Analysis of melting behavior of PCMs in a cavity subject to a non-uniform magnetic field using a moving grid technique

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Abstract

Melting flow and heat transfer of electrically conductive phase change materials subjecting to a variable magnetic field are addressed in a cavity enclosure. The top and bottom walls of the cavity are adiabatic, and the sidewalls are isothermal at different temperatures. The temperature of the hot wall is higher than the fusion temperature of PCM (T_f), and the cold wall is at the fusion temperature or lower. At the initial time, the cavity is filled with a solid saturated PCM. In the vicinity to the hot wall, there is an external line-source magnet, inducing a magnetic field. The location of the magnetic source (Y_0) can be changed along the hot wall. The cavity domain is divided into two parts of the liquid domain and the solid domain. The moving grid method is utilized to track the phase change interface at the exact fusion temperature of T_f. The governing equations for continuity, flow and heat transfer associated with the Arbitrary Lagrangian-Eulerian (ALE) moving mesh technique are solved using the finite element method. The results are investigated for the melting behavior of PCM by the study of Hartmann number ($0 \leq Ha \leq 50$) and the location of the magnetic source ($0 \leq Y_0 \leq 1$). Outcomes show that the effect of the magnetic field on the melting behavior of PCM is negligible at the initial stages of the melting ($Fo < 1.15$). However, after the initial stages of the melting, the effect of the presence of a magnetic field becomes significant. Moreover, the location of the magnetic source induces a feeble effect on the melting front at the initial melting stages, but its effect on the shape of the melting front increases by the increase of the non-dimensional time. The location of the magnetic source also significantly affects the streamlines patterns. Changing the position of the magnetic source from the bottom of the cavity ($Y_0 = 0.2$) to the almost middle of the cavity ($Y_0 = 0.6$) would decrease the required non-dimensional time of full melting from $Fo = 10.4$ to $Fo = 9.0$.

Keywords: Variable magnetic field; line-source magnet; melting heat transfer; phase change heat transfer; moving mesh method.

Paper type: Research paper

Nomenclature

Symbols	Description

B	magnetic induction vector
c_p	specific heat in constant pressure (J/kgºC)
Ec	Eckert number
F	volumetric force (N/m^3)
Fo	non-dimensional time (Fourier number)
g	gravitational acceleration (m/s^2)
H	the strength of the magnetic field
Ha	Hartmann number
J	voltage field (V)
k	thermal conductivity coefficient (W/mK)
L	cavity size
P	pressure (Pa)
Pr	Prandtl number
Q	Joule heating source term
Ra	Rayleigh number
S	dimensionless stream function
Ste	Stefan number
t	time (s)
T	temperature (ºC)
u	velocity (m/s)
u	velocity component in the x-direction (m/s)
U	non-dimensional velocity component in the x-direction
v	velocity component in the y-direction (m/s)
V	non-dimensional velocity component in the y-direction
x	Cartesian coordinate in the horizontal direction (m)
X	non-dimensional Cartesian coordinate in the horizontal direction
y	Cartesian coordinate in the vertical direction (m)

Y	the non-dimensional Cartesian coordinate in the vertical direction
Greek symbols	
μ	dynamic viscosity (kg s/m)
μ_0	magnetic constant
α	thermal diffusivity (m^2/s)
β	thermal expansion coefficient (1/K)
γ	the strength of the magnetic source
θ	non-dimensional temperature
ρ	density (kg/m^3)
σ	the electrical conductivity of the liquid (Ω·m)
Subscript	
$_0$	location of the magnetic source
B	buoyancy force
c	cold
f	fusion
h	hot
L	Lorentz force
l	liquid
s	solid
Superscript	
*	dimensional values

1. Introduction

The Phase Change Materials (PCMs) are capable of storing or releasing a large amount of latent energy during solidification or melting. Therefore, PCMs have been subject of various practical applications in the body of domestic buildings and building envelopes such as walls, roofs, ceilings, and floors [1, 2]. Moreover, PCMs have found applications for thermal energy storage in concentrated solar thermal power plants [3]. In thermal storage applications, PCMs are packed in solid flat plates, solid cylinders, spheres, rods and various forms of enclosures [4-7]. Farah et al. [8] studied some practical applications of using PCMs in energy storage systems.

By using PCMs, the renewable sources of energy can be stored in the form of the latent heat of phase change if the time of demand does not coincide with the time of production. In fact, the amount of latent heat that can be stored in a unit volume of a PCM is much larger than that of the sensible heat. Hence, a large amount of energy can be stored in the form of phase change latent heat. Fokaides et al. [9] and Silva et al. [10] reviewed the application of phase change materials in transparent elements for buildings usage. The transparent PCMs have found essential applications in windows cavities, decorated walls, and light walls.

Tay et al. [11] point out one of the very important advantages of PCMs thermal storage for time shift of energy usage. PCMs can be charged in off-peak electricity tariffs with low-cost electricity and later be used as a heating or cooling source in regular times. Malik et al. [12] studied the application of PCMs in battery thermal management for electric and hybrid electric vehicles. Chandel and Agarwal [13] reviewed the application of PCMs as energy storage coolants for enhancing the efficiency of photovoltaic power systems. The literature review shows that PCMs are packed and contained in closed enclosure units. In this regard, Kylili and Fokaides [14] performed a review study on numerical analysis of PCMs in cavity enclosures. Due to the importance of natural convection heat transfer in enclosures, this phenomenon has been addressed in many of the recent studies such as Alsabery et al. [15, 16], Janagi et al. [17], Pop et al. [18], Zargartalebi et al. [19], and Sheikholeslami [20, 21].

The magnetic field can be the result of high-power transformers, current in batteries or microwave systems with a transient electrical load. In such systems, a phase change heatsink can be utilized for thermal management of the device in environments with low ventilation. The heatsink is the cavity enclosure with a potential of energy storage/release and the magnetic field is the device. A magnetic field can affect the convective heat transfer in an enclosure as a controlling mean for control of heat transfer rate. Considering the presence of a magnetic field, most of the available works have addressed the effect of the presence of a uniform or an inclined uniform magnetic field on the single-phase convection applications with no phase change. The presence of a uniform magnetic field induces volumetric forces on the moving electrical conducting fluid proportional to the fluid velocity. For example, in the case of natural convection of a liquid fluid (with no phase change), Rashad et al. [22] investigated the entropy generation and heat transfer in an inclined cavity subject to a uniform magnetic field. Dogonchi et al. [23] conducted the numerical analysis of natural convection inside in the cavity containing inclined elliptical heater under shape factor of nanoparticles and magnetic field. Also, the natural convection heat transfer in a square enclosure with a wavy circular heater under magnetic field is studied by Dogonchi et al. [24]. Chamkha and Selimefendigil [25] investigated the Magnetohydrodynamic natural convection and entropy generation in a corrugated porous cavity using finite element method. Sheremet et al. [26] addressed the effect of magnetic field on the flow and heat transfer of nanofluids in a cavity filled with a porous media. Reddy and Murugesan [27] analyzed the influence of an inclined uniform magnetic field on the double-diffusive natural convection in a square cavity with the temperature difference at the vertical walls. The outcomes reveal that the increase of the magnetic-field-intensity decreases the heat and mass transfer in the cavity.

Considering the melting and an inclined uniform magnetic field effect, Bondareva and Sheremet [28] addressed the melting heat transfer in a 2D square cavity subject to a uniform magnetic field. The vertical walls were at a constant cold temperature, and there was a constant temperature heat source at the bottom of the cavity. The other parts of the cavity walls were well insulated. At the initial time, the cavity was filled with a solid phase change material. Later, PCM started to phase change from the solid to liquid due to the energy of the heat source. The results showed that a symmetrical thermo-hydrodynamic structure was observed in the liquid at the beginning of the melting. The structure of this initial region is not under the influence of the magnetic field. However, as the molten region around the heat source starts to extend, the effect of the magnetic field becomes significant. The inclination angle of the magnetic field tends to reduce the symmetry of the molten zone.

Later, Bondareva and Sheremet [29] extended their previous work presented in [28] to the case of a 3D cavity. They investigated the effect of an inclined uniform magnetic field on the melting heat transfer of gallium in a 3D cubic cavity. They studied a cavity bounded by two opposite isothermal cold vertical-walls while the other cavity walls were well insulated. There was a heat source with constant hot temperature, mounted at the bottom of the cavity. At the initial state, the gallium in the cavity was in a solid phase. So that, the melting commenced from the below of the cavity due to the energy of the heat source. The results show that the increase in the intensity of the magnetic field reduces the convective heat transfer. For large values of the magnetic field, the natural convection can be suppressed, which leads to a stratified liquid around the heat source. Sheikholeslami and Rokni [30] investigated the melting behavior of CuO-water nanofluid in a cavity in the presence of an inclined uniform magnetic field.

