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1 **Seismic behavior of the cube of Zoroaster tower**  
2 **using the discrete element method**

3

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# 1 **Seismic behavior of the cube of Zoroaster tower** 2 **using the discrete element method**

3 There are several ancient stone masonry structures of great archeological significance in  
4 earthquake prone areas around the world. Ka'ba-ye Zartošt (Cube of Zoroaster) is a 14.2 m square  
5 in shape tower, which was built using white limestone blocks and dry joints. The tower dates  
6 back to the Achaemenid empire era and is located in the earthquake prone area of Nagsh-e  
7 Rostam in Fars, Iran. Although, after approximately 2,500 years the tower is still standing, it is  
8 now in a severely deteriorated condition and may be vulnerable against future large in magnitude  
9 earthquakes. This paper presents the application of a previously developed three dimensional  
10 numerical model based on the discrete element method of analysis to investigate the seismic  
11 behavior of the Cube of Zoroaster tower. The tower was represented by a series of distinct blocks  
12 separated by zero thickness interfaces. The developed model allows finite displacements and  
13 rotations of distinct blocks while new contacts between the blocks are automatically recognized  
14 and updated as the calculation progresses. A series of non-linear dynamic analysis have been  
15 performed. To this end, the behavior of the tower to different ground shaking motions is discussed  
16 and the possible failure modes for each case are explored. The significant advantage of this  
17 numerical approach is that discontinuity among stone blocks can be obtained. In addition, reliable  
18 prediction of the dynamic behavior of such historic constructions can allow a better  
19 understanding of their seismic vulnerability and inform decisions for their maintenance and  
20 preservations.

21 Keywords: stone masonry; ancient monument; Discrete Element Method (DEM); seismic  
22 behavior; Ka'ba-ye Zartošt

## 23 **Introduction**

24  
25 There are many ancient monuments (e.g. towers, multi-drum columns and colonnades)  
26 of great architectural and archeological significance around the world. Most of these have  
27 experienced major earthquakes during their life. Although some of them are still standing, there  
28 are examples where collapses occurred in the past (e.g. the 2,500 years old citadel of the city  
29 of Bam, the frescoed vault of the Basilica of St. Francis of Assisi in Perugia, Italy etc.). For  
30 engineers, it is important to understand the behavior of historic masonry structures when  
31 subjected to strong ground excitations in order to propose options for repair and strengthening  
32 for their preservation. There are several historic masonry structures which have been

1 constructed with dry joints (i.e. no mortar between the masonry units) around the world and  
2 are composed of rigid blocks stack on top of each other and have been subjected to strong  
3 earthquake excitations. According to Psycharis et al. (2011) and Bui et al. (2017), during strong  
4 earthquakes, the behavior of masonry constructions with dry joints can be characterized by  
5 high non-linearity which is governed by sliding, rocking, wobbling and/or complete  
6 detachment of adjacent masonry units. Thus, failure is usually at the block-to-block joint  
7 interface rather than in the blocks itself (e.g. cracking or crushing of the masonry units). In  
8 addition, in most cases, it is impossible to undertake destructive tests on such constructions  
9 since most of them are historical and there are restriction to access and testing on them (Omorfi  
10 1900). Therefore, efforts to develop full-scale experimental tests of scaled historical buildings  
11 and monuments or their components have been undertaken over the last years to understand  
12 their behaviour when subjected to seismic loading. For example, a series of seismic shaking  
13 investigations in historic buildings and monuments is presented at the Krstevska et al. (2010).  
14 Tests carried out at the IZIIS' Dynamic Testing Laboratory and the aim was to assess the  
15 vulnerability of the structures as well as test the experimental verification of different  
16 methodologies for seismic strengthening. Experimental tests to assess the seismic performance  
17 of ancient columns were also performed by Drosos and Anastasopoulos (2015) to understand  
18 the factors affecting their response. Physical models of multi-drum columns were constructed  
19 at reduced scale and tested at the shaking table of the NTUA Laboratory of Soil Mechanics.  
20 The difficulties associated with performed such experimental testing was highlighted in their  
21 manuscript. Moreover, a state of the art review on the testing of historic masonry elements and  
22 buildings is presented by Vintzileou (2010). In particular experimental studies about the  
23 behaviour of historic masonry elements in compression, in diagonal compression, in in-plane  
24 shear and simultaneous compression, out-of-plane bending, as well as publications related to  
25 the behaviour of subassemblies and building models subjected to monotonic, pseudo-dynamic

