. INTRODUCTION

In this paper the mathematical representation of the integrated land use-transport model described elsewhere (Mackett, 1979b) is presented. This is done on a topic-by-topic basis in the same order as the other paper. In each topic the basic principle is described, then the relevant equations are presented. In this way the individual mechanisms can be seen and so compared with those in models of individual sectors. The whole model and its behaviour over time is described in Mackett (1979b), together with some comments on the use of the model. This paper supercedes descriptions of earlier versions of the model (Mackett, 1974,1976a) and it is intended to make understanding of the behaviour of the individual components much clearer.

2. NOTATION

The model is represented by notation that has been derived with the twin objectives of defining every variable explicitly and uniquely, and being easily comprehensible. With such a complex model this is not a trivial task. The following rules are used:

a) superscripts describe or qualify the main variables; the may be used singly or in combination;
b) subscripts are integer labels to which the variables refer, for example zones or social groups;
c) superscripts are used to indicate the particular subscript being used where this is necessary to ensure unique definition, so that when a particular value is assigned to a subscript the variable is still unique;
d) whenever possible a variable is defined by its initial letter, otherwise some other appropriate letter is used.

Main variables

Zonal variables

D - density of housing

E - employment

F - car ownership level

H - housing

J - jobs

L - land area

N - number of cars owned

P - population

R - employed residents

W - attraction factor

X - expenditure

Y - accessibility

h - housing demolition factor

Spatial variables

C - cost of travel (or parking)

M - number of buses used (between a pair of zones)

S - sales or demand for non-retail tertiary economic activity

T - trips

d - travel distance

t - travel time

Other factors

A - balancing factor

B - balancing factor

G - growth factor for commuting and car ownership

I - number of iteration

K - balancing factor

Q - survival or birth rate

U - rate (for conversion from one variable to another)

V - value of time

Z - zonal sets (see below)

b - employment - demand ratio (for tertiary activities)

c - operating costs for cars

e - bus fare boarding element

f - bus fare distance element

g - inverse activity rate

o - occupancy rate (for housing and cars)

s - speed of travel

v - relative weighting on the value of time

x - expenditure (or demand) per head

δ - maximum proportion of housing in a zone occupied by a particular social group that is demolished within a time period

Superscripts

A - agricultural (land)

B - built (housing)

C - converted (housing)

D - demolished (housing)

E - employment

F - fixed (over time)

G - developed (land)

H - housing

I - intrazonal (trips)

K - mode of transport

L - vacated by out-migrants

M - car ownership group for shopping

N - car ownership group for work

O - occupied (housing)

P - primary and secondary economic activity

Q - social group

R - residential

S - tertiary activity

T - total

U - residential mover set

V - employment mover set

W - work

X - exogenously located

Y - survival

Z - zonal

a - available (land and housing)

b - born

c - car (for occupancy rate)

f - out- commuting (from internal zones)

h - held off market (land)

g - released for development (land)

l - left at home (cars)

o - off-peak

p - peak

r - car ownership (growth factor)

t - in- commuting (to internal zones)

u - unusable (land)

v - vacant

w - walking (time or speed)

x - waiting (time)

y - parking (cost)

z - operating (cost)

<u>max</u> - maximum (population)

<u>min</u> - minimum (population)

<u>inc</u> - increment (over time)

<u>Subscripts</u>

N - number of internal zones

M - number of all zones (internal and external)

i - zone

j - zone

c - city (that is, summed over all internal zones)

k - mode

m - car ownership group for shopping

n - car ownership group for work

p - primary and secondary economic activity

q - social group

s - tertiary economic activity

u - residential mover set

v - employment mover set

Mover sets

$u = 1$ newly located residents

$u = 2$ non-moving residents

$v = 1$ newly located workers

$v = 2$ non-moving workers

Calibrated parameters

α - parameter representing non-linearities between a located variable and the independent factors determining its location

β - parameter representing the deterrence effect of distance upon trip length

Zonal sets

Z_{q1} residential zones with constrained population for social group q

Z_{q2} residential zones with unconstrained population for social group q

Z_{q3} work zones with constrained employment for social group q

Z_{q4} work zones with unconstrained employment for social group q

Z_{q5} zones with population located exogenously

Z_6 zones with no houses demolished

Modal sets

$\gamma(n)$ modes of transport available to those in car ownership group n

Time

(t) time point t

(t-1,t) time period from t-1 to t

3. TRANSPORT

3.1 Basic principle

One of the main purposes of this model is to permit the examination of the effect of changes in the transport system upon land use. Transport is represented by the cost of travel between each pair of zones by private and public modes of travel. In the model public transport includes not only bus and train but all modes except car and motor-cycle (which are the private modes). In a study concerned only with the behaviour of vehicular trips it is quite permissible to ignore those who work at home, have no job or fixed workplace, but in this study in which the location of everybody living in the city is being considered, these other cases cannot be ignored. In the main data source being used (the Census of Population) those trips are classified as intrazonal and so they are treated as such here. Similarly, it has not been found practical to model walking trips explicitly (because of lack of data, lack of information on walking behaviour, insufficiently detailed networks and computational constraints). Thus other modes are taken into account by applying suitable factors to the intrazonal costs for the public transport mode.

The interzonal cost functions are of the standard form used in many transportation studies (for example, WYTCONSULT, 1976). There are many weaknesses in such formulations, but the fact that they are so widely used suggests that these are in agreement with the general concensus in the field.

Different values are used for peak and off-peak. In the model the former are used for work journeys the latter for shopping. Clearly, there are exceptions to these assumptions but again this is in line with the usual thinking.

3.2 Private mode - peak (C^p_{ijl})

Three elements make up the cost of travel by car - the travel time, the monetary cost of travel and the cost of parking (C^{yp}). The monetary cost of travel is a function of distance (d_{ijk}), the perceived operating cost per unit distance (c^z) and the car occupancy rate (o^{cp}). Money costs are converted to time units by dividing by the value of time, thus:

$$C^p_{ijk} = t^p_{ijk} + \frac{d_{ijk}c^z}{o^{cp}V} + \frac{C^{yp}_j}{V} \qquad (1)$$

3.3 Private mode off-peak (C^o_{ijl})

This is similar to the peak function but different values of travel time, car occupancy rate and parking charges are used:

$$C^{o}_{tjk} = t^{o}_{ijk} + \frac{d_{ijk}c^{z}}{{}_{o}c^{o}V} + \frac{C^{yo}_{j}}{V} \qquad (2)$$

3.4 Public mode - peak (except intrazonal) (C^{p}_{ij2})

The cost of travel by public transport consists of four elements - the travel time, the walking time at each end of the trip (t^{w}_{i}, t^{w}_{j}) the waiting time (t^{xp}) taking into account the number of buses used (M_{ij}) and the fare paid ($e+fd_{ij}$), thus:

$$C^{p}_{ijk} = t^{p}_{ijk} + v^{w}\,(t^{w}_{i} + t^{w}_{j}) + v^{x}M_{ij}\,t^{xp} + \frac{e+fd_{ij}}{V} \quad i\neq j \qquad (3)$$

The terms v^{w} and v^{x} allow for the fact that waiting and walking times are generally held to be assigned greater values than in-vehicle time. The use of the term M_{ij}, representing the number of buses used means that in effect, a penalty is imposed if more than one bus is used, since each must be waited for in turn.

3.5 Public model off-peak (except intrazonal) (C^{o}_{ij2})

This is similar to the peak function except for the travel time (t^{o}_{ijk}) and waiting time (t^{xo}):

$$C^{o}_{ijk} = t^{o}_{ijk} + v^{w}(t^{w}_{i} + t^{w}_{j}) + v^{x}\,M_{ij}t^{xo} + \frac{e+fd_{ij}}{V} \quad i\neq j \qquad (4)$$

3.6 Public mode - intrazonal

As discussed above the public transport intrazonal costs are designed to represent walking trips and those with no fixed workplace or working at home. This is done by averaging over the costs of travel by bus, walking and in the case of peak, zero costs.

The peak intrazonal costs are given by:

$$C^{p}_{ijk} = \frac{[t^{p}_{ii2} + 2v^{w}t^{w}_{i} + v^{x}M_{ii}t^{xp} + (e+fd_{ii})/V]T^{IK}_{ii2} + d_{ii}T^{IK}_{ii3}/s^{w}}{\sum_{k=2}^{4} T^{I}_{iik}} \qquad (5)$$

where T_{iik}^{IK} are the actual intrazonal trips by mode k, where k = 2 represents motorised public transport, k = 3 represents walking and k = 4 represents those who work at home (and hence have no journey to work cost); s^w is the mean walking speed.

The equivalent off-peak costs are given by:

$$C_{iik}^{o} = \frac{\left[t_{ii2}^{o} + 2v^{w}\, t_{i}^{w} + v^{x}M_{ii}\, t^{xo} + (e+fd_{ii})/v\right]T_{ii2}^{IK} + d_{ii}T_{ii3}^{IK}/s^{w} + T_{ii4}^{IK}}{\sum_{k=2}^{4} T_{iik}^{IK}} \qquad (6)$$

Since those who work at home make no trip as such they are excluded from the peak cost calculation, but are included in the off-peak value since they will make other trips.

The intrazonal travel times in minutes are calculated by applying a mean speed (peak or off-peak) to the intrazonal distance d_{ii}:

$$t_{iik}^{p} = 60\ d_{ii}/s_{k}^{p} \qquad (7)$$

$$t_{iik}^{o} = 60\ d_{ii}/s_{k}^{o} \qquad (8)$$

where s_k^p and s_k^o are the peak and off-peak speeds by motorised public transport respectively in kilometres per hour.

The intrazonal distances are computed based upon the area of urban development in each zone (L_i^G) in hectares:

$$d_{ii} = \frac{2}{30}\ \frac{L_i^G}{\pi} \qquad (9)$$

The value of L_i^G is kept constant over forecasts since the slight change in its value makes very little difference to the value of d_{ii}. The use of the square root of two thirds the area is based upon assuming that all the area of urbanization is circular and that trips are evenly spread through the zone. See Bonsall (1975) for further details.

The intrazonal distance is used in the calculation of the mean walking time to public transport, which is taken to be half the mean intrazonal distance up to a maximum of ten minutes:

$$t_i^w = 0.5\left(\frac{60d_{ii}}{s^w}\right) \Bigg\} \text{ whichever is less} \qquad (10)$$
$$= 10$$

It is quite recognised that some of the assumptions made here are rather sweeping. Nonetheless this is a very difficult field and it would be a mistake to be diverted from the main task of producing an operational integrated land use - transport model by detailed considerations that are common to other models.

4. HOUSING LOCATION

4.1 Basic principle

In reality the location of housing is largely a function of the decisions by the local authority; consequently the model facilitates the exogenous location of housing to represent a particular policy. However, housing locations may not have an explicit role in some policies being tested, so a method has been developed making the endogenous location of housing a function of the area of land available for housing, the area of land already used by housing and the relative increase (or decrease) in the accessibility of other residential areas and workplaces. The two area terms mean that new housing is most likely to go to a zone that is about half-developed, since a zone on the fringe with little development is less likely to have the necessary physical service provision in terms of sewers, water, and so on, and a zone that is nearly full is unable to accomodate many more housing units. The accessibility terms are included to represent the effects of substantial changes in travel costs associated with a new road or bus service. The land market is an extremely complex process and it would be impossible to represent it accurately within this type of model, but by incorporating the accessibility effects an approximation to the impact of new transport facilities on the location of new housing is included. Using the ratio of accessibilities at the current time point to that previously ensures that only large changes have a significant effect.

In the short run it is difficult to forecast accurately the location of houses to be demolished, since this is largely a political matter. Consequently the location of demolition is made a function of the existing housing stock and a factor representing the propensity of a zone to have the houses in it demolished. The allocation of demolished houses to social groups is based upon the hypothesis that the lower the social status of the occupant of a house, the greater the probability that it will be demolished.

4.2 New housing (H_i^B)

This is located using the factors indicated above. The total new housing in the city (H_c^B) is given, and a balancing factor (B_i^H) used to ensure that the zonal allocations sum to this total:

$$H_i^B = H_i^{BX} + (H_c^B - \sum_j H_j^{BX}) B^H \frac{L_i^{Ha}(t) L_i^H(t-1)}{L_i^T} \left(\frac{Y_i^R(t)\ Y_i^E(t)}{Y_i^R(t-1)\ Y_i^E(t-1)} \right)^{\alpha^H} \qquad (11)$$

$$\text{where } B^H = \left[\sum_{i=1}^{N} \frac{L_i^{Ha}(t)\ L_i^H(t-1)}{L_i^T} \left(\frac{Y_i^R(t)\ Y_i^E(t)}{Y_i^R(t-1) Y_i^E(t-1)} \right)^{\alpha^H} \right]^{-1} \qquad (12)$$

$$\text{to ensure } \sum_{i=1}^{N} H_i^B = H_c^B \qquad (13)$$

where H_i^{BX} is the new housing being located exogenously in zone i, Y_i^R is the accessibility to residential areas and Y_i^E is the accessibility to employment areas. α^H is a parameter designed to take into account non-linearities between the values of H_i^B and the locational factors.