The uniform magnetic field can be produced as the result of a solenoid magnetic field. However, there are many cases, in which the magnetic field is variable in space. For example, the magnetic field around a wire is variable with the distance from the wire. The variable magnetic field, induced by a line magnetic source, has been addressed in some of the recent studies regarding the natural convective heat transfer in cavities with no phase-change heat transfer. Sheikholeslami et al. have addressed the effect of a point source magnetic field on the flow and heat transfer of Fe_3O_4-water nanofluid in a semi-annulus enclosure [31], a lid-driven semi annulus enclosure [32], a semi-rectangle enclosure [33] and circular cavity [34]. The outcomes show that the presence of the magnetic field decreases the heat transfer rate. Later, some researchers studied the natural convection heat transfer of nanofluids in a square cavity [35], a cavity with a hot pipe [36] and a wavy wall cavity [37] in the presence of a variable magnetic field. These authors [35-37] studied the effect of Rayleigh and Hartmann numbers on flow and heat transfer of Fe_3O_4-water nanofluid. The results show that the heat transfer is a decreasing function of the Lorentz force.

From the theoretical point of view, the modeling of phase-change heat transfer in enclosures has been subject of two major approaches, the enthalpy-porosity methods and the phase change interface tracking methods. In the enthalpy-porosity method, it is assumed that the phase change occurs in a temperature range instead of an exact fusion temperature. So, the latent heat of phase change is included in the heat capacity of the phase-change medium. The momentum equation in liquid and solid regions is controlled by using source terms. The source terms affect the momentum in a way to allow free fluid motion in liquid regions, but they force the velocity to zero in solid regions. It should be noted that dealing with a temperature range instead of an exact fusion temperature is a modeling approximation which adds some modeling errors. This approximation error can be overcome by reducing the fusion temperature range; however, a narrow fusion temperature range results in some instability and convergence problems. Besides, a very high grid resolution is also required to capture the temperature gradients at the phase change interface.

Another approach to model the phase-change is dividing the domain of the solution into pure solid and pure liquid phases, and hence, the phase change can occur at the interface of the two regions at an exact fusion temperature. In this approach, the solid region shrinks and the liquid region expands as the melting process continues in time. Thus, an interface tracking system is required to follow the melting interface. Viswanath and Jaluria [38], Wintruff et al. [39] and Li et al. [40] discussed the advantage and drawbacks of these solution methods in details.

Considering the enthalpy-porosity method, Sushobhan and Kar [41] studied the melting of nano-based phase change materials in a cavity for the applications of thermal energy storage. Yang et al. [42] analyzed the melting of gallium in a cavity with a temperature difference between the vertical walls. Ye [43] investigated the effect of the cavity aspect ratio on the melting behavior of phase change materials. The vertical walls of the cavity were subject to in isothermal hot temperature, and the top and bottom walls were adiabatic. The outcomes reveal that the aspect ratios significantly affect the time scale of melting phenomenon and the structure of the convection currents inside the cavity. Bondareva and Sheremet [28, 29] utilized the enthalpy-porosity method differently. They solved the momentum equations solely in grid cells with liquid, and they considered the cells with the solid as the boundaries for momentum equations. This way, they do not need to deal with source teams and convergence problems. However, this method has some drawbacks. For example, criteria for deciding the cell is solid or liquid are required. In every time step, the grid domain for momentum equations should be updated. In contrast, Hossain et al. [44] and Al-Jethelah et al. [45] utilized an interface tracking approach to model the melting flow and heat transfer in a porous space.

The literature review shows that the line-source variable magnetic field has been studied in enclosures with single phase natural convective heat transfer in recent years. The uniform melting heat transfer (two-phase) has also been investigated in the literature. However, the effect of the line-source magnetic field on the melting heat transfer has not been addressed yet. To the best of the author’s knowledge, the present study is the first work to analyze the effects of the presence and location of a non-uniform magnetic source on the melting rate of an MHD phase change material utilizing advanced tracking method (moving grid method).

2. Mathematical model

Fig. 1 illustrates the schematic view of the domain of interest selected for this study. A phase-change solid substance with initial temperature T_f has filled the enclosure. As can be seen in Fig. 1, the right wall and the left wall are in the hot and the cold isothermal temperatures of T_h and T_c, respectively. The top and bottom bounds are well insulated. There is a magnetic source at the location of (x_0, y_0) outside the cavity. The intensity of the magnetic source is variable and decreases as the square of the distance with the source. The gravity and non-uniform magnetic body forces influence the entire of the enclosure domain. Due to the thermal buoyancy effects, there is a natural convection flow in the molten region. It is assumed that the thermo-physical properties in the molten and solid regions are independent of temperature except for the density which is modeled by the Boussinesq approximation. It is also assumed that the flow of the melted substance is laminar and Newtonian.

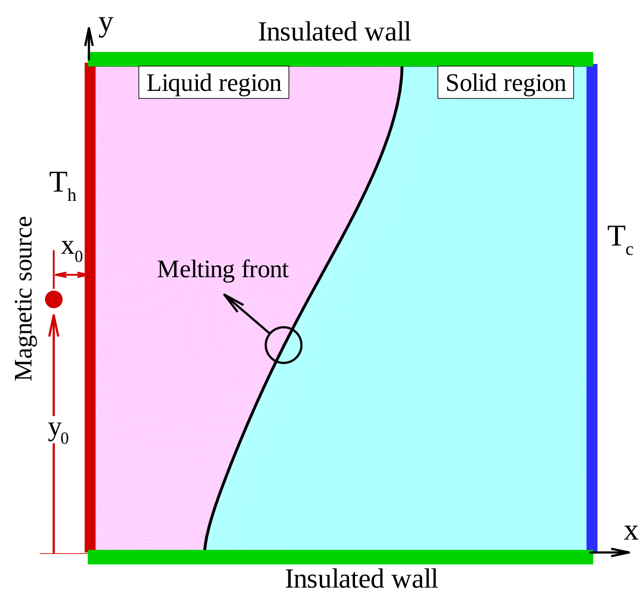


Fig. 1. Schematic view of the physical model and computational domain

Following the studies of Sheikholeslami et al. [31] and Sheikholeslami and Vajravelu [35], and the components of the intensity of the magnetic field along x and y axes introduced by H_x^* and H_y^*, respectively. The strength of the magnetic field, H^*, can be written as follow:

$$H_x^* = \frac{\gamma}{2\pi} \frac{(y - y_0)}{(x - x_0)^2 + (y - y_0)^2} \quad (1\text{-a})$$

$$H_y^* = -\frac{\gamma}{2\pi} \frac{(x - x_0)}{(x - x_0)^2 + (y - y_0)^2} \quad (1\text{-b})$$

$$H^* = \left(H_x^{*2} + H_y^{*2}\right)^{0.5} \quad (1\text{-c})$$

In the relations introducing the magnetic field, γ is the strength of the magnetic source in (x_0, y_0). Applying the above assumptions to derive the governing equations leads to the equations given below:

Continuity equation:

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

where **u** is the velocity vector including u and v components along x and y directions.

Momentum equation:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{F} \quad (3)$$

In this equation, **F**, the volume force, can be explained as follow

$$\mathbf{F} = \mathbf{F_L} + \mathbf{F_B} \quad (4)$$

where $\mathbf{F_L}$ is Lorentz force that is related to the velocity field so that $\mathbf{F_L} = \mathbf{J} \times \mathbf{B}$. Here **B** is the magnetic induction vector with components of $B_x = \mu_0 H_x^*$ and $B_y = \mu_0 H_y^*$. **J** is also the voltage field. Three vectors of the magnetic induction, velocity and voltage are correlated by the equation given below:

$$\mathbf{J} = \sigma \left(-\nabla \phi + \mathbf{u} \times \mathbf{B} \right) \quad (5)$$

The buoyancy force applied to the flow field is $\mathbf{F_B}$ and is written as:

$$\mathbf{F_B} = \rho g\beta\left(T - T_{\mathrm{f}}\right) \tag{6}$$

Energy equation of liquid PCM:

$$\left(\rho c_p\right)_l\left(\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T\right) = k_l\nabla^2 T + Q \tag{7}$$

The magnetic field can lead to heat generation Q as known joule heating:

$$Q = \sigma\left(uB_y - vB_x\right)^2 \tag{8}$$

Energy equation of solid PCM:

$$\left(\rho c_p\right)_s\frac{\partial T}{\partial t} = k_s\nabla^2 T \tag{9}$$

The boundary conditions of the domain are presented as:

$$x = 0,\ 0 \le y \le L,\ t > 0 \ \rightarrow\ u = v = 0, T = T_h \tag{10-a}$$

$$x = L,\ 0 \le y \le L,\ t > 0 \ \rightarrow\ u = v = 0, T = T_c \tag{10-b}$$

$$y = 0,\ 0 \le x \le L,\ t > 0 \ \rightarrow\ u = v = 0, \partial T/\partial y = 0 \tag{10-c}$$

$$y = L,\ 0 \le x \le L,\ t > 0 \ \rightarrow\ u = v = 0, \partial T/\partial y = 0 \tag{10-d}$$

$$t = 0,\ 0 \le x \le L,\ 0 \le y \le L,\ t > 0 \ \rightarrow\ u = v = 0, T = T_f \tag{10-e}$$