1 or dynamic tests on earthquake simulator were presented. However, although experimental  
2 studies are important to understand how physical models and material will behave when  
3 subjected to different types of loading, from the above experimental studies it was evident that  
4 experimental studies to represent historical masonry construction are both very expensive and  
5 time consuming. Therefore, it is fundamentally important to have available computational tools  
6 to predict the in-service and near-collapse behavior of such complex structures with sufficient  
7 reliability. Once such a tool has been established, a range of complex problems and scenarios  
8 can be investigated in detail.

9 Today, there are several approaches to model the mechanical behavior of masonry  
10 structures in the literature. An extensive literature review of the different numerical modeling  
11 approaches to simulate the mechanical behavior of masonry can be found at Asteris et al.  
12 (2015). These approaches include: limit analysis method, finite-element method (FEM),  
13 discrete/distinct element method (DEM) and finite-discrete element method (FDEM). Among  
14 these approaches, the DEM and FDEM approaches are the best ones to simulate the static and  
15 dynamic behavior of jointed discrete blocks media like masonry structures.

16 Research undertaken in the past (Bui et al. 2017; Azevedo et al. 2000; Psycharis et al.  
17 2003) demonstrated that the discrete/distinct element method (DEM) can be effectively used  
18 to simulate the in-plane and out-of-plane nonlinear behavior of masonry structures constructed  
19 with dry joints. Discrete/distinct element method developed by Cundall (1971) to investigate  
20 the behavior of jointed rocks where continuity between the separate blocks of rock did not  
21 exist. Over the last two decades, DEM has been employed with success to study the mechanical  
22 behaviour of historic masonry structures and monuments subjected to static and dynamic loads  
23 (Azevedo et al. 2000, Psycharis et al. 2000, 2003, 2013, Papantonopoulos et al. 2002,  
24 Komodromos et al. 2008, Papaloizou and Komodromos 2009, 2012, DeJong et al. 2012,  
25 Sarhosis et al. 2015, Sarhosis et al. 2016 a,b, Çakti et al. 2016, Pulatsu et al. 2017). For example,

1 Psycharis et al. (2000), Papantolopoulos et al. (2002, Psycharis et al. (2013) successfully  
2 investigated the seismic behavior of freestanding multi-drum columns. It was found that multi-  
3 drum columns are more vulnerable to long-period ground motions than short-period ones.  
4 DeJong and Vibert (2012) made use of the three-dimensional software 3DEC based on the  
5 DEM to investigate the seismic response of a damaged stone masonry spire in UK. Nayeri  
6 (2012) performed a series of parametric DEM analyses under far field and near field  
7 earthquakes on the stone Roman Temple in Évora. Later, Çakti et al. (2016) used DEM to  
8 simulate the seismic behavior of a scaled stone masonry mosque with dry joints tested in a  
9 shaking table. From the results analysis, it was found that the numerical results were in a good  
10 agreement with those obtained from the experiment. They also observed that for lower levels  
11 of the input motion, the frequency characteristics of the DEM model correlate fairly well with  
12 the tested model response characteristics.

13         Finite-discrete element method is an approach that combines the advantages of the FEM  
14 and FEM approaches (Smoljanović, 2013). In this approach, deformable blocks are represented  
15 using a mesh of triangular elements between which contact elements can be inserted. Therefore,  
16 the discretized elements may split and separate during the analysis. Furthermore, similar to the  
17 FEM, nonlinear material models are employed in this approach to simulate the masonry units  
18 and mortar failure. Munjiza (2004) developed a method based on fracture mechanics criteria  
19 to simulate fracturing problems called the combined finite-discrete element method. Due to the  
20 possibility of modelling the fragmentation of each block in the combined FDEM, Smoljanović  
21 et al. (2013) recently adopted and developed the FDEM to simulate the monotonic and cyclic  
22 shear behaviour of in-plane dry stone masonry walls and monuments. They showed that the  
23 FDEM has the ability to simulate the main features of dry stone masonry assemblages such as  
24 joints' shear behavior and rocking motion. Smoljanović et al. (2015) using the FDEM proposed  
25 a new material model in the contact element between masonry units and mortar. This contact

