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THE DAY-TO-DAY DYNAMICS OF ROUTE CHOICE

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ABSTRACT

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This paper reviews methods proposed for modelling the day-to-day dynamics of route choice, on an individual driver level. Extensions to within-day dynamics and choice of departure time are also discussed. A new variation on the approaches reviewed is also described. Simulation tests on a simple two-link network are used to illustrate the approach, and to investigate probabilistic counterparts of equilibrium uniqueness and stability. The long-term plan is for such a day-to-day varying demand-side model to be combined with a suitable microscopic supply-side model, thereby producing a new generation network model. The need for such a model - particularly in the context of assessing real-time transport strategies - has been identified in previous working papers.

KEY-WORDS: Route choice; day-to-day variability; traffic network model; stochastic process; departure time choice.

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THE DAY-TO-DAY DYNAMICS OF ROUTE CHOICE

1. INTRODUCTION

This note describes some possible models of the day-to-day dynamics of driver route choice. The motivation for the work is the study of dynamic route guidance systems. The models considered include approaches which have been proposed in the literature, as well as a new variation on these models.

The purpose of this review is to determine the most appropriate model, which will be used as a component in a much larger network model. This particular component is concerned only with the *demand* side of route choice decisions made at the trip origin - it is intended that it may be used with any supply model (although the expectation is that a stochastic, capacity-restrained supply model will be used). For the main part of the note, attention will be restricted to the case where demand is constant over the modelled period in any given day, departure times are fixed, and the attractiveness of competing routes is constant within a day. However, towards the end of the note, a possible specification is given for the day-to-day evolution of the within-day dynamic problem of joint route and departure time choice.

2. REVIEW

The first notable piece of work on this subjects seems to be that of Horowitz (1984). His work related to the stability of stochastic equilibrium. In the study of dynamic route guidance systems, the notion of equilibrium is of questionable use (Watling and Van Vuren, 1992) and so stability issues are not of great concern to us. Of greater interest are the various models proposed by Horowitz for describing the day-to-day adjustment in perceived travel costs (and hence route choice). He describes three basic models; all are studied in conjunction with a deterministic supply model (link performance functions), and in all cases drivers choose the minimum perceived cost route. They work at a macroscopic (route flow) level of detail.

Horowitz's first model (Model 1) assumes that the travel cost on day k is made up of a mean value (same for all drivers) and a driver-dependent error term. The mean value is obtained as a weighted average of the *measured* costs for all previous days. These weights may vary with respect to the day k, but for simplicity are assumed to be constant for all links. On any given day k, there are no restrictions on the weights, except that they must sum to one. For example, they could be chosen to put a successively decreasing weight on previous times, the further in the past they were experienced - thus, costs in the recent past have the greatest effect on perception. Alternatively, the weights could be highest in the more distant past, suggesting that habits are formed early on in drivers' experiences. Horowitz suggests some specific forms of weighting scheme, based on a moving averages adjustment process.

The error term is a random variable whose distribution is independent of the day k; thus, the magnitude of the errors does not tend to decrease with experience. The error term is said to represent a number of factors:

i)perception errors in evaluating travel costs;
ii)omission of variables relevant to route choice in the cost definition; and
iii)differences in cost definition between individuals.

In spite of this we shall henceforth refer to such errors as `perception errors', with the intention that this embraces points (i) and (iii); the model mis-specification errors of point ii) are not of concern at the moment.

Horowitz's Model 3 differs from Model 1 in that route choices are based on weighted averages of *perceived* route costs on previous days. In this case, however, the notion of a `perceived cost' becomes somewhat ambiguous - we should distinguish between:

i)*perceived experienced cost on day k* - that is, what a driver perceives the experienced travel cost was on links used during his day k trip; and
ii)*perceived predicted cost on day k* - what the driver, prior to his day k trip, perceives the link costs to be.

The route choice on day k can only be based on perceived predicted costs. Model 3 appears to be somewhat strange in that in forming the perceived predicted cost, the weighting process seems to act upon the sum of the actual experienced cost (the deterministic, measured cost component of (i)) and the perception error in the predicted cost (the random component of (ii)). [NB: The weighting scheme leaves unchanged, perceived predicted costs for links *not* used on day k]. A more justifiable scheme would seem to be forming perceived predicted costs from a weighted average of previous days' perceived experienced costs. This latter scheme is almost certainly what Horowitz intended.

Model 2 differs from Model 3 only in the sense that drivers are assumed to form a day k perceived experienced cost even for links they did not use on day k. The justification is that they received information on unused links from broadcasts or conversations with other travellers. The main reason for Horowitz considering Model 2 appears to be that it is possible to prove a number of theoretical results regarding stability. However, it has some relevance to the modelling of dynamic route guidance systems, and the representation of the increased information available.

Van Berkum and Van Der Mede (1990) describe a framework for a model of route, mode and departure time choice - we shall only consider the route choice component of this framework. It is a microscopic, stochastic model with no `within-day dynamics', and may be described in three steps. For each driver on each day:

a)Define the set of known routes. The subset of routes assumed to be available to each driver for each origin-destination movement is specified externally to the program, and is assumed not to vary with time (i.e. day).

b)Form the perceived costs of each route. Each driver remembers costs previously experienced on each route - this memory is stored as an experienced travel cost mean and variance per route. The perceived (predicted) route cost is formed by generating a pseudo random number from a Normal distribution with parameters given by the experienced travel cost mean and variance. This randomisation is used to represent drivers' uncertainty about the route cost. For routes not previously used by an individual, the mean experienced travel cost is set to the free-flow value and the variance set arbitrarily high.

c)Choose a route. The choice process takes account of the costs currently perceived and of habit effects. For each previously-used route, a probability is calculated of a driver using that route again merely out of habit (independent of currently perceived costs). This probability depends upon how long ago the route was last chosen and how many times it has been chosen. The authors suggest a possible functional form for this probability - although they do not justify it and it appears to depend on a number of unknown quantities which they do not suggest how to estimate. If the driver chooses to use none of the routes out of habit, then he chooses the minimum perceived cost one.

A final comment to make on the approach of Van Berkum and Van Der Mede is the fact that the model is based entirely on routes, rather than on links. The reason given is that they believe drivers perceive route costs and not link costs - an opinion which may well have some justification. However, dealing with routes requires somewhat more care because of correlation issues. For example, consider two competing routes which coincide except for a very small part of their length. Because drivers' memory is built up in terms of route costs in the model, a driver may have a great deal of experience of one route but be assumed to have no idea of travel costs on the very similar, competing route. Furthermore, even if a driver has experienced very similar travel costs on these two routes, the model would still allow him to *perceive* the route costs as very different (since perceived route costs are independent between routes). The latter problem could be overcome by use of a Multivariate Normal for route costs; the former problem is somewhat more difficult to solve in terms of routes, without disaggregating to links.

Cascetta (1989) describes a general framework for studying day-to-day dynamics, in which the state occupied by the transportation system on any particular day is the realisation of a stochastic process. The `state' of the system can be defined at a number of levels - at the most detailed, it represents individual route choices. Horowitz, in the work described earlier, investigated `stability' in the context of equilibrium models - that is, the existence of a unique self-reproducing state, which the system attains independent of the starting conditions. Cascetta considers a probabilistic counterpart of this property, in which a system is said to be `dynamically stable' if there exists a unique, steady-state probability distribution which gives the long-term probability of the system occupying any feasible state, independent of the initial state. This arises from the premise that no system remains in the same state over successive `days', due to (random) variations in demand and supply conditions as well as users' choices.

Cascetta considers the case in which the network and the potential demand remain unchanged for a large enough number of days for a steady-state evolution eventually to take place. Under this assumption, sufficient conditions on the driver route choice decision process to allow `dynamic stability' were deduced. These are that the route choice probabilities

i) depend on not more than a *finite* number of *previous* states;
ii) are non-zero for all feasible paths (the set of feasible paths is defined a priori); and
iii) depend only on the time at which previous states occurred *relative* to the current state (i.e. the driver decision *rules* do not vary with time, even though the measures used by the rules are time-dependent).