In order to evaluate the displacement velocity, and hence, the grid movement of the interface, the interfacial energy balance or Stefan condition is utilized:

$$k_l\left.\frac{\partial T}{\partial x}\right|_l - k_s\left.\frac{\partial T}{\partial x}\right|_s = \rho u h_{sf} \tag{11-a}$$

$$k_l\left.\frac{\partial T}{\partial y}\right|_l - k_s\left.\frac{\partial T}{\partial y}\right|_s = \rho v h_{sf} \tag{11-b}$$

The dimensionless parameters, introduced as below, are employed to transfer the dimensional equations to dimensionless X-Y coordinates:

$$
\begin{aligned}
&X=\frac{x}{L},\; Y=\frac{y}{L},\; U=\frac{uL}{\alpha_l},\; V=\frac{vL}{\alpha_l},\; \theta=\frac{T-T_c}{T_h-T_c},\; P=\frac{L^2 p}{\rho\alpha_l^{\,2}},\\
&H=\frac{H^*}{H_0^*},\; H_X=\frac{H_x^*}{H_0^*},\; H_Y=\frac{H_y^*}{H_0^*},\; Fo=\frac{t\alpha_l}{L^2}
\end{aligned}
\tag{12}
$$

where $H_0^* = \gamma/2\pi L$. Substituting these parameters for the governing equations result in the appearance of the following dimensionless equations:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{13}$$

$$\frac{\partial U}{\partial Fo}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-Ha^2 Pr H_Y\left(UH_Y-VH_X\right) \tag{14}$$

$$\frac{\partial V}{\partial Fo}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-Ha^2 Pr H_X\left(VH_X-UH_Y\right)+Ra Pr\,\theta \tag{15}$$

$$\frac{\partial \theta}{\partial Fo}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+Ec Ha^2\left(UH_Y-VH_X\right)^2 \tag{16}$$

$$\frac{\partial \theta}{\partial Fo}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{17}$$

where

$$
\begin{aligned}
&Ra=\frac{g\beta\left(T_h-T_f\right)L^3}{\mu_l\alpha_l},\; Pr=\frac{\upsilon}{\alpha_l},\; Ha=\mu_0 H_0 L\sqrt{\frac{\sigma_l}{\mu_l}}\\
&,Ec=\frac{\mu_l\alpha_l}{\rho_l\, c_{p,l}\left(T_h-T_f\right)L^2},\; Ste=\frac{h_{sf}\left(T_h-T_f\right)}{k_l}
\end{aligned}
\tag{18}
$$

The boundary conditions in dimensionless coordinates X-Y are:

$$X=0,\; 0\le Y\le 1,\; Fo>0 \;\rightarrow\; U=V=0, \theta=1 \tag{19-a}$$

$$X=1,\; 0\le Y\le 1,\; Fo>0 \;\rightarrow\; U=V=0, \theta=\left(T_c-T_f\right)/\left(T_h-T_f\right) \tag{19-b}$$

$$Y=0,\; 0\le X\le 1,\; Fo>0 \;\rightarrow\; U=V=0, \partial\theta/\partial Y=0 \tag{19-c}$$

$$Y=1,\; 0\le X\le 1,\; Fo>0 \;\rightarrow\; U=V=0, \partial\theta/\partial Y=0 \tag{19-d}$$

$$Fo=0,\; 0\le X\le 1,\; 0\le Y\le 1 \;\rightarrow\; U=V=0, \theta=0 \tag{19-e}$$

Here, it is assumed that $T_c=T_f$, and hence, $\theta=0$ at X=1. Besides, the dimensionless equations of movement of the solid-liquid interface can be written as

$$U = Ste\left(\left.\frac{\partial\theta}{\partial X}\right|_l - \left.\frac{\partial\theta}{\partial X}\right|_s\right) \tag{20-a}$$

$$V = Ste\left(\left.\frac{\partial\theta}{\partial Y}\right|_l - \left.\frac{\partial\theta}{\partial Y}\right|_s\right) \tag{20-b}$$

The moving grid method requires an initial small region of liquid to be defined as the liquid region. Hence, at the initial state, one present of the length of the cavity is assumed in the melting phase with an initial non-dimensional temperature of zero. This assumption is required for dividing of the cavity into two domains of liquid and solid and commencing of the melting process.

3. Numerical approach

To solve the coupled and non-linear Eqs. (13)-(17) and the boundary conditions of Eqs. (19) and (20), the Galerkin finite element method is employed. The details of this method are well discussed in [46, 47]. The pressure term in momentum equations can be eliminated by a penalty function defined as follows:

$$P = \chi\left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y}\right) \tag{21}$$

It is known that the continuity equation is satisfied, if χ, namely penalty number, is a large value. Substituting this penalty function for pressure term gives the following equations:

$$\frac{\partial U}{\partial Fo} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial}{\partial X}\left(\chi\left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y}\right)\right) + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \\ -Ha^2 Pr H_Y\left(UH_Y - VH_X\right) \tag{22}$$

$$\frac{\partial V}{\partial Fo} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial}{\partial Y}\left(\chi\left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y}\right)\right) + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \\ -Ha^2 Pr H_X\left(VH_X - UH_Y\right) + RaPr\,\theta \tag{23}$$

Employing a basis set $\{\xi_k\}_{k=1}^{N}$, the velocity components, and the temperature can be expanded such as:

$$U \approx \sum_{k=1}^{N} U_k \xi_k (X, Y), V \approx \sum_{k=1}^{N} V_k \xi_k (X, Y), \theta \approx \sum_{k=1}^{N} \theta_k \xi_k (X, Y) \tag{24}$$

Since the basic functions of the variables ξ are the same, the total grids number for all of the variables is N=3. The heat and fluid equations were coupled using the Newton method, and they were solved simultaneously using a MUltifrontal Massively Parallel Sparse (MUMPS) direct solver [48, 49]. The use of Galerkin finite element approach results in the non-linear residuals as below:

$$\begin{aligned} R_i^1 \approx & \sum_{k=1}^{N} U_k \gamma \int \frac{\partial \xi_k}{\partial \mathrm{Fo}} \xi_i \, dXdY + \sum_{k=1}^{N} U_k \int \left[\left(\sum_{k=1}^{N} U_k \xi_k \right) \frac{\partial \xi_k}{\partial X} + \left(\sum_{k=1}^{N} V_k \xi_k \right) \frac{\partial \xi_k}{\partial Y} \right] \xi_i \, dXdY \\ & + \gamma \left[\sum_{k=1}^{N} U_k \int \frac{\partial \xi_i}{\partial X} \frac{\partial \xi_k}{\partial X} dXdY + \sum_{k=1}^{N} V_k \int \frac{\partial \xi_i}{\partial X} \frac{\partial \xi_k}{\partial Y} dXdY \right] \\ & + \mathrm{Pr} \sum_{k=1}^{N} U_k \int \left[\frac{\partial \xi_i}{\partial X} \frac{\partial \xi_k}{\partial X} + \frac{\partial \xi_i}{\partial Y} \frac{\partial \xi_k}{\partial Y} \right] dXdY + \mathrm{Pr}\mathrm{Ha}^2 H_Y^2 \int \left(\sum_{k=1}^{N} U_k \xi_k \right) \xi_i \, dXdY \\ & - \mathrm{Pr}\mathrm{Ha}^2 H_X H_Y \int \left(\sum_{k=1}^{N} V_k \xi_k \right) \xi_i \, dXdY \end{aligned} \tag{25}$$

$$\begin{aligned} R_i^2 \approx & \sum_{k=1}^{N} V_k \gamma \int \frac{\partial \xi_k}{\partial \mathrm{Fo}} \xi_i \, dXdY + \sum_{k=1}^{N} V_k \int \left[\left(\sum_{k=1}^{N} U_k \xi_k \right) \frac{\partial \xi_k}{\partial X} + \left(\sum_{k=1}^{N} V_k \xi_k \right) \frac{\partial \xi_k}{\partial Y} \right] \xi_i \, dXdY \\ & + \gamma \left[\sum_{k=1}^{N} U_k \int \frac{\partial \xi_i}{\partial Y} \frac{\partial \xi_k}{\partial X} dXdY + \sum_{k=1}^{N} V_k \int \frac{\partial \xi_i}{\partial Y} \frac{\partial \xi_k}{\partial Y} dXdY \right] \\ & + \mathrm{Pr} \sum_{k=1}^{N} V_k \int \left[\frac{\partial \xi_i}{\partial X} \frac{\partial \xi_k}{\partial X} + \frac{\partial \xi_i}{\partial Y} \frac{\partial \xi_k}{\partial Y} \right] dXdY - \mathrm{Ra}\mathrm{Pr} \int \left(\sum_{k=1}^{N} \theta_k \xi_k \right) \xi_i \, dXdY \\ & + \mathrm{Pr}\mathrm{Ha}^2 H_X^2 \int \left(\sum_{k=1}^{N} V_k \xi_k \right) \xi_i \, dXdY - \mathrm{Pr}\mathrm{Ha}^2 H_X H_Y \int \left(\sum_{k=1}^{N} U_k \xi_k \right) \xi_i \, dXdY \end{aligned} \tag{26}$$

