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Between the impossible and the everyday: optimisation-driven conceptual design of a large transfer truss†

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Synopsis

This article describes the use of structural optimisation at the conceptual design stage to identify materially-efficient solutions which incorporate buildability considerations. In the proposed approach a minimum weight solution is first identified, providing a benchmark against which other designs can be judged. However, as this solution will often be complex in form (and effectively impossible to construct in practice), additional constraints are then gradually introduced to rationalise it, and to explore the regions of the solution space which separate it from simpler, more familiar solutions. This enables the designer to balance material-efficiency and complexity in a more informed manner. The efficacy of the approach is demonstrated via application to a case study transfer truss design.

Introduction

Recent advances in affordable computing power means that optimisation techniques, once primarily the preserve of aerospace and high end automotive designers, are becoming accessible to the wider structural design community. The most general forms of structural optimisation are *topology optimisation* or, for frame structures, *layout optimisation*. Both of these involve starting with a blank space, the *design domain*, and generating a geometry for the structure based on mathematical rules. These techniques have the potential to achieve the most materially-efficiency designs possible, though are not currently commonly used in structural engineering practice, partly due to the complexity of the forms they generate.

For example, consider the 50m span basement transfer truss required for a hotel development project (Figure 1). A range of designs for the problem are presented in Figure 2. In Figure 2a the basic truss structure derived manually is shown, which resembles a Warren truss. In contrast the solution shown in Figure 2c, obtained using numerical layout optimisation, would in theory consume much less material. However, this comprises many thin members, each of different length and cross section. This is a close approximation to the true theoretical solution for this problem, which can be obtained based on the principles set out by Michell in 1904[1], and which includes an infinite number of infinitely thin members.

Application of new manufacturing methods have the potential to remove some barriers to fabrication. For example, additive manufacturing, or '3D printing', techniques are being developed for a range of materials, including concrete[2] and steel joints[3]. Alternatively, steel joints cast using 3D printed moulds can provide a means of avoiding issues with scale and certification. The use of 3D printing in the production of joints for a truss may reduce the costs associated with unusual joint configurations and may ultimately allow more tailored designs to be considered.

[†] This paper is based on a poster prepared by the first author that was awarded First Prize at the 2017 IStructE Young Researchers' Conference.