It is noted that property (i) still allows route choice probabilities to depend on a possibly large number of previous states which need not be the same for all drivers.

These properties also ensure that the resulting process is `ergodic'. This means that expectations and higher order moments can be estimated from a single, pseudo-realisation of the process. Although the properties given are with respect to the highest level of detail (individual route choices), if they are satisfied then dynamic stability and ergodicity also hold at the route flow and link flow levels. Cascetta suggests a scheme for computing link flow means and variances from a single pseudo-realisation of the process, by which moment estimates are formed as soon as the `steady state' hypothesis is not rejected by a suitable statistical test (e.g. t-test on flows from successive days).

As well as proposing a general framework for studying day-to-day evolution, Cascetta goes on to suggest a specific model known as STODYN. This is a macroscopic model, with link flows as output. In a similar vein to Horowitz's model 2, drivers base their route choice on a weighted average of travel costs from a given number of previous days. This weighted average gives the mean of the perceived predicted cost distribution. The supply model is a stochastic one, based on randomised link impedance functions (alternatively, the stochastic element can be regarded as a perception error in evaluating experienced travel times). Cascetta carried out a theoretical comparison of this model with a conventional stochastic user equilibrium (SUE) model - that is, he compared expected steady state STODYN route flows with those produced by SUE. He found that the quality of the approximation given by SUE to STODYN expected route flows could be reasonable, but deteriorated with an increase in day-to-day variability of route flows. At the link flow level, the approximation may be expected to be better; however, Cascetta notes that `higher variances and an autocorrelation structure should be expected for STODYN link flows'.

Cascetta and co-workers (1991a) later extended the above simple model, to produce `STODYN 2'. The most important differences with STODYN are that STODYN 2:

i)simulates day-do-day evolution in terms of within-day variable route flows;
ii)within-day dynamic link flows are then obtained from these route flows via a dynamic network loading model (supply model);
iii)models the combined choice of route and departure time using a nested logit formulation;
iv)includes a crude `habit' effect, by which only a pre-specified proportion of drivers consider their previous day's choices;
v)forms the mean perceived predicted travel cost (assumed to be equal to mean perceived predicted travel time) for each route and departure time interval as a weighted average of the previous day's perceived predicted travel time and the previous day's average `experienced' travel time (i.e. the average travel time for drivers who used that route and departure time);
vi)models en route path switching;
vii)is able to simulate the effect of route guidance/information systems.

STODYN 2 appears to be at present the most complete model for simulating the effect of DRG systems. However, we shall remain - for the moment - with the issue of modelling day-to-day dynamics in the route choice process.

A final comment we make in passing regarding Cascetta's work is that he appears earlier to have proposed a somewhat different way of modelling a driver's "habit" (Cascetta & Cantarella, 1991). In this, it was assumed that drivers follow the route and departure time used on the previous day, as long as the previous day's actual "experienced" disutility (or, alternatively, they suggest the *perceived* experienced disutility) is within a pre-specified percentage tolerance of the previous day's perceived predicted disutility. [This has some passing resemblance to Iida et al's (1992) experiments on route choice behaviour, described later in this review, in which the proximity of predictions and experience is also a factor in day-to-day choices].

An approach which is similar to STODYN has been independently proposed by Ben-Akiva et al (1991), for modelling driver information systems. They were specifically concerned, as in this note, with the impact of pre-trip information. The choice dimensions available to the user are route and departure time. The primary objective of Ben-Akiva et al's work was to propose a general framework for simulating the day-to-day adjustment of these decisions when pre-trip driver information is provided. The work focuses, therefore, on a `demand side' formulation, with no en route path switching.

There are two components to this part of the model, for each individual:

i)the updating of historic perceptions of travel times (for each route and departure time) and the prediction of times (and hence costs) for the current journey;
ii)a decision based on the predicted costs.

Prior to a trip on any particular day, a driver computes his current perceived *historic* travel times as a weighted average of (with pre-specified, fixed weights):

a)the previous day's perceived historic travel times and the previous day's experienced travel times (for the route and departure time chosen on the previous day);
b)the previous day's perceived historic travel times and the travel times deduced from any exogenous information acquired regarding the previous day (for other route and departure time combinations).

The exogenous information includes weather conditions, accidents, media reports and advice from the driver information system *for the previous day*. (Compare with Horowitz's Model 2, mentioned earlier).

Having computed the perceived historic travel times, each driver then associates a perceived *predicted* travel time with each route/departure time combination. This latter is computed as a weighted average (again with pre-specified, day-independent weights) of the current perceived historic travel time and the travel time information supplied exogenously *for the current day*. These perceived predicted travel times are then used as a component in the perceived predicted generalised travel costs. A cost is then associated with each route and departure time, as a combination of the generalised travel cost and the schedule delay.

On any given day, the drivers' choice mechanism is as follows.

i)A pre-specified proportion of drivers update their perceived historic travel times (in the manner described above). The remainder do not update their historic perceptions.
ii)A pre-specified proportion of drivers receive media reports about the trip to be made, and combine them with their historic perceptions to obtain perceived predicted costs. The remainder have perceived predicted costs which are equal to their perceived historic costs.
iii)A pre-specified proportion of drivers who by this stage have different perceived predicted costs to their previous day's perceived predicted costs then review their choice of route and departure time. The remainder make the same choice as the previous day.
iv)Of the drivers reviewing their travel choice, a pre-specified proportion seek and acquire driver information and revise their predicted costs accordingly.
v)The drivers reviewing their travel choice then choose a route / departure time combination according to a random utility model, where the random error terms have a fixed probability distribution.

A number of points of criticism could be levelled at the approach of Ben-Akiva et al. In particular it suffers from the route-based learning deficiencies mentioned earlier in relation to Van Berkum and Van der Mede's approach; the learning process only affects the <u>mean</u>

perceived predicted travel times, but has no effect on the perception error variance; and the pre-specified proportions mentioned above do little to make the model more general, but would be expected to be rather difficult to estimate.

An interesting series of tests were conducted by Iida et al. (1992), their work being in the realms of part simulation, part behavioural experimentation. A simple, artificial network was considered, consisting of one origin-destination pair and two parallel, uni-directional routes. Forty participants were involved in the study, and were told that: they were making a daily journey to work; they had a choice between two routes, but the departure time was fixed; and they were to choose the route with the minimum (perceived predicted) travel time. Journeys were made over a series of twenty days. The participants made one trip per day - thus the origin-destination demand was constant from day to day. On any given day, the actual travel time on a given route was computed from a link performance function. Travel times thus did not vary between individuals using the same route on the same day, nor did the performance function vary from day to day. The participants performed each of two experiments. In the first, each participant was told - prior to his trip - the actual travel time for the route he used on the previous day. In the second experiment, each participant was given the entire history of actual and perceived predicted travel times for all his previous trips (the perceived predicted travel times had previously been stated by the individual before the start of each trip). The main conclusions were as follows.

i)Route flows in both experiments were quite unstable, more so for experiment 1. Notably, after about ten days, they appeared to be stabilising but later became unstable again.
ii)There appeared to be evidence that some drivers anticipate the effect of other drivers switching route for the forthcoming day, by taking into account the previous day's travel times (and the fact that they know the O-D flow to be constant).
iii)Each day, the participants stated their predicted travel times before selecting a route. Iida et al attempted to fit a regression relationship between the current day's predicted travel time and the previous day's actual and predicted travel times (experiment 1) or the previous three days' actual and predicted times (experiment 2). In both cases the previous day's actual and predicted times had a significant effect on the current prediction. However, the regression models were found to provide a poor fit to the data.

