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Time domain analysis of structures with hysteretic vibration suppression systems

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Abstract. The use of viscous damping terms to simplify the damping of a vibrating system has been well established for decades. For solid materials whose energy dissipated per cycle is frequency-independent, an equivalent viscous damping has often been used. However, this may give inaccurate results, especially at higher excitation frequencies. Alternatively, a complex stiffness term can be used. In this case, a challenge arises for the time domain analysis due to the unstable poles in the resulting model. Several methods have been proposed to deal with this issue. The use of an analytic signal along with Hilbert transform and a time reversal technique is one of the first introduced methods. In this paper, we extend the method so that it can be used for solving the system equations of motion using the numerical integration algorithm solvers that are available in MATLAB. We also present the application of this extended method to simulate a multi-degree-of-freedom (MDOF) structure with supplemental passive vibration suppression systems using linear hysteretic damping in the time domain.

1. Introduction

In civil engineering applications, solid materials, such as rubber, are one of the most widely used materials for dampers and base isolations. In this regards, most of the linear analyses in the literature modeled the damping of these materials in the form of viscous damping. However this is not strictly an accurate representation due to the fact that these types of solid materials exhibit a hysteresis in their force-displacement behavior. It has also been shown experimentally that the energy dissipated by a solid material is frequency-independent [1]. On the other hand, energy dissipated by viscous damping is linearly proportional to the excitation frequency. To represent more realistic physical behavior, a complex stiffness model can be used. However, the time domain analysis of this type of damping is challenging due to its non-causality.

Some methods have been proposed to solve the equation of motion of a system with hysteretic damping in the time domain. One of the first was introduced by Inaudi and Makris [2]. In this method, the hysteretic characteristic of the material damping in the equation of motion is modelled by using the Hilbert transform. Using the state space formulation, the equation of motion can be solved by using time-reversal technique to avoid the instability problems associated with the unstable pole. Some improvement of this method were given by Bae et al. [3, 4]. The application of this method to a free and transient response of a hysteretic damping system was presented in [5, 6, 7].

In general these previous works were limited to forced vibration cases, where the external force is applied to the mass of the structure. For civil engineering application, it is also important to study the system subjected to ground motion. In this scenario it becomes even more important to consider the role of hysteretic damping, because such systems frequently possess vibration absorbers or isolators that incorporate hysteretic damping components such as laminated rubber bearing base isolation [8] and hysteretic dynamic vibration absorber (HDVA) [9].

In this paper, an extended method is developed for analysis of structures with a hysteretic vibration absorber in the time domain. This extended method makes it possible to solve the system equation of motions in the time domain using numerical integration algorithm solvers that are available in MATLAB.

2. Description of the technique

A generalized n-DOF structure subjected to base excitation $r(t)$ is given in Figure 1. The structure is separated into three parts: bottom storey, $i = 1$; middle storeys, i^{th}, where $i \in [2 : n-1]$; and top storey, $i = n$.

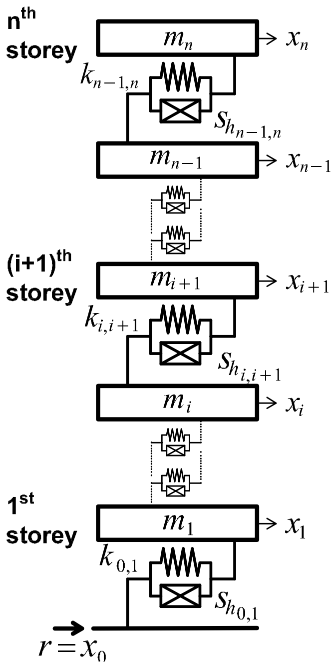

Figure 1. n-DOF structure with hysteretic damping

The equation of motion of the above structure can be written using analytic functions, given that $x_{i_a}(t) = x_i(t) + jH[x_i(t)]$, the equations can be written as follows:

$$\begin{cases} m_1\ddot{x}_{1_a}(t) + k_{0,1}(1+j\eta_{0,1})(x_{1_a}(t) - r_a(t)) - k_{1,2}(1+j\eta_{1,2})(x_{2_a}(t) - x_{1_a}(t)) = 0 \\ \vdots \\ m_i\ddot{x}_{i_a}(t) + k_{i-1,i}(1+j\eta_{i-1,i})(x_{i_a}(t) - x_{(i-1)_a}(t)) - k_{i,i+1}(1+j\eta_{i,i+1})(x_{(i+1)_a}(t) - x_{i_a}(t)) = 0 \\ \vdots \\ m_n\ddot{x}_{n_a}(t) + k_{n-1,n}(1+j\eta_{n-1,n})(x_{n_a}(t) - x_{(n-1)_a}(t)) = 0 \end{cases} \tag{1}$$

where m_i and $x_i(t)$ represent the mass concentrated on the i^{th} storey and its displacement response; $H[x_i(t)]$ is the Hilbert transform of $x_i(t)$; $k_{i-1,i}$ and $s_{h_{i-1,i}}$, $i \in [1 : n]$ represent the stiffness and a parameter with unit of stiffness characterising the damping between storeys $i-1$

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and i; $\eta_{i-1,i}$ is the loss factor of the linear hysteretic damping between storey $i-1$ and i, given by $\eta_{i-1,i} = \frac{s_{h_{i-1,i}}}{k_{i-1,i}}$; $r(t)$ represents ground displacement input signal; subscript a denotes an analytic signal; and $j = \sqrt{-1}$. In state-space formulation, Equation 1 can be expressed as:

$$\dot{\mathbf{x}}_a(t) = \mathbf{A}\mathbf{x}_a(t) + \mathbf{B}r_a(t) \tag{2}$$

where

$$\mathbf{x}_a(t) = \begin{bmatrix} x_{1_a}(t) \\ \dot{x}_{1_a}(t) \\ \vdots \\ x_{n_a}(t) \\ \dot{x}_{n_a}(t) \end{bmatrix} \quad ; \quad \mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n-1} & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n-1} & a_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n-1,1} & a_{n-1,2} & \cdots & a_{n-1,n-1} & a_{n-1,n} \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n-1} & a_{n,n} \end{bmatrix} \quad ; \quad \mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} \tag{3}$$

The eigenvalues of $\mathbf{A}$ are given by s_z, where $z = 1, 2, ..., n-1, n$. Note also that

$$s_z = s_{z_{re}} + s_{z_{im}}j \tag{4}$$

Here, im stands for "imaginary" and re for "real". Defining the analytic modal coordinates $q_{z_a}(t)$

$$\dot{\mathbf{x}}_a(t) = \mathbf{\Phi} \begin{bmatrix} q_{1a}(t) \\ q_{2a}(t) \\ \vdots \\ q_{(n-1)_a}(t) \\ q_{na}(t) \end{bmatrix} \quad ; \quad \mathbf{\Phi} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,n-1} & \phi_{1,n} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,n-1} & \phi_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{n-1,1} & \phi_{n-1,2} & \cdots & \phi_{n-1,n-1} & \phi_{n-1,n} \\ \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,n-1} & \phi_{n,n} \end{bmatrix} \tag{5}$$

where $\mathbf{\Phi}$ is the modal matrix, and $\phi_{l,z} = \phi_{l,z_{re}} + \phi_{l,z_{im}}j$, where $l = 1, 2, ..., n-1, n$. Now we have

$$\mathbf{\Phi} \begin{bmatrix} \dot{q}_{1_a}(t) \\ \dot{q}_{2_a}(t) \\ \vdots \\ \dot{q}_{(n-1)_a}(t) \\ \dot{q}_{n_a}(t) \end{bmatrix} = \mathbf{A}\mathbf{\Phi} \begin{bmatrix} q_{1a}(t) \\ q_{2a}(t) \\ \vdots \\ q_{(n-1)_a}(t) \\ q_{na}(t) \end{bmatrix} + \mathbf{B}r_a(t) \tag{6}$$