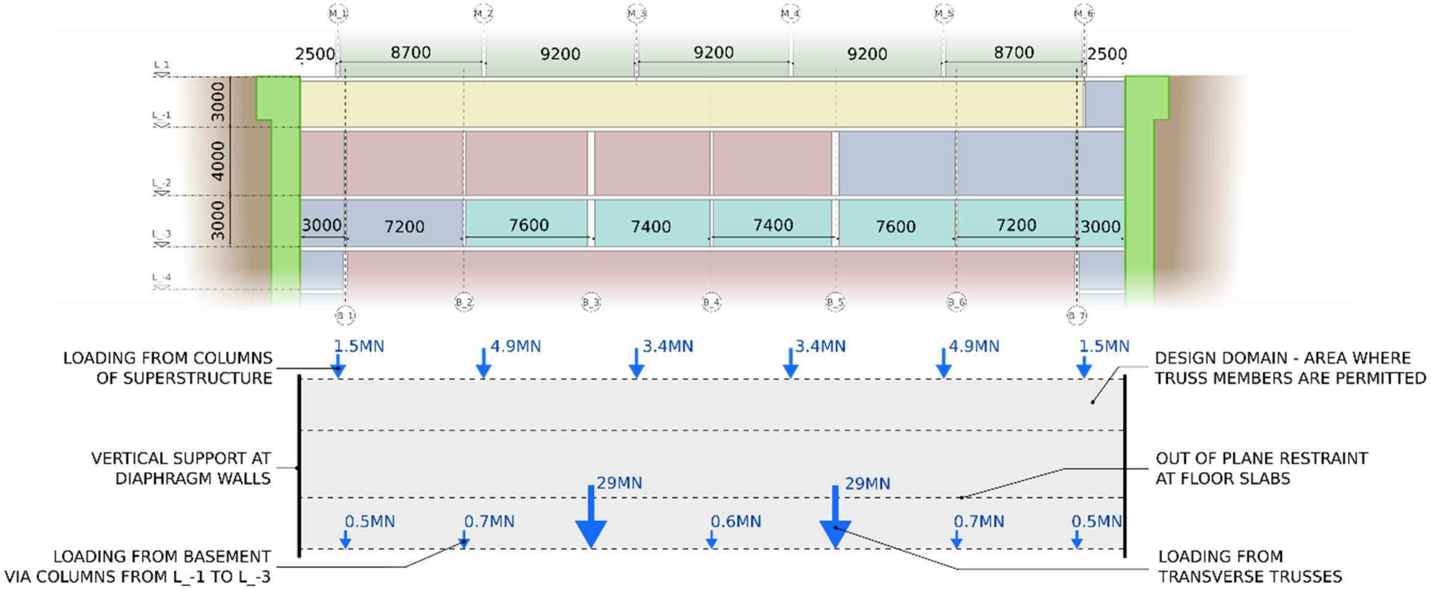


Figure 1: Basement transfer truss: configuration and applied load magnitudes

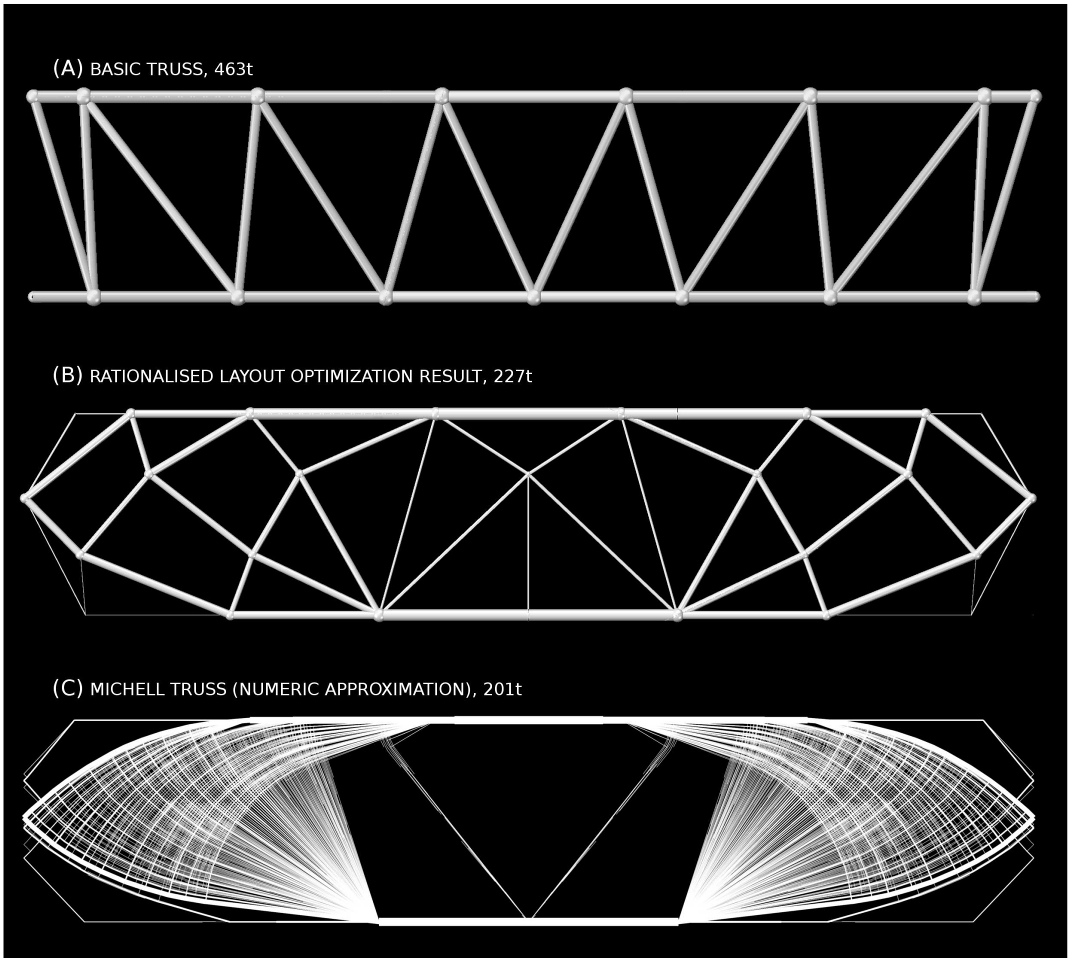


Figure 2: Basement transfer truss: sample discrete designs for the problem shown in Fig. 1

Alternatively, it is possible to rationalise optimum forms to make them easier to fabricate. Previous studies have shown that simplified structures that are similar in form to the optimal layout have the potential to retain a significant level of material-efficiency with much lower complexity[4]. This is explored here. One such simplified structure is shown in Figure 2b; this has a weight which is little higher than the identified optimum form, and less than half that of the manually derived truss (Figure 2a).

Based on this observation, the method described here involves first finding a minimum weight structure without imposing limits on complexity, and then gradually rationalising the design by accounting for practical considerations. Here the term 'structural complexity' is used to refer a range of features; considering minimum weight structures, the number of members and the number of different cross sections are both commonly identified as major issues. Therefore the method progressively removes members and/or standardizes cross-sections, to lead to a gallery of candidate designs.

However, it is unlikely to be possible to find forms that are both very simple and very lightweight. Instead a choice will need to be made concerning the trade-off between material-efficiency and complexity, as illustrated on Figure 3. A demonstration of this will be provided, using the hotel basement transfer structure problem already referred to, from a project by Expedition Engineering.

To achieve this, a number of numerical layout optimization methods developed at the University of Sheffield are employed. These methods are implemented in the Limitstate:FORM[5] design optimisation software; this is now commercially available. A feature of the software is that the user can manually remove or modify individual structural members during the optimisation process. The study demonstrates the potential to achieve greater material-efficiency through exploration of the solution space, the set of all possible feasible solutions to a problem.

It is found that a wide range of possibilities lie between the mathematically optimal, but impractical, forms at one extreme, and the forms typically used in practice today at the other extreme. This allows the design team to make informed decisions on the implications of using a less conventional layout and to indicate where an investment in more complex detailing may be justified. Conversely, it will also indicate when benefits are marginal, and therefore when a more standardised design is likely to provide the most economical choice.