Two final remarks will be made in this review. Smith (1984) considered a dynamical system formulation of day-to-day route choice, in terms of route flows. Drivers were assumed to switch between routes according to the difference in route travel costs; Smith showed that such a system converged to a stable equilibrium under the usual assumptions. Mahmassani and co-workers (1991) have proposed a route choice model which appears to be particularly suited to modelling day-to-day evolution. In fact, Cascetta et al (1991a) suggest the possible use of Mahmassani's model within their stochastic process framework; as far as the author is aware, this is a possibility presently being investigated by Mahmassani. The route choice model is derived from the premise that drivers base their decisions on minimum perceived travel time differences, or thresholds. This is known as `boundedly rational' behaviour, with a boundedly rational user equilibrium occurring when every driver is satisfied with his current choice of route (eg there is no alternative route which is perceived more than the threshold percentage quicker than the current choice). The model also makes sense in non-equilibrium contexts. The threshold values may vary across the population, according to driver characteristics and the propensity to switch; thus, a kind of habit effect is achieved.

3. MODELLING REQUIREMENTS

The purpose of this note is the specification of the demand side of a day-to-day route choice sub-model, to be used in a much larger model for evaluating dynamic route guidance and dynamic information systems. The modelling requirements of a dynamic route guidance model were discussed at length by Watling and Van Vuren (1992). The conclusions they drew have the following implications for a `demand side' day-to-day route choice model (ignoring, for the moment, within-day dynamic effects and the operation of the route guidance control system). Such a model must:

i)be behaviourally driven, recognising the effect of habit, the driver learning process and uncertainty;
ii)accommodate a diverse range of behaviour, according to (for example) personal and trip attributes and familiarity with the network. The response to the provision of route guidance information may be expected to vary significantly with respect to these characteristics;
iii)represent day-to-day variations in demand;
iv)be able to handle `normal' day-to-day variations in network supply conditions (e.g. due to weather or lighting), as well as `incident' conditions.

The review of past work in this area revealed some interesting approaches which address some of the above issues, but it is the author's belief that none adequately addresses *all* of the above points. The primary reasons for this are as follows:

a)The process by which a driver `learns' about network conditions is an *individual* one - it is a function of the driver's own personal experience. Drivers may have some rough idea of travel costs on unused routes, but they will not learn about these costs in the same way that they do for used routes. Thus, the approaches of Horowitz's model 2 and Cascetta's STODYN and STODYN 2 are difficult to justify. Indeed, providing increased information about unused (or infrequently used) routes is surely one of the aims of a dynamic route guidance/information system. Further, since journey times on a route vary from trip to trip on a given day, a user will only learn about his own individual travel cost for that day, rather than the average of all users travelling that day. Therefore, Horowitz's model 3 is also unsuitable. The conclusion is that a proper simulation of the learning experience can only be gained by a microscopic model, taking account of individual route choice decisions.

b)Uncertainty about network conditions arises because these conditions will vary from trip to trip (on the same day) and from day-to-day, even in incident free conditions. The behavioural model must therefore be sufficiently flexible to account for such variability in the information acquisition process of the driver (an issue not addressed in Ben-Akiva et al's model). Uncertainty is also a function of a driver's experience of travelling along different parts of the network, again supporting the need for a microscopic model. Van Berkum and Van der Mede go some way to representing these effects, but their approach has some serious flaws for general networks. In particular, they treat routes as *independent* entities in terms of the learning process and in terms of their perceived attributes (this point will be discussed at greater length later, when the proposed modelling approach has been introduced).

c)A driver's propensity to form a `habit' has an essential role in the context of this model. Firstly, a driver may not divert away from his usually chosen route just because of one bad experience on the previous day (e.g. an accident). Secondly, when providing dynamic route guidance information, a driver's habit may be too strong for him to be persuaded to switch to an alternative route, even though it may reduce his travel cost (either in actual or perceived terms). Simplistic notions of habit-like effects can easily be introduced, such as in the approaches of Van Berkum and Van der Mede, Ben-Akiva et al and Cascetta et al. The question remains as to whether such devices have any behavioural foundation, are sufficiently driver specific and can be calibrated. There is little use in developing a `flexible' model with a number of new parameters, if calibration in practical situations is impossible or requires a good deal of guesswork. Mahmassani's concept of bounded rationality appears to be particularly useful in this context.

4. THE MODEL DEVELOPMENT IN CONTEXT

As has been mentioned earlier, the day-to-day route choice modelling considered in this paper is intended to provide a component to a much larger network simulation model. This simulation model will be a microscopic, dynamic, stochastic representation of the day-to-day evolution of supply and demand in the network. It will incorporate effects such as daily fluctuations in network conditions (eg due to weather) and travel demand, incidents (eg accidents), day-to-day and within-day dynamics of route and departure time choice, en route path switching, and a detailed supply model incorporating the dynamics of queuing, junction interactions, lane choice behaviour and any controls in operation (eg traffic signals). It should also ultimately be able to handle the introduction of a dynamic route guidance system. The development and implementation of such a model is clearly a huge task - the first aim is to decide upon a suitable model specification. In developing this specification, a number of factors need to be borne in mind (aside from the modelling requirements outlined in the previous section):

i)A great deal of research has been carried out individually into many of the model components described above, and there is even work in the literature on day-to-day simulations. It is not the purpose of the research described in this note to propose new theoretical constructs purely for their own sake, but rather to develop a practical simulation tool. Therefore, when suitable specifications exist in the literature these will be adopted or modified.

ii)The model as a whole is certain to put great demands on computer processing time and memory, and so the efficiency of each component is of prime concern.

iii)For the model to be eventually used in practice, there will certainly be the need to collect a large amount of new data - for example, on variability and behavioural characteristics. This, coupled with point (ii), suggests that each component should be as *simple* as possible. Additional parameters should be avoided as far as possible, particularly when they require more data to be collected. The introduction of parameters which are difficult to estimate from basic data (because, for example, they are highly correlated with other unknown factors) should be avoided, since uncertainty in the values assigned to them would make interpretation of such a large, complex model extremely difficult. [An example of such a parameter is the link travel time variance used in stochastic user equilibrium models].

iv)The issues of convergence and the existence of a unique solution, which notably arise in network equilibrium models, are less apparent within the modelling framework proposed. However, it will be important to ensure that any outputs which will be used in evaluation (e.g. twenty-day average link flows) are unaffected - within the bounds of sampling variability - by the particular pseudo-random numbers generated in any single run. Ultimate stability (in some sense) of the process generated and independence of the final state from the initial state are not seen as pre-requisites of the model. Traffic networks may indeed exhibit instability. Almost certainly after a new measure is introduced into a network, driver behaviour is dependent to some extent on conditions (e.g. habits formed) before the measure was introduced. Nevertheless, the importance of these issues for evaluation make them worthy of consideration.

v)It would be expected that such a simulation model would be applied over a similar evaluation time-scale to existing network models, in which the underlying pattern of travel demand and level of service in the network are approximately constant (notwithstanding daily and seasonal fluctuations.

vi)Without a good deal of data, it may not be possible to decide upon a single, best specification for each component of the model - so for the moment a number of candidate specifications will be considered.

5. THE MODEL

The modelling requirements ideally indicate the need for a blend of psychology and statistics; the proposed model, however, owes much more to the latter, with little psychological foundation. The proposed model is based upon a statistical estimation procedure, in an analogous way to which the behavioural element in equilibrium techniques is essentially an economic model. It assumes, to some extent, that drivers are able to process (albeit in a subjective way) the information they learn from their travel experiences in some kind of optimal way. The process used is a simple standard Bayesian method of combining subjective beliefs with objectively measured data (experienced travel costs, in this case). Route choice decisions are then made on the basis of posterior expected travel costs.

