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# Multi-task Projected Embedding for Igbo

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**Abstract.** NLP research on low resource African languages is often impeded by the unavailability of basic resources: tools, techniques, annotated corpora, and datasets. Besides the lack of funding for the manual development of these resources, building from scratch will amount to the reinvention of the wheel. Therefore, adapting existing techniques and models from well-resourced languages is often an attractive option. One of the most generally applied NLP models is word embeddings. Embedding models often require large amounts of data to train which are not available for most African languages. In this work, we adopt an alignment based projection method to transfer trained English embeddings to the Igbo language. Various English embedding models were projected and evaluated on the *odd-word, analogy* and *word-similarity* tasks intrinsically, and also on the diacritic restoration task. Our results show that the projected embeddings performed very well across these tasks.

Keywords: low-resource, Igbo, diacritics, embedding models, transfer learning

# 1 Background

The core task in this paper is embedding-based diacritic restoration. Training embedding models requires large amounts of data which are unavailable in low resource languages. Web-scraped data are often relied upon but they are of poor quality. Languages with diacritics have most of the words wrongly written with missing diacritics. Diacritic restoration helps to improve the quality of corpora for NLP systems.

This work focuses on Igbo, mainly spoken in the south-eastern part of Nigeria and worldwide by about 30 million people. Igbo has diacritic characters (Table ??) which often determine the pronunciation and meaning of words with the same latinized spelling.

## 1.1 Previous Approaches

Key studies in diacritic restoration involve word-, grapheme-, and tag-based techniques [?]. Earlier examples include Yarowsky's works [?,?] which combined decision list with morphological and collocational information. POS-tags and language models have also been applied by Simard [?] to well resourced languages (French and Spanish). Hybrid of techniques are common with this task e.g. Yarowsky [?] used decision list, Bayesian

Char	Ortho	Tonal
а	-	à,á, ā
е	_	è,é, ē
i	i	ì, í, ī̄, ị̀, í́, ī̄
0	ò	ò, ó, ō, ò, ó, ō
и	ų	ù, ú, ū, ù, ú, ū
т	_	m̀,ḿ, m̄
n	'n	'n,ń, ñ
Table 1.	İgbo dia	acritic complexity

classification and Viterbi decoding while Crandall [?] applied Bayesian- and HMMbased methods. Tufiş and Chiţu [?] combined the two approaches by backing off to character-based method when dealing with "unknown words".

However, these methods are mostly on well-resourced languages (French and Spanish) with comparatively limited diacritic complexity. Mihalcea *et al* [?] proposed an approach that used character based instances with classification algorithms for Romania. This inspired the works of Wagacha *et al* [?], De Pauw *et al* [?] and Scannell [?] on a variety of relatively low resourced languages. However, it is a common position that the word-based approach is superior to character-based approach for well resourced languages. Diacritic restoration can also be modelled as a classification task. For Maori, Cocks and Keegan [?] used naïve Bayes algorithms with word *n*-grams to improve on the character based approach by Scannell [?].

For Igbo, however, one major challenge to applying most of the techniques mentioned above that depend on annotated datasets is the lack of these datasets for Igbo e.g tags, morph-segmented or dictionaries. This work aims to apply a resource-light approach that is based on a more generalisable state-of-the-art representation model like word-embeddings which could also be tested on other tasks.

## 1.2 Igbo Diacritic Restoration

Igbo was among the languages in a previous work [?] with 89.5% accuracy using a version of their *lexicon lookup* methods, *LL2*. This technique used the most frequent word and a bigram model to determine the right replacement. However, we could not directly compare their work to ours as the task definitions are slight different. While their accuracy is based on the restoration of every word in a sentence, our work focuses on only the ambiguous words. Besides, their training corpus was too little (31k tokens and 4.3k types) to be representative and there was no language speaker in their team to validate their results. However, we re-implemented a version of the *LL2* and bigram model as our baseline for the restoration task reported in this work.

