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**Article:**

Ko, Youngwook and Choi, Jinho (2019) Unsupervised Machine Intelligence for Automation of Multi-Dimensional Modulation. *IEEE Communications Letters*. ISSN 1089-7798

<https://doi.org/10.1109/LCOMM.2019.2932417>

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# Unsupervised Machine Intelligence for Automation of Multi-Dimensional Modulation

Youngwook Ko, and Jinho Choi

**Abstract**—In this letter, we propose a new unsupervised machine learning technique for a multi-dimensional modulator that can autonomously learn key exploitable features from significant variations of multi-dimensional wireless propagation parameters, followed by a real-time prediction of the best multi-dimensional modulation mode to be used for the next resilient transmission. The proposed method aims to embrace the potential of the unsupervised  $K$ -means clustering into the physical layer of non-coherent multi-dimensional transmission. Simulation results show that the proposed scheme can outperform the benchmarks at a cost of simple offline training.

**Index Terms**—Unsupervised machine learning, non-coherent prediction, autonomous multi-dimensional modulation

## I. INTRODUCTION

Due to the limited resources of machine type communications (MTC) devices, new analytics techniques such as machine learning and data analytics are emerging to support reliable connectivity of such MTC networks, which involve big data typically in unknown locations [1].

Since the 3rd Generation Partnership Project (3GPP) systems' MTCs are based on a group of multi-carriers [2], the index modulation for orthogonal frequency division multiplexing (IM-OFDM) that was studied to enhance the reliability and energy efficiency (e.g., see [3], [4]) can be employed for MTC. In IM-OFDM, both index and sub-carrier domains were used in modulation and various approaches have been proposed to improve the reliability. For example, [5] proposed the transmit diversity for IM-OFDM, while [6] studied the adaptive modulation for IM-OFDM with the use of channel information. The cooperative IM-OFDM for relay networks was investigated in [7], while the potentials of spread IM-OFDM were studied to increase the reliability and efficiency in [8], [9]. However, the existing works only consider pre-defined transmission protocols that do not adapt to their deployment environments. Furthermore, since MTC devices are usually affected by diverse physical layer features, which are difficult to be uniquely characterized by a few parameters across diverse MTC's surrounding, the existing works may hardly guarantee the expected reliability with limited knowledge of surrounding.

Machine learning techniques have been very recently explored at the physical layer of wireless communications [10]–[13]. For example, [11] developed a deep-learning (DL) auto-encoder for single-input multiple-output (SIMO) communication systems with deep neural networks (DNNs). In OFDM

systems, the DL was also utilized to design a joint channel estimation and signal detection receiver [12]. [13] used the DL for single-dimensional modulation recognition at the receiver.

In this paper, we focus particularly on the potential of unsupervised machine learning techniques in the physical layer of multi-carrier communications. To the best of our knowledge, this work is the first attempt to embrace the potential of machine learning into an autonomous IM-OFDM in specific, and generally, autonomous multi-dimensional modulation (MDM) where data bits are mapped to several domains. We develop multiclass clustering, as a primary task in machine learning, for automation of MDM transmission in a non-coherent manner. So-called, Auto-MDM aims to automatically adapt a modulation mode by clustering variations of multi-dimensional training channel vectors, which is not feasible at the classical adaptive modulation under limitation of an established one-dimensional threshold between modes. From such training, we learn exploitable physical layer features and offline obtain a clustering set that is used to cluster a new multi-dimensional observation and to autonomously predict the best mode for the next resilient communications (i.e., learning-driven prediction). Over the existing multicarrier transmissions that are limited to the fixed modulation with lack of knowledge on the surroundings, novel contribution of the Auto-MDM is three-fold: (i) to embrace the unsupervised learning (i.e., a simple end-to-end training and unsupervised  $K$ -means clustering) into the multi-dimensional modulation design; (ii) to automatically predict the best mode for Auto-MDM across variations of multiple sub-carrier signals with a non-coherent observation; and (iii) to obtain a better reliability with variable rates, outperforming the existing IM-OFDM alternatives at the cost of simple, offline training. The proposed work provides insights into a convergence of machine learning and MDM transceiver.