The idea of using a statistical estimation process, as opposed to a true behavioural model, may be defended in a number of ways. Nisbett et al (1982) discussed the use of probabilistic models in everyday thinking. They stated `the suspicion that in most cases where a formal probabilistic model can be usefully applied by a statistician there are analogues in the everyday world in which a similar intuitive use of probabilistic thinking occurs frequently in intelligent laypeople'. In fact the equations that arise for combining information in the proposed model will be seen to be reasonably intuitive. Secondly - and more importantly - the aim of the model is to provide a useful *representation* of the way in which route choice decisions occur, rather than directly to simulate the driver's cognitive process. The model has all the necessary tools for this purpose - a representation of daily perceptions, learning and uncertainty. Furthermore, it satisfies the requirement mentioned in section 4 of being a reasonably simple formulation.

We shall firstly make a number of basic assumptions:

i)the total demand for travel between origin p and destination q on day k (during modelled period) is d_{pq}^k - this may vary from day to day, and in that case it is natural to model d_{pq}^k as a realisation of a random variable (ignore `seasonal' effects);

ii)given the demand d_{pq}^k, the group of individuals who actually travel on day k,

$: i \in I_k \}$

are drawn at random from the complete set of individuals who could possibly travel, I;

iii)there is no en route path switching;

iv)the random components of actual travel time are distributed independently between links (ie there is no between-link correlation in the day-to-day variability in supply conditions, as may occur in adverse weather conditions, for example).

Under these assumptions, a number of different model specifications - along the same theme - are proposed. Each specification is characterised by an additional set of assumptions

$(A_1, A_2, A_3, A_4, A_5)$

where

A_1 =Sif there are no within-day dynamics (`S' for static)

Dif the attractiveness of routes varies within a day and drivers make a choice of departure time (`D' for dynamic)

A_2 =∞if all previously experienced travel times contribute to a driver's current perception of travel costs

Nif only at most a pre-specified number of the most recently experienced travel times contribute to the perceived predicted costs

A_3 =DEif distance (link length) is perceived exactly by all drivers, regardless of their experience

DLif drivers learn about distance through their own travel experiences

A_4 =MINif the choice process is a conventional utility maximising / cost minimising one

BRif choices are made based on the bounded rationality principle

A_5 =DTNif departure time and route choice is handled in a nested process

DTSif drivers simultaneously choose a route and departure time

0if departure time choice is irrelevant as there are no within-day dynamics

The possible specifications are introduced below, and then a number of comments are made on them.

5.1 MODEL A: (S, ∞, DE, MIN, 0)

DAY O

For each individual i (in I) and for each link a, select a `normalised perception error' ε_{ia}, by generating pseudo-random numbers from a Normal (0,1) distribution.

DAY k (k≥1)

Step 1. Generate the total demand d_{pq}^k for day k and each origin-destination pair (p,q) from some specified probability distribution. Randomly select the set of individuals I_k who actually travel on day k, according to the total demands.

Step 2. For each individual i in I_k (travelling on day k):

The predicted journey time y_{ia}^k on link a is *perceived* as an observation of a Normal random variable with mean μ_{ia}^k and standard deviation ρ_{ia}^k. ρ_{ia}^k is a measure of the current `uncertainty' for individual i on link a.

Step 2a. Values for μ_{ia}^k and ρ_{ia}^k are obtained from subjective evaluations of (i) prior beliefs/information, (ii) previously experienced journey times.

According to a Bayesian analysis (details given in Appendix), these are given by:

$\mu_{ia}^k = E\ [\mu\ |$ Prior beliefs + Previously experienced times by individual i on link a]

$\rho_{ia}^k = \text{var}\ (\mu\ |\ \ldots)$.

Since ρ_{ia}^k depends on the journey time variance σ_{ia}^k for individual i on link a, also form:

$\sigma_{ia}^k = E\ [\sigma\ |\ \ldots]$.

Step 2b. To represent correlation in perception errors between days (for a given i and a), in fact form perceived journey times from:

$$: \mu_{ia}^k + \varepsilon_{ia}\ \rho_{ia}^k$$

where ε_{ia} was determined at `DAY O' above.

Step 3. Individual i selects his minimum cost route, according to his perceived predicted link costs $\{U_{ia}(y_{ia}^k): \forall a\}$, where $U_{ia}(y)$ is the perceived predicted generalised cost for individual i and link a corresponding to a perc. predicted link travel time of y.

In the Appendix, it is shown how values for μ_{ia}^k and ρ_{ia}^k may be computed, given suitably specified prior distributions. It is also shown how "recursion formulae" may be set up to compute the posterior parameters on day k from their values on day k-1. (μ_{ia}^k and ρ_{ia}^k are functions of these posterior parameters).

5.2 MODEL B: (S, N, DE, MIN, 0)

This is identical to Model A, except that Step 2(a) on day k is replaced by:

Step 2a. Values for μ_{ia}^k and ρ_{ia}^k are obtained from subjective evaluations of (i) prior beliefs/information, (ii) the most recently experienced journey times.

According to a Bayesian analysis, these are given by:

$\mu_{ia}^k = E\ [\mu\ |$ Prior beliefs + Previously experienced times by individual i on link a in the last r_i [journeys on that link]

$\rho_{ia}^k = \text{var}\ [\mu\ |\ \ldots]$.

Since ρ_{ia}^k depends on the journey time variance σ_{ia}^k for individual i on link a, also form:

$$\sigma_{ia}^k = E[\sigma \mid \ldots].$$

Note that if on a particular day, individual i has made less than r_i journeys previously on link a, then *all* previously experienced times by i on link a are used to form μ_{ia}^k and ρ_{ia}^k.

For links a and individuals i where the number n_{ia}^k of journeys made before day k is less than or equal to r_i, the formulae given in the Appendix for μ_{ia}^k and ρ_{ia}^k, as well as the recursion relationships, are still valid. When $n_{ia}^k > r_i$, the recursion formulae are replaced by:

i)If no journey made on (previous) day k using link a:

Set m_{ia}^{k+1}, τ_{ia}^{k+1}, υ_{ia}^{k+1} and ω_{ia}^{k+1} to their values at day k.

ii)If a journey was made on day k on link a, with a link journey time of s, then:

If s_{oia}^k is the `most distantly remembered' journey time for individual i on link a at day k (i.e. the journey time corresponding to the journey made r_i trips ago), then:

$$m_{ia}^{k+1} = \frac{\tau_{ia}^k m_{ia}^k + s - s_{oia}^k}{\tau_{ia}^k}$$

$$\tau_{ia}^{k+1} = \tau_{ia}^k$$

$$\nu_{ia}^{k+1} = \nu_{ia}^k$$

$$= \frac{\nu_{ia}^k \ \omega_{ia}^k + \tau_{ia}^k \ (m_{ia}^k)^2 + s^2 - (s_{oia}^k)^2 - \tau_{ia}^{k+1} (m_{ia}^{k+1})^2}{\nu_{ia}^k}$$

It is not clear that this recursion is as useful as for the $n_{ia}^k \leq r_i$ case considered in the Appendix, due to the need to keep track of the most distantly remembered times s_{oia}^k.

5.3 MODEL C: (S, ∞, DE, BR, 0)

This is identical to Model A, except that Step 3 is replaced by the following boundedly rational choice process:

*Step 3.*Individual i first forms his current perceived predicted link costs $\{U_{ia}(y_{ia}^k): \forall a\}$, where $U_{ia}(y)$ is the perceived generalised cost for individual i and link a corresponding to a perceived predicted link travel time of y.

Based on these costs, he compares the attractiveness of the minimum cost route with that of the route he used on his previous trip between that origin-destination pair. If the travel cost on the minimum cost route is at least a given `threshold' percentage less than the cost of the route used on his previous trip, he chooses to use the minimum cost route. Otherwise, he chooses the route used on his previous trip.