4.3 Housing conversions

Beside being built, dwellings can be created by conversion, for example, splitting a house into two or more flats. Like new housing this can be specified exogenously, or located endogenously on the basis of the existing housing pattern. The total is specified exogenously. Thus the number of dwellings created by conversion (H_i^C) is given by:

$$H_i^C(t-1,t) = H_i^{CX}(t-1,t) + \left(H_c^C(t-1,t) - \sum_{j=1}^{N} H_j^{CX}(t-1,t)\right) \frac{H_i(t-1)}{\sum_{j=1}^{N} H_j(t-1)} \qquad (14)$$

where H_i^{CX} is the number of housing conversions being located exogenously.

Housing conversions are assumed to take up no extra land.

4.4 Housing demolition

Housing demolition is part of the urban renewal process, and as such, part of the function of the local authority. Consequently, it would normally be expected that the number of homes being demolished is an exogenous input to the model. However, in a long run forecast only the total demolitions in the study area may be known, so in this case the number of demolitions in each zone (H_i^D) is given by:

$$H_i^D(t-1,t) = H_i^{DX}(t-1,t) + \left[\bar{H}_c^D(t-1,t) - \sum_j H_j^{DX}(t-1,t)\right] B^D H_i^X(t-1) h_i \qquad (15)$$

$$\text{where } B^D = \left[\sum_{i=1}^{N} H_i(t-1) h_i\right]^{-1} \qquad (16)$$

to ensure that

$$\sum_{i=1}^{N} H_i^D(t-1,t) = H_c^D(t-1,t) \qquad (17)$$

and h_i is a factor representing the propensity of houses in a zone to be demolished, since some zones will contain older houses and so more likely to have houses demolished. Ideally data on the age of housing would be used, but in the absence of such data a factor (h_i) is calculated in the first time period, for which it is assumed that the pattern of all demolition ~~is~~ known, as is the case in this study.

Thus:

$$h_i = H_i^{DX}(1,2)/H_i(1) \qquad (18)$$

In reality many houses may be vacant for a long period prior to demolition. In order to represent this effect, the proportion of houses which are occupied in zones with no demolitions is found and compared with the proportion in zones with houses being demolished. Any excess vacant houses in the latter are assumed to be due to the effects of blight and so are included in the demolished set; the other houses demolished are taken to be occupied (the occupants being forced to move out). Examination of the housing demolition policy being pursued by Leeds M.D.Council shows that, in general, relatively small areas are demolished at any one time (compared with the size of the zones). Whilst the model explicitly ensures that the houses occupied by those of lowest social status are the most likely to be demolished, a factor is applied to ensure that no one particular social group completely removed from a zone.

We define Z_6 as the set of zones with no houses demolished, then the number of vacant houses demolished (H_i^{DV}) is given by:

$$H_i^{DV}(t-1,t) = H_i(t-1)\,\frac{\sum_{i\varepsilon Z_6} H_i^O(t-1)}{\sum_{i\varepsilon Z_6} H_i(t-1)} - H_i^O(t-1) \quad \text{whichever is greater for } i \notin Z_6 \qquad (19)$$

$$\text{or } H_i^{DV}(t-1,t) = 0 \qquad (20)$$

The number of occupied houses that is demolished (H_i^{DO}) is given by

$$H_i^{DO} = H_i^D - D_i^{DV} \qquad (21)$$

As described above, a factor (δ) is applied to prevent the removal of a complete social group because of demolition. If the total proportion of occupied housing demolished is less than the value of δ, the number demolished occupied by each social group is the total demolished up to the proportion given by δ, starting with the bottom social group (q=3) and working up towards the top (q=1) :

$$\text{If } H_i^{DO}\ (t-1,t) \leqslant \delta\ H_i^{O}(t-1) \qquad (22)$$

$$\left.\begin{array}{ll} \text{then } H_{iq}^{DQ} = H_i^{DO} - \sum_{q'>q} H_{iq'}^{DQ} & (23) \\ \text{or } \quad H_{iq}^{DQ} = \delta\ H_{iq}^{O}\ (t-1) & (24) \end{array}\right\} \text{whichever is the less}$$

If the total proportion of occupied houses in a zone demolished is greater than the value of δ, it is assumed that widespread redevelopment is taking place and that the proportion of houses occupied by each social group that are demolished are identical:

$$\text{If } H_i^{DO}\ (t-1,t) > \delta\ H_i^{O}(t-1) \qquad (25)$$

$$\text{then } H_{iq}^{DQ} = H_i^{DO} \frac{H_{iq}^{OQ}\ (t-1)}{\sum_q H_{iq}^{OQ}\ (t-1)} \qquad (26)$$

4.5 Housing availability (H_{iq}^{a})

Within the model houses become available for occupation for two reasons, either because they are newly built or because they have been vacated by out-migrants. Vacated housing (H_i^L) is given by:

$$H_i^L(t) = H_i(t-1) - H_i^D\ (t-1,t) - \sum_q P_{iq2}^{QU}(t)/o_{iq}^{HQ}(t) \qquad (27)$$

The demolished housing is removed because it is no longer available for occupation. The final term represents the number of houses being occupied by non-moving population. The current housing occupancy rate is used to reflect the fact that the actual number of people in the home may have changed (due to birth, death or migration) while still being occupied by the same family.

The total number of houses available (H_i^a) is given by:

$$H_i^a = H_i^B + H_i^L + H_i^C \qquad (28)$$

In the model vacant houses are made available to the top (q=1) social group; those houses that are still unoccupied are made available to the next (lower) social group, and finally the bottom (q=3) social group have to choose from what is left, thus:

$$H^{aQ}_{iq} = H^{a}_{i} - \sum_{q'<q} P^{QU}_{iq'1} / o^{HQ}_{iq'} \tag{29}$$

where the second term represents the number of houses newly occupied by members of higher social groups.

Total housing in each zone (H_i) is given by

$$H_i(t) = H_i(t-1) + H^{B}_{i}(t-1,t) - H^{D}_{i}(t-1,t) + H^{C}_{i}(t-1,t) \tag{30}$$

4.6 Housing occupancy

In the model, the housing occupancy rate is allowed to vary between social groups and across zones. It can also vary over time although this is largely a scaling process, that is, the ratio of values in zones tends to be maintained. The overall housing occupancy rate (o^{HQ}_{cq}) is found from:

$$o^{CQ}_{cq}(t) = \frac{o^{HQ}_{cq}(t-1)}{H_c(t)} \sum_{q} \frac{P^{Q}_{cq}(t)}{o^{HQ}_{cq}(t-1)} \tag{31}$$

This formula ensures that the housing occupancy rate rises with an increase in population or reduction in the number of houses.

This is disaggregated by zone to give the zonal values (o^{HQ}_{iq}):

$$o^{HQ}_{iq}(t) = o^{HQ}_{iq}(t-1) \frac{o^{HQ}_{cq}(t)}{o^{HQ}_{cq}(t-1)} \tag{32}$$

This is subject to there being sufficient housing in a zone to accomodate the population, as computed by equation (134). If not equation (191) is used.

5. PRIMARY ECONOMIC ACTIVITY LOCATION

5.1 Basic principle

Primary economic activities are those activities that are site-dependent and so not likely to be influenced significantly by changes in transport costs. There is a strong case for the exogenous location of any large changes in these sectors, otherwise keeping them in much the same location over time. The three industries in this category are agriculture, fishing and forestry, mining and quarrying, and gas, water and electricity. Agriculture is being subjected to many pressures from urban development and this is incorporated within the model by using the area of agricultural land (found after other activities have been distributed) as a locational factor. The other factor is the employment in the sector at the previous point in time, but using a calibrated parameter to reflect the fact that much 'rationalisation' is going ahead in this industry, so growth(or decline) may well occur differentially in areas with large and small levels of employment. This last factor is also used for the mining and gas, water and electricity industries.

5.2 Agriculture (p=1)

The distribution of agricultural jobs (J^P_{i1}) is given by:

$$J^P_{ip}(t) = J^{PX}_{ip}(t) + (J^P_{cp}(t) - \sum_j J^P_{jp}(t))B^P_p \frac{L^A_i(t)}{L^A_i(t-1)} \left[J^{PX}_{jp}(t-1)\right]^{\alpha^P_p} \qquad (33)$$

A balancing factor (B^P_p) is introduced to ensure that the sum of the allocated values seems to the exogenously given total, thus:

$$B^P_p = \left[\sum_i \frac{L^A_i(t)}{L^A_i(t-1)} (J^P_{ip}(t-1)^{\alpha^P_p}\right]^{-1} \qquad (34)$$

to ensure $\sum_i J^P_{ip} = J^P_{cp}$ $\qquad$ (35)

5.3 Mining and quarrying (p=2) and gas, water and electricity (p=3)

As indicated above this is a simpler mechanism than the location of agricultural jobs:

$$J_{ip}^{P}(t) = J_{ip}^{PX}(t) + \left[J_{cp}^{P}(t) - \sum_{j} J_{jp}^{PX} \right] B_{p}^{P} / J_{ip}^{P}(t-1) \Big]^{\alpha_p^P} \qquad (36)$$

Again a balancing factor is used. Thus:

$$B_{p}^{P} = \left[\sum_{i} (J_{ip}^{P}(t-1))^{\alpha_p^P} \right]^{-1} \qquad (37)$$

to ensure $\sum_{i} J_{ip}^{P} = J_{cp}^{P}$ (38)

6. SECONDARY ECONOMIC ACTIVITY LOCATION

6.1 Basic principle

These economic activities are manufacturing, construction, transport and communication, and public administration and defence. These are located in a similar manner to the primary economic activities, but because these sectors are more responsive to changes in transport availability, accessibility factors are incorporated. Two accessibility factors are included - to a supply of labour and to other industry. It would be unrealistic to expect a change in transport costs to have very much effect so the ratios of the accessibilities to these two activities at the current time point to those at the previous time point are used.

6.2 Allocation to zones

The distribution of jobs in these sectors are given by:

$$J_{ip}^{P}(t) = J_{ip}^{PX}(t) + \left[J_{cp}^{P}(t) - \sum_{j} J_{jp}^{PX}(t) \right] B_{p}^{P} \left[J_{ip}^{P}(t-1) \frac{Y_{i}^{R}(t)\, Y_{i}^{E}(t)}{Y_{i}^{R}(t-1)\, Y_{i}^{E}(t-1)} \right]^{\alpha_p^P} \qquad (39)$$

Again a balancing factor, B_p^P is used. Its value is given by:

$$B_p^P = \left[\sum_i \left(J_{ip}^P(t-1) \frac{Y_i^R(t)\ Y_i^E(t)}{Y_i^R(t-1)\ Y_i^E(t-1)} \right)^{\alpha_p^P} \right]^{-1} \qquad (40)$$

to ensure that

$$\sum_{i=1}^{N} J_{ip}^P(t) = J_{cp}^P \qquad (41)$$

7. TERTIARY ECONOMIC ACTIVITY LOCATION

7.1 Basic principle

Tertiary economic activities are the retail and service sectors, and the most responsive to changes in the transport networks. In fact, the interzonal costs are incorporated directly into a model of the form derived by Lakshmanen and Hansen (1965). For the retail sectors the flows of cash from residential zones to shopping zones are derived. From this, the total sales and employment in each zone can be found. For the non-retail service sectors the employment is calculated in terms of the demand for the activity generated by the population in all zones. These sectors are business services, educational services and personal services. Much less is known about trip making behaviour for these sectors than for retail activities, but it seems reasonable to regard business services, which tend to be located in the central area, as similar to the durable retail sector, and the other two to behave more like the convenience retail sector with a more dispersed distribution. These assumptions are unlikely to cause serious errors since the model begins with the correct distribution of employment in each sector and this is a major determinant of the future location of the activities. Thus the calibrated parameters for the retail sectors are applied to the non-retail service sector.

It is useful to be able to incorporate some policy changes for these sectors, for example the location of a new hypermarket, so a term is included for this exogenous location.

7.2 Residential demand for tertiary activity (X_{ism})

The residential demand for retail activity is measured in terms of cash, found by multiplying the mean expenditure per head by the population, disaggregated by social group, and summing, thus:

$$X_{ism} = \sum_q x_{sq} P^{QM}_{iqm} \qquad (42)$$

where x_{sq} is the mean expenditure per head by members of social group q, and P^{QM}_{iqm} is the population in social group q in zone i in car availability group m. As explained below a distinction is made between a car available for work and one available for shopping. The field of car availability is another which is full of problems, with much more research required.