$$\begin{aligned} R_i^3 \approx & \sum_{k=1}^{N} \theta_k \gamma \int \frac{\partial \xi_k}{\partial \mathrm{Fo}} \xi_i \, dXdY + \sum_{k=1}^{N} \theta_k \int \left[\left(\sum_{k=1}^{N} U_k \xi_k \right) \frac{\partial \xi_k}{\partial X} + \left(\sum_{k=1}^{N} V_k \xi_k \right) \frac{\partial \xi_k}{\partial Y} \right] \xi_i \, dXdY \\ & + \sum_{k=1}^{N} \theta_k \int \left[\frac{\partial \xi_i}{\partial X} \frac{\partial \xi_k}{\partial X} + \frac{\partial \xi_i}{\partial Y} \frac{\partial \xi_k}{\partial Y} \right] dXdY - \mathrm{Ec}\mathrm{Ha}^2 \left(H_Y \int \left[\sum_{k=1}^{N} U_k \xi_k \right] - H_X \int \left[\sum_{k=1}^{N} V_k \xi_k \right] \right)^2 \xi_i \, dXdY \end{aligned}$$

(27)

The Laplace equation was utilized to compute the grid motion. The motion of the phase change interface was controlled using the Stefan boundary condition. The motion of the vertical walls was fixed in a horizontal direction, but they were allowed to move in the vertical direction. In the same way, the motion of the horizontal walls was fixed in the vertical direction, but they were allowed to move in the horizontal direction. The fixed boundary conditions for horizontal and vertical walls forces the cavity to remain in its original form, but it also provides enough flexibility for the grid to move smoothly. A systematic re-meshing based on the general mesh quality is also employed to ensure the quality of the grid. The solution from the previous mesh was interpolated into the new mesh. The time step is automatically controlled using the Backward Differentiation Formula (BDF). The time step selected based on a free time steps scheme within BFD order in the range of one and two [50]. The following chart representing the algorithm of the numerical approach is depicted in Fig. 2.

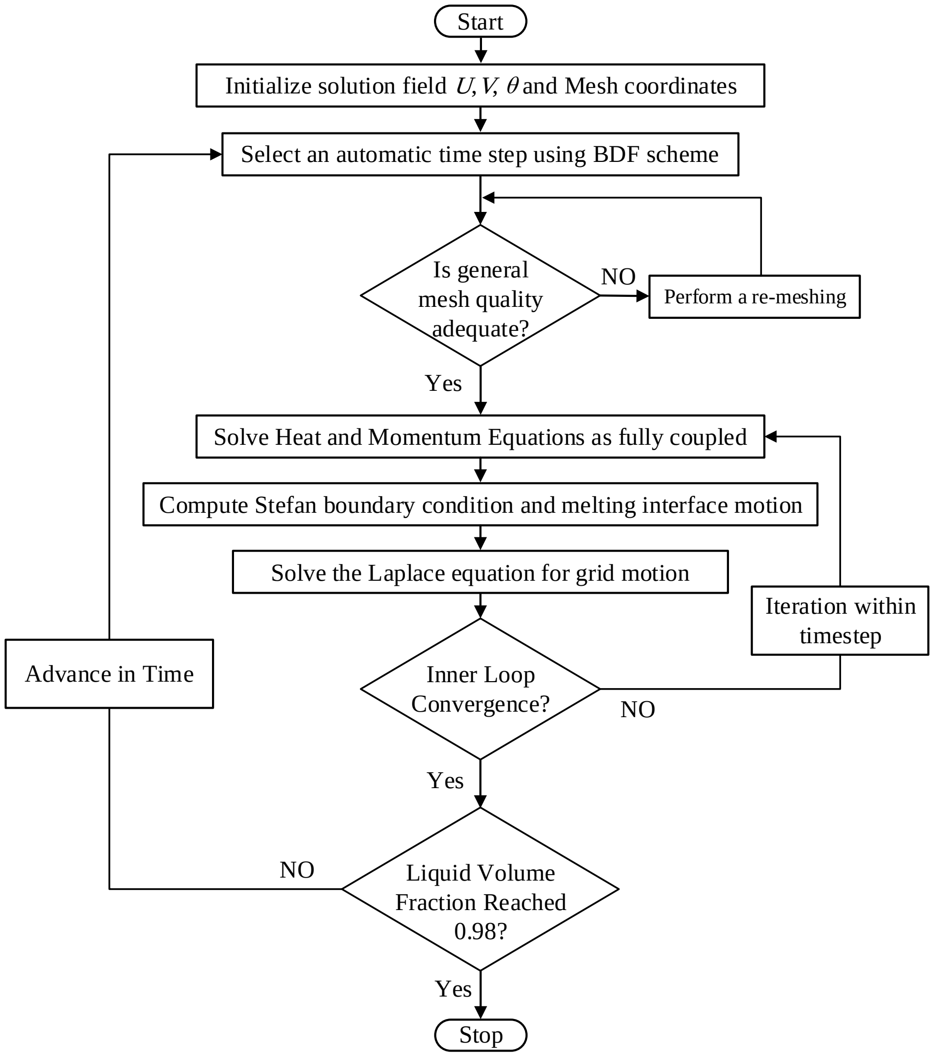


Fig. 2. Flow chart of the utilized numerical approach

3.1 Grid test and verification

In the numerical calculations, the study of grid independence is momentous so that the study is incomplete without doing this operation. For this purpose, the computational domain is divided into two regions: the melted liquid and the solid substances. A structured mesh with quadratic elements is used to discretize the melted liquid region while another region is discretized by employing unstructured mesh and triangular elements. The number of elements for both the liquid and solid regions is presented Table 1 for Ra = 2.1×10^5, Pr = 0.021, Ste = 0.039, Ha = 10, $Y_0 = 0.5$, Ec = 10^{-6}. The liquid fraction, corresponding to various grid sizes of Table 1, is plotted in Fig. 3. It is observed that case III provides accurate results for the solution.

Table 1. The grid sizes in the liquid and the solid regions

Cases	Case I	Case II	Case III	Case IV	Case V	Case VI
Grid size in solid	183	686	974	1597	1941	2275
Grid size in liquid	20×20	40×40	60×60	80×80	100×100	120×120

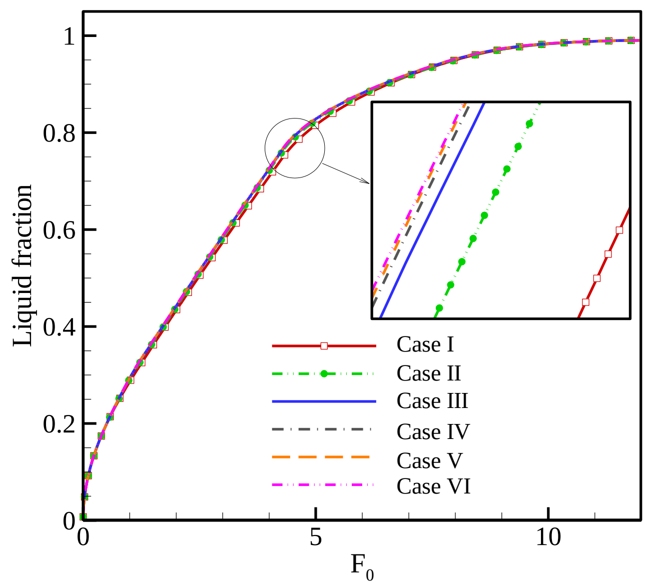


Fig. 3. The dependency of the liquid fraction on the grid size

The accuracy and correctness of the utilized code are evaluated by comparing the outcomes of the present work and those mentioned in [35, 47, 51–53]. Figs. 4–7 show these verifications and validations. The results show excellent accommodations between the outcomes of the present study and previous works.