Ezeani *et al* [?] implemented a more complex set of *n*-gram models with similar techniques on a larger corpus but though they reported improved results, their evaluation method assumed a closed-world by training and testing on the same dataset. While a more standard evaluation method was used in [?], the data representation model was akin to *one-hot* encoding which is inefficient and could not easily handle large vocabulary sizes.

Another reason for using embedding models for Igbo is that diacritic restoration does not always eliminate the need for sense disambiguation. For example, the restored word *àkwà* could be referring to either *bed* or *bridge*. Ezeani *et al* [?] had earlier shown that with proper diacritics on ambiguous wordkeys (e.g. *akwa*), a translation system like *Google Translate* may perform better at translating Igbo sentences to other languages. This strategy, therefore, could be more easily extended to sense disambiguation in future.

Statement	Google Translate	Comment
O ji <i>egbe</i> ya gbuo <i>egbe</i>	He used his gun to kill gun	wrong
O ji <b>égbè</b> ya gbuo <b>égbé</b>	He used his <b>gun</b> to kill <b>kite</b>	correct
Akwa ya di n'elu akwa ya	It was on the <b>bed</b> in his room	fair
Ákwà ya di n'elu àkwà ya	his <b>clothes</b> on his <b>bed</b>	correct
<i>Oke</i> riri <i>oke</i> ya	Her addiction	confused
<b>Òké</b> riri <b>òkè</b> ya	Mouse ate his share	correct
O jiri <i>ugbo</i> ya bia	He came with his <i>farm</i>	wrong
O jiri <b>ugbo</b> ya bia	He came with his <b>car</b>	correct

 Table 2. Disambiguation challenge for Google Translate

# 2 Experimental Setup

Our experimental pipeline follows four fundamental stages:

- 1. pre-processing of data (Section ??);
- 2. building embedding models (Section ??);
- 3. enhancing embedding models (Section ??);
- 4. evaluation of models (Section ??)

Models are intrinsically evaluated on the *word similarity*, *analogy* and *odd-word identification* tasks as well as the key process of diacritic evaluation.

#### 2.1 Experimental Data

We used the Igbo-English parallel bible corpora, available from the *Jehova Witness* website<sup>1</sup>, for our experiments. There are 32,416 aligned lines of text, bible verses, and chapter headings, from both languages. Total token sizes, without punctuations, are 902,429 and 881,771 with vocabulary lengths of 16,084 and 15,000 for Igbo and English respectively.

Over 50% of both the Igbo tokens (595,221) and vocabulary words (8,750) have at least one diacritic character. There are 550 ambiguous *wordkeys*<sup>2</sup>. Over 97% of the ambiguous wordkeys have 2 or 3 variants.

<sup>&</sup>lt;sup>1</sup> jw.org

<sup>&</sup>lt;sup>2</sup> A *wordkey* is a word stripped of its diacritics if it has any. Wordkeys could have multiple diacritic variants, one of which could be the same as the wordkey itself.

#### 2.2 Embedding Models

Inspired by the concept of the universality of meaning and representation (Figure ??) in distributional semantics, we developed an embedding-based diacritic restoration technique. Embedding models are very generalisable and therefore will constitute essential resources for Igbo NLP work. We used both trained and projected embeddings, as defined below, for our tasks.

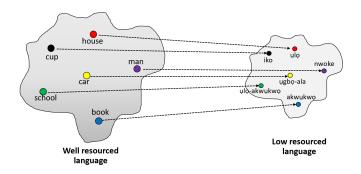


Fig. 1. Embedding Projection

**Embedding Training** We built the **igBbltrain** embedding from the data described in Section **??** using the Gensim *word2vec* Python libraries [**?**]. Default configurations were used apart from optimizing *dimension* and *window\_size* parameters to 140 and 2 respectively on the **Basic** restoration method described in Section **??** <sup>3</sup>.