## II. SYSTEM MODEL

Consider a multi-carrier system which consists of a block of  $N_c$  sub-carriers in a time division duplexing (TDD)-based MTC application [2]. Each block can be divided into  $G$  groups of  $N$  sub-carriers, i.e.,  $N_c = GN$ . Denote by  $\mathcal{M}$  and  $\mathcal{C}$  a set of complex data constellations and a codebook of  $N$  codewords, respectively, where  $\mathcal{M} = \{\mathcal{M}_0, \mathcal{M}_1, \dots, \mathcal{M}_T\}$ ,  $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_N\}$  and  $M_t = |\mathcal{M}_t|$  denotes the cardinality of  $\mathcal{M}_t$ -ary constellation,  $\forall t$ .

For every transmission,  $p (= p_1 + p_2)$  data bits per group are divided to two parts such that  $p_1$  and  $p_2$  bits are used to modulate  $k (\leq N)$  data symbols ( $\mathbf{s} = [s_1, \dots, s_k]^T$ ) and

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indices of  $k$  active codewords ( $\theta_i = \{i_1, \dots, i_k\}$ ), respectively, where for a given  $\mathcal{M}_t$ ,  $s_a \in \mathcal{M}_t$  and  $i_a \in \{1, \dots, N\}$  for  $a = 1, \dots, k$ . Denote by  $\mathbf{x}$  the signal vector to be transmitted over  $N$  sub-carriers with the aid of precoder matrix  $\mathbf{C}_\theta = [\mathbf{c}_{i_1}, \dots, \mathbf{c}_{i_k}]$ , where  $\mathbf{x} = \mathbf{C}_\theta \mathbf{s}$  and details on the choice of  $k$  are addressed in Section III. Notice that  $p_1 = k \log_2(M_t)$  bits correspond to  $k$  data symbols (DS) while  $p_2 = \lfloor \log_2 |\Sigma_\theta| \rfloor$  bits to codeword index symbol (CIM),  $\theta_i$ , where  $\Sigma_\theta$  denotes the constellation of  $\theta_i$ 's. Due to the fact that each group performs independently, without loss of generality, we present only one group hereinafter. The downlink received signal for each group can be given, in the frequency domain, by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}, \quad (1)$$

where  $\mathbf{x} = [x_1, \dots, x_N]^T$ ,  $x_n = \sum_{a=1}^k c_{i_a n} s_a$ ,  $c_{i_a n}$  represents the  $n$ -th entry of  $\mathbf{c}_{i_a}$ ,  $\mathbf{H} = \text{diag}(H_1, \dots, H_N)$  denotes the fading channel matrix of  $N$  sub-carriers,  $H_n$  represents the channel coefficient of sub-carrier  $n$ , and  $\mathbf{v} = [v_1, \dots, v_N]^T$  is the additive white Gaussian noise vector, i.e.,  $v_n \sim \mathcal{CN}(0, 1)$ .

Assume the followings: A1) the transmitter does not know the channel matrix  $\mathbf{H}$ , while the uplink received signal energy per sub-carrier is available to the transmitter; A2) the transmitter adapts a modulation order  $M_t$  of each group, learning multi-dimensional physical layer features in a non-coherent manner, with no use of pre-defined switching thresholds; and A3) codewords  $\mathbf{c}_n$  are orthogonal to each other with  $\|\mathbf{c}_n\|^2 = 1$  and  $\|\mathbf{c}\|_0 = N, \forall n$ . Even through non-coherent manner and without any pre-defined switching thresholds, the transmitter intends to automatically predict and choose  $M_t$  used for the transmit vector  $\mathbf{x}$ , which differs from the existing multi-carrier transceivers that use a fixed modulation mode.

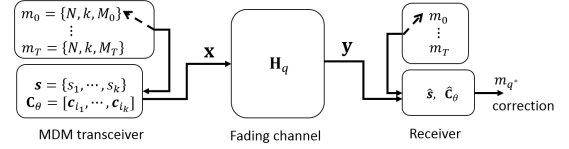
Due to the fading channel ambiguity and A1)- no channel state information at the transmitter (CSIT),  $M_t$  cannot be uniquely selected from the variations of uplink signal across  $N$ -dimensional sub-carriers, even in the noiseless case. In such unknown real-time wireless environments, multi-dimensional randomness and fading channel ambiguity are the dominating factors that limit the performance of non-coherent multi-dimensional transceiver. This is the main problem to resolve through the unsupervised machine learning, jointly developed by the non-coherent multi-dimensional modulator.