Figs. 4 and 5 depict admissible agreement with the results of the current work and those reported in the literature. The main reason for the discrepancies between the results is the fact that the phase change occurs at a fixed fusion temperature; however, using a fixed fusion temperature results in a discontinuity in the heat equation. Therefore, in the enthalpy-porosity method, the fusion occurs in a narrow temperature range. Capturing accurate fusion interface requires a narrow band of $\delta\theta$ and a large value of A_{mush} parameter. However, employing a narrow band of $\delta\theta$ and a large value of A_{mush} significantly reduced the stability of the governing equation. Using a narrow band for fusion temperature demands a very fine grid. To reduce the computational cost, some researchers have used a larger fusion temperature and a small value of A_{mush}. Therefore, the phase change interface reported by various researchers is not unique. In the present study, we utilized a new approach based on the deformed grid method, in which the phase change occurs at the exact fusion temperature with no need of using a phase change temperature band. Moreover, most of the experimental works have used mechanical methods to capture the phase change interface. For instance, Gau and Viskanta [53]utilized a mechanical probe to capture the melting interface. At the onset of phase change, the melting interface is unstable, and the measurement of the phase change interface using mechanical probes involves some degrees of deviation.

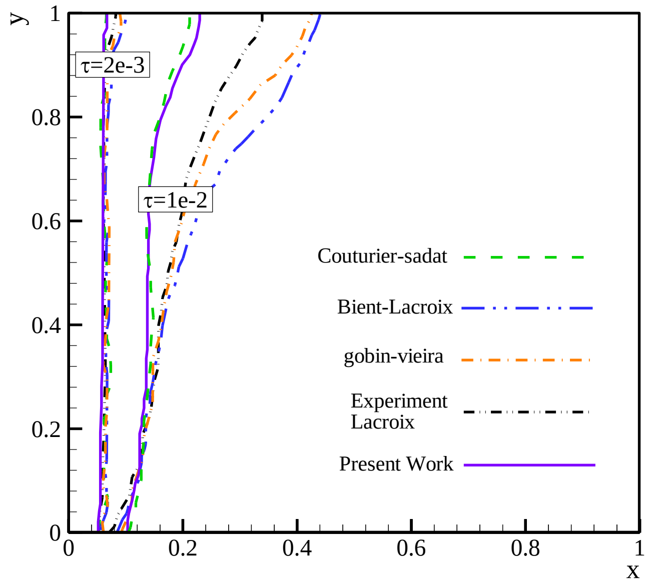


Fig. 4. Comparison between the results of current work and those reported in Bertrand et al. [52]

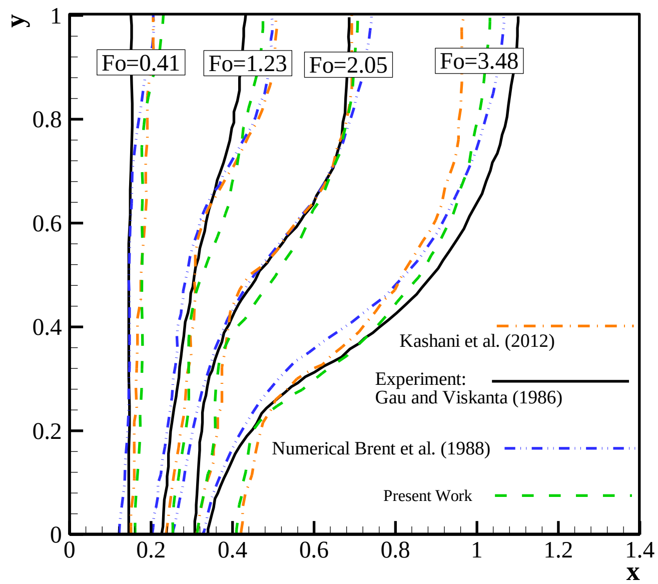


Fig. 5. Comparison between the results of current work and those reported in literature [53]

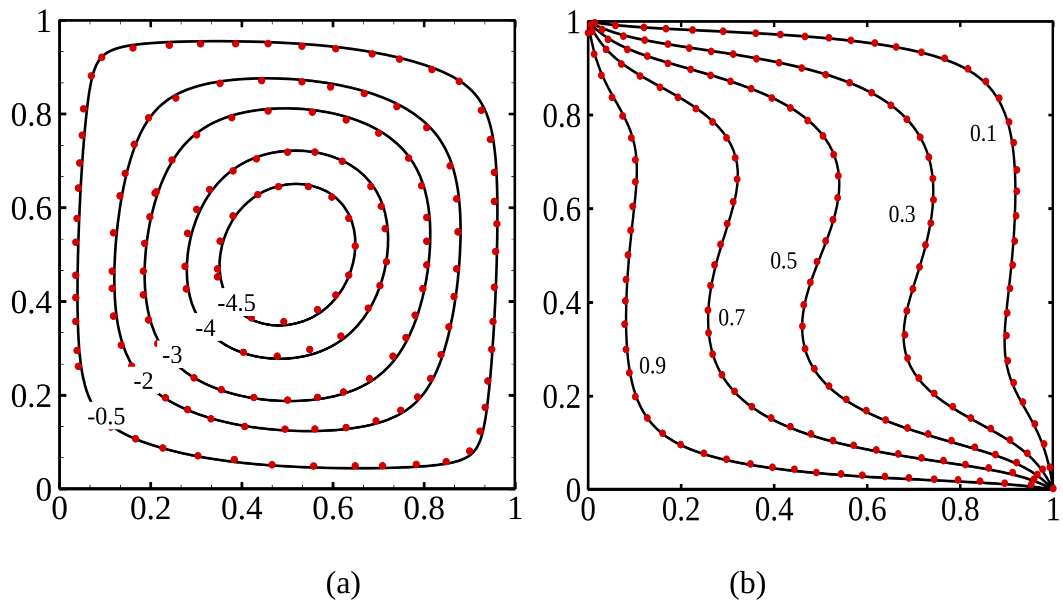
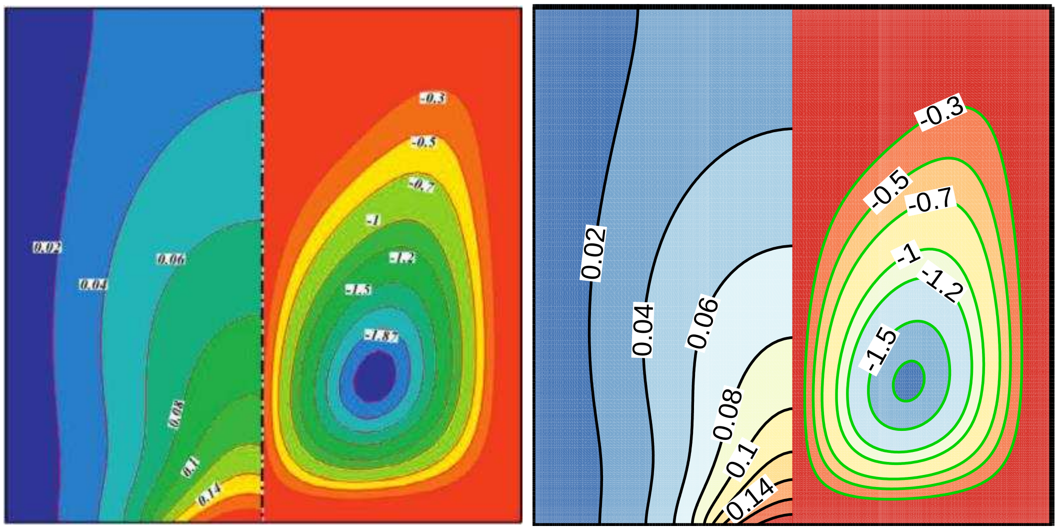


Fig. 6. Comparison between (a): streamlines and (b): isotherms of the current work (solid lines) and those reported (points) in Sathiyamoorthy and Chamkha [47]



(a) (b)

Fig. 7. temperature and streamlines of the work done by (a): Sheikholeslami and Vajravelu [35] and (b): the current work

4. Results and discussion

This work aims to study the existence effects of the non-uniform magnetic field on the melting process driven by the natural convection. The impacts of strength ($0 \leq Ha \leq 50$) and location ($0 \leq Y_0 \leq 1$) of the magnetic field on the mass fraction of the melted liquid are perused while the other parameters are kept constant so that $Ra = 2.1\times10^5$, $Pr = 0.021$, $Ste = 0.039$ and $Ec = 10^{-6}$.

Fig. 7 depicts the deformable grid patterns during the melting process for various Fourier number when $Ha = 30$ and $Y_0 = 0.5$. As depicted, the structured and unstructured grids are used to discretizing melted fluid and solid substances, respectively. It is worth mentioning that the employed code utilizes the re-meshing technique during melting progress to satisfy the accuracy of the results. As seen in this figure, a large grid-size is utilized in the solid region because there is no significant temperature gradient in the solid region. Moreover, the temperature of the interface and the cold wall are equal in solid region, and hence, the non-dimensional temperature in the solid domain is zero.