**Embedding Projection** We adopt an alignment-based projection method similar to the one described in [?]. It uses an Igbo-English alignment dictionary  $A^{I|E}$  with a function  $f(w_i^I)$  that maps each Igbo word  $w_i^I$  to all its co-aligned English words  $w_{i,j}^E$  and their counts  $c_{i,j}$  as defined in Equation ??.  $|V^I|$  is the vocabulary size of Igbo and n is number of co-aligned English words.

$$A^{I|E} = \{w_i^I, f(w_i^I)\}; i = 1..|V^I|$$
  
$$f(w_i^I) = \{w_{i,j}^E, c_{i,j}\}; j = 1..n$$
(1)

The projection is formalised as assigning the weighted average of the embeddings of the co-aligned English words  $w_{i,j}^E$  to the Igbo word embeddings  $\text{vec}(w_i^I)$  [?]:

$$\operatorname{vec}(w_i^I) \leftarrow \frac{1}{C} \sum_{w_{i,j}^E, c_{i,j} \in f(w_i^I)} \operatorname{vec}(w_{i,j}^E) \cdot c_{i,j}$$
(2)

<sup>4</sup> I. Ezeani, M. Hepple, I. Onyenwe, C. Enemuo

<sup>&</sup>lt;sup>3</sup> The pre-trained Igbo model from *fastText Wiki word vectors* project [?] was also tested but its performance was so bad that we had to drop it.

where  $C \leftarrow \sum_{c_{i,j} \in f(w_i^I)} c_{i,j}$ 

Using this projection method, we built 5 additional embedding models for Igbo:

- igBblproj from a model we trained on the English bible.
- **igGNproj** from the pre-trained *Google News*<sup>4</sup> *word2vec* model.
- **igWkproj** from *fastText* Wikipedia 2017, UMBC webbase corpus and statmt.org news dataset.
- igSwproj from same as igWkproj but with subword information.
- igCrlproj from fastText Common Crawl dataset

Table ?? shows the vocabulary lengths ( $Vocabs^L$ ), and the dimensions (*Dimension*) of each of the models used in our experiments. While the pre-trained models and their projections have vector sizes of 300, our trained **IgboBible** performed best with vector size of 140 and so we trained the **IgboEnBbl** with the same dimension.

Model	Dimension	$Vocabs^{I}$	$Vocabs^E$	Data
igBbltrain	140	4968	-	902.5k
igBblproj	140	4057	6.3k	881.8k
igGNproj	300	3046	3m	100bn
igWkproj	300	3460	1m	16bn
igSwproj	300	3460	1m	16bn
igCrlproj	300	3510	2m	600bn

Table 3. Igbo and English models: vocabulary, vector and training data sizes

## 2.3 Enhancing embedding models

For this experiment, our dataset consists of 29 ambiguous *wordkeys*<sup>5</sup> from our corpus. For each wordkey, we keep a list of sentences (excluding punctuations and numbers), each with a place-holder (see Table **??**) to be replaced with the correct variant of the wordkey.

Variant	Left context I	Meaning						
àkwá	ka okwa nke kpokotara		o na-eyighi eyi otu	egg				
ákwà	a kpara akpa mee		ngebichi nke onye na-ekwe	cloth				
ákwá	ozugbo m nuru mkpu		ha na ihe ndi a	cry				
Table 4. Instances of the wordkey akwa in context								

In both trained and projected embedding models, vectors are assigned to each word in the dictionary, and that includes each diacritic variant of a wordkey. The **Basic** 

<sup>&</sup>lt;sup>4</sup> https://code.google.com/archive/p/word2vec/

<sup>&</sup>lt;sup>5</sup> Highly dominant variants or very rarely occurring *wordkeys* were generally excluded from the datasets.

### 6 I. Ezeani, M. Hepple, I. Onyenwe, C. Enemuo

restoration process (Section **??**) uses this initial embedding model *as-is*. The models are then refined by "learning" new embeddings for each variant that correlate more with its context words embeddings.