A number of comments may be made about this choice process. In proposing it, Mahmassani and colleagues have recommended that the decision to switch route from that previously used should only be made subject to the additional constraint that the *absolute* saving in travel cost is greater than some fixed minimum value (any smaller savings being imperceptible to the user). Furthermore, the question arises as to how this choice process should be implemented for the individual's *first* trip between this origin-destination pair; this is an important issue, as it has been shown that the final state of a network under such decision rules is very likely to be dependent on the initial conditions. The most obvious starting condition of choosing the minimum perceived cost route may therefore be inadvisable (It may prove difficult to move away from an unrealistic initial pattern). An alternative would be to apply a straight cost minimising rule for a number of days, as a start-up period, before introducing boundedly rational choice. When the aim of the study is a before-and-after assessment of some measure, then a sensible starting point for the simulation of the `after' situation are the choices which prevailed from the `before' scenario.

5.4 MODEL D: (D, ∞, DE, MIN, DTS)

For the purposes of this model, the study period (typically, say, of an hour's duration) is divided into a number of smaller time periods of equal length (of around five minutes, say). A time period is denoted by the additional superscript t.

*Step 1.*As per model A

*Step 2.*For each individual i in I_k:

The predicted journey time y_{ia}^{kt} on link a when entering the upstream end of link a in time interval t, is perceived as an observation of a Normal random variable with mean μ_{ia}^{kt} and standard deviation ρ_{ia}^{kt}.

*Step 2a*Values for μ_{ia}^{kt} and ρ_{ia}^{kt} are obtained separately for each interval t, according to the same Bayesian analysis used in model A (with an additional superscript t added to all prior and posterior parameters). That is, they are formed from a subjective evaluation of prior beliefs and previously experienced journey times when entering link a in time period t.

*Step 2b*Perceived predicted link journey times are then formed from

$$= \mu_{ia}^{kt} + \varepsilon_{ia} \rho_{ia}^{kt}$$

*Step 3.*For each i in I_k, form perceived predicted *route* journey times for each route r and departure time interval t:

*Step 3a*Choose a representative departure time d(t) within the interval t. (For example, the centre of the interval or a time-point chosen at random within the interval).

*Step 3b*Without loss of generality, label the links on route r in the order in which they would be traversed: a=1,2,...,R. Thus define inductively the perceived predicted travel time Y_{air}^{kt} for each link a on route r when departing from the origin in the interval t and using route r:

$$Y_{1ir}^{kt} = y_{1i}^{kt}$$

$$= \sum_{s} y_{ai}^{ks} H_s(\_d(t) + \sum_{b=1}^{a-1} Y_{bir}^{kt})$$

for a=2,3,...,R, where

$H_s(x)$= 1if the time x is in the interval s
= 0 otherwise

*Step 3c*Finally, form the perceived predicted travel time w_{ir}^{kt} on route r for a departure in time interval t:

$$= \sum_{a} Y_{air}^{kt} \delta_{ar}$$

where

δ_{ar}= 1if route r uses link a
= 0otherwise.

Step 4.For each i in I_k, form the perceived predicted generalised travel cost for route r and departure time t:

$$= G_{ir}^{kt}(w_{ir}^{kt}) \ .$$

For example, if the only components of cost are time and distance, then

$$= \theta_i w_{ir}^{kt} + \phi_i \sum_{a} l_a \delta_{ar}$$

where l_a is the length of link a, and φ_i and Θ_i are generalised cost weightings.

Step 5.For each i, r and t, define the perceived predicted disutility $-U_{ir}^{kt}$ of individual i using route r and departing at time t as a sum of the generalised travel cost and a penalty for not arriving at the desired time:

$$ {}^{t} = \alpha_i \, G_{ir}^{kt} + P_i(\_B_{ir}^{kt} \,|\, D_i) $$

where

P_i is the early/late arrival penalty function
D_i is the desired arrival time of individual i
B_{ir}^{kt} is the arrival time at the destination for individual i when using route r and departure time interval t
α_i is a constant.

In the notation of step 3, it follows that

$$ = d(t) + \sum_{a} Y_{air}^{kt} \, \delta_{ar} \quad . $$

Step 6. Each individual i in I_k selects the route and departure time combination with the smallest perceived predicted disutility.

Comments

i) The most straightforward approach for handling dynamic travel costs is to deal with route costs by departure time from the origin. In the learning process (step 2) above, a *link*-based method is used, in order to overcome problems with route-based techniques identified earlier in this note, of correlations being ignored between partially overlapping routes. In reality, drivers may not be able to perceive and store information on experienced dynamic link travel times. However, the above scheme could be regarded as a route-based learning process (with the learning taking place in steps 2 and 3 combined) with perceived predicted route travel times correlated between departure time periods. The disaggregation into perceived link travel times is then regarded only as a convenient means of implementation.

ii) The penalty function P_i introduced in Step 5 could take one of a number of forms (it would be expected that the general form would be the same for all individuals, but with individual-specific parameters). The approach is a generalisation of the schedule-delay concept of Vickrey (1969), which has later been adopted in a number of different modelling schemes (Vythoulkas, 1990, Ben-Akiva et al., 1991; Cascetta et al., 1991a). A direct application of this work would involve a penalty function of the form:

$$
\begin{aligned}
3_{ir}^{kt} \,|\, D_i) &= \beta_i \, [(D_i - \varepsilon_{1i}) - B_{ir}^{kt}\,] && \text{for } B_{ir}^{kt} < D_i - \varepsilon_{1i} \\
&= 0 && \text{for } D_i - \varepsilon_{1i} \le B_{ir}^{kt} \le D_i + \varepsilon_{2i} \\
&= \gamma_i \, [\, B_{ir}^{kt} - (D_i + \varepsilon_{2i})] && \text{for } B_{ir}^{kt} > D_i + \varepsilon_{2i}
\end{aligned}
$$

where

$- \varepsilon_{1i} \ , D_i + \varepsilon_{2i}]$

is a tolerance interval around the desired arrival time, and β_i and τ_i are constants. Thus, the penalty increases linearly with the extent to which the predicted arrival time is outside the tolerance interval. It is reasonable to expect that late arrivals will be valued differently to early ones, thus β_i and ε_{1i} may not be the same as τ_i and ε_{2i}.

Clearly, many other forms could be considered - for example, a general power law case with no tolerance interval:

$$B_{ir}^{kt} \mid D_i) = \beta_i (D_i - B_{ir}^{kt})^{u_i} \quad \mathit{for}\ B_{ir}^{kt} < D_i$$

$$= \gamma_i (B_{ir}^{kt} - D_i)^{v_i} \quad \mathit{for}\ B_{ir}^{kt} \geq D_i \ .$$

In practice, the penalty relationship may be calibrated from the results of a stated preference experiment - choosing the form which gives the best overall fit. For the meantime, the relationship is just considered in a general form.

iii)Steps 3 to 6 are useful for introducing the general concept of the model, but would make very inefficient use of computer power and storage. By using a sensible search strategy in step 6, route/departure time disutilities (steps 3 to 5) and possibly even link travel times (step 2) should only need to be calculated as required. Should it not prove possible to develop an exact method to achieve this, an efficient yet approximate technique would be more than adequate. Indeed, in practice drivers have to resort to heuristics when comparing the utilities of different choices. For example, a strategy could be (for trips after the first one): From the choice on the previous trip, consider only the time interval chosen then and the interval before and after and select a route from a set of reasonable ones. The reasonable ones could be defined `a priori', or defined relative to the previous route chosen (cf Dial's method). The minimisation problem of step 6 is similar to that arising in the iterations of a conventional static equilibrium model. the differences are that the problem of step 6 has an extra choice dimension and is specific to each individual. Thus, for each individual a cost minimisation problem is solved for a single origin and single destination. The tree-building algorithms used in static equilibrium methods, whereby one origin and all destinations are simultaneously considered, therefore appear to be of limited use. It is noted that techniques for determining minimum time (though not minimum cost) routes in the dynamic case have been proposed (Hall, 1987; Chen and Underwood, 1991; Kaufman et al, 1991).