Using

$$\mathbf{\Phi}^{-1}\mathbf{A}\mathbf{\Phi} = \begin{bmatrix} s_1 & \cdots & \cdots & \cdots & 0 \\ \vdots & s_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & s_{n-1} & \vdots \\ 0 & \cdots & \cdots & \cdots & s_n \end{bmatrix} \quad ; \quad \mathbf{\Phi}^{-1}\mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix} \tag{7}$$

where

$$B_z = B_{z_{re}} + B_{z_{im}}j \tag{8}$$

we obtain

$$\begin{cases} \dot{q}_{1_a}(t) = s_1 q_{1a}(t) + B_1 r_a(t) \\ \dot{q}_{2_a}(t) = s_2 q_{2a}(t) + B_2 r_a(t) \\ \vdots \\ \dot{q}_{(n-1)_a}(t) = s_{n-1} q_{(n-1)_a}(t) + B_{n-1} r_a(t) \\ \dot{q}_{n_a}(t) = s_n q_{n_a}(t) + B_n r_a(t) \end{cases} \tag{9}$$

In [2], a similar form of Equation (9) was solved by using zero-order hold method which subsequently required a special integration formula. In this paper, this equation is separated into real and imaginary parts, so that $q_{z_a}(t) = q_z(t) + jH[q_z(t)]$ and $B_1 = B_{1_{re}} + B_{1_{im}}j$. A similar technique was introduced in [6] for single-degree-of-freedom (SDOF) structures for force excitation problems. Here, we extend the technique for multi-degree-of-freedom (MDOF) structures subjected to base displacement and show that it can be easily implemented by using various ode-family options available in MATLAB (i.e. ode45). Note that the forcing function $r(t)$ is treated in its real value only. Separating Equation (9) into real and imaginary parts, we have

$$\begin{cases} \dot{q}_1(t) = s_{1_{re}} q_1(t) - s_{1_{im}} H[q_1(t)] + B_{1_{re}} r(t) \\ H[\dot{q}_1(t)] = s_{1_{re}} H[q_1(t)] + s_{1_{im}} q_1(t) + B_{1_{im}} r(t) \\ \dot{q}_2(t) = s_{2_{re}} q_2(t) - s_{2_{im}} H[q_2(t)] + B_{2_{re}} r(t) \\ H[\dot{q}_2(t)] = s_{2_{re}} H[q_2(t)] + s_{2_{im}} q_2(t) + B_{2_{im}} r(t) \\ \vdots \\ \dot{q}_{n-1}(t) = s_{(n-1)_{re}} q_{n-1}(t) - s_{(n-1)_{im}} H[q_{n-1}(t)] + B_{(n-1)_{re}} r(t) \\ H[\dot{q}_{n-1}(t)] = s_{(n-1)_{re}} H[q_{n-1}(t)] + s_{(n-1)_{im}} q_{n-1}(t) + B_{(n-1)_{im}} r(t) \\ \dot{q}_n(t) = s_{n_{re}} q_n(t) - s_{n_{im}} H[q_n(t)] + B_{n_{re}} r(t) \\ H[\dot{q}_n(t)] = s_{n_{re}} H[q_n(t)] + s_{n_{im}} q_n(t) + B_{n_{im}} r(t) \end{cases} \tag{10}$$

Equation (10) can be solved using a standard differential equation solver that is available in MATLAB or other standard computational software packages (i.e. ode45). The equations containing unstable poles can be solved by integrating the equations backward in time [2]. From Equation (5), $\dot{\mathbf{x}}_a(t)$ can be written as:

$$\dot{\mathbf{x}}_a(t) = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,n-1} & \phi_{1,n} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,n-1} & \phi_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{n-1,1} & \phi_{n-1,2} & \cdots & \phi_{n-1,n-1} & \phi_{n-1,n} \\ \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,n-1} & \phi_{n,n} \end{bmatrix} \begin{bmatrix} q_{1_a}(t) \\ q_{2_a}(t) \\ \vdots \\ q_{(n-1)_a}(t) \\ q_{n_a}(t) \end{bmatrix} \tag{11}$$