For example, let  $mcw_v$  contain the top n (say n = 20) of the most co-occurring words of a certain variant, v and their counts, c. The diacritic embedding is derived by replacing each diacritic variant vector with the weighted average of the vectors of its most co-occurring words (see Equation (??)).

$$\operatorname{diac}_{\operatorname{vec}} \leftarrow \frac{1}{|mcw_v|} \sum_{w \in mcw_v} w_{vec} * w_c \tag{3}$$

where  $w_c$  is the 'weight' of w i.e. the count of w in  $mcw_v$ .

## 2.4 Model Evaluation

We evaluate the models on their performances on the following NLP tasks: *odd-words*, *analogy* and *word similarity* and diacritic restoration. As there are no standard datasets for these tasks in Igbo, we had auto-generate them from our data or transfer existing ones from English. Igbo native speakers were used to refine and validate instances of the dataset or methods used.

**The odd word** In this task, the model is used to identify the odd word from a list of words e.g. breakfast, cereal, dinner, lunch  $\rightarrow$  "cereal". We created four simple categories of words Igbo words (Table ??) that should naturally be mutually exclusive. Test instances were built by randomly selecting and shuffling three words from one category and one from another e.g. okpara, nna, ogaranya, nwanne  $\rightarrow$  ogaranya.

category	Igbo words				
nouns(family) e.g. father, mother	ada, okpara, nna, nne, nwanna, nwanne, di, nwunye				
adjectives e.g. tall, rich	ocha, ogaranya, ogbenye, ogologo, oji, ojoo, okenye, oma				
nouns(humans) e.g. man, woman	nwaanyi, nwoke, nwata, nwatakiri, agbogho, okorobia				
numbers e.g. one, seven	otu, abụọ, atọ, anọ, ise, isii, asaa, asatọ, itoolu, iri				
Table 5. Word categories for odd word dataset					

**Analogy** This is based on the concept of analogy as defined by [?] which tries to find  $y_2$  in the relationship:  $x_1 : y_1$  as  $x_2 : y_2$  using vector arithmetic e.g  $king - man + woman \approx queen$ . We created pairs of opposites for some common nouns and adjectives (Table ??) and randomly combined them to build the analogy data e.g. di (husband) – nwoke (man) + nwaanyi(woman)  $\approx nwunye$ (wife) ?

category	opposites
oppos-nouns	nwoke:nwaanyi, di:nwunye, okorobia:agbogho, nna:nne, okpara:ada
oppos-adjs	agadi:nwata, ocha:oji, ogologo:mkpumkpu, ogaranya:ogbenye
	Table 6. Word pair categories for analogy dataset

**Word Similarity** We created Igbo word similarity dataset by transferring the standard *wordsim353* dataset [?]. Our approach used *Google Translate* to translate the individual word pairs in the combined dataset and return their human similarity scores. We removed instances with words that could not be translated (e.g. cell  $\rightarrow$  *cell* & phone $\rightarrow$ *ekwenti*,7.81) and those with translations that yield compound words (e.g. situation  $\rightarrow$  *onodu* & conclusion  $\rightarrow$  *nkwubi okwu*,4.81)<sup>6</sup>.

**Diacritic restoration process** The restoration process computes the cosine similarity of the variant and context vectors and chooses the most similar candidate. For each wordkey, wk, candidate vectors,  $D^{wk} = \{d_1, ..., d_n\}$ , are extracted from the embedding model on-the-fly. C is defined as the list of the context words and  $vec_C$  is the context vector of C (Equation (??)).

$$\operatorname{vec}_{\mathbf{C}} \leftarrow \frac{1}{|C|} \sum_{w \in C} vec_w$$
(4)

$$\operatorname{diac}_{\operatorname{best}} \leftarrow \operatorname{argmax}_{d_i \in D^{wk}} sim(\operatorname{vec}_{\mathbf{C}}, d_i)$$
(5)

## **3** Results and Discussion

Our results on the odd-word, analogy and word-similarity tasks (Table ??, Figure ??) indicate that the projected embedding models, in general, capture concepts and their relationships better. This is not surprising as the trained model, **igBible**, and the one from its parallel English data, **igEnBbl** are too little and cover only religious data. Although **igWkSbwd** includes subword information which should be good for an agglutinative language like Igbo, these subword patterns are different from the patterns in Igbo. Generally, the models from the news data, **igGNews, igWkNews**, did well on these tasks.