5.5 MODEL E (D, ∞, DL, MIN, DTS)

In reality a driver will probably experiment with only a fairly small number of departure time and route combinations - thus, in the above model, many of the disutilities will be computed based only on the prior information. However, drivers in fact *do* learn - from their experience at one time of day - about travel costs in other time periods (because the travel cost on a link is correlated between the two periods). One possibility would be to assume that with experience, a driver builds up some general, network wide picture of the temporal variation of travel costs, and uses this to modify his prior information for unused links (this would also require the ability to project this temporal variation through the network, in much the same way as route travel times are deduced from link travel times in step 3 of Model D). Unfortunately, this would seem to be rather complicated to implement and difficult to calibrate. As an alternative, the modification to Model D proposed below assumes that the temporal correlation between travel costs on a given link is due only to the *distance* component of generalised cost (for simplicity, it is assumed that generalised cost is defined solely in terms of time and distance). That is, drivers do not perceive distance precisely when they travel along a route/link, but learn about it with experience.

Model E is the same as Model D, except replace Step 4 by:

*Step 4.*The perceived predicted length l_{ai}^{k} of link a by individual i on day k is obtained from the following adjustment process:

*Step 4a*For day 0:

$$\frac{\eta_{ai}}{q_{ai}}$$

$$= q_{ai}$$

*Step 4b*If individual i did not travel on link a on day k-1:

$$l_{ai}^{k-1}$$

If individual i used link a on day k-1 and perceived the link length to be L:

$$l_{ai}^{k} = q_{ai}^{k-1} + 1$$

$$\frac{q_{ai}^{k-1} \, l_{ai}^{k-1} + L}{q_{ai}^{k}} .$$

In the process above, η_{ai} and q_{ai} represent prior information. The perceived predicted generalised travel cost on route r when departing in time interval t is then:

$$= \theta_i \, w_{ir}^{kt} + \phi_i \sum_{a} l_{ai}^{k} \; \delta_{ar} \; .$$

Comments

i)The above modification requires that perceived link lengths are generated at some stage during the application of the supply model. For example, a Normal distribution could be used for this purpose, with the true link length as the mean.

ii)Unlike the link travel time learning process, perceptions errors are *not* made in forming the perceived predicted link length. The errors in the former (travel time) case were due to inherent variability in those times, whereas the link length is fixed. The errors in the process above are solely due to the drivers inability to estimate accurately the *experienced* link length. In order to be consistent, perceived experienced link travel times ought also to incorporate a mis-perception component, since these too may be estimated with error.

5.6MODEL F: (D, ∞, DE, MIN, DTN)

This is the same as Model D, except that the choice rule (Step 6) is replaced by some kind of nested process. For example:

Step 6.For each individual i in I_k:

Step 6aSelect a departure time according to the perceived predicted disutility of each, defined as the *average* perceived predicted disutility over all routes in individual i's *current route consideration set* R_i^{kt} at departure time t for the moment individual i wishes to make on day k, ie.

$$= \frac{1}{|R_i^{kt}|} \sum_{r \in R_i^{kt}} (-U_{ir}^{kt})$$

where |A| is the number of elements in the set A. Individual i selects the departure time with the minimum average perceived predicted disutility.

*Step 6b*Given the departure time t determined in (a), select the minimum cost route according to the dynamic route costs G_{ir}^{kt}.

Comments

i)-U_i^{kt} is a rather crude measure of the attractiveness of departure time interval t, since it ignores any correlation between the disutilities for different routes. Again, the justification may be given that such an approximation may, however, be an adequate representation of the human choice process.

ii)In two respects it could be claimed that this nested approach is behaviourally sounder than the simultaneous approach of Model D. Firstly, it is unlikely that drivers are able to process the large amount of information on all route/departure time combinations in order to determine an optimal choice. Secondly, when a driver is dissatisfied with his current choice, it would be expected that he would consider a change of route *before* attempting a change of departure time (which disrupts his daily schedule). The averaging process by which departure time disutilities are calculated means that departure time choice is less sensitive than route choice to a change in the disutility associated with a particular route.

iii)The definition of the route consideration set R_i^{kt} could take one of a number of forms:

(1)$R_i^{kt} = R_i$ (for all k, t), where R_i is the set of *all* possible routes for the movement to be made by individual i.

(2)$R_i^{kt} = R_i^* \subseteq R_i$ (for all k, t), where R_i^* is a set of a priori "reasonable" routes.

(3)$R_i^{kt} = R_i^k$ (for all t), where R_i^k is the set of all previously used routes (at any departure time) by individual i (but what to do before a trip is made?).

(4)R_i^{kt} is the n `shortest' routes at departure time t based on the current perceived predicted route costs G_{ir}^{kt} (for some small, given n).

Of the above, (1) is unlikely to be feasible. (4) is quite appealing, but a rather difficult problem, although there do exist methods for determining the n shortest routes when link costs are static (Shier, 1979; Lee and Ho, 1992).

iv)A nested approach to route and departure time choice has previously been proposed by a number of authors (eg Cascetta et al., 1991a; Ben-Akiva et al., 1991). In these cases, a probabilistic discrete choice model was used (eg nested logit or probit). It is straightforward to adapt the above departure time choice sub-model to include a random error term in the perceived predicted disutilities; for the moment, however, a simple deterministic choice process is considered.

5.7GENERAL COMMENTS ON THE MODELLING APPROACHES

a)The additional restriction could be made in any of the proposed models that the subjective estimates of the perceived predicted journey times are formed using experienced journey times from no more than M days ago (for some large, given M). If the network and potential demand are constant for a sufficiently long period of time that a steady state is reached, then the results of Cascetta (1989) may be applied. That is, the process generated by the above scheme is `dynamically stable' and ergodic. Thus, in the steady state case, link flow means and variances (for example) could be estimated from a single pseudo-realisation of the process, using a statistical test to determine when steady state conditions had been reached. With regard to this latter comment, we note that Cascetta et al (1991b) in applying the STODYN2 model, used a t-test on link flow means over successive ten-day periods (since, as they mention in their later Cascetta et al (1991a) paper, using a larger number of days to produce the means just delays `acceptance' of the stationarity hypothesis). In their tests, stationarity was reached after 50 days (although they do not mention to what significance level, and whether this had to be achieved across all links), and `a further 50 days of simulation were needed to keep link flow mean estimates within a pre-specified sampling error'.

b)In order to carry out the subjective evaluation of experienced travel times, it is necessary for the modeller to specify appropriate values of (in the within day static case) the prior parameters m_{ia}, τ_{ia}, υ_{ia} and ω_{ia} for each link a and individual i (and in the within day dynamic case, additionally for each departure time interval t). When such models are first used to study a network, the prior parameters are probably of little value. In this case, the parameters should be chosen so that the variance in the prior estimates is large. In this way, after the first one or two days, the route choice process will be based almost entirely on experienced travel costs. The interpretation of the prior parameters is much more natural, however, when the aim is a before-and-after study of some measure or strategy (e.g. traffic management schemes, dynamic route guidance). Here, the prior expected values in the `after' situation may be set equal to the posterior expectations at the end of the model run in the `before' case. Values controlling the prior variances in the `after' situation (e.g. $\tau_{ia}$) the may be treated as calibration parameters or, in the case of $\tau_{ia}$ specifically, can be set to some reasonable number of days over which conditions simulated in the `before' situation have in reality prevailed (which may not be the same as the number of days actually simulated in the `before' case).