Separating the real and imaginary parts, this equation now can be solved using:

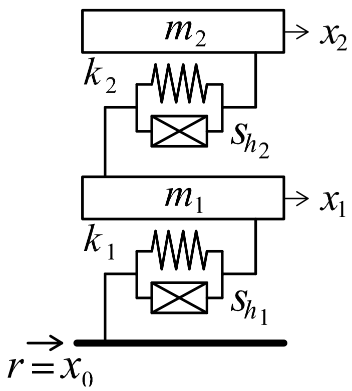
$$\begin{bmatrix} x_1(t) \\ \dot{x}_1(t) \\ \vdots \\ x_n(t) \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} \phi_{(1,1)_{re}} & \phi_{(1,2)_{re}} & \cdots & \phi_{(1,n-1)_{re}} & \phi_{(1,n)_{re}} \\ \phi_{(2,1)_{re}} & \phi_{(2,2)_{re}} & \cdots & \phi_{(2,n-1)_{re}} & \phi_{(2,n)_{re}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{(n-1,1)_{re}} & \phi_{(n-1,2)_{re}} & \cdots & \phi_{(n-1,n-1)_{re}} & \phi_{(n-1,n)_{re}} \\ \phi_{(n,1)_{re}} & \phi_{(n,2)_{re}} & \cdots & \phi_{(n,n-1)_{re}} & \phi_{(n,n)_{re}} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_{n-1}(t) \\ q_n(t) \end{bmatrix} + \begin{bmatrix} \phi_{(1,1)_{im}} & \phi_{(1,2)_{im}} & \cdots & \phi_{(1,n-1)_{im}} & \phi_{(1,n)_{im}} \\ \phi_{(2,1)_{im}} & \phi_{(2,2)_{im}} & \cdots & \phi_{(2,n-1)_{im}} & \phi_{(2,n)_{im}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{(n-1,1)_{im}} & \phi_{(n-1,2)_{im}} & \cdots & \phi_{(n-1,n-1)_{im}} & \phi_{(n-1,n)_{im}} \\ \phi_{(n,1)_{im}} & \phi_{(n,2)_{im}} & \cdots & \phi_{(n,n-1)_{im}} & \phi_{(n,n)_{im}} \end{bmatrix} \begin{bmatrix} H[q_1(t)] \\ H[q_2(t)] \\ \vdots \\ H[q_{n-1}(t)] \\ H[q_n(t)] \end{bmatrix} \tag{12}$$

where $q_z(t)$ and $H[q_z(t)]$ were obtained from Equation (10).

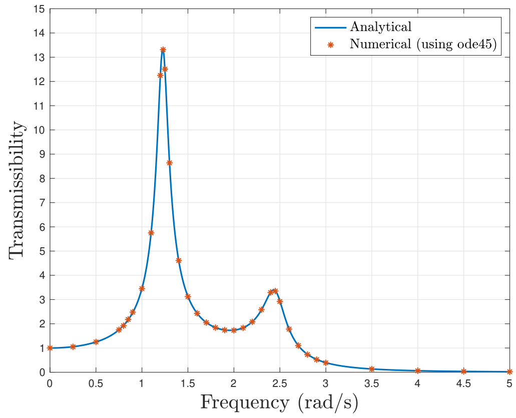
Figure 2 (a) shows an example of a 2-storey structure with linear hysteretic damping on each storey. Note that in this case, as the system is relatively simple, a comparison can be made between the numerical method proposed in this paper and the analytical solution. The two methods are in very close agreement as can be seen in Figure 2 (b), and therefore we have confidence in using the method on more complex examples.

3. Numerical examples

To the authors' knowledge, the application of the above method for passive vibration suppression systems in civil structures has never been investigated. Therefore in this paper, we use the method for time domain analysis of MDOF structures with passive vibration suppression systems



(a)



(b)

Figure 2. (a) 2-storey structure with hysteretic damping and (b) its top storey transmissibility when the structure is subjected to base displacement $r(t)$

containing a hysteretic damping element. Two devices are considered, namely a tuned-mass-hysteretic-damper (TMhD) and a tuned-inerter-hysteretic-damper (TIhD), as shown in Figure 3. The host structure is considered to be undamped with $m_1 = m_2 = m_3 = 1\text{kNs}^2/\text{m}$ and $k_{0,1} = k_{1,2} = k_{2,3} = 1500\text{kN/m}$.

