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Modelling orthogonal and oblique cutting via discontinuity layout optimization

Thomas Pritchard[a*], Colin Smith[b], Hassan Ghadbeigi[c], Marco Galindo-Fernandez[c], Matthew Gilbert[b], Sabino Ayvar-Soberinas[d]

[a]LimitState, The Innovation Centre, 217 Portobello, Sheffield, S1 4DP, UK

[b]University of Sheffield, Department of Civil and Structural Engineering, Sir Frederick Mappin Building, Mappin Street, Sheffield, S1 3JD, UK

[c]University of Sheffield, Department of Mechanical Engineering, Sir Frederick Mappin Building, Mappin Street, Sheffield, S1 3JD, UK

[d]University of Sheffield, The Advanced Manufacturing Research Centre with Boeing, Advanced Manufacturing Park, Wallis Way, Catcliffe, Rotherham, S60 5TZ,UK

[*] Corresponding author. Tel.: +44 (0) 114 224 2240; *E-mail address: t.j.pritchard@limitstate.com*

Abstract

The discontinuity layout optimization (DLO) numerical modelling procedure is used to model the slip-line fields associated with plastic deformation and chip formation in metal cutting operations. A series of orthogonal and oblique cutting operations at low and high cutting speeds are simulated, with modelling results validated against key cutting parameter data from machining trials (cutting forces, chip morphology etc.). Reasonably good correlation between the predicted and experimental outputs is obtained, suggesting that the DLO method may provide a viable alternative to traditional methods such as incremental finite element analysis.

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Keywords: Predictive Model, Cutting, Discontinuity Layout Optimization

1. Introduction

Mechanical cutting is used in all metal machining processes (e.g. turning, milling etc.) and lies at the heart of the global manufacturing industry. However, modelling the physics of metal cutting reliably and quickly has always been difficult. This means that designing efficient manufacturing processes has remained an inexact science, generally relying on collected experience and/or time consuming and costly trial and error testing.

Many efforts have been made since the middle of the last century to reliably model the chip formation process. Early attempts employed simplistic analytical methods involving use of an assumed shear plane, with all deformation taking place along this single plane [1-3]. Subsequently simplified plasticity models were employed, considering a plastic zone ahead of the tool tip [4, 5]. However, it was found that the physics of the metal cutting process is more complex than could be faithfully represented.

More realistic models were therefore developed. Based on the application of slip-line field theory [6, 7], these models were used to predict the chip formation process and material performance during the cutting action, considering the essential physics of the process and material properties under similar deformation conditions [8].

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Palmer and Oxley [6] considered the work hardening characteristics of the material using slip-line field theory, while Lin and Oxley [7] introduced a shear zone model. Here, the shear zone is not simply a thin plane, but also has thickness which varies with cutting speed. This is a fan shape zone for low cutting speeds, being thinner near the cutting tool [9] and changing to a parallel sided region at higher cutting speeds [10].

Fracture mechanics concepts can also be applied to the problem of chip formation by considering the metal cutting process as a class of ductile fracture [11]. It has been argued that, since earlier models did not consider the work done in the chip formation process, they neglected the huge amount of surface work associated with chip separation.

Developing the required slip-line field model for a machining operation is a difficult and labour intensive process and the solutions obtained are only applicable for a limited range of problem parameters, e.g. cutting and tool geometry parameters [12-14]. Finite element analysis (FEA) modelling approaches have therefore been developed to overcome this. However, success has been limited as in order to achieve reliable and accurate simulation of chip formation the advanced FEA models required are computationally expensive due to the need for incremental solvers and non-linear material models; also the modelling process can be unreliable due to mesh dependency of the solutions [15-20]. In recent years, Childs et al. [21-23] have presented and extended an FEA approach to simulating machining operations and chip morphology over a range of cutting speeds.