The assumption here seems a reasonable one - that choices made in the `before' situation affect choices made in the `after' situation, independently of the conditions actually experienced in the `after' case (a kind of habit effect). This may be contrasted with equilibrium methods of analysis (whether static or dynamic), where it is seen as a pre-requisite that the final modelled situation is independent of the starting conditions. Within the framework proposed here, the extent to which the prior affects later states of the system depends upon the specific model used. For example, if *all* previously experienced journey times contribute to a driver's current perceptions (as in Model A, for example), then as a link is used more times the effect of the prior diminishes, until it ultimately may only be negligible. For the modification proposed in Model B, the prior potentially has a larger effect on later states. Likewise, when a boundedly rational decision rule is used (Model C), the later states of the system depend upon the initial state.

c)At the first step of the schemes described above, the normalised perception errors ε_{ia} are drawn and kept fixed during the simulation. This achieves two things. Firstly, if a link is not traversed by individual i on day k (and during time period t, in the dynamic case), then the perceived predicted travel time on day k+1 will be the same as it was on day k. That is, perceptions only change when a link is used. Secondly, it means there is a correlation in the perception errors between days (and time periods t) for a given link and individual, even when additional experience is gained of that link. Thus, an individual will tend to always either over- or under-estimate the travel time on a particular link.

d)The weighting scheme used for forming the mean perceived predicted journey time is somewhat cruder than Horowitz's. All experienced times which contribute to the perceived predicted time are given equal weight - whether this means all experiences (as in Model A, for example) or only the more recent ones (e.g. Model B), in which case less recent ones are essentially given zero weight. Whilst it is intuitively appealing to have a scheme in which greater weight is given to experienced travel times, the more recently they occurred, there is no evidence as yet to suggest a suitable form for the weights.

e)A driver's `uncertainty' (and hence perception errors) is related directly to (i) his experience in different parts of the network; (ii) travel time variability. This may be contrasted with Van Berkum and Van der Mede, where perception errors are primarily caused by (ii); experience only has the effect of changing the mean perceived cost. Thus, in this latter model, if a driver increases his experience by travelling on a link, and the experienced travel time is approximately the same as his previous mean perceived travel time, then his uncertainty is not reduced. In the new model, on the other hand, the perception error will be smaller with the additional travel time experienced (until the limit r_i is reached, in the case of Model B type of modifications).

f)Although drivers make choices in terms of routes, their perceptions and predictions are built up in terms of links. This overcomes the problems of a route-based learning process as given by Van Berkum and Van der Mede and by Ben-Akiva et al. Firstly, a driver travelling on one route will learn something about conditions on an overlapping route. Secondly, there will be a correlation in perception errors between overlapping routes. For example, two routes which overlap for most of their length will be compared only on the basis of their non-overlapping parts, the remainder being perceived as identical.

g)In the departure time choice process, it has been assumed that drivers discretise the possible departure times into a finite number of alternatives. Intuitively, this is almost certainly the kind of heuristic used by drivers. Continuous time models, such as that used by Ben-Akiva et al (1984) are difficult to justify behaviourally.

h)In the microscopic modelling framework proposed, a number of individual -specific parameters arise. In comment (b) above, it has been suggested how parameters of the prior may in certain cases be estimated from a previous simulation. Other parameters which have arisen include: the number of previous trips which contribute to current perceptions (as in Model B); generalised cost weightings; those related to the distance learning mechanism of Model E; and the departure time choice parameters (eg desired arrival time, Model F). It is not expected that information will be available down to such a fine level of detail, but two approaches are possible:

1.For each attribute (to which a number of parameters may relate), divide the population into a small number of groups, and assume that each individual within a group takes an average, estimated group value. Note that these groups need not be defined in the same way for all attributes.

2.Assume some statistical distribution of parameter values, and use data to estimate this distribution (eg assume a Normal distribution of desired arrival times across the driver population, and use data to estimate the mean and variance). Then, during the modelling stage, use a pseudo-randomisation process to select values for each individual. Of course, this approach could be combined with the grouping approach mentioned in point 1.

i)For a network of a realistic size, the model proposed will put a great demand on computational power. In the dynamic case, such as in Model F, the most demanding stage will be the determination of a minimum cost departure time and route over a network with dynamic link costs, which must be carried out separately for each individual on each simulated day. A natural first stage would be to attempt to express this as a shortest route problem with static link costs over an extended network. Since for each individual, the interest will only be in `feasible' routes for a single origin-destination movement, intuition suggests that it should be possible to restrict such an extended network to a manageable size. Existing algorithms for determining dynamic minimum time routes are also worthy of investigation. Approximate methods for determining a near-optimal solution should also be investigated, possibly by restricting the range of choice open to the user relative to the previous day's choice. If one of these latter approaches led to a significant saving in computer processing time, then they should be preferred, even if the solution is not exact. In any case, if a computer has difficulty finding an optimal-choice with a purpose-built algorithm, then an individual will surely have similar difficulty. If the approximation methods used in the model in any way reflect human heuristics, it could be claimed that the model is then more realistic.

j)If the notation [ ] is used to denote `the number of ...', the computer storage space required at any one time to store perceived travel costs is of the order:

[links] x [time periods] x [individuals] x [history parameters]

where the history parameters may be the parameters of the posterior distribution (in fact, three would be adequate) or recently experienced travel times (in the case of Model B formulation). In order to ease the burden on storage space (and possibly save some time in retrieving the relevant array elements), an individual-independent `typical' range for each of the history parameters could be stored, with the parameters relating to a particular individual only saved if they fall outside the typical range. Equally, the history parameters could be stored only approximately, according to a grouped data approach. In addition, computer memory compression techniques may be adopted to make best use of the resources. The quality of these approximations need, of course, to be investigated. If the storing of all such experiences proves infeasible, then the use of some kind of weighting scheme (à la Horowitz) may be the only alternative, despite the problem of calibrating the weights. It is notable that Cascetta (1989) appears to have found it feasible to implement an averaging process of the type described in Model B in the current paper, based on seven previous days. This was achieved using the within-day static model STODYN, applied to a realistic size network. Later, however, in thw within-day dynamic (yet still macroscopic) STODYN2, experiences were formed using weighting schemes. It is not clear to what extent this change of methodology was due to computational/storage considerations.

k)Possible extensions/modifications to the approaches include:

•In its current state, the driver `uncertainty' modelled represents some kind of lower bound, with the only source being supply/demand variability. This could be extended in two ways, to represent

-taste variation, with possibly an additional, additive variance component for perceived predicted travel cost;

-errors in evaluating experienced travel costs, with perceived experienced costs being stochastic instead of deterministic.

•An investigation into exploiting the learning process of perceived travel time *variability* - with route choice optionally based on risk minimising or minimax principles.

•Introduce dynamic route guidance into the choice process.

6.SIMPLE SIMULATION TESTS

To gain an insight into the workings of the models proposed, a small number of tests were carried out on a simple, two-link (two parallel routes) network. The Model B formulation was used - that is, conditions were assumed to be static within a given day (and there was consequently no departure time choice) and only a pre-specified number of previous experiences of a link contributed to the perceived predicted travel times.

In addition to the specification given in Model B, the following particular assumptions were made.

1.Drivers assess generalised cost purely in terms of travel time.
2.In order to ensure that the model satisfied Cascetta's (1989) conditions for `dynamic stability' and ergodicity of the process, the additional restriction was made that at most M previous states contribute to the current perceived predicted costs, for some given M.
3.The demand randomisation process was implemented as follows: The number of drivers in the whole population was specified. Furthermore, the lower limit on the probability of a driver making a trip on any given day was given, and assumed to be the same for all drivers. Denoting this lower limit P_{min}, then on any given day, the probability of travelling is generated as a random number on the interval $[P_{min}, 1]$. Then on that day, each driver travels with this day-specific probability.
4.Network supply conditions were generated via randomised link performance functions. Power law performance functions were used in conjunction with Normal random error terms. The standard deviation of the error terms was defined as a link-specific constant multiplied by the user equilibrium travel costs. The distribution of error terms was the same for all individuals and all days.
5.The randomisation elements were made repeatable by the specification of a `seed' value.
6.The stability of the process was monitored by use of a two-sample t-test, with an exact significance level computed. This was achieved by comparing the mean flow on one of the links over the last n days with the mean flow for n days previous to this (ie the flows which were between 2n and n+1 days ago).
7.The parameters of the prior were assumed to be the same across all individuals.