### III. LEARNING-DRIVEN AUTO-MDM

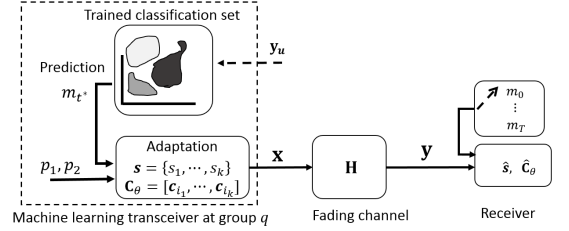
We propose new unsupervised learning-driven algorithms for Auto-MDM. To this end, the manipulation of end-to-end training sets to be embraced for the proposed Auto-MDM transceiver is firstly presented, followed by multi-dimensional clustering and online prediction test for Auto-MDM.

#### A. End-to-End Training Set of Auto-MDM

The structure of the proposed Auto-MDM is depicted in Fig. 1. It consists of training and testing procedures, taking into account clustering and learning-driven MDM automation, respectively. Specifically for the clustering in training procedure, we propose an end-to-end training structure, the key advantage of which is that the training becomes straightforward. In the training structure, the channel observation at a given time is



(a) Classification model of Auto-MDM



(b) Learning-driven Auto-MDM framework

Fig. 1: The structure of machine learning Auto-MDM

assumed to be known at the receiver. Consider  $Q$  simulated training data ( $\mathbf{H}_1, \dots, \mathbf{H}_Q$ ) based on a stochastic channel model, and denote by  $m_t = (N, k(t), \mathcal{M}(t))$  the  $t$ -th possible modulation mode that is related to  $k(t) \in \{1, \dots, k\}$  active CIMs and  $\mathcal{M}(t) \in \mathcal{M}$  constellation. For each simulative observation  $\mathbf{H}_q$ , the transmitter and receiver collaborate to supervise a right choice of  $m_t$  so that the bit error probability (BEP) (denoted by  $P_e$ ) remains less than or equal to the target level  $\mu$  (i.e.,  $P_e \leq \mu$ ), at the largest data bits ( $R_t$ ) that rely on  $m_t$ . Note that the value for  $Q$  trades off complexity against learning accuracy, as the common overfitting problem.

In particular, prior to the clustering of training procedure, we need to constitute a training data set. Denote by  $\mathbf{z} = \{\mathbf{z}_1, \dots, \mathbf{z}_Q\}$  the training data set, where  $\mathbf{z}_q = (|H_{1q}|^2, \dots, |H_{Nq}|^2, q^*)$  determines the  $q$ -th simulated training data,  $|H_{nq}|$  is a real-valued amplitude of  $n$ -th sub-carrier of  $\mathbf{H}_q$ , and  $q^* \in \{0, \dots, kT\}$  an index for one of  $(1 + kT)$   $m_t$ s. The key idea is to train the automated MDM over substantial variations across the  $N$ -dimensional boundaries of each simulative channel vector and then to adopt the index of most potential modulation mode (denoted by  $m_{q^*}$ ). Here, we train by the channel vectors, not the received signals  $\mathbf{y}$ , because  $\mathbf{y}$  are additionally distorted by the background noise in less relation to surrounding. So, after the  $Q$  simulative training observations, we will obtain the data set  $\mathbf{z}$  based on a stochastic channel model and the collaboration between transmitter and receiver. Intuitively, such training data set can help relate  $N$  dimensional variations of simulated fading channel coefficients in  $\mathbf{H}_q$  to a supervised index of the chosen  $m_{q^*}$ , for each training observation.

To obtain a better training data set in relation to the prediction accuracy of  $m_t$ ,  $\mathbf{y}$  and  $\mathbf{H}_q$  can be processed offline before the clustering. Referring to the simulative knowledge of the non-adaptive MDM (e.g., the existing IM-OFDM [4]), firstly, the well-known maximum likelihood (ML) detector is employed to estimate the transmitted vector as follows:  $\hat{\mathbf{x}} = \arg \min \|\mathbf{y} - \mathbf{H}_q \mathbf{x}\|^2$ . Based on  $\hat{\mathbf{x}} = \hat{\mathbf{C}}_\theta \hat{\mathbf{s}}$ , this optimum ML scheme is expected to improve the detection of index symbol  $\theta_i$  from the active codeword indices in  $\hat{\mathbf{C}}_\theta$ , as well

as the  $k$  non-zero  $M$ -ary data symbols from  $\hat{s}$ . Based on such pre-processing before the clustering, estimated data bits can result in  $P_e$ . Secondly, the simulative results of  $P_e$  can be used to reveal the choice of  $m_{q^*}$  whose index satisfies  $q^* = \arg \max_t \{R_t | P_e(m_t) \leq \mu\}$  for  $t = 0, \dots, kT$ .