The demand for non-retail service sectors is expressed in terms of population:

$$X_{ism} = \sum_q P^{QM}_{iqm} \qquad (43)$$

7.3 Residential demand from external zones

Less information is available for the external zones. The population in each internal zone is generated within the model, but not for external zones. This is specified for the base year and, if desired can be kept constant over time. The balancing factor K^S_s is used to ensure the 'correct' total demand from external zones is met from internal zones; keeping the population constant in external zones means that the relative residential demand from each external zone remains in the same proportion over time. If desired the proportion can be changed (for example to represent a policy of a new town within an external zone) by adding on a suitable population increment, thus

$$P_i(t) = P_i(t-1) + P_i^{inc}(t-1,t) \quad \text{for } i = N+1, M \tag{44}$$

where $P_i^{inc}(t-1,t)$ is this exogenous change in population in zone i. Equation (44) could be expressed in terms of individual social groups, or the existing rate be applied to the total:

$$P_{iq}^{Q}(t) = P_i(t)\, U_{qi}^{QZ} \tag{45}$$

where U_{qi}^{QZ} is the proportion of population in zone i who are in social group q.

The population is used in equations (42) and (43) above.

7.4 Interzonal demand (S_{ijskm})

As indicated above, the interzonal demand is expressed in a fairly standard form, but with two differences - the use of external zones, for which there is less information than the internal zones, and the possible exogenous location of economic activity.

The interzonal demand (S_{ijskm}) is given by:

$$S_{ijksm} = A_{ism}^{S} X_{ism} \left[(W_{js}^{S})^{\alpha_{sm}^{S}} + B_{js}^{S} S_{js}^{SX} \right] \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \quad \begin{array}{l} i = 1, N \\ j = 1, M \end{array} \tag{46}$$

$$S_{ijksm} = K_{s}^{S} X_{ism} \left[(W_{js}^{S})^{\alpha_{sm}^{S}} + B_{js}^{S} S_{js}^{SX} \right] \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \quad \begin{array}{l} i = N+1, M \\ j = 1, N \end{array} \tag{47}$$

where W_{js}^{S} is the attraction of zone j for tertiary activity s, S_{js}^{SX} is the exogenously located tertiary activity in zone j (and may be zero), A_{ism}^{S}, B_{js}^{S} and K_{s}^{S} are balancing factors, C_{ijk}^{o} is the off-peak cost of travel between zone i and zone j, and α_{sm}^{S} and β_{sm}^{S} are parameters obtained by calibration.

7.5 Balancing factors (A_{ism}^{S}, B_{js}^{S}, K_{s}^{S})

The balancing factor A_{ism}^{S} is used to ensure that the flows of interzonal originating from each internal zone sum to the residential demand given by equations (41) and (42). Thus:

$$A_{ism}^{S} = \left[\sum_{j=i}^{M} \sum_{k\varepsilon\gamma(m)} (\{W_{js}^{S}\}^{\alpha_{sm}^{S}} + B_{js}^{S} S_{js}^{SX}) \exp(-\beta_{sm}^{S} C_{ijk}^{o})\right]^{-1} \qquad (48)$$

to ensure that

$$\sum_{j=1}^{M} \sum_{k\varepsilon\gamma(m)} S_{ijskm} = X_{ism} \qquad (49)$$

The balancing factor B_{js}^{S} is used to ensure that the correct additional exogenous demand for activity s is generated in zone j. Thus

$$B_{js}^{S} = \left[\sum_{i=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} A_{ism}^{S} X_{ism} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) + \sum_{i=N+1}^{M} \sum_{m} \sum_{k\varepsilon\gamma(m)} K_{s}^{S} X_{ism} \exp(-\beta_{sm}^{S} C_{ijk}^{o})\right]^{-1} \qquad (50)$$

to ensure that

$$\sum_{i=1}^{M} \sum_{m} \sum_{k\varepsilon\gamma(m)} S_{ijskm} - \sum_{i=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} A_{ism}^{S} X_{ism} (W_{sm}^{S})^{\alpha_{sm}^{S}} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) - \sum_{i=N+1}^{M} \sum_{m} \sum_{k\varepsilon\gamma(m)} K_{s}^{S} X_{ism} (W_{sm}^{S})^{\alpha_{sm}^{S}} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) = S_{js}^{SX} \qquad (51)$$

Equation (51) says that the total sales or demand located in zone j less that located endogenously (represented by the second and third terms) should equal the exogenously located activity.

The balancing factor K_{s}^{S} is used to ensure that the model generates the 'correct' flow of demand from the external zones. It is very difficult to obtain a figure for the flow of cash (or demand) across a boundary, but it is quite easy to obtain a suitable figure for the total sales in the city or population served by a non-retail service and to incorporate this into the equation. Thus K_{s}^{S} is given by:

$$K_{s}^{S} = \left[S_{cs}^{SZ} - \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} S_{ijskm}^{S}\right] \Big/ \left[\sum_{i=N+1}^{M} \sum_{j=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} (\{W_{js}^{S}\}^{\alpha_{sm}^{S}} + B_{js}^{S} S_{js}^{SX}) \exp(-\beta_{sm}^{S} C_{ijk}^{o})\right] \qquad (52)$$

to ensure that

$$\sum_{i=N+1}^{M} \sum_{j=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} S_{ijskm} = S_{cs}^{SZ} - \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{m} \sum_{k\varepsilon\gamma(m)} S_{ijskm} \qquad (53)$$

S^{S}_{cs} is the total sales (or demand generated) in the city. A value for this can be obtained in the base year; to ensure that it rises with population growth (and real income growth for the retail sectors) it is scaled by the total change in residential demand in the city. Thus for retail sectors:

$$S^{SZ}_{cs}(t) = S^{SZ}_{cs}(t-1) \frac{\sum_q \sum_m x_{sqm}(t) P_{cqm}(t)}{\sum_q \sum_m x_{sqm}(t-1) P_{cqm}(t-1)} \tag{54}$$

For non-retail tertiary sectors:

$$S^{S}_{cs}(t) = S^{SZ}_{cs}(t-1) \frac{P_c(t)}{P_c(t-1)} \tag{55}$$

7.6 Attraction factors (W^{S}_{js})

The attraction factors for these sectors are the employment in the sector at the previous timepoint, modified by the change in accessibility to residential areas and to economic activity. The accessibility terms mean that an improvement in the transport facilities will increase the amount of tertiary activity in the zones with the increased accessibility. This represents the increase in the number of shops (or services) in the zones. The change in the off-peak costs in equations (46) and (47) represent an increase in the interzonal demand for the activity.

The attraction factor (W^{S}_{js}) is given by:

$$W^{S}_{js}(t) = E^{S}_{js}(t-1) \frac{Y^{R}_{i}(t)\, Y^{E}_{i}(t)}{Y^{R}_{i}(t-1)\, Y^{E}_{i}(t-1)} \tag{56}$$

7.7 Tertiary employment (E^{S}_{js})

The total demand generated in each zone can be found by summation:

$$S^{SZ}_{js} = \sum_{i=1}^{M} \sum_{m} \sum_{k \epsilon \gamma(m)} S_{ijskm} \quad \text{for } j=1, N \tag{57}$$

For retail sectors S^{SZ}_{js} is the total sales of sector s in zone j. Employment in sector s in each zone (E^{S}_{js}) can be found from:

$$E^{S}_{js} = b_s S^{SZ}_{js} \tag{58}$$

where b_s is the employment-demand ratio for section s. Its value is given by

$$b_s = E^{S}_{cs} / S^{SZ}_{cs} \tag{59}$$

E^{S}_{cs} is given exogenously, and S^{SZ}_{cs} is given by equations (54) and (55) above.

8. JOB LOCATION

8.1 Basic principle

The model distributes jobs to zones by sector. These have to be converted to social groups using suitable rate variables.

8.2 Job summation and conversion

The location of primary and secondary economic activity jobs have been found as described above. It is not practical using this formulation for tertiary activity to make a distinction between jobs and employment for these sectors, that is:

$$J^{S}_{js} = E^{S}_{js} \tag{60}$$

Where J^{S}_{js} is the number of jobs in tertiary sector s in zone j.

The spatial distribution of jobs by social group (J^{Q}_{jq}) is given by:

$$J^{Q}_{jq} = \sum_{p} U^{QP}_{qp} J^{P}_{jp} + \sum_{s} U^{QS}_{qs} J^{S}_{js} \tag{61}$$

where U^{Q}_{qp} is the proportion of jobs in primary or secondary sector p that is in social group q and U^{QS}_{qs} is the proportion of jobs in tertiary sector s that is in social group q.

9. SURVIVAL

9.1 Basic principle

Survival means that a person remains in the same residence or job from one time point to the next. It is calculated by applying survival rates to the population or employment at one time point to find those surviving to the next.

9.2 Survival in residences

During a time period those who live in a zone at the beginning of the period can do three things - die, move out or survive. (We are ignoring those who move out and back). Thus defining a survival rate means that those who do not survive have either died or moved out. Also during a time period people may be born. The number being born can be calculated by applying a suitable rate to the population at the beginning of the time period. These principles are basic to demographic modelling where the populations at risk are usually disaggregated by age and sex. In this model it is felt more useful to disaggregate the population by zone and social group (not normally done in demographic modelling) so that the results are compatible with the rest of the model. It is not practical to obtain survival data disaggregated in both ways. A further important point that has some bearing on this argument is that many people are forced to move when their houses have been demolished, and these must be taken into account in calculating the number of survivors to prevent double counting.

The number of people in zone i, in social group q who have not moved in during the preceding time period, that is, they have survived or been born (P^{QU}_{iq2}) is given by

$$P^{QU}_{iq2}(t) = \left[\frac{\left(H^{OQ}_{iq}(t-1) - H^{D}_{iq}(t-1,\, t)\right) Q^{YQ}_{iq}}{H^{OQ}_{iq}(t-1)} + Q^{bQ}_{iq}\right] P^{Q}_{iq}(t-1) \qquad (62)$$

where Q^{bQ}_{iq} is the birth rate (persons born per head of the population) in zone i for social group q and Q^{YQ}_{iq} is the survival rate for members of social group q in zone i. The housing terms are included to ensure that those living in houses that have been demolished are not included as survivors.

The number of employed residents surviving (R^{QU}_{iq2}) is found by using the inverse activity rate:

$$R^{QU}_{iq2} = P^{QU}_{iq2} \,/\, g^{QU}_{iq2} \qquad (63)$$

where g^{QU}_{iq2} is the inverse activity rate for surviving population in social group q in zone i, given by equation (130).

9.3 Survival in jobs

Survival in jobs is calculated in a similar way to that for residences except that, because of the paucity of data on this topic, the rates are not spatially disaggregated. Also, there is no equivalent to births in the model. Thus the number of people remaining in the same job in social group q in zone j (E^{QV}_{jq2}) is given by:

$$E^{QV}_{jq2}(t) = Q^{EQ}_{q}\, E^{Q}_{jq}\,(t-1) \qquad (64)$$

where Q^{EQ}_{q} is the employment survival rate for social group q.

The number of jobs available in new locations in each social group (J^{aQ}_{jq}) can be found by deducting the number of jobs filled by surviving workers from the total number of jobs given by equation (61):

$$J^{aQ}_{jq} = J^{Q}_{jq} - E^{QV}_{jq2} \tag{65}$$

10. TOTAL EMPLOYED RESIDENTS, IN- AND OUT- COMMUTERS

10.1 Basic principle

The total population in each social group and the total number of jobs in each sector are specified exogenously. The number of employed residents in each social group is computed within the model along with the number of in- and out- commuters, that is the number of people who live in external zones and work in internal zones and those who work in external zones and live in internal zones respectively. A change in the level of employment can be met by a change in the activity rate or the number of in- or out- commuters. Since trip lengths are becoming longer on average, the number of in- and out- commuters is also increasing, and so a suitable growth factor technique has been applied. This is done by assuming that the rate of change from not commuting across the boundary of a city, to being a commuter is constant over time. This reflects the fact that the number of commuters in each direction has been increasing over time, but, because the factor is applied to a small remaining number of people, there will not be exponential growth emptying the city of employed residents and workers as would happen with some other methods. The number of employed residents as well as the numbers in- and out-commuting is found.

10.2 Methodology

The growth factor for in-commuting for social group q (G^{t}_{q}) is defined as:

$$G^{t}_{q} = \frac{E^{Q}_{cq}(t) - R^{t}_{q}(t)}{E^{Q}_{cq}(t)} \Bigg/ \frac{E^{Q}_{cq}(t-1) - R^{t}_{q}(t-1)}{E^{Q}_{cq}(t-1)} \tag{66}$$

where E^{Q}_{cq} is total employment in social group q in the city, R^{t}_{q} is the number of in-commuters into the city. The factor for out-commuting G^{f}_{q} is defined as:

$$G_q^f = \frac{R_{cq}^Q(t) - R_q^f(t)}{R_{cq}^Q(t)} \Bigg/ \frac{R_{cq}^Q(t-1) - R_q^f(t-1)}{R_{cq}^Q(t-1)} \tag{67}$$

where R_{cq}^Q is the number of employed residents in social group q living in the city, R_q^f is the number of out-commuters in social group q. The following relationship can be specified:

$$R_{cq}^Q - R_q^f = E_{cq}^Q - R_q^t \tag{68}$$

since both sides define those who both live and work in the city.