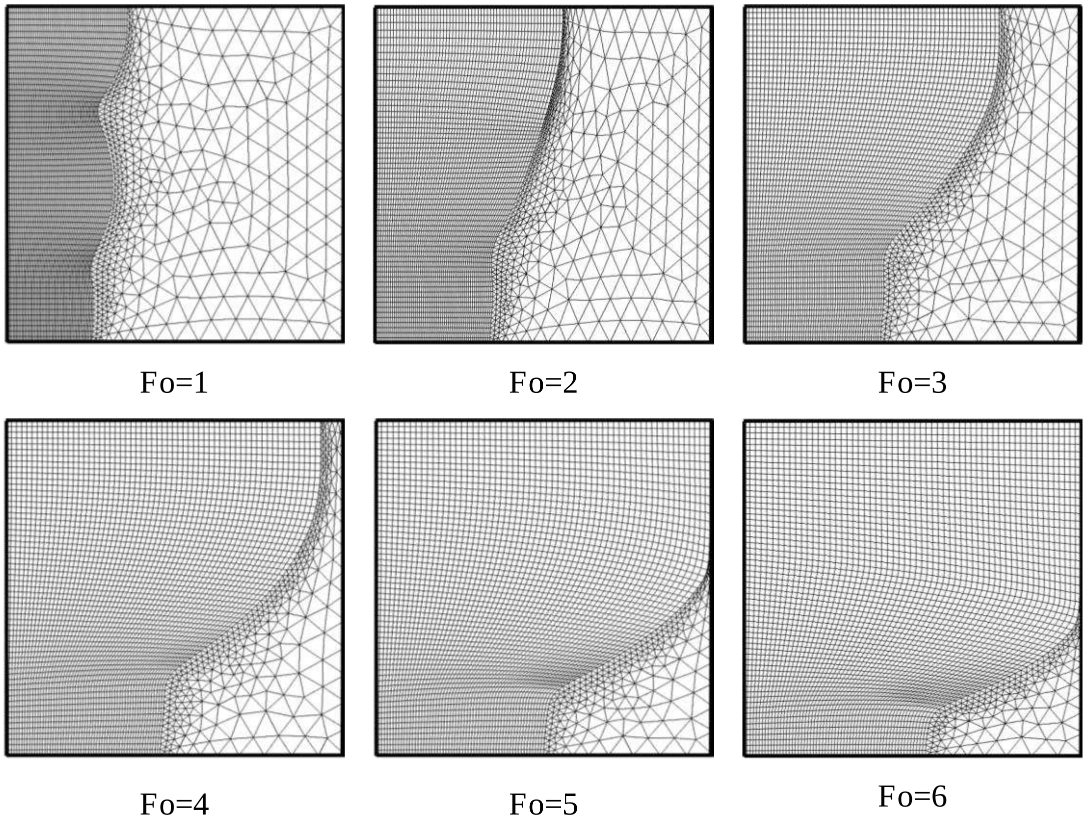


Fig. 7. Display of the deformable mesh during the melting process

Figs. 8-10 depict the melting front surface, streamlines and isotherms patterns of the melted liquid for the different Fourier and Hartman numbers when $Y_0 = 0.5$. Increasing Fourier number develops the melted liquid region and the depth of the melting-front surface. It is obvious that an increase in Hartman number Ha descends the melted liquid space. The Lorenz force is acting as a resistance force and decreases the strength of the convection mechanism in the melting region, and hence, the increment of Hartman number descends the rate of melting. Additionally, the force of the magnetic source acts as a strong barrier for fluid motion next to the hot wall. However, the strength of the magnetic force drastically decreases as the distance between the magnetic source and the fluid increases. This phenomenon can decline the convection heat transfer from the hot wall.

(a) (b) (c)

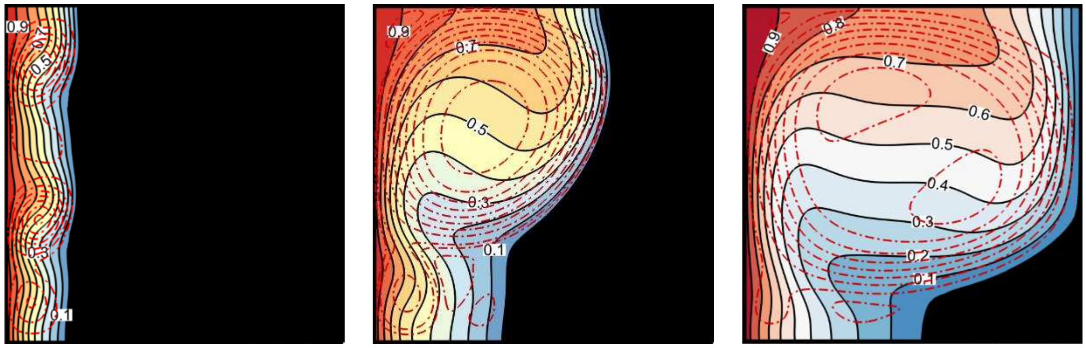


Fig. 8. Display of streamlines and isotherms for Ha=0 and Y_0 = 0.5 when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

(a) (b) (c)

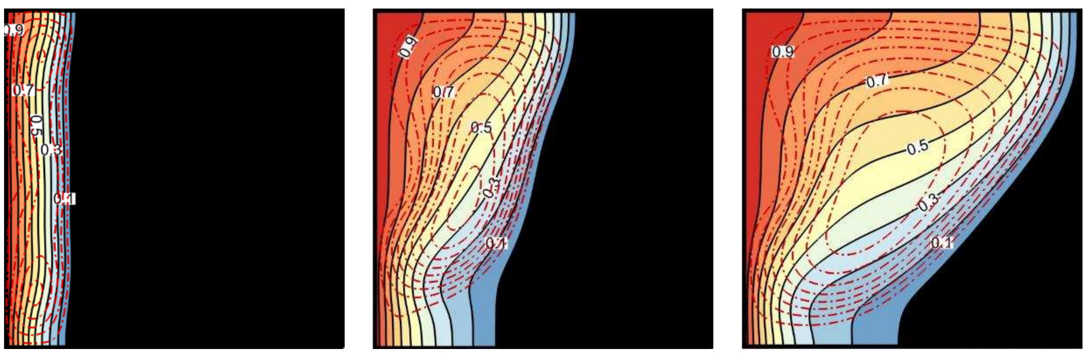


Fig. 9. Display of streamlines and isotherms for Ha=25 and Y_0=0.5 when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

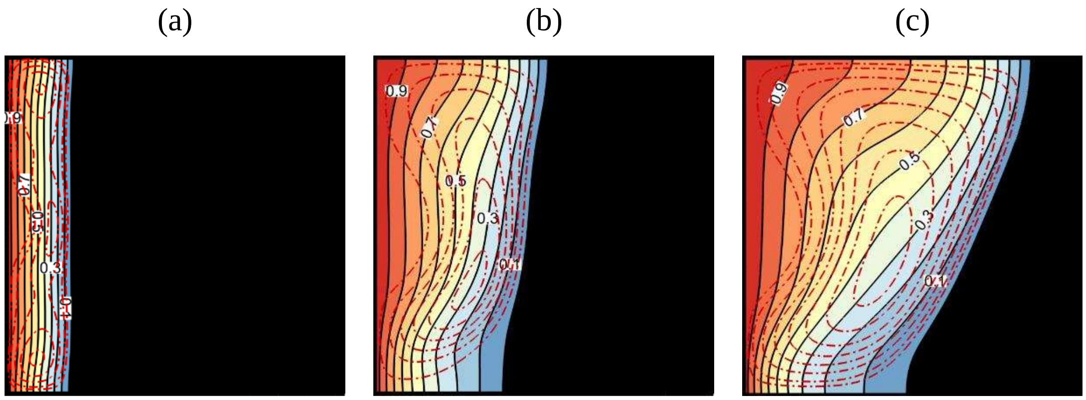


Fig. 10. Display of streamlines and isotherms for Ha = 50 and $Y_0 = 0.5$ when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

The impact of the Hartman number (Ha) on the melting process is studied using the melting-front surfaces. Here, the melting-front surfaces are depicted as curves in 2D. Fig. 11 shows the melting-front surfaces. When Fo = 0.5, the variation of Hartman number does not show a significant effect on the melted liquid space. However, the effect of Hartman number on the melting front progress is notable when the Fourier number is high. The reason is that a direct relationship exists between the Lorentz force and the melted liquid velocity. Indeed, when the molten fraction is low, the motion of the fluid is bounded by the zero velocity at nearby walls. By the increase of Fourier number, the melted region expands, and the fluid has more freedom to move.