### B. Multi-Dimensional Clustering by Manipulated Training

In order to run the proposed automated MDM, we need to train the transceiver with the data set collected from the training observations. We employ the simple but effective unsupervised learning algorithm (i.e.,  $K$ -means clustering) because of its linear complexity, suitable to low-complexity device applications. Once the training data set  $\mathbf{z}$  is collected, the Auto-MDM is trained offline by clustering variations of  $N$ -dimensional training data into one of the clusters that represents the best modulation mode.

Given the set  $\mathbf{z} = \{\mathbf{z}_1, \dots, \mathbf{z}_Q\}$ , each observation  $\mathbf{z}_q$  is a  $(N+1)$ -dimensional real-valued vector.  $K$ -means clustering aims to partition the  $Q$  observations into  $K_c$  clusters, as shown in Algorithm 1.

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#### Algorithm 1 $K$ -means clustering algorithm

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- 1: Select  $K_c (\leq Q)$ , and generate a set of input data  $\mathbf{z}_q, \forall q$ .
  - 2: **procedure** CLUSTERING( $\mathbf{z}, \mathbf{u}, K_c$ )
  - 3:   **repeat**
  - 4:      $i = i + 1$ .
  - 5:     Form  $K_c$  clusters by assigning all points  $\mathbf{z}_q, \forall q$  into the closest centroid.
  - 6:     Recompute the centroid of each cluster,  $\mathbf{u}_{(l)}, \forall l$ .
  - 7:     **until**  $\Sigma_i = \Sigma_{i-1}$    ▷ We have the answer if the centroids don't change
  - 8:   **return**  $\Sigma_i$
- 

Notice that in the above algorithm, the initial centroids are chosen randomly. Clusters produced vary from one iteration to another. In every iteration, the centroid indicates the mean of the points within the cluster. Specifically at step 5 of the algorithm, the closest centroid is determined by measuring the squared Euclidean distance, which is written, for given  $\mathbf{z}_q$ , by

$$d_{e,l}^2 = \|\mathbf{u}_{(l)} - \mathbf{z}_q\|^2, \quad \text{for } l = 1, \dots, K_c. \quad (2)$$

Thus, the index of the closest cluster that  $\mathbf{z}_q$  will belong to at each iteration becomes  $(l) = \arg \min_l d_{e,l}^2$ . Accordingly, at step 6, if cluster  $(l)$  contains  $n_{(l)}$   $(N+1)$ -dimensional points at a given iteration, then the centroid of cluster  $(l)$  is

$$\mathbf{u}_{(l)} = \frac{1}{n_{(l)}} \sum_{j=1}^{n_{(l)}} \mathbf{z}_{j,(l)}, \quad (3)$$

where  $\mathbf{z}_{j,(l)}$  is the  $j$ -th point within the cluster  $(l)$ .

In Fig. 2, an exemplary clustering set along with four centroids is depicted, experimentally finding multiple boundaries between clusters in the two-dimensional space. Notice that once the iterations are over, the  $(N+1)$ -th entries of  $\mathbf{u}_{(l)}$  at each cluster  $(l)$  for  $l = 1, \dots, K_c$  are re-computed to provide an integer index of the modulation mode that appears most likely within the cluster  $(l)$ , denoted by  $\mathbf{u}_{(l),N+1}$ .

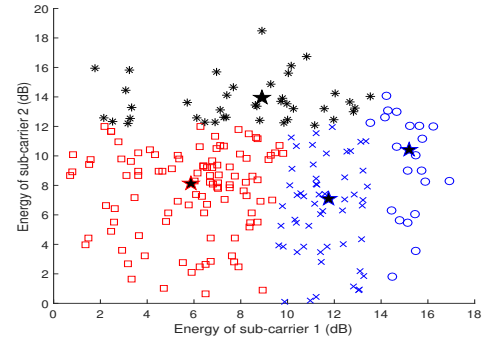


Fig. 2: An exemplary clustering set for the Auto-MDM when  $(N, k) = (2, 1)$ ,  $M_t \in \{0, 2, 4\}$ , and  $K_c = 4$ . For illustration, only two dimensional sub-carrier vector is used: each 'star' marker represents the centroid of each cluster.