By manipulation of the above equations, the following are determined:

$$R_q^t(t) = E_{cq}^Q(t) - \left[G_q^t\ E_{cq}^Q\ (t)(E_{cq}^Q(t-1)-R_q^t(t-1))\right] \Big/ E_{cq}^Q(t-1) \tag{69}$$

$$R_{cq}^Q(t) = \frac{R_{cq}^Q(t-1)\ \left[E_{cq}^Q(t) - R_q^t\ (t)\right]}{G_q^f\ (R_{cq}^Q(t-1) - R_q^f\ (t-1))} \tag{70}$$

$$R_q^f\ (t) = R_{cq}^Q(t) - E_{cq}^Q(t) + R_q^t(t) \tag{71}$$

11. CAR OWNERSHIP

11.1 Basic principle

The forecasting of car ownership levels is fraught with difficulties and there is no generally accepted methodology. Tanner (1974) has devised one of the most well known. One of the basic principles of his and other methodologies is the use of some form of logistic curve with a saturation level of car ownership as asymptote. It is not practical to include income which is one of the main components of other car ownership models within the model in this paper. Because it is desired to include differences in car ownership level between small areas, which is not done in car ownership forecasting, a method has been devised here that is simple, uses a similarly-shaped curve to Tanner's and permits forecasts to be made for each zone.

11.2 Methodology

The rate of growth of change from not owning a car to owning a car is input at each time point for each social group (but may be zero if desired). Since the level of car ownership is rising the factor is applied to a smaller number over time so there is a gradual slowing down in the rate of growth towards some level, which will be when every household owns a car, that is, saturation.

The proportion of non-car owning households (F_{iq2}) is given by:

$$F_{iq2}(t) = (1 - G^r_q(t-1,t))F_{iq2}(t-1) \tag{72}$$

where G^r_q is the proportional change in the non-car ownership rate for social group q. The proportion of households owning a car (F_{iq1}) is given by:

$$F_{iq1}(t) = 1.0 - F_{iq2}(t) \ . \tag{73}$$

This may be converted to the equivalent population:

$$P^{QN}_{iqn}(t) = F_{iqn}(t)P^Q_{iq}(t) \tag{74}$$

when P^{QN}_{iqn} is the population in social group q in car ownership group n in zone i.

As indicated previously car ownership and car availability are very complex fields, even in terms of definition, and even more so in terms of modelling. In this study it has been assumed that members of households that own cars have one available for the journey to work. It is then assumed that a car used for work is not available for shopping. This is felt to be more realistic than assuming that any car owned is available for shopping purposes. Thus, the proportion of cars left at home in each zone (U^L_i) is given by:

$$U^L_i = 1.0 - \frac{R^K_{i1}}{N_i O^{cp}} \tag{75}$$

where R^K_{i1} is the number of employed residents in zone i travelling to work by car, N_i is the number of cars owned in zone i and O^{cp} is the peak occupancy rate.

The number of people in households with a car available for shopping (P^{QM}_{iq1}) is given by:

$$P^{QM}_{iq1}(t) = U^{L}_{i} \; P^{QN}_{iq1}(t) \qquad (76)$$

The number of people with no car available for shopping is given by:

$$P^{QM}_{iq2}(t) = P^{Q}_{iq}(t) - P^{QM}_{iq1}(t) \qquad (77)$$

12. POPULATION AND EMPLOYMENT LOCATION

12.1 Basic principle

The methodologies for the location of jobs and housing have already been specified above. Here people are allowed to choose their workplaces and residences. This is a fairly complex problem, because the equation system has to be specified so that the solution agrees with information from elsewhere. Since people's residential and workplace distributions will be known, the journey to work pattern is an immediate product. This is disaggregated by the mode of transport to work on the basis of the relative cost of travel by each mode, and the car ownership group at the residential end. As in the rest of the model a social group disaggregation is used and capacity constraints representing planning policies are incorporated. As well as these constraints, others ensure the correct numbers of in- and out-commuters, and that some people retain the same residential and/or employment location.

All these conditions lead to some rather complicated equations.

12.2 Living and working in the city

In this section equations will be defined for those people who both live and work within the city.

12.2.1 New locations of both home and job (u=1, v=1)

As described above, some capacity constraints may be applied. For those living and working in zones with no such constraints ($i \varepsilon Z_{q2}$, $j \varepsilon Z_{q4}$) the journey to work pattern is given by:

$$T^{W}_{ijqknuv} = K^{R}_{q} \; K^{E}_{q} \; W^{R}_{iqn} \; W^{E}_{jq} \; \exp(-\beta^{W}_{qn} \; C^{P}_{ijk}) \quad u=1,\ v=1 \quad i\varepsilon Z_{q2} \quad j\varepsilon Z_{q4} \qquad (78)$$

where K^{R}_{q} is to constrain on the total number of newly located employed residents in social group q, K^{E}_{q} is to constrain on the total number of newly located workers in social group q, W^{R}_{iqn} is the attraction of zone i for residents in social group q in car ownership group n and W^{E}_{jq} is the attraction of zone j for workers in social group q.

The equivalent equation for trips by newly located residents in zones with capacity restraints is given by:

$$T^{W}_{ijqknuv} = K^{E}_{q} A^{W}_{iqnu} R^{QNU}_{iqnu} W^{E}_{jq} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \quad u=1 \; v=1 \; i\varepsilon Z_{q1} \; j\varepsilon Z_{q4} \qquad (79)$$

where R^{QNU}_{iqnu} is the number of employed residents in social group q, in car ownership group n and mover group u living in zone i and A^{W}_{iqnu} is the equivalent balancing factor.

The trip pattern for new locators working in a constrained employment zone is given by:

$$T^{W}_{ijqknuv} = K^{R}_{q} B^{W}_{jq} W^{R}_{iqn} E^{QV}_{jqv} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \quad u=1 \; v=1 \; i\varepsilon Z_{q2} \; j\varepsilon Z_{q3} \qquad (80)$$

where E^{QV}_{jqv} is the number of people in social group q and employment mover set v working in zone j, and B^{W}_{jq} is the equivalent balancing factor.

There may be some new locators both living and working in zones in which constraints are applied:

$$T^{W}_{ijqknuv} = A^{W}_{iqnu} B^{W}_{jqv} R^{QNU}_{iqnu} E^{QV}_{jqv} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \quad u=1 \; v=1 \; i\varepsilon Z_{q1} \; j\varepsilon Z_{q3} \qquad (81)$$

12.2.2 <u>New residential locations, job fixed (u=1, v=2)</u>

Since the employment end of the trip has not changed for these workers there are only two equations - one for the residence zone in the constrained set and one for it in the unconstrained set. Those with both ends of the trip unchanged are considered later. To distinguish the non-moving workers with a new residential location from those with a fixed residential location another subscript (u) has to be added. The equation for residents living in a zone in the unconstrained set is:

$$T^{W}_{ijqknuv} = K^{R}_{q} B^{W}_{jqv} W^{R}_{iqn} E^{QUV}_{jquv} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \quad u=1 \; v=2 \; i\varepsilon Z_{q2} \; j=1,N \qquad (82)$$

where E^{QUV}_{jq12} is the number of people in social group q who have a new residential location but have the same employment location in zone j. The equation for residents living in a zone in the constrained set is:

$$T^{W}_{ijqknuv} = A^{W}_{iqnu} B^{W}_{jqv} R^{QNU}_{iqnu} E^{QUV}_{jquv} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \quad u=1 \; v=2 \; i\varepsilon Z_{q1} \; j=1,N \qquad (83)$$

12.2.3 Residence fixed but new employment location (u=2, v=1)

These equations are the converse of the equations above:

$$T^{W}_{ijqknuv}=K^{E}_{q}A^{W}_{iqnu}W^{E}_{jq}R^{QNUV}_{iqnuv}\exp(\beta^{W}_{qn}C^{p}_{ijk}) \quad u=2 \quad v=1 \quad i=1,N \quad j\varepsilon Z_{q4} \tag{84}$$

$$T^{W}_{ijqknuv}=A^{W}_{iqnu}B^{W}_{jqv}R^{QNUV}_{iqnuv}E^{QV}_{jqv}\exp(-\beta^{W}_{qn}C^{p}_{ijk}) \quad u=2 \quad v=1 \quad i=1,N \quad j\varepsilon Z_{q3} \tag{85}$$

where R^{QNUV}_{iqn21} is the number of people in social group q and car ownership group n who have retained the same residential location in zone i but have a new employment location.

12.2.4 Both residential and employment location fixed (u=2, v=2)

Those who have the same residential and employment location have the same journey to work pattern as at the previous time point, but may have changed car ownership group and mode of transport, so a fairly tight constraint is imposed:

$$T^{W}_{ijqknuv}=B^{WF}_{ijqn}T^{F}_{ijqn}\exp(-\beta^{W}_{qn}C^{p}_{ijk}) \quad u=2 \quad v=2 \quad i=1,N \quad j=1,N \tag{86}$$

where T^{F}_{ijqn} is the number of people in social group q who are now in car ownership group q who have retained the same residential location i and employment location j.

This mechanism is rather different from that devised by Wilson (1970) because, as shown in Mackett (1976b), that method would be inconsistent if transport costs change.

12.3 Workers living in city but employed outside

The total number of workers who live in the city but work outside (R^{f}_{q}) has been computed using equation (71). These are allocated to zones using the mechanism described in Mackett (1974). There are three cases at the residential end - a new residential location in a zone in either the constrained or unconstrained set or an unchanged residential location. No such distinction is made at the workplace location because the employment characteristics of these people is a function of factors not being considered for the external zones, and so all are located at the current time point.

12.3.1 New residential location (u=1)

The equation for those living in zones in the unconstrained set is:

$$T^{W}_{ijqknuv} = K^{f}_{q}\ K^{R}_{q}\ W^{R}_{iqn}\ W^{E}_{jq}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=1\ v=1\ i\varepsilon Z_{q2}\ j=N+1,\ M \qquad (87)$$

where K^{f}_{q} is a balancing factor corresponding with the term R^{f}_{q}. The equivalent equation for those in zones in the constrained set is:

$$T^{W}_{ijqknuv} = K^{f}_{q}\ A^{W}_{iqnu}\ R^{QNU}_{iqnu}\ W^{E}_{jq}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=1\ v=1\ i\varepsilon Z_{q1}\quad j=N+1,\ M \qquad (88)$$

12.3.2 Fixed residential location (u=2)

The equation for those who commute out of the city, but have not changed their residential location is:

$$T^{W}_{ijqknuv} = K^{f}_{q}\ A^{W}_{iqnu}\ R^{QNUV}_{iqnuv}\ W^{E}_{jq}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=2\ v=1\ i=1,\ N\ j=N+1,\ M \qquad (89)$$

12.4 Workers employed in the city but living outside

These workers journey to work behaviour is modelled in a similar manner to the out-commuters, with the value of R^{t}_{q} given by equation (69) above.

12.4.1 New employment location (v=1)

The equation for those working in zones in the unconstrained set is:

$$T^{W}_{ijqknuv} = K^{t}_{q}\ K^{E}_{q}\ W^{R}_{iqn}\ W^{E}_{jq}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=1\ v=1\ i=N+1,\ M\ j\varepsilon Z_{q4} \qquad (90)$$

where K^{t}_{q} is a balancing factor corresponding to R^{t}_{q}. The equivalent equation for those working in zones in the constrained set is:

$$T^{W}_{ijqknuv} = K^{t}_{q}\ B^{W}_{jqv}\ W^{R}_{iqn}\ E^{QV}_{jqv}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=1,\ v=1,\ i=N+1,\ M\quad j\varepsilon Z_{q3} \qquad (91)$$

12.4.2 Fixed employment location

The equation for those who commute into the city and have retained the same employment location is:

$$T^{W}_{ijqknuv} = K^{t}_{q}\ B^{W}_{jqv}\ W^{R}_{iqn}\ E^{QUV}_{jquv}\ \exp(-\beta^{W}_{qn}\ C^{p}_{ijk})\quad u=1\ v=2\ i=N+1,\ M\ j=1,\ N \qquad (92)$$

12.5 Attraction factors

Two attraction factors are used above for the allocation of newly locating employed residents and workers to zones of residence and employment. These are designed to reflect the relative attractiveness of zones for these activities.