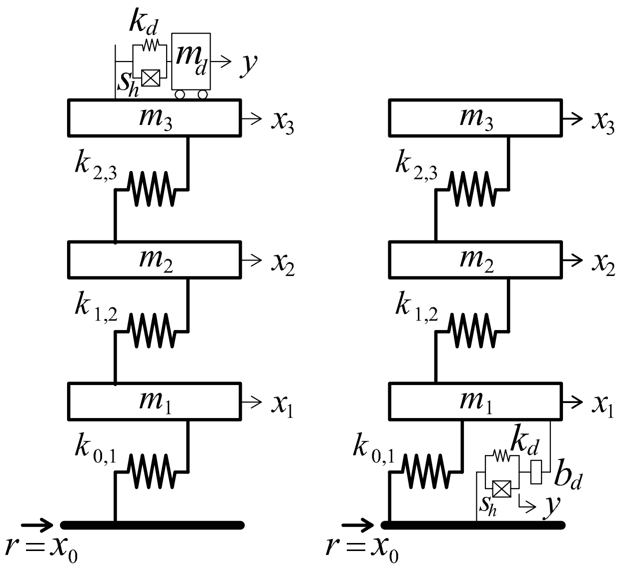


Figure 3. 3-storey structure with (a) a TMhD at the top storey (b) a TIhD at the base storey

3.1. Example 1: Tuned-mass-hysteretic-damper (TMhD)

Many strategies have been introduced for protecting structures from unwanted vibrations. Using the tuned-mass-damper (TMD) is one of the established strategies that has been used in many structures. Wong [9] has investigated the TMD with hysteretic damping and showed how the

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tuning procedure based on fixed-point theory in the frequency domain can be used. However, its time-domain analysis has not yet been presented. In the present study, a 3-storey undamped structure is equipped with a TMhD at the top storey subjected to base displacement as shown in Figure 3(a), and is then investigated in the time-domain. The optimum parameters of the TMhD are found to be $m_d = 0.102\text{kNs}^2/\text{m}$, $k_d = 27.6\text{kN/m}$ and $\eta = s_h/k_d = 0.2336$. m_d and k_d represent the mass and stiffness of the TMhD, and η is the loss factor of the linear hysteretic damping of the TMhD. The equation of motions of the system can be written as:

$$\begin{cases} \ddot{x}_{1_a}(t) = \dfrac{-(k_{0,1}+k_{1,2})x_{1_a}(t) + k_{1,2}x_{2_a}(t) + k_{0,1}r_a(t)}{m_1} \\ \ddot{x}_{2_a} = \dfrac{-(k_{1,2}+k_{2,3})x_{2_a}(t) + k_{2,3}x_{3_a}(t) + k_{1,2}x_{1_a}(t)}{m_2} \\ \ddot{x}_{3_a} = \dfrac{-(k_{2,3}+k_d(1+j\eta)x_{3_a}(t)) + k_{2,3}x_{2_a}(t) + kd(1+j\eta)y_a(t)}{m_3} \\ \ddot{y}_a(t) = \dfrac{-k_d(1+j\eta)(y_a(t)-x_{3_a}(t))}{m_d} \end{cases} \tag{13}$$

Figure 4(a) shows the performance of the structure with a TMhD on the top storey in the frequency domain by assuming the base displacement input is harmonic. A further novelty of the method is, for example, that the top storey response of the structure can also be considered when subjected to other time domain signals as shown in Figure 5. Three different input signals were considered: sine wave (Figure 5(a)), white noise (Figure 5(c)), and El Centro 1940 (Figure 5(e), its acceleration data were taken from https://strongmotioncenter.org).