1 element takes into consideration the possibility of failure and softening behaviour of masonry  
2 in tension and shear. Smoljanović et al. (2018) also presented the application of the combined  
3 FDEM to simulate the behaviour of 3D dry stone masonry walls and structures.

4 The purpose of this paper is to investigate the 3D nonlinear seismic behavior of the dry  
5 joint stone masonry tower of Ka'ba-ye Zartošt (Cube of Zoroaster) in Nagsh-e Rostam in Fars,  
6 Iran. To this end, a previously developed three-dimensional discrete element model was  
7 utilized. Use made of the commercial software 3DEC (Itasca, 2013). The intention was to  
8 investigate the structural behavior and failure mode of the Cube of Zoroaster tower when  
9 subjected to earthquake vibrations with different frequency content. The observations  
10 presented herein provide insights into the non-linear dynamic response of masonry structures  
11 constructed with dry joints and lead to suggestions for informed decisions for their maintenance  
12 and preservation.

13

#### 14 **Description of the monument**

15

16 Ka'ba-ye Zartošt (Cube of Zoroaster), as shown in Figure 1, is a square tower built of  
17 white limestone blocks placed on top of each other. The tower was built during the pre-  
18 Persepolitan phase of the Achaemenid monumental architecture (ca. 559-511 B.C.) (Schmidt  
19 1970) in Nagsh-e Rostam, Fars, Iran. The tower has been constructed in two levels. The lower  
20 level has no voids in it. Stone unit were placed in regular interval and contain the entire space  
21 of the lower level of the tower. The upper part of the tower contains a thick stone wall around  
22 the perimeter of the tower as well as some false/blind windows of dark gray limestone and an  
23 entrance in the north side. The slightly pyramidal roof of the tower was made of stone blocks  
24 tied together using metal clamps. However, there is no steel clamp in its current state. The  
25 foundation of the tower is an approximately square terraced pyramid of three steps made of

1 large stone slabs (Schmidt 1970). Although the foundation slabs have different thicknesses,  
2 they have been laid in level surface.

3         Prior to the archeological excavations of Prof. Schmidt from the Oriental Institute,  
4 University of Chicago in June 1939, the lower part of the Tower was buried beneath debris  
5 (Schmidt 1970). The staircase which gave access to the main room of the Tower, was built of  
6 stone blocks. However, the exposed blocks of the upper part of the staircase have been  
7 demolished. Today, only part of the staircase is preserved. The portion of the north wall behind  
8 the upper part of the staircase has been mostly destroyed. The Cube of Zoroaster tower is square  
9 in plan with dimensions of 7.30 m on each side (Schmidt 1970). However, there are four  
10 ornamental piers in its corners of approximately 1.06 m on each side. The total maximum  
11 height of the Cube of Zoroaster tower including the steps of the base is 14.12 m. The average  
12 height of the stone courses is 0.5 m. The upper room of the tower is almost square with  
13 dimensions of 3.72m × 3.74 m. The walls' thickness ranges from 1.54 to 1.62 m. The height  
14 of the tower room is 5.54 m.