12.5.1 Residential attraction - internal zones (W^R_{iqn})

The main factor representing the attraction of a zone for residential use is the availability of housing. The factor for members of social group q and car ownership n is

$$W^R_{iqn}(t) = H^{aQ}_{iq}(t)\ F_{iqn}(t)\ \frac{H^{OQ}_{iq-1}(t)\ P^Q_{iq}(t-1)}{H^{aQ}_{iq-1}(t)\ \sum_q P^Q_{iq}(t-1)}\ .\ a_i \qquad \text{for } i=1,N \qquad (93)$$

$H^{aQ}_{iq}(t)$ is the number of houses available for members of social group q in zone i, since at a macro level, this is the main determinant of the attraction of a particular zone. If no houses are available the attraction factor automatically becomes zero so no more people are located there. The definition of housing availability in equation (29) means that the higher one's social status the wider the choice of housing - a fairly realistic approach. F_{iqn} is the car ownership level for members of social group q in zone i. This is included to reflect that fact that those with no car available will tend to be attracted to areas with a low level of car ownership since these are more accessible by public transport. Similarly those with a car available will tend to choose a location less accessible by public transport. The mode actually used for the work trip will depend upon the relative cost of travel by each mode. The third term has a dual purpose. Firstly it represents people's aspirations to locate in the most attractive housing available, by making them choose housing that has been attractive to those of higher social status since the term is the porportion of available housing that has been occupied by the next highest social group. The second purpose is rather more pragmatic since the term helps ensure that it is the more attractive housing that is filled first; if it were not used two zones with an equal number of houses still available, but one of which had had most of its housing already filled, would tend to be equally attractive, whereas the zone in which more houses have been filled is more attractive than the one in which few have chosen to locate. The fourth term ensures that people tend to locate near those of similar social status. The final term (a_i) represents the environment, that is, it is designed to represent the quality of the area other than the factors already discussed. Clearly quantification of the environment is fairly subjective. Because of this two different measures have been used in this study. In the earlier work used for forecasts from 1966 to 1971 the form used was:

$$a_i = Y_i^S (t) \ / \ Y_i^S (t-1) \tag{94}$$

where Y_i^S is the accessibility to shops. Its value is given by equation (138). Because the ratio of the accessibility at two different time points is being used, the values will, in general, be close to unity. If new shops have been introduced into an area, the value will tend to be greater than one, and so, it can be argued, making the area more attractive for residence. The disadvantage of this formulation is that because transport costs are included in the factor, a change in such costs tends to be over-emphasised, that is making areas which have become relatively more accessible disproportionately attractive. Accessibility is already taken into account in the residential location process. An improved version is used in later forecasts. It takes the form:

$$a_i = \frac{H_i(t-1) - H_i^D(t-1,t)}{H_i \ (t-1)} \bigg/ \frac{L_i^G}{L_i^T} \tag{95}$$

where H_i is the housing stock in zone i, H_i^D is the number of houses demolished in zone i, L_i^G is the area of zone i that has been developed and L_i^T is the total area of zone i. That is, the larger the proportion of houses not being demolished and the smaller the proportion of area of land that has been developed the better the environment of that zone. The proportion not being demolished is used rather than the inverse of the proportion being demolished because a value of H_i^D of zero would lead to the value of a_i being undefined.

12.5.2 Residential attraction - external zones (W_{iqn}^R)

Since only some of the employed residents are being located in external zones (those who work in the city) a different attraction factor is used. This causes no problem since the total number of in-commuters in each social group (R_q^t) has already been calculated in equation (69) and so employed residents are being distributed between a set of zones on the relative value of the attraction factor. Following on from this, since it is only an attraction factor the value can be kept constant over time, implying no change in the relative values over time. However if desired, the value can be changed, particularly to test the effect of overall growth in one or more external zones. The value is given by:

$$W_{iqn}^{R}(t) = P_{iqn}^{QN}(t) \text{ for } i=N+1, M \tag{96}$$

where either P_{iqn}^{QN} has been kept constant over time or the total population increased:

$$P_{iqn}^{QN}(t) = F_{iqn}(t)\, U_{qi}^{QZ}(t) \left[P_i(t-1) + P_i^{inc}(t-1,t)\right] \text{ for } i=N+1,M \tag{97}$$

where $F_{iqn}(t)$ is the car ownership level given by equations (72) and (73). $U_{qi}^{QZ}(t)$ is the proportion of population of zone i in social group q and $P_i^{inc}(t-1,t)$ is the population increment in zone i from time t-1 to time t. In practice it may well be that $U_{iq}^{ZQ}(t)$ is kept constant over time. This is unlikely to lead to serious error.

12.5.3 <u>Employment attraction - internal zones</u> (W_{jq}^{E})

The employment attraction factor is much simpler; it is the number of jobs available in each zone, as given by equation (65):

$$W_{jq}^{E} = J_{jq}^{aQ} \text{ for } j=1, N \tag{98}$$

12.5.4 <u>Employment attraction - external zones</u> (W_{jq}^{E})

The employment attraction factor for external zones is the number of jobs for the appropriate social group:

$$W_{jq}^{E} = J_{jq}^{Q} \text{ for } j=N+1, M \tag{99}$$

The total number of jobs in each social group is not calculated endogenously so may either be kept constant over time for the same reasons as the external residential attraction factor, or may be increased exogenously, to represent an increase in the number of jobs in one or more external zones. This may be done for all jobs in aggregate or for individual sectors:

$$J^{Q}_{jq}(t) = \sum_{p} \left[J^{P}_{jp}(t-1) + J^{Pinc}_{jp}(t-1,t)\right] U^{QP}_{qp} + \sum_{s} \left[J^{S}_{js}(t-1) + J^{Sinc}_{js}(t-1,t)\right] U^{QS}_{qs}$$

$$\text{for } j=N+1,M \qquad (100)$$

where J^{P}_{jp} J^{S}_{js} are the number of jobs in each primary or secondary, and tertiary sector respectively, J^{Pinc}_{jp} and J^{Sinc}_{js} are the increments in the number of jobs in each primary or secondary, and tertiary sector respectively and U^{QP}_{qp} and U^{QS}_{qs} are the proportions of the jobs in the respective sectors in each social group.

12.6 Capacity constraints

As indicated in the description of the trip distribution certain capacity constraints can be applied. These serve two purposes - to permit the incorporation of land use policies for testing with the model and to remove assumptions about the equality of supply and demand. The inequalities work by finding the appropriate distribution with all zones in the unconstrained set, comparing the distribution with the relevant capacity value, moving zones in which the constraint is violated to the constrained set and then computing a new distribution. Since the excesses will have been redistributed to other zones, further constraints may be violated so the whole process is repeated until no constraints are violated.

12.6.1 Population constraints

The main constraint on the location of population is the availability of housing. This is particularly important since population is being located by social group and only a limited quantity may be left available for a particular social group; thus

$$P^{Q}_{iq} \leqslant H^{aQ}_{iq}\, o^{HQ}_{iq} \qquad (101)$$

where H^{aQ}_{iq} is the number of houses available, as given by equation (29) and o^{HQ}_{iq} is the appropriate housing occupancy rate as given by equation (32).

It may be desired to test the effect of a particular population level for one or more social groups, hence:

$$P^{Q}_{iq} = P^{QX}_{iq} \tag{102}$$

where P^{QX}_{iq} is the exogenously defined population in social group q.

Similarly, it may be desired to ensure that a particular maximum or minimum population is located in a zone, that is:

$$P^{Q}_{iq} \leqslant P^{max}_{iq} \tag{103}$$

$$P^{Q}_{iq} \geqslant P^{min}_{iq} \tag{104}$$

where P^{max}_{iq} and P^{min}_{iq} are the maximum and minimum desired populations in social group q respectively. These can be used to ensure a particular social mix in a zone.

12.6.2 Employment constraints

The employment constraint equivalent to that representing the availability of housing is the availability of jobs:

$$E^{Q}_{jq} \leqslant J^{Q}_{jq} \tag{105}$$

where J^{Q}_{jq} is the number of jobs, given by equation (61).

12.7 Non-moving workers

In equation (86) above the number of people in each social group who have retained the same residential and employment locations over the preceding time period was considered. The number of people in this category has to be calculated. As described above, only those who live and work in the city are considered in this category; these are calculated by assuming that the decision to retain either the residential or employment location fixed is independent of the decision about the location at the other end for those living and working in the city at the previous time point. This arises from the lack of information upon the relationship between residential and employment relocation. Thus the number of people in each social and car ownership group with a fixed residential and employment location is given by:

$$T^{F}_{ijqn}(t)=\frac{R^{FQN}_{iqn}(t)}{R^{Q}_{iq}(t-1)}\ \frac{E^{FQ}_{jq}(t)}{E^{Q}_{jq}(t-1)}\ \sum_{u}\sum_{v}\sum_{n}\sum_{k\epsilon\gamma(n)} T^{W}_{ijqknuv}(t-1) \text{ for } i=1,N\ j=1,N \quad (106)$$

where R^{FQN}_{iqn} is the number of employed residents who worked in the city at the previous time point who have kept the same residential location, and are in social group q and car ownership group n. Its value is given by:

$$R^{FQN}_{iqn}=\frac{R^{QNU}_{iqn2}(t)}{R^{Q}_{iq}(t-1)}\ \sum_{j=1}^{N}\ \sum_{u}\sum_{v}\sum_{n}\sum_{k\epsilon\gamma(n)} T^{W}_{ijqknuv}(t-1) \quad (107)$$

It will be noted that the current car ownership level is being used since people in this category may well have purchased (or sold) a car. E^{FQ}_{jq} is the number of workers who lived in the city at the previous time point and have kept the same employment location and are in social group q. Its value is given by:

$$E^{FQ}_{jq}(t) = \frac{E^{QV}_{jq2}(t)}{E^{Q}_{jq}(t-1)}\ \sum_{i=1}^{N}\ \sum_{u}\sum_{v}\sum_{n}\sum_{k\epsilon\gamma(n)} T^{W}_{ijqknuv}(t-1) \quad (108)$$

12.8 Balancing factors

Various balancing factors have been mentioned above. In general these are designed to ensure that the final solution is consistent with a particular set of values. Because this part of the model is fairly complex the balancing factors have also to be complex, although their basic form is simple.

12.8.1 Residential balancing factors (A^{W}_{iqnu})

The balancing factor for newly locating employed residents in zones in the constrained set (A^{W}_{iqn1}) is:

$$A^{W}_{iqn1} = \Big[\sum_{j\epsilon Z_{q3}} B^{W}_{jq1}E^{QV}_{jq1}\exp(-\beta^{W}_{qn}C^{p}_{ijk}) + \sum_{j\epsilon Z_{q4}} K^{E}_{q}W^{E}_{jq}\exp(-\beta^{W}_{qn}C^{p}_{ijk})$$

$$+ \sum_{j=1}^{N} B^{W}_{jq2}E^{QUV}_{jq12}\exp(-\beta^{W}_{qn}C^{p}_{ijk}) + \sum_{j=N+1}^{M} K^{f}_{q}W^{E}_{jq}\exp(-\beta^{W}_{qn}C^{p}_{ijk})\Big]^{-1} \quad (109)$$

to ensure that

$$\sum_{j=1}^{M} \sum_{k\epsilon\gamma(n)} \sum_{v} T^{W}_{ijqknuv} = R^{QNU}_{iqnu} \text{ for } i\epsilon Z_{q1} \tag{110}$$

The balancing factor for non-moving employed residents has one term fewer than the one for newly locating residents, because those who have retained the same employment location are considered elsewhere. The balancing factor is given by:

$$A^{W}_{iqn2} = \Big| \sum_{j\epsilon Z_{q3}} B^{W}_{jq1} E^{QV}_{jq1} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{j\epsilon Z_{q4}} K^{E}_{q} W^{E}_{jq} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{j=N+1}^{M} K^{f}_{q} W^{E}_{jq} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \Big|^{-1} \tag{111}$$

to ensure that

$$\sum_{j=1}^{M} \sum_{k\epsilon\gamma(n)} T^{W}_{ijqkn21} = R^{QNUV}_{iqn21} \quad \text{for } i=1, N \tag{112}$$

12.8.2 Employment balancing factors (B^{W}_{jq})

The employment balancing factors are similar to those for residents. For those working in zones in the constrained set the factor is:

$$B^{W}_{jq1} = \Big[\sum_{i\epsilon Z_{q1}} A^{W}_{iqn1} R^{QNU}_{iqn1} \exp(-\beta^{W}_{qn} C^{P}_{ijk}) + \sum_{i\epsilon Z_{q2}} K^{R}_{q} W^{R}_{iqn} \exp(-\beta^{W}_{qn} C^{P}_{ijk}) + \sum_{i=1}^{N} A^{W}_{iqn2} R^{QNUV}_{iqn21} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{i=N+1}^{M} K^{t}_{q} W^{R}_{iqn} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \Big]^{-1} \tag{113}$$

to ensure that

$$\sum_{i=1}^{M} \sum_{n} \sum_{k\epsilon\gamma(n)} \sum_{u} T^{W}_{ijqknu1} = E^{QV}_{jq1} \text{ for } j\epsilon Z_{q3} \tag{114}$$

The factor for non-moving workers is:

$$B^{W}_{jq2} = \Big[\sum_{i\epsilon Z_{q1}} A^{W}_{iqn1} R^{QNU}_{iqn1} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{i\epsilon Z_{q2}} K^{R}_{q} W^{R}_{iqn} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{i=N+1}^{M} K^{t}_{q} W^{R}_{iqn} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) \Big]^{-1} \tag{115}$$

to ensure that

$$\sum_{i=1}^{M} \sum_{n} \sum_{k\varepsilon\gamma(n)} T^{W}_{ijqkn12} = E^{QUV}_{jq12} \quad \text{for } j=1, N \tag{116}$$

12.8.3 <u>Fixed journey to work pattern</u> (B^{WF}_{ijqn})