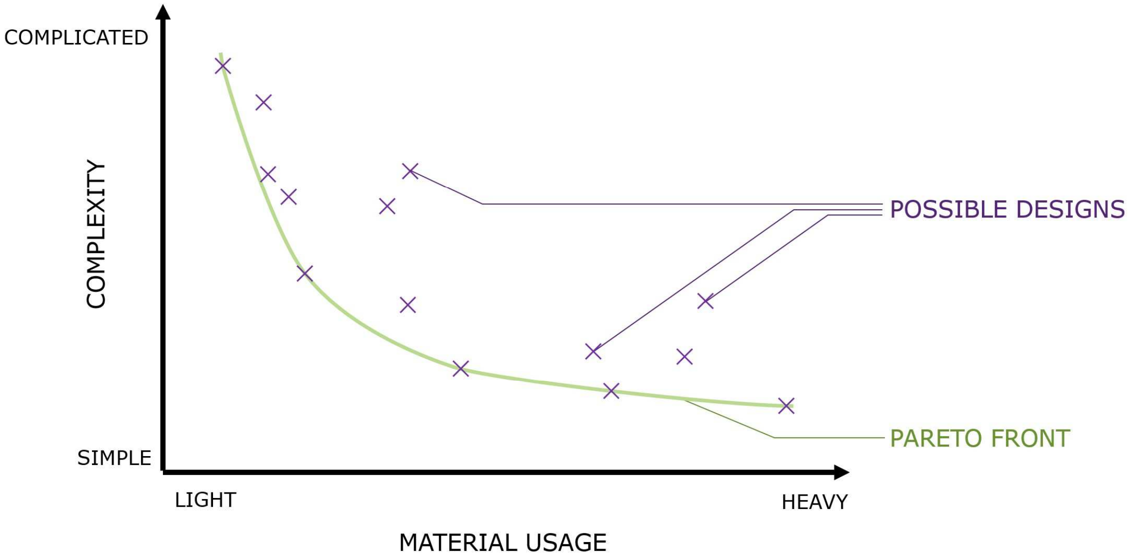


Figure 3: Example of trade-off between material usage and structural complexity

Available optimisation techniques

Heuristic optimisation methods such as genetic algorithms have proved popular in recent years, largely due to the ease with which a non-specialist user can include real-world constraints; a description of several such approaches and their application has been given by Debney[6]. However, these methods have severe drawbacks. For example, the starting points for these methods are usually randomly generated, or based on intuition, but will often influence the final output, likely leading to a sub-optimal design being obtained (i.e. corresponding to a “local hollow”, which may be much higher than the “valley bottom” shown in Figure 4). Additionally, with these methods there is no way of knowing how much further benefit is possible once a solution has been found. This may lead either to unnecessarily inefficient designs being accepted, or alternately to substantial effort being wasted attempting to improve on a design which is already optimal or near-optimal.

Here, an alternative two step optimisation approach is proposed. In the first step the problem is simplified so that only the essential physics is modelled, in this case meaning that only equilibrium and stress limit considerations remain. This allows a reference solution to be obtained, providing a lower bound on the structural volume (or weight). The second step is to apply various methods to move through the solution space to locate promising solutions, if possible in close proximity to the reference solution, as shown in Fig. 4b.

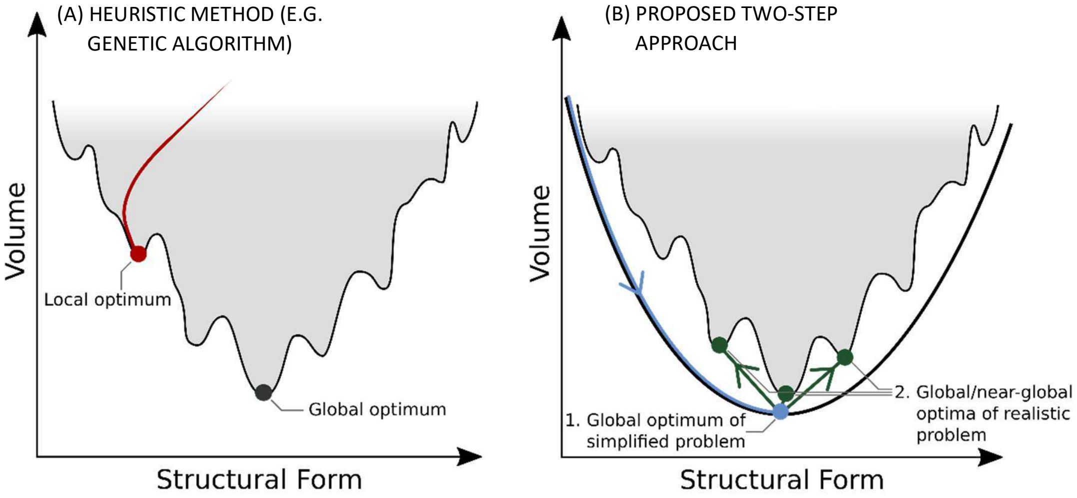


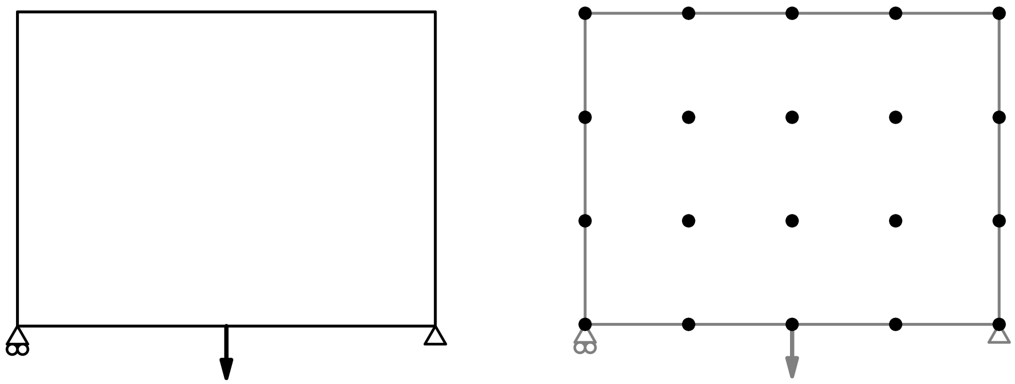
Figure 4: Methods of treating real-world, non-convex, design problems

For the identification of minimum weight structures, continuum based topology optimisation methods are commonly used. These routines are now included in many general purpose finite element software packages, and are widely used in the aerospace and automotive industries. However, they are less obviously useful in the design of building structures. This is because a typical building structure comprises a sparse assembly of discrete elements, with a very low associated volume fraction (the proportion of original design domain which is occupied by structure after the optimisation), beyond the normal working range of continuum topology optimisation routines. Figure 5 shows an example continuum optimisation result for the present case study, produced using the freely available MATLAB script developed by Sigmund[7].