15

### 16 **Overview of the discrete element method for modeling dry masonry**

17

18         3DEC is a numerical modeling code based on DEM for discontinuous modeling and  
19 can simulate the three dimensional response of discontinuous media, such as masonry,  
20 subjected to either static or dynamic loading. When used to model masonry, the units (i.e.  
21 stones) are represented as an assemblage of rigid or deformable blocks which may take any  
22 arbitrary geometry. Typically, rigid blocks are adequate for structures with stiff, strong units,  
23 in which deformational behavior takes place at the joints. For explicit dynamic analysis, rigid  
24 block models run significantly faster. For static problems, this computational advantage is less  
25 important, so deformable blocks are preferable, as they provide a more elaborate representation  
26 of structural behavior. In the analyses reported herein, rigid blocks were used. Joints are



1 represented as interfaces between blocks. These interfaces can be viewed as interactions  
2 between the blocks and are governed by appropriate stress-displacement constitutive laws.  
3 These interactions can be linear (e.g. spring stiffness) or non-linear functions. Interaction  
4 between blocks is represented by set of point contacts, of either vertex to face or edge to edge  
5 type (Itasca 2013). In 3DEC, finite displacements and rotations of the discrete bodies are  
6 allowed. These include complete detachment between blocks and new contact generation as  
7 the calculation proceeds. Contacts can open and close depending on the stresses acting on them  
8 from the application of the external load. Contact forces in both the shear and normal direction  
9 are considered to be linear functions of the actual penetration in shear and normal directions,  
10 respectively (Itasca 2013). In the normal direction, the mechanical behavior of joints is  
11 governed by the following equation:

$$\Delta\sigma_n = - JK_n \cdot \Delta u_n \quad (1)$$

12  
13  
14 where  $JK_n$  is the normal stiffness of the contact,  $\Delta\sigma_n$  is the change in normal stress and  
15  $\Delta u_n$  is the change in normal displacement. Similarly, in the shear direction the mechanical  
16 behavior of mortar joints is controlled by a constant shear stiffness  $JK_s$  using the following  
17 expression:

$$\Delta\tau_s = - JK_s \cdot \Delta u_s \quad (2)$$

18  
19  
20 where  $\Delta\tau_s$  is the change in shear stress and  $\Delta u_s$  is the change in shear displacement.  
21 These stress increments are added to the previous stresses, and then the total normal and shear  
22 stresses are updated to meet the selected non-elastic failure criteria, such as the Mohr-Coulomb  
23 model.

1           The calculations are made using the force-displacement law at all contacts and the  
2 Newton's second law of motion at all blocks. The force-displacement law is used to find contact  
3 forces from known displacements, while the Newton's second law governs the motion of the  
4 blocks resulting from the known forces acting on them. Convergence to static solutions is  
5 obtained by means of adaptive damping, as in the classical dynamic relaxation methods.

## 7 **DEM modeling of the tower**

### 8 *Development of the geometric model and material properties*

11           A geometric model to represent the current condition of the tower has been developed  
12 (see Figure 2). Blocks assumed to behave as rigid elements connected together by a zero  
13 thickness interface. Thus, the nonlinear behavior of the structure is caused by geometrical  
14 nonlinearity and friction between the stone blocks in contact and not with the material  
15 nonlinearity. The generated model had about 2,433 rigid blocks.

16           The unit weight of the limestone masonry blocks was considered to be 2,680 kg/m<sup>3</sup>.  
17 The joints' normal and shear stiffness parameters (i.e.,  $JK_n$  and  $JK_s$ ) can be evaluated from the  
18 stiffness of the real joint under the assumption of stack bond (Lourenço et al. 2005). However,  
19 due to the lack of experimental data, typical values of  $4 \times 10^9$  and  $2 \times 10^9$  N/m<sup>3</sup> for the joints'  
20 stiffness in the normal and shear directions respectively have been used (Sarhosis et al. 2016b).  
21 The zero thickness interfaces between adjacent blocks were modelled using the Mohr-Coulomb  
22 slip model (Itasca 2013). The angle of internal friction at the block-to-block interfaces were  
23 assumed 30°, which is typical for low bond strength limestone construction with dry mortar.  
24 Since the tower has been constructed with dry joints, both the cohesion and tensile strength at  
25 the interfaces were set to zero.

### 26 *Seismic input and damping*

1 Initially, gravitation load has been assigned into the tower system. Then, the system  
2 brought into equilibrium under its own weight where the unbalance forces checked whether  
3 they are constant and almost equal to zero. Strong ground motions obtained from the Tabas  
4 earthquake in three different stations (i.e. Tabas, Ferdows and Kashmar) were applied at the  
5 base of the tower.