A balancing factor has to be defined to ensure the correct trip distribution for those who have retained the same residential and employment location for use in equation (86).
It is given by:

$$B^{WF}_{ijqn} = \left[\sum_{k\varepsilon\gamma(n)} \exp(-\beta^{W}_{qn}\, C^{p}_{ijk})\right]^{-1} \tag{117}$$

to ensure that

$$\sum_{k\varepsilon\gamma(n)} T^{W}_{ijqkn22} = T^{F}_{ijqn} \quad \text{for } i=1, N \quad j=1, N \tag{118}$$

12.8.4 <u>Total newly locating employed residents</u> (K^{R}_{q})

The number of newly locating employed residents in zones in the unconstrained set can be found from:

$$R^{QU}_{cq1} = R^{Q}_{cq} - \sum_{i\varepsilon Z_{q1}} R^{QU}_{iq1} - \sum_{i=1}^{N} R^{QU}_{iq2} \tag{119}$$

that is, it is the total employed residents in the social group less those living in zones in the unconstrained set and non-moving employed residents. The equivalent balancing factor K^{R}_{q} is calculated from:

$$K^{R}_{q} = R^{QU}_{cq1} \Big/ \Big[\sum_{i\varepsilon Z_{q2}} \sum_{n} \sum_{k\varepsilon\gamma(n)} W^{R}_{iqn} \Big(\sum_{j\varepsilon Z_{q2}} B^{W}_{jq1} E^{JQV}_{jq1} \exp(-\beta^{W}_{qn}\, C^{p}_{ijk})$$

$$+ \sum_{j\varepsilon Z_{q4}} K^{E}_{q}\, W^{E}_{jq} \exp(-\beta^{W}_{qn} C^{p}_{ijk}) + \sum_{j=1}^{N} B^{W}_{jq2} E^{JQUV}_{jq12} \exp(-\beta^{W}_{qn}\, C^{p}_{ijk})$$

$$+ \sum_{j=N+1}^{M} K^{f}_{q}\, W^{E}_{jq} \exp(-\beta^{W}_{qn} C^{p}_{ijk})\Big)\Big] \tag{120}$$

to ensure that:

$$\sum_{i\varepsilon Z_{q2}} \sum_{j=1}^{M} \sum_{n} \sum_{k\varepsilon\gamma(n)} \sum_{v} T^{W}_{ijkn1v} = R^{QU}_{cq1} \tag{121}$$

12.8.5 Total newly locating employment (K_q^E)

The number of newly locating workers in zones in the unconstrained set can be found from:

$$E_{cq1}^{QV} = E_{cq}^{Q} - \sum_{j\varepsilon Z_{q3}} E_{jq1}^{QV} - \sum_{j=1}^{N} E_{jq2}^{QV} \tag{122}$$

The balancing factor to ensure that the total number of trips in this category sums to this total (K_q^E) is defined by:

$$K_q^E = E_{cq1}^{QV} / \Big[\sum_{j\varepsilon Z_{q4}} \sum_{n} \sum_{k\varepsilon\gamma(n)} W_{jq}^{E} \Big(\sum_{i\varepsilon Z_{q1}} A_{iqn1}^{W} R_{iqn1}^{QNU} \exp(-\beta_{qn}^{W} C_{ijk}^{p})$$

$$+ \sum_{i\varepsilon Z_{q2}} K_q^R W_{iqn}^{R} \exp(-\beta_{qn}^{W} C_{ijk}^{p}) + \sum_{i=1}^{N} A_{iqn2}^{W} R_{iqn21}^{QNUV} \exp(-\beta_{qn}^{W} C_{ijk}^{p})$$

$$+ \sum_{j=N+1}^{M} K_q^t W_{iqn}^{R} \exp(-\beta_{qn}^{W} C_{ijk}^{p}) \Big) \Big] \tag{123}$$

to ensure that:

$$\sum_{i=1}^{M} \sum_{j\varepsilon Z_{q4}} \sum_{n} \sum_{k\varepsilon\gamma(n)} \sum_{u} T_{ijqknuv}^{W} = E_{cq1}^{QV} \tag{124}$$

12.8.6 Out-commuters from the city (K_q^f)

A constraint is imposed to ensure that the trips across the city boundary by those living inside sum to the total given by equation (71). Its value is given by:

$$K_q^f = R_q^f / \Big[\sum_{j=N+1}^{M} W_{jq}^{E} \sum_{n} \sum_{k\varepsilon\gamma(n)} \Big(\sum_{i\varepsilon Z_{q2}} K_q^R W_{iqn}^{R} \exp(-\beta_{qn}^{W} C_{ijk}^{p})$$

$$+ \sum_{i\varepsilon Z_{q1}} A_{iqn1}^{W} R_{iqn1}^{QNU} \exp(-\beta_{qn}^{W} C_{ijk}^{p}) + \sum_{i=1}^{N} A_{iqn2}^{W} R_{iqn21}^{QNUV} \exp(-\beta_{qn}^{W} C_{ijk}^{p}) \Big) \Big] \tag{125}$$

to ensure that:

$$\sum_{i=1}^{N} \sum_{j=N+1}^{M} \sum_{n} \sum_{k\varepsilon\gamma(n)} \sum_{u} T_{ijqknul}^{W} = R_q^f \tag{126}$$

12.8.7 In-commuters into the city (K_q^t)

A similar constraint is imposed to ensure that the trips across the city boundary by those working in the city sum to the total given by equation (69). Its value is given by:

$$K_q^t = R_q^t / \Big[\sum_{i=N+1}^{M} \sum_n W_{iqn}^R \sum_{k\varepsilon\gamma(n)} \Big(\sum_{j\varepsilon Z_{q4}} K_q^E W_{jq}^E \exp(-\beta_{qn}^W C_{ijk}^P)$$

$$+ \sum_{j\varepsilon Z_{q3}} B_{jq1}^W E_{jq1}^{QV} \exp(-\beta_{qn}^W C_{ijk}^P) + \sum_{j=1}^{N} B_{jq2}^W E_{jq12}^{QUV} \exp(-\beta_{qn}^W C_{ijk}^P) \Big) \Big] \qquad (127)$$

to ensure that

$$\sum_{i=N+1}^{M} \sum_{j=1}^{N} \sum_n \sum_{k\varepsilon\gamma(n)} \sum_v T_{ijqknul}^W = R_q^t \qquad (128)$$

12.9 Inverse activity rate

The inverse activity rate is the ratio of the population to the employed residents in the same area. In most applications of this type of model a single value is used. In this study not only is the value disaggregated by social group but also by zone. This is done by scaling the values at the previous time period by the ratio of the total value at the current time point to the previous value. It is necessary to allow for slightly different values for newly locating employed residents from those for non-movers to ensure that the total number of employed residents sums to the total given by equation (70) and the sum of the population equals the exogenously defined total. The differences arise from the spatial variation in the inverse activity rate and have been found to be very small.

The overall inverse activity rate for the city is:

$$g_{cq}^Q(t) = P_{cq}^Q(t)/R_{cq}^Q(t) \qquad (129)$$

The inverse activity rate for non-movers in each zone (g_{iq2}^{QU}):

$$g_{iq2}^{QU}(t) = g_{iq}^Q(t-1)\; g_{cq}^Q(t)/g_{cq}^Q(t-1) \qquad (130)$$

The value of $g^{Q}_{iq}(t-1)$ can be found from:

$$g^{Q}_{iq}(t-1) = P^{Q}_{iq}(t-1)/R^{Q}_{iq}(t-1) \tag{131}$$

The zonal value for newly locating employed residents (g^{QU}_{iq1}) is

$$g^{QU}_{iq1}(t)=g^{QU}_{iq2}(t)\ \frac{\left(P_{cq}(t)-\sum_{i=1}^{N} P^{QU}_{iq2} - \sum_{i\varepsilon Z_{q1}} P^{QU}_{iq1}\right)}{\sum_{i\varepsilon Z_{q2}} R^{QU}_{iq2}\ g^{QU}_{iq2}(t)} \tag{132}$$

12.10 Population location

The number of newly located employed residents in each social and car ownership group (R^{QNU}_{iqn1}) can be found by summation:

$$R^{QNU}_{iqn1} = \sum_{j=1}^{M} \sum_{k\varepsilon\gamma(n)} \sum_{v} T^{W}_{ijqknuv} \text{ for } i=1,N \tag{133}$$

The population in each zone in each social group (P^{Q}_{iq}) is given by:

$$P_{iq} = g^{QU}_{iq1} \sum_{n} R^{QNU}_{iqn1} + P^{QU}_{iq2} \quad \text{for } i=1,\ N \tag{134}$$

12.11 Employment location

The employment in each zone for each social group (E^{Q}_{jq}) is obtained by adding together the surviving workers from equation (64) and the newly located workers obtained by summation:

$$E^{Q}_{jq} = E^{QV}_{jq2} + \sum_{i=1}^{M} \sum_{n} \sum_{k\varepsilon\gamma(n)} \sum_{u} T^{W}_{ijqknuv} \tag{135}$$

13. ACCESSIBILITIES

13.1 Basic principles

The concept of accessibility has been used in several places above as the link between transport and land use. It is computed implicitly within the previously defined equations.

13.2 Accessibility to residents (Y_i^R)

The accessibility to residents is used in the location of housing and industry. It could be calculated especially, but this is not necessary because examination of the term B_{jq1}^W in equation (113) shows that not only does this contain the size and the distance terms but also A_{iqnu}^W which can be interpreted as the competition from others for the same employment opportunities. The value used is that for the middle social group since this is by far the largest. Thus:

$$Y_i^R = 1.0/B_{j21}^W \tag{136}$$

13.3 Accessibility to employment (Y_i^E)

The A_{iqn1}^W term may be interpreted as the inverse of the accessibility to economic activity. The value for car owners (n=1) is used because it contains the cost of travel by both modes. Thus:

$$Y_i^E = 1.0/A_{i211}^W \tag{137}$$

13.4 Accessibility to shops (Y_i^S)

The accessibility to shops used in the location of residents in houses is found by using the inverse of the balancing factor in equation (48) for those with a car available (so that both modes are included). The values for convenience and durable shopping are multiplied together:

$$Y_i^S = 1.0/A_{i11}^S \ A_{i21}^S \tag{138}$$

14. LAND TAKE

14.1 Basic principle

The availability of land is a very important factor in the location of activities in the city. This is taken into account in the model by allocating activities in a particular sequence; the sequence is exogenous new housing, primary and secondary economic activity (except agriculture), tertiary economic activity, endogenous new housing and finally agriculture. Exogenous housing, if any, is given priority to represent the allocation of land for this specific purpose. Economic activity is given priority over other housing because, in general, it can outbid housing for a particular parcel of land which is why economic activity tends to be concentrated at the most accessible part of the city, that is, the centre. On some occasions there is insufficient land available to locate the activity at the density being used in the model, so like Lowry (1964) in his model, overcrowding is assumed to occur, because it is felt that locational factors are more important than the density of a particular activity.

14.2 Density of housing (D_i^H)

The density of housing (D_i^H) is calculated from the previous time point:

$$D_i^H(t) = H_i(t-1)/L_i^H(t-1) \tag{139}$$

where $L_i^H(t-1)$ is the area of land occupied by housing.

The area of land released by the demolition of housing (L_i^D) is found using the density of housing at the previous time point:

$$L_i^D(t) = H_i^D(t-1,t)/D_i^H(t-1) \tag{140}$$

14.3 Exogenous new housing

The density at which new housing is being located (D_i^{HX}) may be specified, otherwise the current value is used:

$$D_i^{HX} = D_i^H \tag{141}$$

The area of land used by exogenously located housing can then be calculated, except in cases when there is insufficient land available for the housing in which case the density is allowed to rise so that housing can be accommodated, that is

$$L_i^{HK} = H_i^{BX}/D_i^{HX} \qquad \text{whichever} \tag{142}$$

or $\qquad$ is

$$L_i^{HX} = L_i^T - L_i^U(t) - L_i^H(t-1) - L_i^h(t) + L_i^D(t) \qquad \text{less} \tag{143}$$

where L_i^T is the total area of land in each zone, L_i^U is the area of unusable land in each zone, L_i^H is the area of land used by housing and L_i^h is the area of land being held off the market. Unusable land is that which cannot be used for any sort of development. Land being held off the market is not currently available, but may become so in the future, or it may be land such as Green Belt that is not developed for political reasons.