### C. Online Prediction Test for Auto-MDM

Given the clustering set of  $(\mathbf{u}_{(1)}, \dots, \mathbf{u}_{(K_c)})$ , we are now ready to test a prediction for the modulation mode to be applied into automated MDM in real-time. According to A1-A2, refer to the energies of new uplink signals  $\tilde{\mathbf{y}}_u = (|y_{u,1}|^2, \dots, |y_{u,N}|^2)$  and the clustering set is applied to firstly determine the closest cluster, whose index satisfies:

$$l^* = \arg \min_l \|\tilde{\mathbf{y}}_u - \mathbf{u}_{(l),1:N}\|^2, \quad (4)$$

where  $\mathbf{u}_{(l),1:N}$  is the vector of the first  $N$  entries of  $\mathbf{u}_{(l)}$  – trained  $N$  dimensional channel energies– and  $l^*$  denotes the index of the cluster which produces the trained channel energies closest to the given uplink energy observation. Based on the choice of cluster  $l^*$ , we secondly predict  $\mathbf{u}_{(l^*),N+1}$  to be the best mode index for the next Auto-MDM transmission. Note that  $\mathbf{u}_{(l^*),N+1}$  represents the one appearing most often within the cluster  $(l^*)$ . Such prediction is highlighted in Algorithm 2. Notice that the complexities of training and testing for the proposed algorithms grow linearly as  $O(NK_cQn_{it})$  and  $O(NK_c)$ , respectively, where  $n_{it}$  denotes the number of iterations for  $K$ -means algorithm.

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#### Algorithm 2 Prediction algorithm for Auto-MDM

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- 1: Refer to the trained clustering set  $\Sigma_{\mathbf{u}} = (\mathbf{u}_{(1)}, \dots, \mathbf{u}_{(K_c)})$ .
  - 2: Set  $t^* = 1$  as an initial modulation mode index.
  - 3: **procedure** ONLINE PREDICTION( $\tilde{\mathbf{y}}_u, \Sigma_{\mathbf{u}}$ )
  - 4:   Non-coherent observation of a new  $\tilde{\mathbf{y}}_u$ .
  - 5:   Find  $l^* = \arg \min_l \|\tilde{\mathbf{y}}_u - \mathbf{u}_{(l),1:N}\|^2$ .
  - 6:   Update  $t^* = \mathbf{u}_{(l^*),N+1}$  where  $t^* \in \{0, \dots, kT\}$ .
  - 7:   **return**  $t^*$    ▷ The prediction for Auto-MDM is made
  - 8: Prediction of the next modulation mode to  $m_{t^*}$ .
  - 9: Adapt  $\mathbf{x}$  based on  $m_{t^*}$  for the transmission.
- 

**Remarks:** Notice that in such a real-time prediction, the learning-driven MDM adaptation benefits from (i) non-coherent automation with no multiple CSI at the transmitter; (ii) low complexity adaptation based on the simple Euclidean distance of real-valued energy vectors; and (iii) high flexibility at runtime with the absence of pre-defined switching mode

thresholds per group, as summarized in Algorithm 2. The ambiguities of multi-dimensional fading channel coefficients and background noises in the prediction for every MDM adaptation are taken into account by performing the unsupervised training of the multiclass clustering set. The accuracy of such learning-driven automation of MDM is simulated in Section IV.

Notice that the size of  $K_c$  can influence the sensitivity of the clustering and thus, the prediction performance of the Auto-MDM. Precisely speaking, a smaller  $K_c$ , a larger distance between each trained centroid and its cluster edge points. This may lead to an ambiguity of the clustering. Therefore, the prediction distortion can be influenced by the ambiguity of the clustering set  $\Sigma_{\mathbf{u}}$ , which can be indicated by the average distance between  $\mathbf{z}_q$  and its cluster's centroid, which is given:

$$D_{K_c} = Q^{-1} \sum_{q=1}^Q \|\mathbf{z}_q - \mathbf{u}_{\langle q \rangle}\|^2, \quad (5)$$

where  $\mathbf{u}_{\langle q \rangle} \in (\mathbf{u}_{(1)}, \dots, \mathbf{u}_{(K_c)})$  denotes the centroid of the cluster to whom  $\mathbf{z}_q$  belongs. Therefore, at  $K_c$  making  $D_{K_c}$  small enough,  $\mathbf{u}_{\langle q \rangle, N+1}$  approaches  $\mathbf{z}_{q, N+1}$ . The prediction of  $\mathbf{u}_{(1), N+1}$  improves at the linear complexity with  $K_c$ .