Figure 5: Basement transfer truss: continuum topology optimisation solution

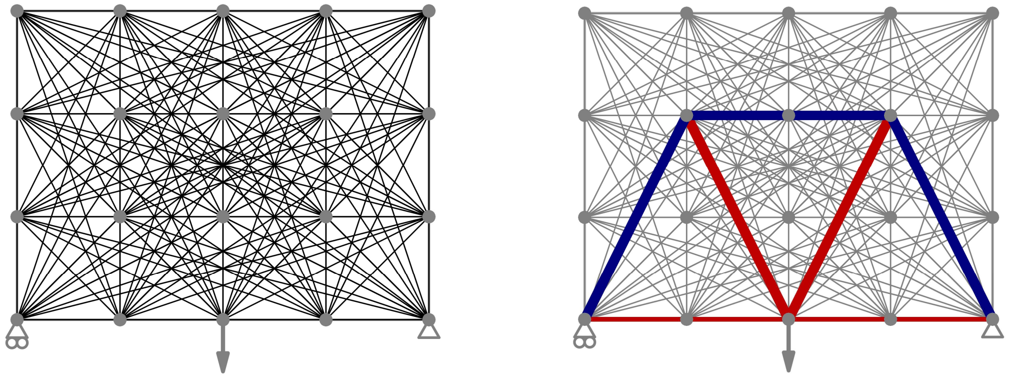
For building frames, it is more appropriate to instead work with discrete structural elements, as is proposed here. In this case numerical layout optimisation employing a *ground structure*[8] can be used, in which the design domain is populated with nodes, then interconnected by potential members as shown in Figure 6. Mathematical routines can then be used to identify the subset of members which form, for example, the structure which consumes the minimum amount of material. In the low resolution example shown in figure 6, this resembles a deep warren truss; at higher resolutions the minimum weight solution closely resembles a half bicycle wheel, with a semi-circular compression member and many radial ties. This demonstrates key characteristics of minimum weight structures; members are axially stressed and bending is eliminated, furthermore, compression and tension members are approximately orthogonal and lie along lines of principal virtual strain.

Here the optimisation problem considered throughout is to minimise the volume of material consumed, subject to equilibrium and strength constraints, using a plastic multiple load case formulation[9]. The mathematical optimisation problem is linear, which ensures that the global optimum for the given problem can be found, and the availability of efficient solvers allows problems with millions of potential members to be solved in seconds on an ordinary desktop computer. A Python script implementing the ground structure-based layout optimisation method is freely available[10].



(a) Design domain with loads and supports

(b) Potential nodes



(c) Potential members forming the ground structure

(d) Resulting optimised (minimum volume) layout

Figure 6: Stages in the numerical layout optimisation method.

Case study: basement transfer truss

The case study concerns the concept design of a steel transfer truss providing a 50m clear span between the retaining walls on opposite sides of a deep basement (Figure 1). This carries the loads from a 5-storey building and a garden at ground level. It also supports a pair of 50m span transverse trusses which apply a substantial load at the bottom of the design domain. The permitted structural depth is 10m, covering three levels below ground floor level. The intermediate floors can be used to provide out-of-plane bracing against buckling of the truss elements.

This provides an ideal application for optimisation methods. The combination of loading from the transverse trusses and the building above provides a specific and unusual configuration, meaning that the minimum volume solution may not be familiar or immediately intuitive. Also, because the basement levels through which the truss passes are primarily plant and 'back of house' spaces, there are far fewer constraints on the locations of structural members than in more visible areas of the project – although the approach described is equally applicable to arbitrary design domains. Finally, the structure is to be constructed using fabricated hollow square sections, constructed from S355 steel plate.

The LimitState:FORM[5] software was used to undertake the optimisation study described; this implements the ground structure based layout optimization procedure described in the previous section. The software also allows deflection limits to be specified and irregularly shaped design domains to be defined, as well as permitting some further basic structural analysis internally and allowing export to external software (e.g. Oasys GSA) to enable more detailed analysis and design to be undertaken.

For the purposes of this exploratory study, the permissible strength in compression was reduced to approximate the effect of member buckling; the value of limiting compressive strength varied from 95% of the yield strength in the shortest members, to zero in members unrestrained for over 19m. Both uniform and pattern loading (with the transverse truss on one side only factored favourably) cases were considered, resulting in three load cases in the optimisation problem.