3.2. Example 2: Tuned-inerter-hysteretic-damper (TIhD)

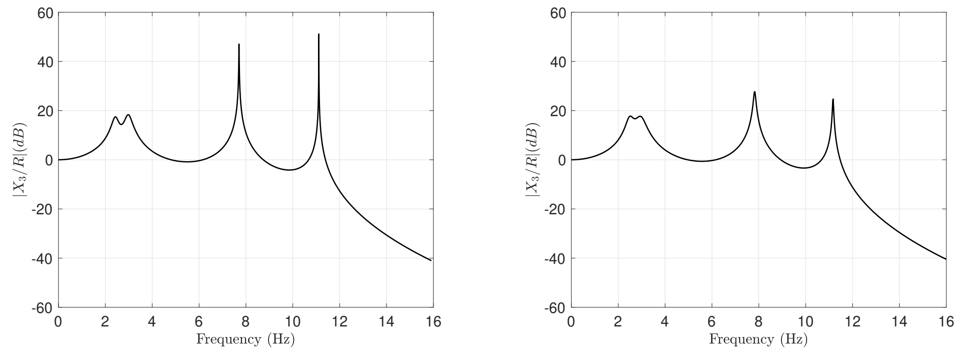
The tuned-inerter-damper (TID) has been introduced in [10] and has been shown to be a better alternative of the TMD in certain circumstances. With its inerter element, not only is a smaller mass required for a large mass-ratio, but also its optimum location is at the base of a structure. Both of these features are often beneficial compared to the TMD.

Despite its promising performance, the TID is still an idealised concept that requires further studies. A more realistic concept of the TID has been studied in [11] by considering the nonlinearities of its damping and inerter elements. In this paper, we introduce the use of a linear hysteretic damping to replace the viscous damping element of the TID. This new device is called the tuned-inerter-hysteretic-damper (TIhD) and is considered in detail in [12]. In this section, the extended method previously discussed was used to solve the system equation of motion of a MDOF structure with a TIhD as shown in Figure 3(b) subjected to base displacement in the time domain.

Considering the same 3-storey structure from the previous example, the optimum parameters of the TIhD are found to be $b_d = 0.48\text{kNs}^2/\text{m}$, $k_d = 138.6\text{kN/m}$ and $\eta = s_h/k_d = 0.306$. b_d and k_d represent the inertance and stiffness of the TIhD, and η is the loss factor of the linear hysteretic damping of the TIhD. The equation of motions of the system can be written as:

$$\begin{cases} \ddot{y}_a(t) = -\dfrac{k_{0,1}}{m_1}(x_{1_a}(t)-r_a(t)) - (\dfrac{k_d}{m_1} + j\dfrac{s_h}{m_1} + \dfrac{k_d}{b_d} + j\dfrac{s_h}{b_d})(y_a(t)-r_a(t)) + \dfrac{k_{1,2}}{m_1}(x_{2_a}(t)-x_{1_a}(t)) \\ \ddot{x}_{1_a}(t) = -\dfrac{k_{0,1}}{m_1}(x_{1_a}(t)-r_a(t)) - (\dfrac{k_d}{m_1} + j\dfrac{s_h}{m_1})(y_a(t)-r_a(t)) + \dfrac{k_{1,2}}{m_1}(x_{2_a}(t)-x_{1_a}(t)) \\ \ddot{x}_{2_a} = -\dfrac{k_{1,2}}{m_2}(x_{2_a}(t)-x_{1_a}(t)) + \dfrac{k_{2,3}}{m_2}(x_{3_a}(t)-x_{2_a}(t)) \\ \ddot{x}_{3_a} = -\dfrac{k_{2,3}}{m_3}(x_{3_a}(t)-x_{2_a}(t)) \end{cases} \tag{14}$$

Figure 4(b) illustrates the frequency response of the considered structure with a TIhD at the base storey. Although the response around the first vibration mode is similar to the TMhD, the structural responses at the higher vibration modes are much better. It is in line with the conclusion discussed in [10] when comparing the TMD and the TID.