6 The Tabas earthquake occurred on the 16th of September 1978 in central Iran. The  
7 earthquake had 7.35 Richter magnitude. The PGA, PGV, PGD and duration of longitudinal and  
8 transverse and vertical components of the selected records are given in Table 1. Figure 3 shows  
9 the longitudinal components of the earthquakes acceleration along with their Fourier amplitude  
10 spectra. The longitudinal and transverse components of the earthquakes were applied parallel  
11 (x-axis) and perpendicular (y-axis) to the staircase direction, respectively (see Figure 2). It  
12 should be mentioned that the peak ground acceleration of the far-filed Ferdows and Kashmar  
13 records were scaled to reach the Tabas record PGA (i.e., L: 0.85g, T: 0.86g and V: 0.64g) as  
14 shown in Table 1. Such scaling was implemented to investigate merely the effect of  
15 earthquake's frequency contents on the tower vulnerability. The dominant frequency of the  
16 records are also given in Table 1. All three components of the earthquake records were applied  
17 simultaneously to the tower base block as velocity records in DEM model. During the dynamic  
18 analysis, no viscous damping was assumed. The only dissipation being due to frictional sliding  
19 on the joints. This conservative assumption is often used in simulating stone masonry structures  
20 containing dry joints (e.g. Papantopoulos et al. 2002). However, it should be mentioned that  
21 considering no viscous damping is just suitable for the analysis of masonry structures in which  
22 frictional sliding is the dominant failure mechanism. This is the case occurred in the monument  
23 investigated in this paper. However, if rocking motion failure is the governing mechanism,  
24 contact-impact energy mechanism should also be considered.

## 25 ***Validation study***

1 Prior to the analysis of the tower under the selected earthquakes, the validity of the  
2 adopted three dimensional DEM model was investigated. The Ka'ba-ye Zartošt (Cube of  
3 Zoroaster) is a historic monument which is impossible to undertake in-situ destructive tests or  
4 to remove and test samples of the tower which are large enough to be representative.  
5 Consequently, the validity of the numerical model undertaken using the shaking table  
6 experimental results of a stone masonry column tested in the laboratory by Mouzakis et al.  
7 (2002). Mouzakis et al. (2002) investigated the seismic behavior of a 1:3 scale model of a stone  
8 multi-drum column of the Pronaos of the Parthenon on a 6-DOFs shaking table at the National  
9 Technical University of Athens. The total height of the column was 3.34 m and was made of  
10 marble. The column had 12 drums and a capital resting on a marble base as shown in Figure  
11 4a. The column was tested under the scaled strong ground motions of Argostoli, Kalamata and  
12 Edessa. For the purpose of this study, numerical investigations of the dynamic behavior of the  
13 column subjected to the earthquake record of Edessa (i.e. specimen EQ17) with a scale factor  
14 of 2.0 and longitudinal, transverse and vertical PGAs of 0.26g, 0.15g and 0.09g, respectively,  
15 was simulated using the distinct element method software 3DEC. It should be noted that the  
16 above mentioned PGAs are the actual input seismic motions given to the shaking table which  
17 are slightly different from the real recorded Edessa motions. The stone blocks were assumed  
18 to be rigid elements connected together by zero-thickness interface elements. The nonlinear  
19 behavior of the column was considered at the block-to-block interfaces using the Mohr-  
20 Coulomb slip model with the angle of internal friction taken equal to  $35^\circ$  and zero cohesion  
21 and zero tensile strengths. The joints' normal and shear stiffness parameters (i.e.,  $JK_n$  and  $JK_s$ )  
22 were assumed to be  $2 \times 10^9$  N/m<sup>3</sup>. The stiffness and mass damping were both assumed equal to  
23 zero as discussed earlier in Section 4.2. It should be mentioned that specimen EQ17 has been  
24 analysed previously in Papantonopoulos et al. (2002) using the same software. In  
25 Papantonopoulos et al. (2002), in order to match better the numerical data with the