On the diacritic restoration task, the embedding based approaches, with semantic information, generally performed comparatively well with respect to the *n*-gram models that capture syntactic details better. **IgBible**'s performance is impressive especially as it outperformed the bigram model<sup>7</sup>.

Expectedly, compared to other projected models, **IgBible** and its parallel, **IgEnBbl**, clearly did better on this task. **IgBible** was originally trained with the same dataset and language of the task and its vocabulary directly aligns with that of **IgEnBbl**. Clearly,

<sup>&</sup>lt;sup>6</sup> An alternative considered is to combine the word e.g.  $nkwubi \ okwu \rightarrow nkwubi-okwu$  and update the model with a projected vector or a combination of the vectors of constituting words.

<sup>&</sup>lt;sup>7</sup> We intend to implement higher level n-gram models.

the enhanced diacritic embeddings improved the performances of all the models which is expected as each variant is pulled to the center of its most co-occurring words.

	Odd-word	Similarity	A	nalogy
Models	Accuracy	Correlation	nouns	adjectives
igBible	78.27	48.02	23.81	06.67
igGNews	84.24	60.00	64.29	56.67
igEnBbl	75.26	58.96	54.76	13.33
igWkSbwd	84.18	58.56	64.29	50.00
igWkCrl	80.72	62.07	78.57	21.37
igWkNews	81.51	59.69	80.95	50.00

Table	7.1	Trained	l and	Proj	ect ]	Embe	ddings	on o	odd-	word	predicti	ion

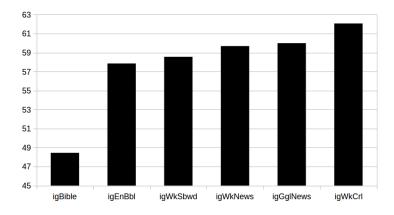


Fig. 2. Worst-to-Best Word Similarity Correlation Performance

# 4 Conclusion and Future Research Direction

This work contributes to the IgboNLP<sup>8</sup> [?] project. The goal of the project is to build a framework that can adapt, in an effective and efficient way, existing NLP tools to support the development of Igbo. In this paper, we demonstrated that projected embedding models can outperform the ones built with small language data on a variety of NLP tasks on low resource languages.

We also introduced a technique for learning diacritic embeddings which could be applied to the diacritic restoration task. Our next focus is to refine our techniques and datasets and train models with sub-word information as well as consider sense disambiguation task.

<sup>&</sup>lt;sup>8</sup> See igbonlp.org

#### Multi-task Projected Embedding for Igbo

Baselines: <i>n</i> -gram models									
		Unigr	·am			Bigram			
		72.25	5%			80.8	4%		
Embedding models									
	Acc	curacy	Prec	ision	Re	call	F1		
	Basic	Diac	Basic	Diac	Basic	Diac	Basic	Diac	
igBible	69.28	82.26	61.37	77.96	61.90	82.28	57.19	76.16	
igEnBbl	64.72	78.71	59.60	75.18	59.65	79.52	50.51	72.93	
igGNews	57.57	74.14	32.20	72.50	49.00	74.56	19.06	62.47	
igWkSbwd	62.10	73.83	13.82	73.81	47.64	74.03	10.65	66.62	
igWkCrl	60.78	73.30	40.07	78.02	49.16	76.24	25.36	68.62	
igWkNews	61.07	72.97	14.16	76.04	46.10	75.14	8.31	65.20	

**Table 8.** Performances of Basic and Diacritic versions of the *Trained* and *Projected* embedding models on diacritic restoration tasks

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