(a) (b)

Figure 4. Frequency response of the MDOF system with (a) TMhD and (b) TIhD subjected to base displacement

The time domain responses obtained by using the previous presented method are shown in Figure 5. Similar to the previous example, three different scenarios were considered: sine wave (Figure 5(b)), white noise (Figure 5(d)), and El Centro 1940 (Figure 5(f)).

4. Conclusions

This paper presents an extended technique for solving equation of motions of structures with linear hysteretic damping in the time domain. This extended technique makes it possible for the equations to be easily solved using the ordinary differential equation solvers that are available in MATLAB subject to the constraint that the hysteretic damping parameter is small. For illustration, two numerical examples are given for a 3-storey structure with supplemental passive vibration suppression systems, namely the TMhD and the TIhD. Three different type of input signals were considered: sine wave, white noise, and earthquake base displacement.

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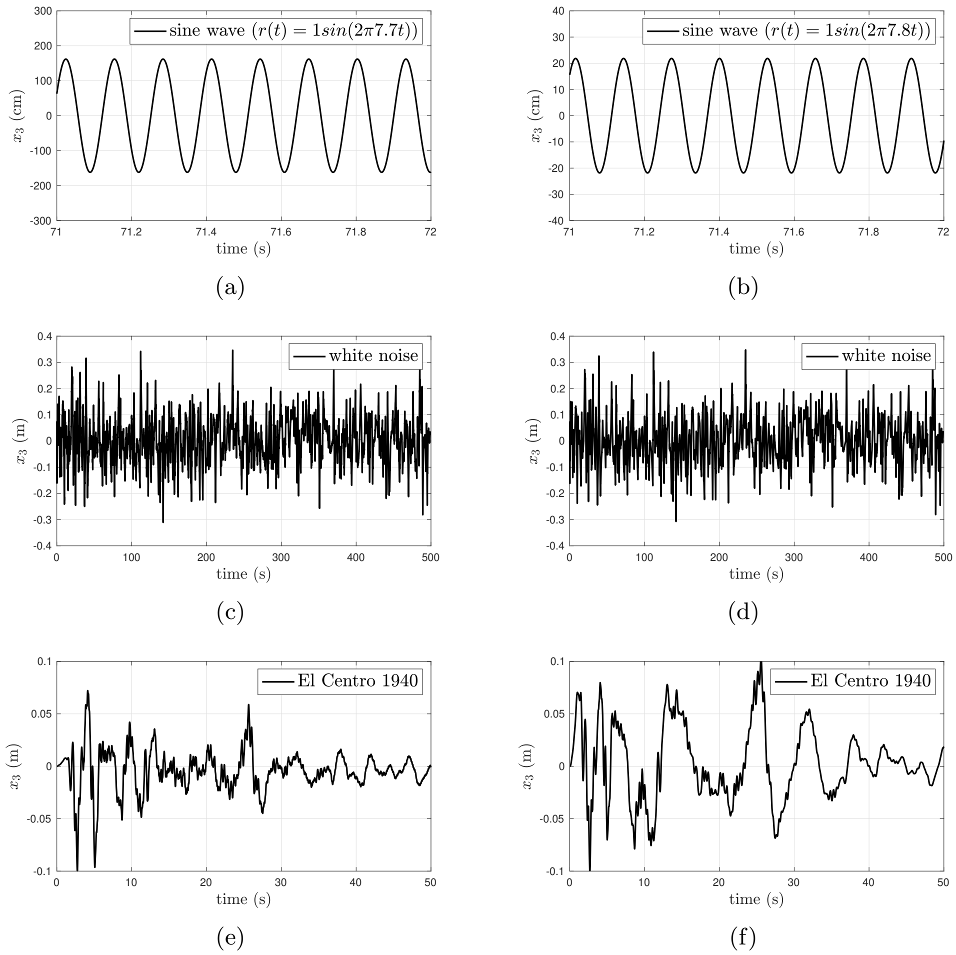

Figure 5. Absolute top storey displacement response subjected to various base displacement input signals: (a),(c),(e) are for TMhD and (b),(d),(f) are for TIhD

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