#### IV. SIMULATION RESULTS

We present simulation results for the proposed Auto-MDM. Provided that  $N_c = 256$  sub-carriers are divided into  $G = 64$  groups of  $N = 4$  sub-carriers, the proposed machine learning trains the Auto-MDM only at the SNR of 10 dB, along with the number of training dataset  $Q = 500$  and the size of  $\mathbf{z}_q$   $N+1 = 5$ . Training only at 10 dB helps to reduce signalling burdens but faces the overfitting problem. Consider two cases of either three or five possible modulation modes. Given  $N = 4$  and  $k = 1, 2$ , for example, there are five modes, among which a choice of the first modulation mode ( $m_0 = (4, 2, 0)$ ) means no transmission, while  $m_1 = (4, 2, 2)$  and  $m_2 = (4, 2, 4)$  produce the data rate of  $p = 4$  bits/cu and  $p = 6$  bits/cu, respectively.

In Fig. 3 the average BEP of the Auto-MDM is depicted for the two cases of either  $k = 1$  or  $k = (1, 2)$ , when  $Q = 500$ ,  $K_c = 100$ ,  $\mu = 10^{-2}$ , and  $n_{it} = 7$  at  $k = 1$  and  $n_{it} = 8$  at  $k = (1, 2)$ . For given  $Q = 500$ , the value for  $K_c = 100$  is numerically found to decrease (5), due to the trade-off between sensitivity and complexity. For comparison, the existing IM-OFDM methods are depicted. As seen in Fig. 3, the Auto-MDM benefits from the capability of automatically adjusting its best mode. For low and moderate SNRs, the average BEP gets higher than  $\mu$  because of the overfitting problem, being still lower than the benchmarks at variable rate less than 6 bits/cu. At high SNRs, the Auto-MDM is equally likely to the spread IM-OFDM because the prediction of  $m_5$  gets dominant over others, obtaining 3 dB power gain over the CI-IM-OFDM for the average BEP of  $10^{-3}$ . The ripples on the BEP results from the varying rate.

#### V. CONCLUSIONS

We have proposed the unsupervised machine learning technique for Auto-MDM. Exploiting the proposed off-line training and clustering algorithms, the Auto-MDM autonomously

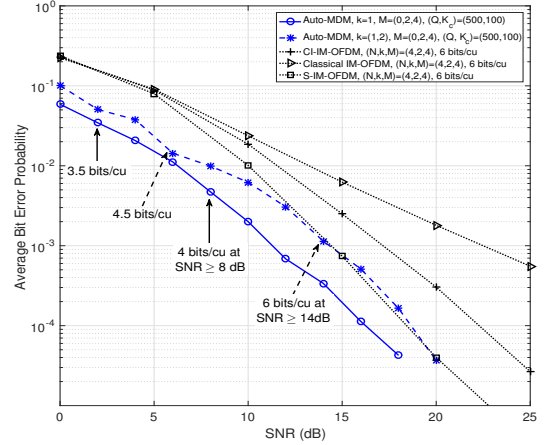


Fig. 3: Average BEP of the Auto-MDM when  $N = 4$ ,  $k = (1, 2)$ ,  $M_t = (0, 2, 4)$ ,  $Q = 500$ , and  $K_c = 100$ . For comparison, classical IM-OFDM [3], Coordinated Interleaving IM-OFDM [5] and spread IM-OFDM [8] are depicted.

learned key exploitable physical layer features from the wireless propagation environment and was able to perform the non-coherent adaptation to multi-dimensional variations of the signal energy vector. Unlike the existing methods that lack details of surrounding contexts and rely on pre-defined modulation rules, the Auto-MDM is flexible to program adaptation rules and predicts the best modulation mode without the channel information. The simulation results showed that the Auto-MDM benefited from the potentials of machine learning.

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