Developed originally for application to soil mechanics problems, discontinuity layout optimization (DLO) is a general purpose engineering analysis procedure which can be used to directly and rapidly establish the collapse state and corresponding magnitude of the maximum load that can be carried by a solid [24, 25]. Using DLO the layout of planes of discontinuity in a deforming solid or structure are identified using mathematical optimization methods, assuming that deformation occurs in a ductile or 'plastic' manner.

The aim of this paper is to demonstrate that the DLO technique can be successfully applied to metal cutting problems, providing accurate solutions in a short timeframe and providing a valuable alternative to traditional analytical methods and/or the empirical data currently provided to industrial users by tool manufacturers and professional engineering associations.

2. Discontinuity layout optimization (DLO)

Discontinuity layout optimization is a numerical limit analysis technique that can be used to obtain accurate upper-bound solutions for a wide range of plastic analysis problems. DLO has already been applied to plane-strain and three dimensional problems [24-27], where a given continuum body is discretized using a suitably large number of interconnected nodes wherein a deformation mechanism forms. This transforms a continuum into a discontinuous layout (analogous to a 'slip-line field'), where the critical arrangement of discontinuities (i.e. slip-lines in metal plasticity) are identified from a vast number of possible alternatives.

The process involves minimization of the total internal energy dissipated due to shearing along the discontinuities in a body subjected to an external action, giving rise to a mechanism in the form of velocity discontinuities (i.e. shear deformations when considering metal plasticity problems). Fig. 1 outlines the steps involved in formulating and solving a metal cutting problem using this technique. Compatibility of displacements is explicitly enforced at each node, and implicitly enforced at locations where discontinuities cross over one another remote from a node. The critical layout of discontinuities is then identified using efficient and rigorous mathematical optimization techniques in order to minimize the overall energy dissipation. Using this approach, highly accurate solutions can be obtained in a short timeframe on a desktop PC.

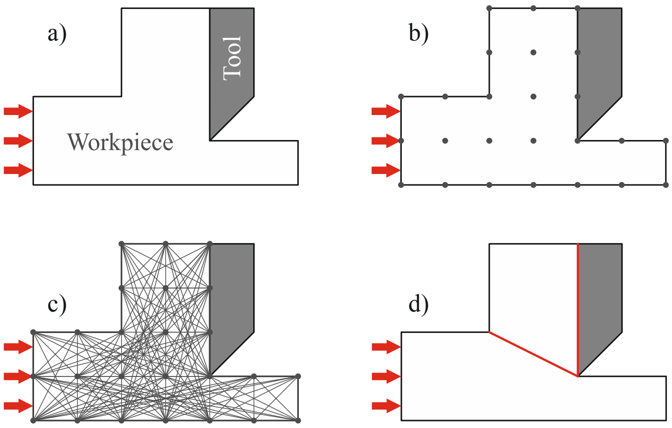

Fig. 1. Steps in the DLO process: a) problem specification; b) discretization using nodes distributed across the problem domain; c) potential slip-line discontinuities interconnecting the nodes; d) optimization used to identify the critical layout of slip-line discontinuities

2.1 General (2D) energy formulation

The general DLO formulation for a 2D plane-strain analysis of a quasi-statically loaded, perfectly plastic cohesive body discretized using m nodal connections (slip-line discontinuities), n nodes and a single load case can be stated in standard matrix-vector form as follows [24]:

$$\min \lambda \mathbf{f}_L^T \mathbf{d} = -\mathbf{f}_D^T + \mathbf{g}^T \mathbf{p} \quad (1)$$