14.4 Land available for primary and secondary economic activity

As indicated above, primary and secondary economic activity (except agriculture) is given next priority. The area available ($\sum_{p=2}^{7} L_i^{Pa}$) is given by:

$$\sum_{p=2}^{7} L_i^{Pa}=L_i^T-L_i^U(t)-L_i^H(t-1)-L_i^h(t)+L_i^D(t)-L_i^{HX}(t) \tag{144}$$

14.5 Land used by primary and secondary economic activity (except agriculture)

Land is used by primary and secondary economic activity at the current density as long as there is sufficient land available; if there is not the density is assumed to rise, that is

$$\sum_{p=2}^{7} L_{ip}^P(t) = \sum_{p=2}^{7} L_{cp}^P(t-1)\, J_{ip}^P(t)/J_{ip}^P(t-1) \tag{145}$$

or:

$$\sum_{p=2}^{7} L_{ip}^P(t) = \sum_{p=2}^{7} L_{ip}^{Pa}(t) \tag{146}$$

whichever is less

The land used by each sector is given by

$$L_{ip}^P(t) = \frac{L_{ip}^P(t-1)J_{ip}^P(t)/J_{ip}^P(t-1)}{\sum_{p=2}^{7} L_{ip}^P(t-1)J_{ip}^P(t)/J_{ip}^P(t-1)} \quad \sum_{p=2}^{7} L_{ip}^P(t) \text{ for } p=2,\ 7 \tag{147}$$

14.6 Land available for tertiary economic activity

Tertiary economic activity is given next priority for location. The land available ($\sum_s L_{is}^{Sa}$) is given by:

$$\sum_s L_{is}^{Sa} = \sum_{p=2}^{7} L_{ip}^{Pa} - \sum_{p=2}^{7} L_{ip}^{P} \tag{148}$$

14.7 Land used by tertiary economic activity

In retailing, and, to some extent, other service activities, there has been an increase in the floorspace per employee ratio, reflecting the move towards supermarkets and discount shopping. It is realistic, therefore not to regard a decrease in tertiary activity in a zone as necessarily leading to a decrease in the area used. Once again, sufficient land must be available to locate the activity, or else the density rises. Because of the difficulty of distinguishing between the various categories on land use maps, these sectors are treated in aggregate. The allocation of land is calculated as follows:

$$\left.\begin{array}{ll}
\text{if} & \sum_s E_{is}^{S}(t) \geqslant \sum_s E_{is}^{S}(t-1) \quad (149)\\
\text{then} & \sum_s L_{is}^{S}(t) = \sum_s L_{is}^{S}(t-1)\ \sum_s E_{is}^{S}(t)/\sum_s E_{is}^{S}(t-1) \quad (150)\\
\text{if} & \sum_s E_{is}^{S}(t) < \sum_s E_{is}^{S}(t-1) \quad (151)\\
\text{then} & \sum_s L_{is}^{S}(t) = \sum_s L_{is}^{S}(t-1) \quad (152)\\
\text{or} & \sum_s L_{is}^{S}(t) = \sum_s L_{is}^{Sa}(t) \quad (153)
\end{array}\right\}\ \text{whichever is less}$$

14.8 Land available for housing

The land available for endogenously located housing (L_i^{Ha}) is given by:

$$L_i^{Ha} = \sum_s L_{is}^{Sa} - \sum_s L_{is}^{S} + L_i^{Hg} \tag{154}$$

This term is used in equation (11) to locate the new housing where L_i^{Hg} is the area of land released for housing development, given by equation (163).

14.9 Land used by housing

The area of land used by new housing (L_i^{HB}) is given by:

$$L_i^{HB}(t)=[H_i^B(t) - H_i^{NX}(t)]/D_i^H(f) + L_i^{HX}(t) \qquad (155)$$

The total area of land used by housing in each zone (L_i^H) is given by:

$$L_i^H(t) = L_i^H(t-1) + L_i^{HB}(t) - L_i^D(t) \qquad (156)$$

14.10 Land developed

Developed land is that occupied by economic activity (other than agriculture) and housing, with the proviso that land once developed cannot become undeveloped, that is, the area developed will not decrease. Hence the area of developed land (L_i^G) is given by:

$$L_i^G(t) = \sum_{p=2}^{7} L_{ip}^P(f) + \sum_s L_{is}^S(t) + L_i^H(t) \qquad \text{whichever} \qquad (157)$$

or $\qquad$ is

$$L_i^G(t) = L_i^G(t-1) \qquad \text{greater} \qquad (158)$$

14.11 Agricultural land

In an urban area, agricultural land is a residual category, gradually being eaten into by urban development. If land being held off the market is, in fact Green Belt, then care must be taken to prevent double counting. If there is land in this category, it is assumed that it is all in agriculture usage, and so not available for urban development but that all other agricultural land is potentially available for development. The quantity of agricultural land that is potentially available for development (L_i^{AGa}) is given by

$$L_i^{AGa}(t) = L_i^A(t-1) - L_i^h(t) \qquad \text{whichever} \qquad (159)$$

or $\qquad$ is

$$L_i^{AGa} = 0 \qquad \text{greater} \qquad (160)$$

where L_i^A is the area of agriculture land and L_i^h is the area of land being held off the market.

Agricultural land that is not Green Belt is assumed to be equally available as other undeveloped land for development, thus the land left for agricultural usage (L_i^A) is given by

$$L_i^A(t) = L_i^A(t-1) - L_i^{AGa}(t) \frac{L_i^G(t) - L_i^G(t-1)}{L_i^T - L_i^U(t) - L_i^h(t) - L_i^G(t-1) + L_i^D(t)} \tag{161}$$

14.12 Land released for development

The model has been designed so that areas of land in each zone can be held off the market, that is development is not permitted there. Land can be released for building during any time period. This can either be specified for each zone, or for the whole city, in which case it is allocated to zones in proportion to the area being held off. Thus, the area of land being held off, L_i^h is given by:

$$L_i^h(t) = L_i^h(t-1) + L_i^{hinc}(t-1,t) - \frac{L_c^g\left[L_i^h(t-1) + L_i^{hinc}(t-1,t)\right]}{\sum_j \left[L_j^h(t-1) + L_j^{hinc}(t-1,t)\right]} \tag{162}$$

where L_i^{hinc} $(t-1,t)$ is the increase in the area of land in zone i being held off the market and L_c^g is the total area of land in the city released for development.

It may be desired to release land for just housing development; this is given by:

$$L_i^{Hg}(t) = L_c^{Hg}(t) \frac{L_i^h(t)}{\sum_j L_j^h(t)} \tag{163}$$

where L_i^{Hg} is the area of land in zone i released for housing development.

14.13 Vacant land

The area of land that could have been used for development, but, in fact, is still available, L_i^V, is given by:

$$L_i^V = L_i^T - L_i^G - L_i^h - L_i^U \tag{164}$$

15 INEQUALITIES IN THE PROVISION OF HOUSING AND JOBS

15.1 Basic principle

One of the main criticisms made of this type of model is that supply is assumed to match demand exactly. This is not so in this particular model. Inequality in the housing market is represented by vacant dwellings or overcrowding. In the job market it is represented by vacant jobs or changes in the activity rate, reflecting marginal workers, for example, housewives or those who are frequently unemployed, entering or leaving the employment market in response to changes in the total number of jobs available. As described above the houses and jobs are located first, then people select from amongst those available. Vacancies are simply those not filled. Demand exceeding supply is less easy to represent, since by definition those who do not have a job or house cannot have a spatial label attached in the same way as those who do.

15.2 Over-provision of housing - vacant housing

The supply of housing (H_i) in each zone is given by equation (30) and the number occupied is given by:

$$H_i^O = \sum_q P_{iq}^Q / o_{iq}^{HQ} \qquad (165)$$

Vacant housing in each zone is then given by:

$$H_i^V = H_i - H_i^O \qquad (166)$$

15.3 Under-provision of housing - overcrowding

Overcrowding is taken to occur when the occupancy rate for one or more zones has to be increased in order to ensure that all the population can be accomodated. Because the lowest social group is located last they are most likely to be overcrowded by this definition (reflecting the realities of the world). The housing occupancy rate is calculated using equations (31) and (32). An increase in the level of population without a concomitant rise in the number of houses will lead to a rise in the occupancy rate. The scaling of the occupancy rate in this manner does not, however, neccesarily lead to equality in the provision of housing because of the spatial variation in the occupancy rate, particularly if not all houses were filled at the previous time point.

Overcrowding occurs if

$$\sum_{i\varepsilon Z_2} g^{QU}_{iql} \sum_n R^{QNU}_{iqnl} > \sum_{i\varepsilon Z_2} H^{aQ}_{iq} o^{HQ}_{iq} \qquad (167)$$

If this occurs the population is located in the housing using

$$P^{QU}_{iql} = \frac{H^{aQ}_{iq} o^{HQ}_{iq}}{\sum_{i\varepsilon Z_2} H^{aQ}_{iq} o^{HQ}_{iq}} \sum_{i\varepsilon Z_2} g^{QU}_{iql} \sum_n R^{QNU}_{iqnl} \qquad (168)$$

thus overriding the value of the housing occupancy rate computed using equation (32).

15.4 Over-provision of jobs - vacant jobs

The total number of jobs in each zone is given by equation (61) and the total number working in each zone by equation (135). Vacant jobs (J^{VQ}_{jq}) can be found by subtraction:

$$J^{VQ}_{jq} = J^{Q}_{jq} - E^{Q}_{jq} \qquad (169)$$

15.5 Under-provision of jobs - underemployment

The model can be run assuming all jobs are filled, that is

$$E^{Q}_{cq} = J^{Q}_{cq} \qquad (170)$$

This may well be true when a declining area is being modelled. This value of E^{Q}_{cq} will then be used in equation (70) to calculate R^{Q}_{cq} and so in equation (129) to calculate the inverse activity rate g^{Q}_{cq}. Consequently a fall in the number of jobs will lead to a fall in the activity rate, reflecting a reduction in the number of persons employed.

16. EXOGENOUS LOCATION AND THE INCORPORATION OF PLANNING CONSTRAINTS

16.1 Basic principle

Several of the location equations defined above have included the variables being located exogenously to represent planning policies, or for testing the model under different assumptions. In this section these will be drawn together and any additional checks that must be made, described.

16.2 Housing locations

The location of housing (H_i^B) is given by equation (11) which can be used to test the impact of a new housing development by giving values to H_i^{BX}. It is important to check that the total new housing being located is greater or equal to the sum of all being located exogenously, that is

$$H_c^B \geq \sum_{i=1}^{N} H_i^{BX} \tag{171}$$

If this is violated the total number of new houses being located is adjusted automatically and the model operator informed.

16.3 Primary and secondary economic activity location

Equation (33), (36) and (39) include terms for the exogenous location of jobs. These would be used to examine the effect of a new industrial estate, for example. A similar check to that for housing is made:

$$J_{cp}^{P} \geq \sum_{i=1}^{N} J_{ip}^{PX} \tag{172}$$

Again the total number of jobs is adjusted if necessary.

16.4 Tertiary economic activity location

Equations (46) and (47) include sales in shops being located exogenously (S_{js}^{SX}). This can be calculated from the equivalent number of jobs using:

$$S_{js}^{SX} = E_{js}^{SX}/b_s \tag{173}$$

This method works for marginal cases, for example the location of a new hypermarket in one zone, or new centres in a few zones. However, if it is desired to test a completely new pattern of tertiary activity a slightly different method is used. Firstly a check is made:

$$\frac{E_{cs}^{S} - \sum_{j=1}^{N} E_{js}^{SX}}{E_{cs}^{S}} \leq 0.001 \qquad (174)$$

that is, all tertiary employment in a sector is being located exogenously. (The term 0.001 is used because equalities cannot be guaranteed to be met with real numbers on a computer). If this inequality is met the tertiary employment is given by:

$$S_{ijskm} = A_{ism}^{S} X_{ism} B_{js}^{S} S_{js}^{SX} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \quad i=1, N, j=1, N \qquad (175)$$

$$S_{ijskm} = K_{s}^{S} X_{ism} B_{js}^{S} S_{js}^{SX} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \quad i=N+1, M, j=1, N \qquad (176)$$

Since only a portion of the sales in external zones are forecast by the model it is not feasible to set these exogenously. Hence equation (46) is used for these. Additional sales could be added in these zones. The balancing factors are given by:

$$A_{ism}^{S} = \left[\sum_{j=1}^{N} \sum_{k\epsilon\gamma(m)} B_{js}^{S} S_{js}^{SX} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) + \sum_{j=N+1}^{m} \sum_{K\epsilon\gamma(m)} (\{W_{js}^{S}\}^{\alpha_{sm}^{S}} + B_{js}^{S} S_{js}^{SX}) \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \right]^{-1} \qquad (177)$$

to ensure that

$$\sum_{j=1}^{m} \sum_{K\epsilon\gamma(m)} S_{ijsKm} = X_{ism} \qquad (178)$$

and

$$B_{js}^{S} = \left[\sum_{i=1}^{N} A_{ism}^{S} X_{ism} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) + \sum_{i=N+1}^{M} K_{s}^{S} X_{ism} \exp(-\beta_{sm}^{S} C_{ijk}^{o}) \right]^{-1} \qquad (179)$$

to ensure that

$$\sum_{i=1}^{M} \sum_{K\epsilon\gamma(m)} S_{ijskm} = S_{js}^{SX} \qquad (180)$$

16.5 Jobs disaggregated by social group

The equations above can be used to locate all jobs by sector, exogenously. However, equation (61) would still be used to determine the social group disaggregation. The proportions defined by the terms U_{qp}^{QP} and U_{qs}^{QS} are kept constant over time, so may not be terribly accurate and so it is possible to override these by specifying the location of jobs disaggregated by social group. It should be noted that this can only be done if the employment in all sectors in a particular zone is being located exogenously. If jobs in all zones are being set exogenously, then the total is used for the number employed:

$$\text{If} \qquad E_{jq}^{QX} \geq 0 \quad \text{for all } i = 1,N \qquad (181)$$

$$E_{cq}^{Q} = \sum_{j=1}^{N} E_{jq}^{QX} \qquad (182)$$

16.6 Population

Equations (103) and (104) can be used to set the population maxima or minima in one or more zones. Clearly the following must hold:

$$\sum_{i=1}^{N} P_{iq}^{max} \geq P_{cq}^{Q} \qquad (183)$$

and

$$\sum_{i=1}^{N} P_{iq}^{min} \leq P_{cq}^{Q} \qquad (184)$$

Also, checks must be made to ensure that the number of survivors does not exceed the exogenous level or maximum for a particular zone:

$$P^{QU}_{iq2} \leqslant P^{XQ}_{iq} \tag{185}$$

$$P^{QU}_{iq2} \leqslant P^{max}_{iq} \tag{186}$$

If either of these is violated the value of P^{QU}_{iq2} is adjusted accordingly and the model operator informed. At first sight this may appear to be inconsistent with equation (62), but because of the nature of the model and the need to assume rates remain constant over time, it is not surprising that the model cannot automatically cope with all possible values of exogenously specified variables. The important point is for the model user to be made aware of what is happening and to act if he so desires, rather than trying the impossible task of producing a foolproof model.