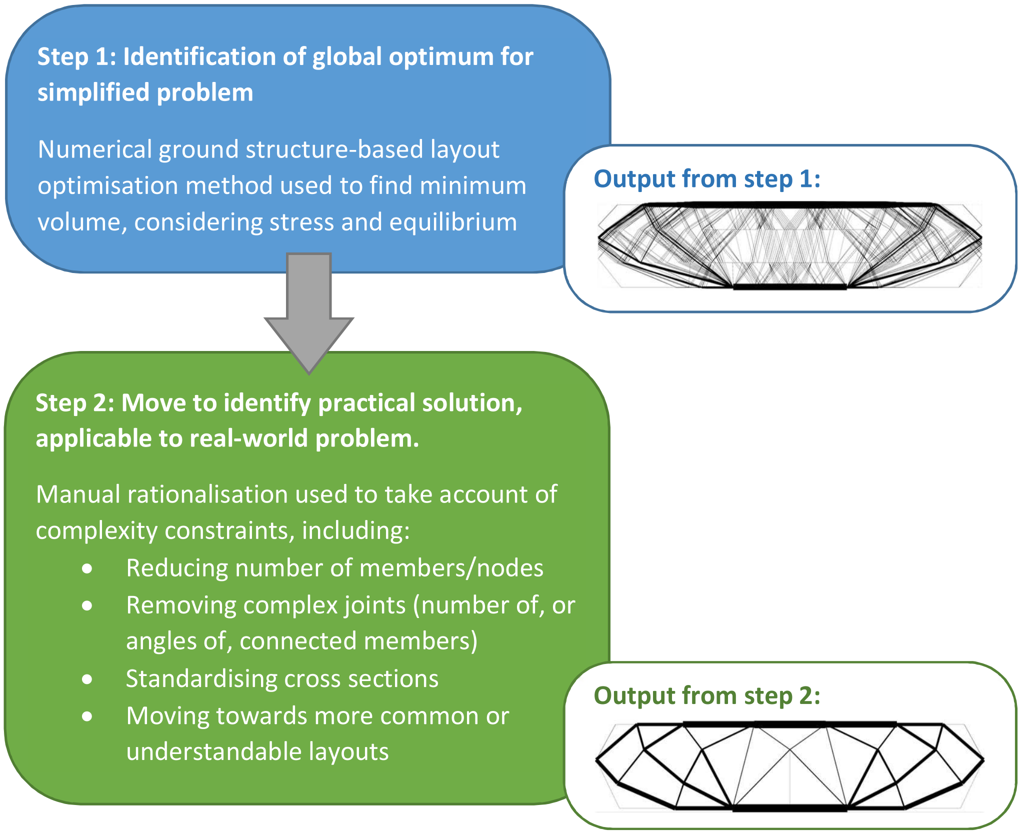
Using layout optimisation the structure is usually modelled as a pin-jointed truss; therefore any node which is not supported out-of-plane by a floor plate or support will be unrestrained against buckling. It would be possible to resist this form of buckling by using nodes/joints with moment capacity. However, in order to ensure all designs were comparable, in this study joints were permitted to lie only on floor levels and up the sides of the design domain.

Although deflections are often critical in the design long span structures, for sake of simplicity in the present study only the ultimate limit state was considered initially (i.e. stress constraints). However, the Limitstate:FORM software can also account for deflections, and that deflection governed problems actually offer greater potential savings with the use of optimisation methods; this is discussed later in the article.

Interactive rationalisation of the structure

The LimitState:FORM software allows the engineer to interactively edit an optimised solution, for example to make it easier to build. This forms the second step of the two step process outlined in figure 4, i.e. moving from the solution of the simplified problem to a design that is a feasible solution for the real-world problem; see figure 7.

Following a manual edit, a secondary optimisation problem is solved to ensure the edited structure is both stable and as lightweight as possible. This uses the same volume minimising objective function and equilibrium/stress constraints as the initial problem; however the reduced ground structure used in the manual editing stage allows simpler structures to be produced. This process can be repeated a number of times if desired. Each secondary optimisation problem includes linear size optimisation and non-linear geometry optimisation steps, where the latter allows joint positions to be adjusted. Each of these optimisation problems can be solved very quickly, in just a few seconds for the problems shown here, permitting rapid exploration of the solution space.



Step 1: Identification of global optimum for simplified problem

Numerical ground structure-based layout optimisation method used to find minimum volume, considering stress and equilibrium

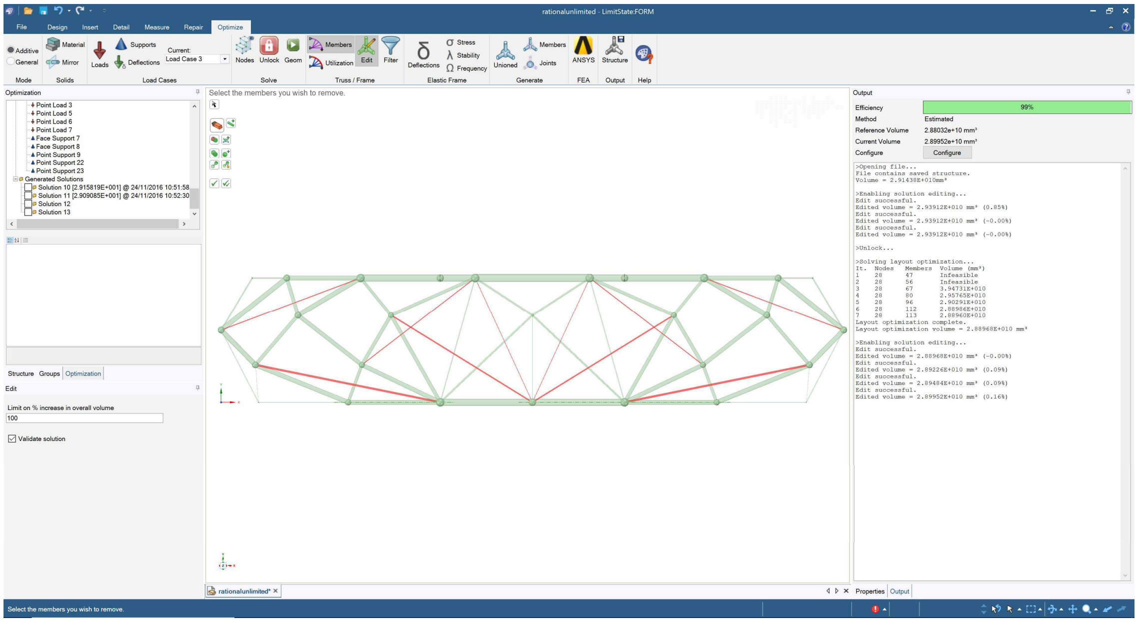
Step 2: Move to identify practical solution, applicable to real-world problem.

Manual rationalisation used to take account of complexity constraints, including:

- Reducing number of members/nodes
- Removing complex joints (number of, or angles of, connected members)
- Standardising cross sections
- Moving towards more common or understandable layouts

Figure 7: The proposed two step process (where step 2 may involve several iterations).