1 experimental one, the friction angle and the stiffness parameters at the joints were chosen  
2 slightly varying for different specimens. In fact in Papantonopoulos et al. (2002), the joints'  
3 normal and shear stiffness parameters were assumed to be 1.0 or  $2 \times 10^9$  N/m<sup>3</sup> and the friction  
4 angle was considered equal to 35 or 37°. However, the exact assumed values of these  
5 parameters used for each specimen model have not been reported in their study. The  
6 experimental and numerical (DEM) lateral displacement histories of the monitoring point K3  
7 (see Figure 4) of the column have been plotted in Figure 5. In Figure 5, the numerical analysis  
8 results conducted by Papantonopoulos et al. (2002) were also plotted for comparison reasons  
9 and denoted as “3DEC”. From Figure 5, the numerical simulation of this study denoted by  
10 “Present study” can capture the overall seismic response of the column with a good agreement  
11 which denotes the validity of our developed model.

12

13

## 14 **Results and discussions**

15

16 During the analyses, the time-history of lateral displacements of eight monitoring points  
17 around the tower at two levels as shown in Figure 6 were recorded. The tower response to each  
18 earthquake record is presented and discussed below.

19

### 20 *Dynamic system identification of the tower*

21 In order to identify the fundamental frequency of the tower, it was first excited by the  
22 transverse component of the Tabas earthquake applied in Y direction, since this was decided  
23 to be the most vulnerable situation. Initially, the record PGA was set to a small value (i.e. 0.2g)  
24 to ensure monolithic and elastic behavior of the tower during the dynamic analysis. During the  
25 analysis, the velocity histories of monitoring point 5 in both X and Y directions (see Figure 6)  
26 were recorded and analyzed. To identify the fundamental frequency, the velocity histories were

1 transferred from the time domain to the frequency domain using the Fourier Amplitude  
2 Spectrum (FAS) as shown in Figure 7. The fundamental frequencies of the tower in the X and  
3 Y directions obtained by the FAS of the recorded velocity histories are presented in Figure 7.  
4 As it is seen, the tower has approximately the same fundamental frequencies in both principal  
5 directions (i.e. 3.75 Hz vs 3.70 Hz). However, it should be noted that if the intensity of input  
6 motion increases, the vibrational frequencies would decrease due to nonlinear behavior of the  
7 tower.

8

### 9 ***Response of the tower to Tabas Earthquake***

10

11 The histories of relative lateral displacements of the monitoring points of the roof slab  
12 along both principal directions of X and Y to the tower base are shown in Figure 8. From Figure  
13 8a, for the first 11 seconds of the earthquake excitation, the X-axis horizontal displacement of  
14 all the monitoring points are the same, while just after the 11th second of the excitation, each  
15 corner of the roof moves independently to each other. This is an indication that the isolated  
16 slabs of the roof has been detached. However, at the end of excitation, the lateral displacements  
17 at the control points 6, 7 and 8 approach more or less to each other, while the X-axis horizontal  
18 displacement for the control point 5 oscillates around a permanent lateral displacement of about  
19 0.064 m to the right.

20 Similarly, from Figure 8b it can be seen that for the first 10 seconds of the  
21 earthquake excitation, the Y-axis horizontal displacement of all the monitoring points are the  
22 same, while just after the 10th second of the excitation, the two corner points located on a  
23 common roof stone slab unit vibrate with the same amplitudes. However, it is apparent from  
24 the Figure 8b that the two parallel roof slabs vibrate independently to each other.

25 The maximum lateral displacement under this record occurred along the Y-axis  
26 direction (i.e. direction perpendicular to the staircase direction). The occurrence of low lateral

1 displacements along the X-axis direction compared to the other direction, may be attributed to  
2 the bracing effect of the damaged staircase along the X-axis direction and the presence of no  
3 opening in this direction, as well.

4

#### 5 ***Response of the tower to Ferdows Earthquake***

6

7 The histories of relative lateral displacements of the monitoring points of the roof slab  
8 under the Ferdows earthquake are plotted in Figure 9. From Figure 9a, it is seen that for the  
9 first 8 seconds of the earthquake excitation, the X-axis horizontal displacement of all the  
10 monitoring points are the same, while just after the 8th second of the excitation, the monitoring  
11 points tend to vibrate independently to each other until the 20th second which they start  
12 approaching to each other again. As the earthquake load progresses, and after the 20th second  
13 of the earthquake, the monitoring points separate again from each other and vibrate  
14 independently around different permanent displacements up to the end of record without  
15 collapse.