The values assumed for the various parameters were:

a)Prior distribution:

Link	m	τ	η	w
1	10	0.01	4.8	1.17
2	20	0.01	7.2	2.89

b) Supply model:

Link	a	b	p	θ	c_{UE}
1	10	1.0	1.0	0.2	50.0
2	20	0.5	1.0	0.2	50.0

where the link performance functions are of the form

$$a + b\, v^p$$

where v is the link flow, and where perceived travel times are the sum of the link performance function and a random element which follows a Normal distribution with mean 0 and standard deviation θc_{UE}.

c)Sundry parameters:

O-D demand = 100
Maximum number of previous trips which contribute to a driver's current perceived predicted cost = 10
Number of days over which averaging takes place, for testing of stability = 20.
Lower limit P_{min} for probability of travelling = 0.8.

The tests consisted of simulating 100 days, for each of three seed values for the randomisation processes. The daily flows on link 1 are given in Figure 1, and the average link 1 flow over the last 20 days is given in Figure 2 (colour, original versions of figure 1, in which the three processes are rather easier to distinguish, are available on request from the author).

Although the simulations offer limited evidence, a number of points are worthy of note. Figure 1 appears to show that the process does indeed settle down to a stable form of oscillation. Furthermore, this stable form does not appear to depend greatly upon the starting conditions (represented by the different seed value used), being within the bounds of sampling variability. Figure 2 gives further evidence of these points. It should be noted that the significance test of dynamic stability was accepted (at the 1% significance level) the first time it was applied (i.e. after 40 simulated days).

7.CONCLUSIONS

This note discusses possible approaches to the modelling of day-to-day dynamic route choice, as well as extensions to within-day dynamics and departure time choice. In order to apply the methods to realistic networks, there are two main areas which need to be considered:

- the feasibility of applying methods such as Model B, with particular regard to computational and storage requirements - see the comments in point (i) in section 5.7;

- the identification and development of suitable algorithms (possibly approximate ones, based on the premise that drivers too use heuristics) for selecting `optimal' departure time and route combinations.

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Figure 1: Flow on link 1 versus day

Figure 2: Average flow (over last 20 days) on link 1 versus day

APPENDIX: Calculation of μ_{ia}^{k} and σ_{ia}^{k}

Data:

Denote previously experienced journey times by individual i on link a up to day k by:

$\{s_{ia}^{kj} : j = 1, 2, \dots n_{ia}^{k}\}$

Assume these times represent a sample of size n_{ia}^{k} from a Normal(μ,σ^2) population.

Prior:

Assume conjugate prior distribution for (μ, σ); that is

$(\mu, \sigma) \sim$ Normal-Gamma $(m_{ia}, \tau_{ia}, \upsilon_{ia}, \omega_{ia})$

with density function

$$\mu, \sigma \,|\, m, \tau, \nu, \omega) = \frac{\sqrt{t}}{\sqrt{2\pi\sigma}} \exp\left\{\frac{-\tau}{2\sigma^2} (\mu - m)^2\right\} \; x \; \frac{2}{\Gamma\left(\frac{\nu}{2}\right)} \left(\frac{\nu\omega}{2}\right)^2 \frac{1}{\sigma^{\nu+1}} \exp\left(-\frac{\nu}{2}\frac{w}{\sigma^2}\right)$$

where $-\infty<\mu<\infty$, $0<\sigma<\infty$, $-\infty<m<\infty$, $\tau>0$, $\upsilon>0$, $\omega>0$.

The first and second moments of such a density are given by:

$$E[\sigma^2] = \frac{\nu\omega}{\nu-2} \quad (\nu > 2)$$

$$\sigma^2) = \frac{2\nu^2\omega^2}{(\nu-4)(\nu-2)} \quad (\nu > 4)$$

$$E[\mu|\sigma^2] = m$$

$$var(\mu|\sigma^2) = \frac{\sigma^2}{\tau}$$

Posterior:

The joint posterior distribution of μ and σ is given by

$$g(\mu, \sigma \mid \{s_{ia}^{kj}\}) \;\alpha\; f(\mu, \sigma)\, L(\mu, \sigma \mid \{s_{ia}^{kj}\})$$

where L is the likelihood. It may be shown that

$$\mu, \sigma \mid \{s_{ia}^{kj}\} \sim \text{Normal-Gamma}\,(m_{ia}^{k}, \tau_{ia}^{k}, \upsilon_{ia}^{k}, \omega_{ia}^{k})$$

where

$$m_{ia}^{k} = \frac{\tau_{ia}\, m_{ia} + \sum_{j} s_{ia}^{kj}}{\tau_{ia}^{k}}$$

$$\tau_{ia}^{k} = \tau_{ia} + n_{ia}^{k}$$

$$\nu_{ia}^{k} = \nu_{ia} + n_{ia}^{k}$$

$$\frac{\nu_{ia}\ \omega_{ia} + \tau_{ia}\ (m_{ia})^{2} + \sum_{j} (s_{ia}^{kj})^{2} - \tau_{ia}^{k}\ (m_{ia}^{k})^{2}}{\nu_{ia}^{k}}$$

Intuitively, one can think of these parameters m_{ia}^k, τ_{ia}^k, v_{ia}^k and w_{ia}^k as subjective evaluations of (respectively) sample mean, sample size, divisor for variance calculations, and average squared deviation from the mean.

Posterior means:

It follows that the posterior expected values of μ and σ are:

$$\mu_{ia}^{k} = E[\,\mu \,|\{\, s_{ia}^{kj} \}] = m_{ia}^{k}$$

$$)^{2} = E[\,\sigma^{2} \,|\, \{\, s_{ia}^{kj} \}] = \frac{\nu_{ia}^{k}\ \omega_{ia}^{k}}{\nu_{ia}^{k}\ -2} \quad (\nu_{ia}^{k}\ > 2)$$

Notes on implementation:

1.In order to make it easier to specify the parameters of the prior, there may be some advantage in reparameterising:

$$(m, \tau, \upsilon, \omega) \rightarrow (m, \tau, \alpha, \gamma)$$

where

$$\chi = \frac{\nu\omega}{\nu - 2}$$

$$\frac{2\nu\omega}{(\nu - 4)(\nu - 2)}$$

$$\gamma = \alpha\beta$$

In this way, α is the prior mean of σ and γ the prior variance of σ. Having specified α and γ, it is possible to return to the original parameterisation by using:

$$= \frac{\gamma}{\alpha}$$

$$\frac{\alpha + 4\beta}{\beta}$$

$$\chi\,(1 - \frac{2}{\nu})$$

2.Recursion formulae may be set up to compute the values of the parameters of the posterior on day k+1 from the values on day k.

NB:These formulae are to be updated every day, even if the individual does not travel.

i)If no journey made on (previous) day k using link a:

Set m_{ia}^{k+1}, τ_{ia}^{k+1}, υ_{ia}^{k+1} and ω_{ia}^{k+1} to their values at day k.

ii)If a journey was made on day k on link a, with a link journey time of s, then:

$$m_{ia}^{k+1} = \frac{\tau_{ia}^{k}\, m_{ia}^{k} + s}{\tau_{ia}^{k} + 1}$$

$$\tau_{ia}^{k+1} = \tau_{ia}^{k} + 1$$

$$\nu_{ia}^{k+1} = \nu_{ia}^{k} + 1$$

$$= \frac{\nu_{ia}^{k}\,\omega_{ia}^{k} + \tau_{ia}^{k}\,(m_{ia}^{k})^{2} + s^{2} - \tau_{ia}^{k+1}\,(m_{ia}^{k+1})^{2}}{\nu_{ia}^{k+1}}$$

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