In the interactive step, the designer can individually choose members to add or remove from the structure (Figure 8), with the influence of such edits on the required volume of material to form the structure clear at all times. The designer can choose their modifications based on any potential criteria, such as minimum angles between members, total numbers of joints etc., but is free to break rules if he/she so wishes. This allows for a flexible definition of complexity, which may be specific to the problem at hand or vary as the designer reacts to solutions identified by the optimiser.



Current: Load Case 3

Select the members you wish to remove.

- Point Load 3
- Point Load 5
- Point Load 6
- Point Load 7
- Face Support 7
- Face Support 8
- Point Support 9
- Point Support 22
- Point Support 23
- Generated Solutions
  - ☐ Solution 10 [2.915819E+001] @ 24/11/2016 10:51:58
  - ☐ Solution 11 [2.909085E+001] @ 24/11/2016 10:52:30
  - ☐ Solution 12
  - ☐ Solution 13

Limit on % increase in overall volume: 100

☑ Validate solution

Efficiency	99%
Method	Estimated
Reference Volume	2.88032e+10 mm³
Current Volume	2.89952e+10 mm³
Configure	Configure

```
>Opening file...
File contains saved structure.
Volume = 2.91438E+010mm³

>Enabling solution editing...
Edit successful.
Edited volume = 2.93912E+010 mm³ (0.85%)
Edit successful.
Edited volume = 2.93912E+010 mm³ (-0.00%)
Edit successful.
Edited volume = 2.93912E+010 mm³ (-0.00%)

>Unlock...

>Solving layout optimization...
It.  Nodes   Members  Volume (mm³)
1    28      47       Infeasible
2    28      56       Infeasible
3    28      67       3.94731E+010
4    28      80       2.95765E+010
5    28      96       2.90291E+010
6    28      112      2.88986E+010
7    28      113      2.88960E+010
Layout optimization complete.
Layout optimization volume = 2.88968E+010 mm³

>Enabling solution editing...
Edit successful.
Edited volume = 2.88968E+010 mm³ (-0.00%)
Edit successful.
Edited volume = 2.89226E+010 mm³ (0.09%)
Edit successful.
Edited volume = 2.89484E+010 mm³ (0.09%)
Edit successful.
Edited volume = 2.89952E+010 mm³ (0.16%)
```

Select the members you wish to remove.

Figure 8: LimitState:FORM user interface - red members manually marked for deletion.

Results

The results of the proposed approach are presented on Figure 9 with an increasing number of constraints applied to the structure.

The output of the layout optimisation technique is shown in Figure 9a. These results use a grid of nodes at 0.25m spacing which results in 32 million possible members. This provides a numerical approximation to the Michell truss for this problem. Whilst this structure would clearly be challenging to construct, it provides a close approximation of the lower bound reference solution, which uses the minimum possible amount of material to carry the applied load.

In Figure 9b the nodes are restricted to lie only on the slab lines where lateral restraint is provided, and in Figure 9c the permitted compressive strength is reduced to account for member buckling. At this point, the structure can be said to be physically possible - although unlikely to be buildable using currently available technology. Figure 9d shows the result when the pattern loading cases are considered.

Figure 9e shows the results of automatic rationalisation using the joint length method[11]. A series of designs following manual rationalisation steps are shown in Figure 9f-h. After this, forms using a limited number of different cross sections are found; the result with 4 cross sections is shown in Figure 9i. This is currently performed as a post processing step with the node locations and layout remaining unchanged.

Figure 9j shows the results of layout optimisation where nodes are only permitted to lie at loading points and at the top and bottom of the supports, and Figure 9k shows the case where these members are not restrained from buckling in the out-of-plane direction by the floor plates that they pass through. In Figure 9l the number of different cross sections used in this case is limited to 4. The manual rationalisation method can be used to force the structure to take on easily recognisable forms, such as the Warren truss like form shown in Figure 9m. For cases where the number of different cross sections is further limited, the outcomes are shown in Figure 9n-p, with the design in Figure 9n allowing 4 cross sections in any configuration, in Figure 9o having equal cross sections along the top and bottom chords, and in Figure 9p also having a single cross section for all diagonals.

The options presented in Figure 9 provide the designer with the information needed to quickly appraise the trade-off between complexity of form and weight of steel required for this particular design problem.

Another way of thinking about the journey through the solution space is to consider the outcome at each step following a given design decision; this is illustrated in Figure 10 for the basement transfer truss case study. This means that the engineer can take an informed view on the consequence of each design decision. For example, moving to a familiar Warren-like truss layout leads to a significant increase in the required steel tonnage, even when compared to other solutions with the same number of joints (e.g. the structure shown directly below has the same number of joints).

Commentary

The proposed method provides a quick way of finding a number of possible design concepts. This allows the impact of different design decisions on the overall structure to be evaluated, informing the decision-making process.

As with all optimisation methods it is important that the problem passed to the optimiser accurately represents the real demands on the design. For example, a structure optimised only for a uniformly distributed loading may not be able to resist pattern loading cases, if these are not also included in the problem specification at the outset. Whilst post-optimisation checks can identify such issues, better solutions will generally be obtained if these are included in the initial model.

As a relatively long span structure, deflections are also likely to be important. Therefore, an understanding of how the stiffness of the optimised designs compares to more traditional layouts is of interest. It has been previously demonstrated that for single load case problems the optimal truss layout will be the same irrespective of whether the design is governed by stress limits or deflection limits[12].

This is based on noting that a minimum weight truss structure for a single load case is statically determinate and fully stressed. This means that the constituent truss members will be uniformly strained and thus that the internal strain energy will be proportional to the volume of material used. Since the external work done by an applied load is proportional to the deflection of that load, and as external and internal work must be equal, the layout of the lowest volume design with stress limits will correspond to the layout with the lowest deflection (when all members are subjected to a given stress).