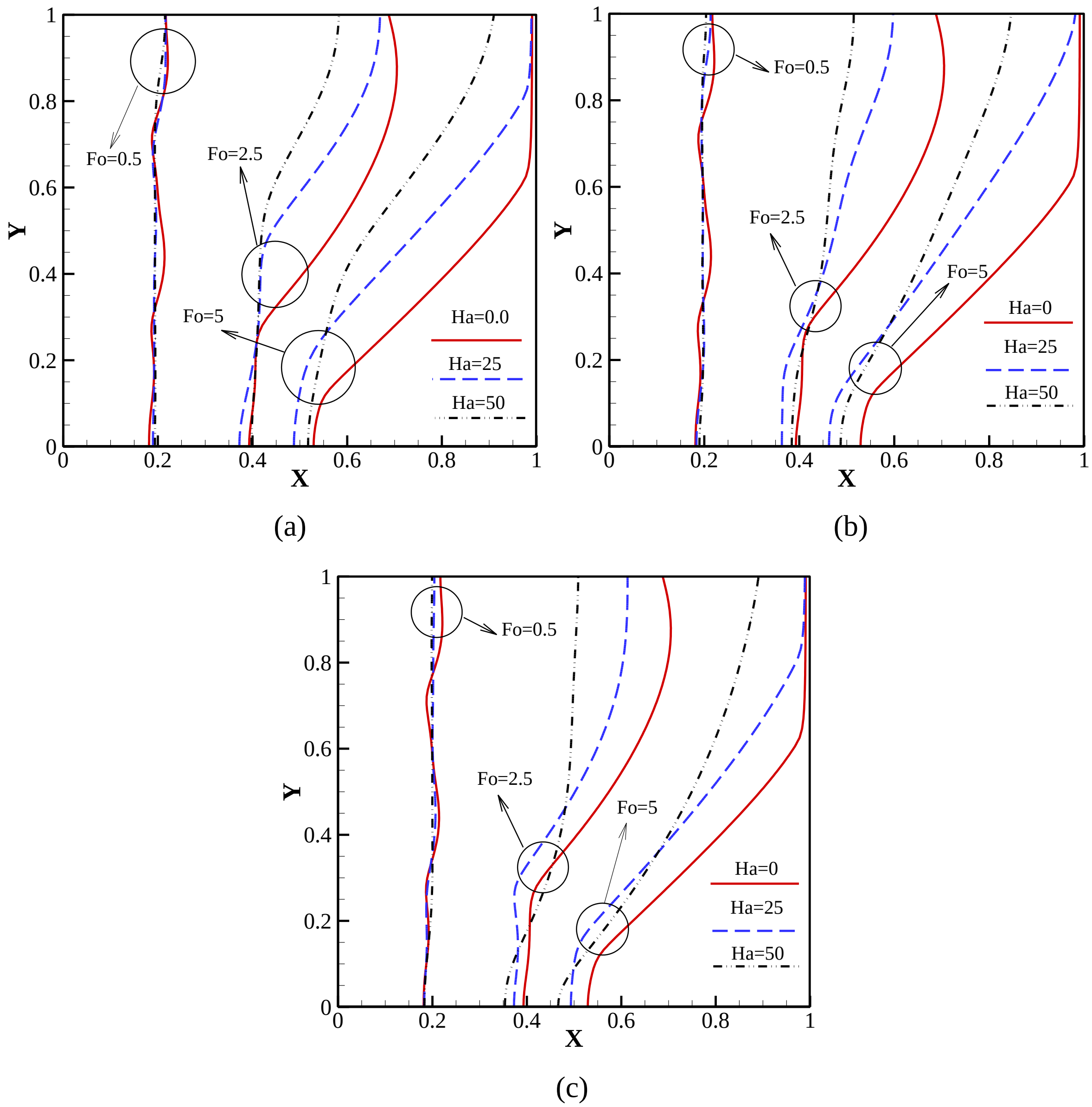


Fig. 11. Melting front surface for various Hartman and Fourier numbers at different locations of the magnetic source: (a): $Y_0 = 0.0$, (b): $Y_0 = 0.5$ and (c): $Y_0 = 1.0$.

Fig. 12 depicts the effects of Hartman number on the liquid fraction as a function of the Fourier number when $Y_0 = 0.5$. As depicted, when Fo is less than 1.15, it can be said that Hartman number does not induce a notable effect on the melted liquid fraction. This is because the Lorentz force is entirely dependent on the velocity. At this stage, the velocities are very low (near to zero), and hence, the conduction heat transfer is dominant, this is the first stage which known as the conduction melting zone. The second stage in the process is dominated by heat convection melting and is utterly dependent on the Hartman number. The convection stage of melting lasts longer at higher Ha. This means that the increment of Ha decreases melted liquid fraction.

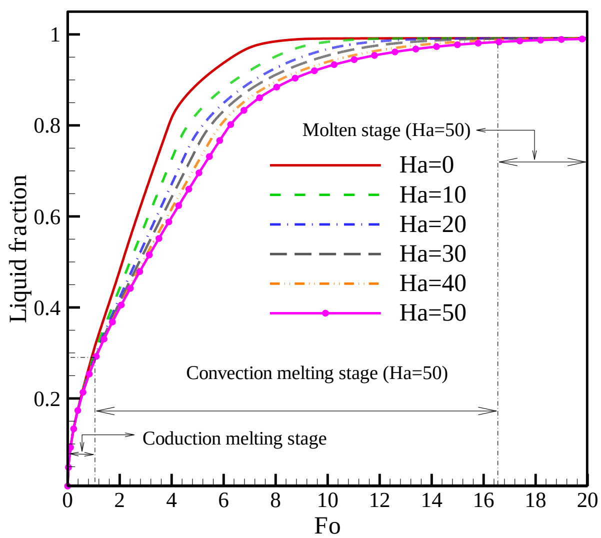


Fig. 12. Liquid fraction as a function of Fourier number Fo for different Hartman numbers Ha

Figs. 13-15 depicts the impacts of the magnetic source locations on the melting process at various Fourier numbers. As can be seen in Figs. 13-15, the magnetic source location does not show a notable effect on the melted liquid space and the depth of melting front for Fo = 0.5; however, the streamlines patterns are entirely affected by the source location. Additionally, it can be seen that the source location significantly impresses the depth of the melting front for further Fourier numbers. As previously mentioned, the force arising from the magnetic field depends on the velocity magnitude, and hence, the effectiveness of the source location is more evident at the higher Fourier numbers.

(a) (b) (c)

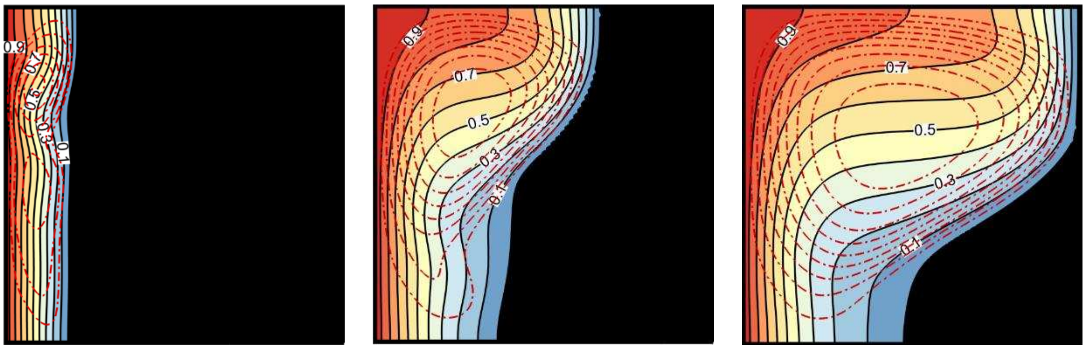


Fig. 13. Display of streamlines and isotherms for $Y_0 = 0.0$, Ha = 25 when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

(a) (b) (c)

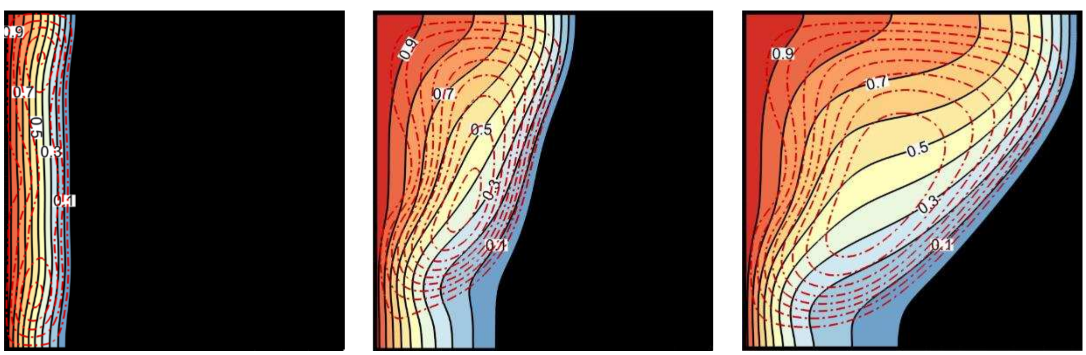


Fig. 14. Display of streamlines and isotherms for $Y_0 = 0.5$, Ha = 25 when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

(a) (b) (c)