16 Similarly, from Figure 9b it can be seen that for the first 13 seconds of the  
17 earthquake excitation, the Y-axis horizontal displacement of all the monitoring points are the  
18 same, while just after the 13th second of the excitation, the two corner points located on a  
19 common roof stone slab vibrate with the same amplitudes. Also, from Figure 9b, it is  
20 apparent that the two parallel roof slabs vibrate independently to each other.

21

#### 22 ***Comparison of inter-story drift demands and failure mode of the tower subjected to Tabas*** 23 ***and Ferdows EQs***

24

25 The maximum inter-story drift demands of the tower corner points under the Tabas  
26 earthquake are shown in Figure 10a. From Figure 10a, the tower sustains considerably large  
27 lateral drifts in each direction with a maximum value of 5.5 % without collapse. While the  
28 ordinary masonry towers can hardly tolerate lateral drifts of more than 1% as reported in the

1 literature (Sarhosis et al. 2018). This large lateral drift may be attributed to the governing failure  
2 mode of the tower which is a sliding shear failure mode throughout the tower height. This mode  
3 is a deformation-controlled (ductile) mode which makes the structure to sustain large lateral  
4 drifts without global instability. On the other hand, the phenomenon of shear sliding along the  
5 bed joints would act like a frictional damper which can benefit the stability of the tower because  
6 of increasing energy dissipation during strong motion reversal.

7         Furthermore, it should be noted that large pieces of stone blocks compared to the tower  
8 overall plan dimensions have been used in its construction. As it was mentioned earlier, the  
9 plan dimension of the tower is 7.30 m by 7.30 m and the horizontal length of smallest and  
10 largest stone blocks are 1.38 m and 4.40 m, respectively. Therefore, the ratio of stone blocks  
11 horizontal dimension ( $b$ ) to the tower horizontal dimension ( $B$ ) varies between 0.19 and 0.6  
12 which is significantly larger to the ratio of conventional masonry towers/buildings constructed  
13 with much shorter masonry units. The ratio of  $b/B$  highlights the “blocks size effect” on the  
14 lateral load behavior of this unique in construction masonry tower. As this ratio increases in a  
15 masonry wall/tower, lateral deformation capacity increases as a result of increasing  
16 interlocking between the adjacent stone blocks. Also, from 12b, during the Ferdows  
17 earthquake, the tower sustains considerably larger lateral drifts in each direction with a  
18 maximum value of 10.1 % without collapse. In all cases, and as expected the maximum inter-  
19 story drift observed at the second story of the tower.

20         Figure 11 shows the deformed geometry of the tower when subjected to Tabas and  
21 Ferdows EQs. From Figure 11 it is evident that the tower undergoes shear sliding failure mode  
22 along bed joints. Furthermore, it is seen that the tower deforms simultaneously in complex  
23 lateral and torsional deformation modes.

24         Figure 12 shows the lateral displacement profile of the tower confirming its complex  
25 and irregular deformed shape at the end of the earthquakes. This complexity may be attributed



1 to the vertical stiffness irregularity of the tower as well to the presence of the door opening in  
2 the upper level. From Figure 12a, the maximum deformation at the tower at the end of the  
3 Tabas earthquake was equal to 45 cm. Also, from Figure 12b, the maximum deformation at the  
4 tower at the end of the Ferdows earthquake was equal to 38 cm. Also, as expected, the  
5 maximum deformation in the tower observed in the second story.