To move between the stress-based solution and the deflection-based one, all cross section areas in the solution can be scaled by an appropriate factor. As the minimum volume layout has both the least material and the highest stiffness, it produces the minimum volume result for a given deflection when scaled (this can be interpreted as the stiffest structure for a given volume). As rationalisation moves a design further from the optimum, its volume increases and also its stiffness reduces. For deflection-governed designs, these effects compound and cause even greater differences in volume.

However, this is not generally the case in multiple load case problems. Whilst this study does have multiple load cases, the fully loaded case dominates the design. The top row of Figure 11 shows stress based designs; the Warren-like truss (right) has both a 40% higher volume than the optimised structure (left) and also a 32% higher deflection. Limitstate:FORM was then used to impose a range of mid-span deflections in the fully loaded case; the results are shown in Figure 11. For each of the deflection-governed cases, the Warren-like truss now requires around 64% higher steel tonnage than the optimal value. The volume penalty on the rationalised form also increases slightly; however, it still offers an increased saving compared to the Warren-like truss. This shows that the potential advantages of layout optimisation are further magnified when the design is governed by deflections.

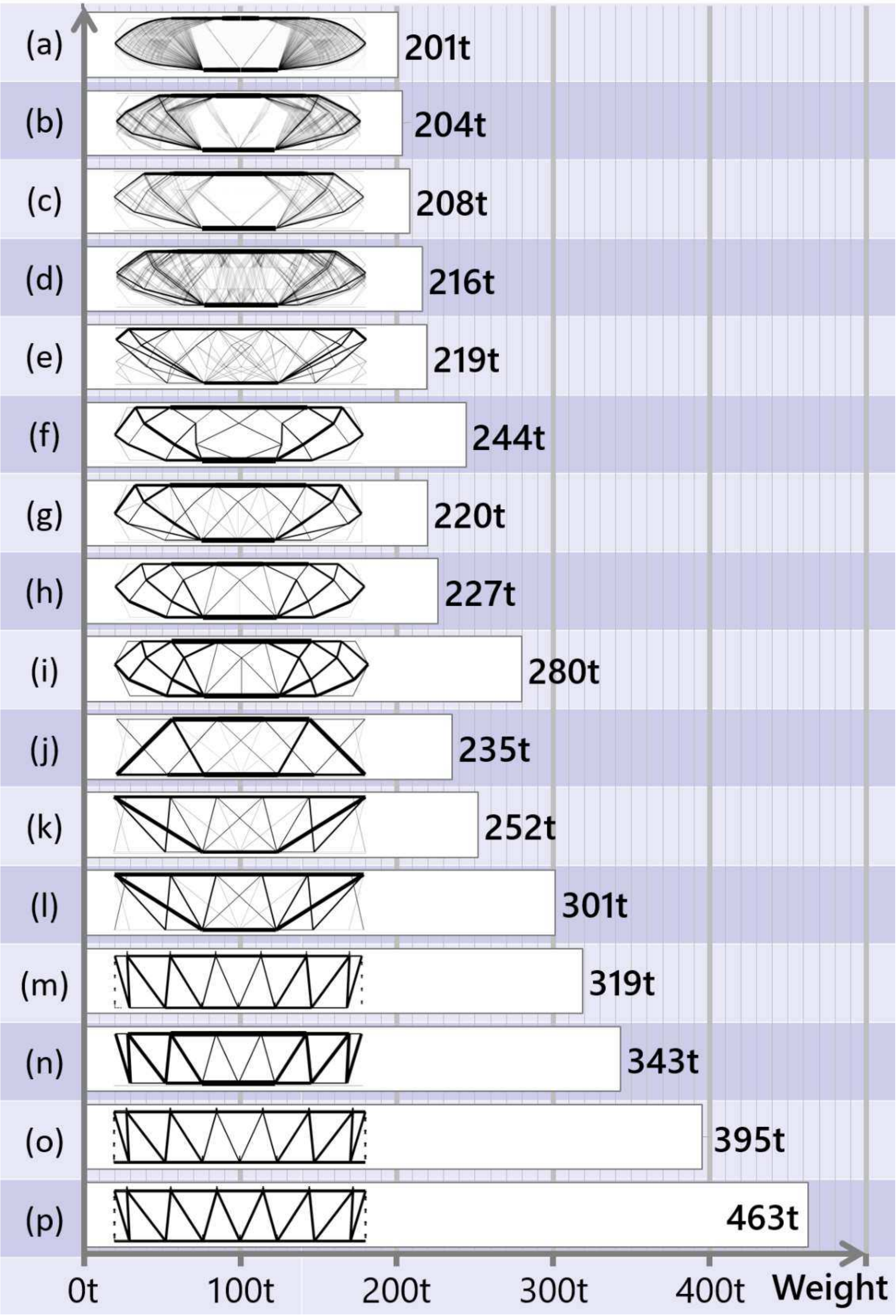


Figure 9: Basement transfer truss: derived concept designs

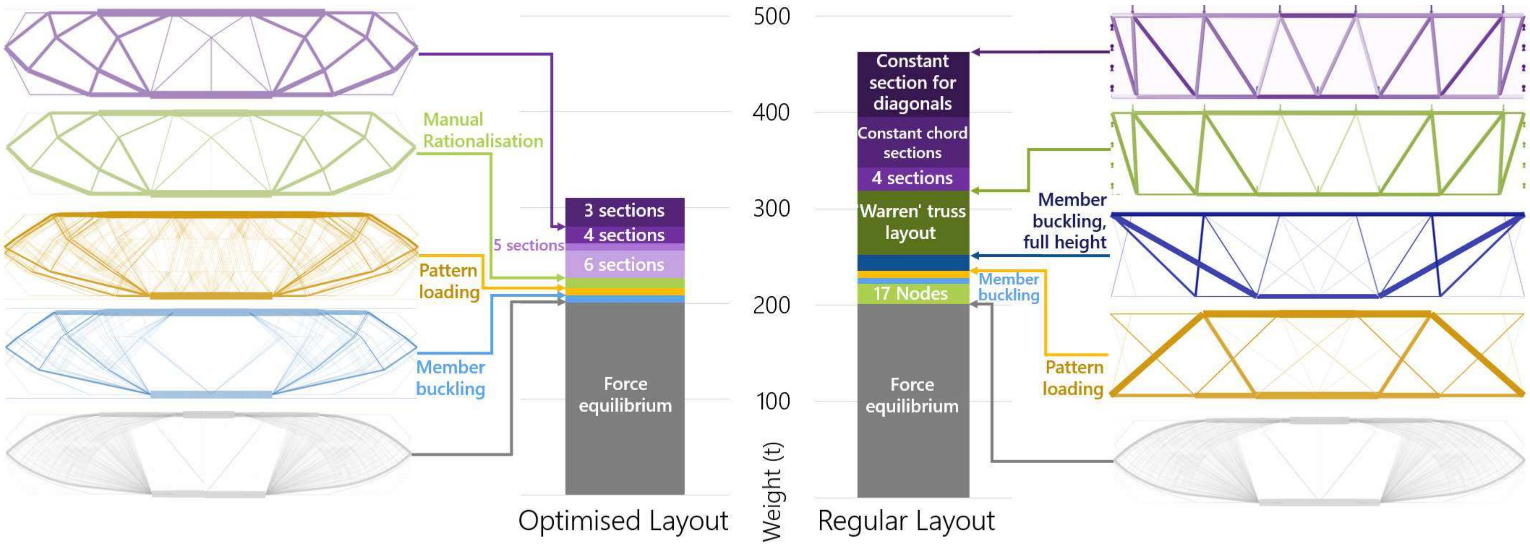


Figure 10: Basement transfer truss: volume increases due to imposed design constraints

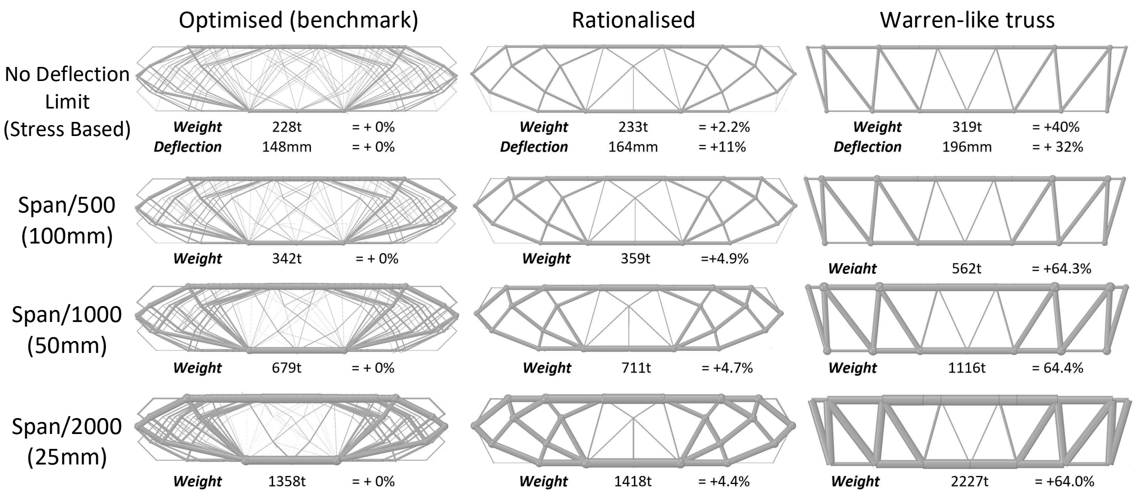


Figure 11: Basement transfer truss: effect of limiting mid-span deflection

Seeking the minimum volume of material is a clear quantifiable goal for use in the optimisation process. This value can potentially be used as a surrogate for other quantities, such as embodied carbon or material cost, although in reality both of these will also be influenced by other factors, including the complexity of the structure.