If the population has been set exogenously there may be a slight problem to allocate all the population at the value of the inverse activity rate defined by equation (132), since the value of P_{iq} may be very different from the value that would be calculated endogenously. Hence if:

$$P^{QX}_{iq} \geqslant 0 \quad \text{for any } i \tag{187}$$

then
$$R^{QU}_{iq1} = \frac{P^{QX}_{iqu}}{g^{QU}_{iq1}} \frac{\left(R^{TQ}_{cq} - \sum\limits_{j \notin Z_{q5}} R^{QU}_{q1} - \sum\limits_{j=1}^{N} R^{QU}_{jq2}\right)}{\sum\limits_{j \in Z_{q5}} P^{QX}_{jq} / g^{QU}_{jq1}} \tag{188}$$

where Z_5 is the set of zones for which $P^{QX}_{iq} \geqslant 0$. The value of g^{QU}_{iq1} is the value computed from equation (132) but a new value can be computed to ensure consistency:

$$g^{QU'}_{iq1} = P^{QX}_{iqu} / R^{QU}_{iq1} \tag{189}$$

An analogous calculation on the housing occupancy rate can be made to ensure that the desired population is located in the available housing:

$$\text{check} \quad \sum_q \frac{P_{iq}^{QX}(t)}{o_{cq}^{HQ}(t-1)} \; \frac{o_{cq}^{HQ}(t-1)}{o_{cq}^{HQ}(t)} \leq H_i \qquad (190)$$

where H_i is the total housing given by equation (30). If this condition is violated, then the housing occupancy rate is given by:

$$o_{iq}^{HQ}(t) = \frac{o_{iq}^{HQ}(t-1)}{H_i(t)} \; \frac{o_{iq}^{HQ}(t)}{o_{cq}^{HQ}(t)} \; \sum_q \frac{P_{iq2}^{QX}}{o_{iq}^{HO}(t-1)} \; \frac{o_{cq}^{HQ}(t-1)}{o_{cq}^{HQ}(t)} \qquad (191)$$

A similar check is carried out on the non-moving population given by equation (62), but this is unlikely to be violated unless there is a very large number of survivors in one or more zones. A check is made on the total number of non-moving employed residents:

$$\text{check} \quad \sum_{i=1}^{N} R_{iq2}^{QU} \leq R_{cq}^{Q} \qquad (192)$$

If not

$$R_{iq2}^{QU} = R_{cq}^{Q} \frac{P_{iq2}^{QU} / g_{iq2}^{QU}}{\sum_{j=1}^{N} P_{jq2}^{QU} / g_{jq2}^{QU}} \qquad (193)$$

then a new value of the non-moving population is found:

$$P_{iq2}^{QU\prime} = R_{iq2}^{QU} \, g_{iq2}^{QU} \qquad (194)$$

and then values are used in preference to those given by equation (62). Once again a flexible approach is being taken if values computed using equation (62) are inconsistent with the total value given by equation (70) an adjustment is made to the values. This does not upset the overall logic of the model, but does allow the model to be used consistently.

The various checks and adjustments outlined in this section have been devised after empirical testing of the model. They only apply under the conditions stated and are a necessary accompaniment to a model that can be used for forecasting under so many different conditions.

17 CONVERGENCE OF THE MODEL

17.1 Basic principle

The complex interactions within the model and the intractability of the solution mean that an iterative solution method has to be used. This means that a convergence test has to be carried out on the outer loop, as well as all those on all the balancing factor calculations. The test is to ensure that none of the zonal values varies from the equivalent value on the previous iteration by more than some small fraction, in this case taken to be 0.1%.

17.2 The convergence test

The test is carried out on the employed residents in each social group, in each zone and may be expressed as:

$$\frac{R_{iq}(I) - R_{iq}(I-1)}{R_{iq}(I-1)} < \Delta \quad \text{for all i and q} \qquad (195)$$

where I is the iteration number and Δ is some small value.

The test is only carried out on the second and subsequent iterations. Experience with the model has shown that it is possible for oscillations to occur. Generally this means that the following sequence is occurring: decentralized employment followed by centralized population, leading to centralized employment, followed by decentralized population which leads to decentralized employment and so on. To check for this the following test is carried out on the eleventh and subsequent iterations:

$$\frac{R_{iq}(I) - R_{iq}(I-2)}{R_{iq}(I-2)} < \Delta \quad \text{for all i and q} \qquad (196)$$

The oscillations generally imply some small error has occurred. If no such test was applied the model would continue running until forcibly stopped by the computer operators.

18. OUTPUT INDICATORS

18.1 Basic principles

A model as complex as the one being described here produces vast quantities of results. These include the zonal distributions of housing population and employment, plus trip patterns. To facilitate interpretation of the changes being examined a set of indicators are computed after the final convergence at each time point. The indicators computed include the modal split, time and money expenditure and various accessibility measures. One of the main uses of the model is for examining the impact of a particular change; consequently it is necessary to be able to compare the results of two runs of the model. This is done by storing the results from one application and then after the second has been run inputting the results from the first and calculating measures of the deviations between the equivalent distributions of activities and trips.

18.2 Zonal distributions

The following items are output for each zone for each time point:

(a) Housing - total
- new
- demolished
- occupied
- vacant

(b) Population - total
- by social group

(c) Employment - total
- by social group

(d) Jobs - total
- by 12 industries
- vacant

(e) Sales in shops - by goods type (convenience and durable)

(f) Land - agricultural
- primary and secondary economic activity (excluding agriculture)
- tertiary economic activity
- housing
- developed
- unused

The zonal values are aggregated to three sets, deferred as inner city, inner suburbs and outer suburbs, so that the decentralizaton effects of policies can be seen.

18.3 Accessibility measures

Accessibility is a useful concept for examining the impact of change upon groups since it includes both transport and land use, and so can reflect a change in either.

One useful accessibility measure was devised by Hansen (1959), which may be shown as

$$Y_i = \sum_j W_j \; f(c_{ij}) \qquad (197)$$

where Y_i is the accessibility of zone i, W_j is a measure of the attraction of zone j and $f(c_{ij})$ is a decreasing function of the cost of travel between zones i and j.

In this model costs are known for each mode, and there is a cost function for each car ownership group. The cost function is that used in the model. Values can be calculated for the appropriate disaggregation, for example, by social group. The accessibility to jobs for each social group is, thus, given by:

$$Y_{iq}^{JQ} = \sum_j J_{jq}^{Q} \sum_n \sum_{k\varepsilon(n)} \exp(-\beta_{qn}^{W} C_{ijk}^{p}) \qquad (198)$$

The accessibility to all jobs is obtained by summation:

$$Y_i^{J} = \sum_q Y_{iq}^{JQ} \qquad (199)$$

Similarly the accessibility to the supply of labour in each social group is given by:

$$Y_{iq}^{RQ} = \sum_j R_{iq}^{Q} \sum_n \sum_{k\varepsilon(n)} \exp(-\beta_{qn}^{W} \; C_{ijk}^{p}) \qquad (200)$$

The total accessibility to all labour is found by summation:

$$Y_i^R = \sum_q Y_{iq}^{RQ} \tag{201}$$

The accessibility to each tertiary economic activity can be found in a similar manner:

$$Y_{is}^{ES} = \sum_j E_{js}^S \sum_m \sum_{k\varepsilon(m)} \exp(-\beta_{sm}^S \, C_{ijk}^o) \tag{202}$$

The accessibility to the population who are customers or users of each tertiary activity is given by:

$$Y_{is}^{RS} = \sum_j P_j \sum_m \sum_{k\varepsilon(m)} \exp(-\beta_{sm}^S \, C_{ijk}^o) \tag{203}$$

The accessibility to some or all the tertiary sectors or users of all tertiary sectors can be obtained by summation:

$$Y_i^E = \sum_s Y_{is}^{ES} \tag{204}$$

and

$$Y_i^R = \sum_s Y_{is}^{RS} \tag{205}$$

In section 13 above the interpretation of the inverse of the balancing factors as accessibility functions containing competition terms was discussed. Because the values are obtained by mutual iteration the values of the factors are not unique, although the product of each pair of A_i and B_j values is. Consequently, no meaningful interpretation can be made of individual values. However, relative comparisons can be made. The first of these is the accessibility advantage of car owners compared to non-car owners found by calculating the ratio of the two A_{iqn} values. Values are obtained for each social group for the journey to work, and for shopping. The second measure is the relative value over space, found by dividing each value by, say, the most central one. This can be found for each car ownership group and social group for the relative accessibility to employment, and for the relative accessibility to labour in each social group. Similar values can be found from the balancing factors found in the calculation of tertiary activity location.

18.4 Modal split

The modal split, expressed as the proportion using each mode, can be found at a variety of spatial levels, and by various disaggregations. It can be found for everybody in the system or just those with a car available.

The three spatial scales are the whole study area, each zone of residence, and the three aggregations of zones, plus the external zone system. The modal split to work can be found for each social group and in aggregate for each of the spatial scales. Similarly the values for each tertiary activity are found at the three spatial scales. In addition the values are found for the two shopping modes together, plus the modal split of arrivals at each zone of shopping.

By way of an example, the modal split for the journey to work for all in the study area is given by:

$$\frac{\sum\sum\sum\sum\sum\sum_{ijqnuv} T^{W}_{ijqknuv}}{\sum\sum\sum\sum\sum\sum\sum_{ijqnkuv} T^{W}_{ijqknuv}}$$

18.5 Expenditures on the journey to work

The generalised cost measure used in this model contains both money and time elements. It is useful to find the implications of expenditure on each; since money expenditure represents a transfer from individuals to organizations, for example petrol companies, bus operators (who may be local government) and national government (via taxes). Time expenditure represents the amount of travel made by individuals. The total expenditure on travel in money and time, by each mode is calculated, plus the mean in money and time, by users of each mode, by members of each car ownership group and by members of each car ownership group using each mode, as well as the overall mean. All these calculations are made for the whole study area, each set of zones and each social group. The total distance travelled and the energy used by each mode are also calculated.

In addition to the money and time expenditures the mean travel cost in generalised cost units is found for the journey to work for the whole study area for each mode, and each car ownership group, both for each social group and overall. The equivalent costs are found for each tertiary activity.

As an example the mean travel time for the journey to work for each social group by each mode in the study area is given by:

$$\bar{t}^{QK}_{qk} = \frac{\sum\sum\sum\sum\sum_{ijnuv} T_{ijqknuv}\, t^{P}_{ijk}}{\sum\sum\sum\sum\sum_{ijnuv} T_{ijqknuv}} \qquad (206)$$

18.6 Comparisons between forecasts

Comparisons between forecasts are made using the following variables:

(a) New housing in each zone
(b) Total population in each zone
(c) Population by social group in each zone
(d) Total employment in each zone
(e) Employment by social group in each zone
(f) Jobs by 12 industrial sectors in each zone
(g) Trips to work between each pair of zones
(h) Trips to work by each mode between each pair of zones.

For each of these variables the percentage deviation from the basic forecast induced by the particular change being tested is found. This can be used either to find how much effect a particular policy has or to see how sensitive a particular variable is to changes in the inputs.

19. CONCLUSIONS

In this paper the mathematical description of a fairly sophisticated model representing the interaction between transport and the location of population, housing, employment, shopping and jobs has been presented. The model has evolved from a long process of consideration of theory and model application. The model may contain some parts that are less sophisticated than is desirable, nonetheless, taken overall it represents an extremely useful planning instrument which has been tested against real data and provides good results (Mackett, 1979a). It will be further developed in a project in which the effects of transport costs on location and commuting patterns in London and South-East England will be examined (Kirkby, Mackett and Nash, 1979). This project will yield further insights into the interaction between transport and land use.

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