6  
7 ***Response to Kashmar Earthquake***  
8

9         The histories of relative lateral displacements of the monitoring points of the roof slab  
10 under the Kashmar earthquake are plotted in Figure 113. From Figure 13a, for the first 6  
11 seconds of the earthquake excitation, the X-axis horizontal displacement of all the monitoring  
12 points are the same, while just after the 6th second of the excitation, the monitoring points of  
13 the different roof slabs separate from each other and their lateral displacement increase rapidly  
14 with time. Similar behavior was observed for the Y-axis displacements, as illustrated in Figure  
15 13b. In fact, this earthquake could destroy the upper level of the tower as shown in Figure 14.  
16 From Table 1, it can be seen that the Kashmar record is the only record which has low-  
17 frequency content in all components. Therefore, it seems that the tower is vulnerable to such  
18 low-frequency content far-field earthquake records. This behavior is somehow similar to the  
19 vulnerability of multi-drum stone columns to low-frequency content earthquakes as reported  
20 in the literature by Psycharis (2013). However, there is a major difference between the failure  
21 modes of this tower and stone columns. According to Psycharis (2003), multi-drum stone  
22 columns under low-frequency content earthquakes fail in a rocking mode like a monolithic  
23 stone block. While, the considered low-frequency content records in this study led to the  
24 collapse of this stocky stone tower in which the upper story blocks displaced excessively and  
25 toppled over (Figure 14).

26

## 1 **Conclusions**

2  
3 Historical masonry constructions have inevitably suffered damage with time.  
4 Earthquakes, soil settlements, material degradation and lack of maintenance are the main  
5 reasons for that. Careful and periodic assessment of such structures is necessary in order to  
6 evaluate their structural capacity and safety levels. However, performing the structural analysis  
7 of historical masonry construction is not an easy task. Historic masonry is a material which is  
8 characterized by high non-linearity and discontinuity; it is the mortar joints in a masonry  
9 structure which act as a plane of weakness. The need to predict the in-service behavior and load  
10 carrying capacity of masonry structures has led researchers to develop several numerical  
11 methods and computational tools which are characterized by their different levels of  
12 complexity. For a numerical model to adequately represent the behavior of a real structure,  
13 both the constitutive model and the input material properties must be selected carefully by the  
14 modeler to take into account the variation of masonry properties and the range of stress state  
15 types that exist in masonry structures. A broad range of numerical methods is available today  
16 ranging from the classical plastic solution methods to the most advanced non-linear  
17 computational formulations (e.g. finite element and discrete element methods of analysis).

18 In this study, an investigation has been undertaken into the seismic behavior of the  
19 Ka'ba-ye Zartošt (Cube of Zoroaster) tower located in Fars, Iran. Use made of the three  
20 dimensional software based on the discrete element method of analysis 3DEC. Since, in-situ  
21 destructive tests or the testing of samples of masonry removed from the structure that are large  
22 enough to be representative, is not usually possible, the model validated against a series of  
23 experimental tests obtained from the literature. Within the model, the tower was represented  
24 by a series of distinct blocks separated by zero thickness interfaces.

25 Ground motions obtained from the Tabas, Ferdows and Kashmar earthquakes were  
26 used and applied at the base of the tower in the longitudinal, transverse and vertical directions,

1 simultaneously. Also, the time-history of horizontal displacements at the corner points in first  
2 story and roof slab of the tower were recorded. From the results analysis, it can be observed that  
3 the seismic behavior of the tower is highly non-linear and influenced by the frequency content  
4 of the earthquake applied. In particular, when the tower subjected to Tabas EQ, it sustained  
5 later drifts up to 5.5% without collapse. However, when the tower subjected to Ferdows EQ, it  
6 sustained considerably larger lateral drifts in each direction with a maximum value of 10.1 %  
7 without collapse. In both cases studied, and as expected the maximum inter-story drift observed  
8 at the second story of the tower. Moreover, the failure mode of the tower was a combination of  
9 both shear and torsional failure. Such failure mode allowed the structure to sustain large lateral  
10 drifts without global instability. This ductile behavior could be due to the relatively large size  
11 of the stone blocks compared to the size of the tower. In addition, when the Kashmar earthquake  
12 applied, which is a much lower frequency earthquake compared to Tabas and Ferdows EQs, it  
13 was found that the stone tower couldn't sustain such load and collapsed. Therefore, from this  
14 present study it is evident that this particular dry joint ancient stone tower requires earthquakes  
15 of low-frequency content to cause global failure of its upper part.

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