In addition to complexity, the manual rationalisation method also allows concerns such as construction sequence and aesthetic considerations to be addressed intuitively. Therefore, it may be necessary to maintain some element of manual control even when more automated methods are used. This also provides the designer with more freedom to explore the design landscape than when using traditional optimisation methods, which output only a single design solution.

The truss-based nature of the layout optimization method means that interaction with solutions should be fairly intuitive to anyone familiar with more conventional frame analysis programs. Developing a feel for the best members to add or remove comes with practice. This intuition may be obtained through experience, though informed by knowledge of the fundamental features of minimum weight structures (such as preferring the use of purely axially loaded members, and noting that tension and compression members should ideally intersect at close to 90 degrees in problems dominated by a single load case). In our increasingly resource-conscious society, the training of future structural engineers should foster a culture of inquisitiveness around optimisation and a desire to improve the efficiency of our structures, making use of effective software tools to facilitate this.

Conclusions

Structural optimisation techniques have the potential to help structural engineering designers to realise significant savings in material usage, embodied carbon and/or cost. Whilst it is important that care is taken in formulating a design problem, and in performing detailed checks on generated design solutions, the 50m span hotel basement transfer truss case study described clearly demonstrates the usefulness of optimisation, and that available tools are now becoming sufficiently mature for use in engineering practice at the conceptual design stage.

The two step optimisation approach presented addresses some of the major concerns that have previously limited uptake of optimisation in practice. Using this approach, a wide range of structural options can be generated, helping the design team to make informed decisions on the balance to be struck between complexity and material-efficiency. The results show the potential for significant material savings, even once fabrication constraints are accounted for. This provides the opportunity for designers to explore options derived from highly efficient structural forms, which may not initially be obvious to the designer.

Acknowledgements

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