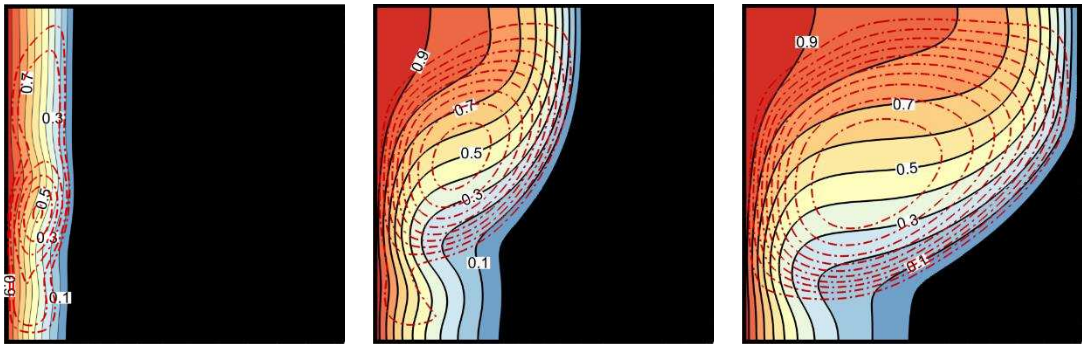


Fig. 15. Display of streamlines and isotherms for $Y_0 = 1.0$, Ha = 25 when Fourier number (Fo) is: (a): 0.5, (b): 2.5 and (c): 5

The melting-front surfaces for the different values of Fourier number and the magnetic source locations are shown in Fig. 16 to demonstrate the effects of the source location on the melting front progress. Once again, when Fo = 0.5, the effect of the increase of Hartman number on the melted liquid space is not notable. However, the effect of Hartman number on the melting front progress is significant when Fourier number is high. The reason is that there exists a direct relationship between the Lorentz force and the melted liquid velocity increasing with Fourier number. Considering Fo = 2.5, when the magnetic source is located next to the bottom of the cavity, the below parts of the melting interface are under the significant influence of the magnetic force, and hence, the melting interface from $Y_0 = 0$ to $Y_0 = 0.5$ is almost a straight line which shows a conduction-dominant mechanism. Relocating the magnetic source to the middle and top of the cavity reduces the effect of the magnetic on the downside parts of the melting interface. When the magnetic source is at the bottom of the cavity, the top parts of the interface are in a large distance from the magnetic source, and hence, the fluid would experience a lower magnetic resistant force at the top parts of the cavity. Hence, the fluid velocity increases and the convection mechanism enhances in this zone. As a result, the top part of the melting interface is further advanced in the solid zone, X ~ 0.6. In contrast, when the magnetic source is in the middle or top of the cavity, the fluid would experience a large magnetic force, and as a result, the convection heat transfer reduces. As seen, in this case, the top section of the melting interface is located at X ~ 0.5.

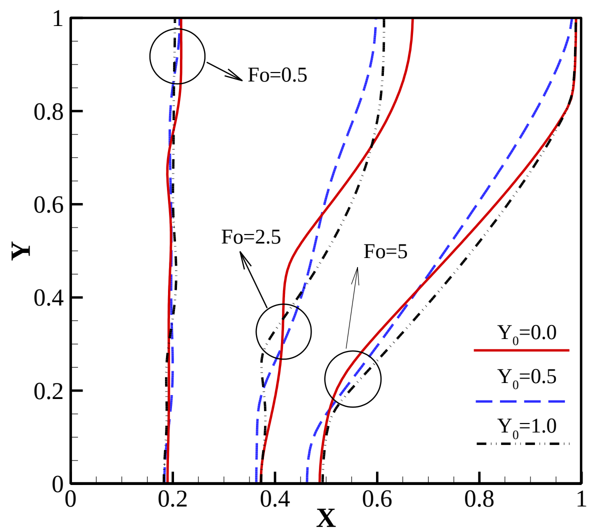


Fig. 16. Melting front surface for different locations of the magnetic source when Ha = 25

Fig. 17 (a) depicts the effects of source magnetic location on the liquid fraction as a function of Fourier number for Ha = 25. As shown, when Fo is less than 2.8, the source location does not alter the melted liquid fraction. As previously mentioned, the first stage is dominated by heat conduction melting. In the convection stage of melting and for $Y_0 < 0.4$, there is not a specific trend as Y_0 approaches 0.4. Whereas, when $Y_0 > 0.4$, an increment of Y_0 increases the liquid fraction. Moreover, Fig. 17 (b) illustrates that the required time for full melting is the highest when $Y_0 = 0.2$. However, the required full melting time is the lowest when the source is located at $Y_0 = 1.0$. It worth noting that the liquid fraction of 0.98 is considered as the criterion in which the PCM is fully melted.

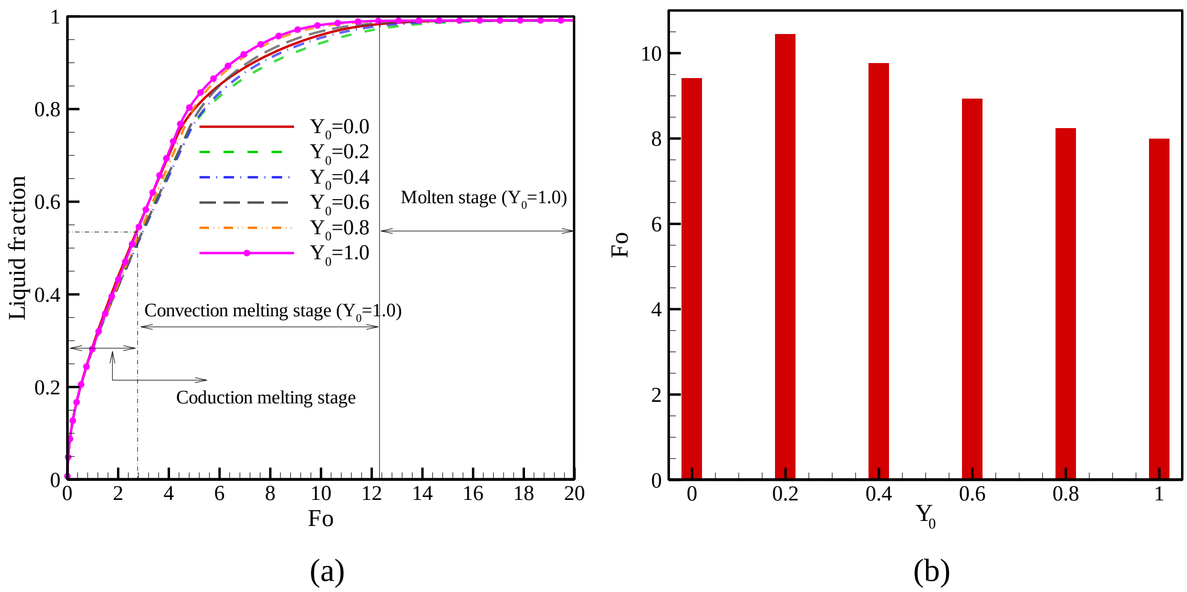


Fig. 17. Melting fraction and required time; (a): Liquid fraction as a function of Fo and (b): The time required to complete melting for different values of Y_0

5. Conclusion

The melting flow and heat transfer in a cavity under the influence of a line-source magnetic field was addressed in this work. The cavity was divided into two domains of liquid and solid PCMs. The governing equations in the melting (liquid) part of the cavity were fluid continuity, the laminar momentum equations, and the heat equation. The governing equation in the solid domain was solely the heat transfer in solids. The phase change effect was introduced at the interface of the liquid and solid domains by considering a constant fusion temperature at the interface. Then, the displacement of the interface was linked to the heat transfer at the domain interface based on interfacial energy balance. The difference of energies reaching the interface from the liquid and solid domains resulted in the phase change and displacement of the interface (Stefan condition).

The governing equations were transformed in a non-dimensional form to generalize the solutions. The finite element method associated with ALE moving grid technique was utilized to solve the governing equations. The grid check was performed, and the results in some limited cases were compared with the available literature and found in good agreement. The effect of the magnitude of the magnetic field and the magnetic source location on the melting behavior of PCMs were investigated. The outcomes can be summarized as follows:

1- The moving grid technique is capable of handling phase change heat transfer in a cavity enclosure.
2- The magnitude and location of the magnetic field source do not show a significant effect on the natural convection heat transfer at the initial melting times, but with the advancement of the melting-front and the increase of the melt volume fraction, the velocity of the fluid increases and the effect of magnetic field boosts.
3- The magnetic field tends to suppress the natural convective flows and make the melting interface uniform. The location of the magnetic source can affect the shape of the melting front.

By utilizing a uniform magnetic field, the control of the melting interface is only possible by suppressing the natural convection in the entire liquid domain. In contrast, the line source variable magnetic field provides a good way of adjusting the location of the magnetic source for controlling the melting process. The focus of the present study was the analysis of the melting process subject to a variable magnetic field; however, the solidification process is also essential in the design of PCM containers and metal casting. Hence, analysis of solidification of PCMs subject to the variable point-source magnetic field can be subject of future studies.

Acknowledgments

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