Subject to:

$$\mathbf{Bd} = 0 \quad (2)$$

$$\mathbf{Np} - \mathbf{d} = 0 \quad (3)$$

$$\mathbf{f}_L^T \mathbf{d} = 1 \quad (4)$$

$$\mathbf{p} \geq 0 \quad (5)$$

Here, the objective is to minimize energy dissipation subject to constraints enforcing energy balance (1), nodal compatibility (2), plastic flow (3) and unit external work by unfactored live loads (4), where λ is the factor on the live load required for failure; $\mathbf{f}_D$ and $\mathbf{f}_L$ are vectors containing specified dead loads (e.g. self-weight, generally disregarded in metal cutting problems) and live (disturbing) loads; $\mathbf{d} = \{s_1, n_1, s_2, n_2, \dots, n_m\}$ where s_i and n_i are the relative shear and normal displacements, respectively, between the blocks along discontinuity i *(i=1, ..., m)*; $\mathbf{g}^{\mathrm{T}} = \{c_1 l_1, c_2 l_2, \dots, c_m l_m\}$, where c_i and l_i are the associated shear strength and length of the discontinuity i, respectively. The compatibility matrix **B** in (2) contains direction cosines. A normalization constraint (4) is required to avoid obtaining a solution in which all displacement jumps **d** (and subsequently plastic multipliers **p**) are trivially zero or unbounded. The flow rule can be enforced with a vector of plastic multipliers (**p**) and a flow matrix (**N**), where **p** contains plastic multipliers (two for the linear yield surface employed here).

Linear optimization is then used to obtain optimal values of the problem variables in **d** and **p**. The subset of slip-line discontinuities that make up the critical mechanism are thus obtained, as seen in Fig. 1d.

In order to achieve accurate solutions the domain of interest should be discretized using a reasonably large number of nodes, with each node in the problem preferably connected to all others (without passing outside the domain) [24]. This provides a large number of possible mechanisms from which the optimization algorithm can select from. Also, while for clarity the problem presented in Fig. 1 depicts only straight discontinuities, curved slip-lines can also be used in the DLO process [26, 28]. This allows the optimization algorithm to choose from an even wider range of potential mechanisms, including those involving full body rotations, frequently identified in metal cutting problems (e.g. associated with chip curl).

3. Experimental work

A number of experimental tests were undertaken in order to capture pertinent cutting data such as the cutting, thrust and out of plane forces as well as, where practicable, chip morphology (see Table 1). For low speed tests, a bespoke cutting rig capable of performing orthogonal and oblique cutting experiments, similar to that reported in [29] was designed and manufactured to be used on a three-axis CNC milling machine (Fig. 2). This ensured high positional and angular accuracy and repeatability of the experiments due to the high available level of control of the machine. The workpieces were square blocks of 25mm by 25mm with 3mm thickness fixed in a specially designed vice that was fixed on a force dynamometer used to measure cutting forces. A TiN coated parting-off insert by SECO Tools (LCGN 160602-0600-FG CP500) with no chip breaker was used for the trials at slow cutting speeds. For high speed tests, a tube and a bar were prepared from the selected materials to represent orthogonal and oblique cutting conditions, respectively. The samples were held in an automatic chuck and had a wall thickness (width of cut) of 4mm. Tube cutting setup were used to conduct high speed orthogonal trials. The tube was initially machined to ensure the wall thickness of 4 mm is achieved along the cut length. A standard uncoated carbide (TCGT16T308F-Al KX) cutting insert by SECO tools was used for these trials and cutting forces were measured using a Kistler force dynamometer.

The cutting edge geometries were quantified using an Alicona vary focal microscope and implemented in the models accordingly with the cutting edge radius of 10 μm measured consistently across all the tools. A fresh cutting edge was used for each cut during the experiments, in order to remove uncertainties associated with the tool wear, where three repeats were conducted for each cut.

Materials were chosen to minimize as far as possible the influence of temperature and strain rate effects (especially at high speed) and to allow focus on the core mechanical processes. Two aluminium alloys were selected; Al6061-T6 and Al2024-T351. Tensile tests were conducted on a number of samples of each and the results extrapolated to account for the much higher (typically 10% to 100%) strains that are likely to be achieved in practice when compared to those obtained experimentally (1% to 15%). The value of stress at 100% strain was used as an indicative strength of the material in each case – 675MPa for Al2024 and 425MPa for Al6061.

Repeated cuts of depth 0.1mm and 0.2mm were made in the specimens and the results averaged. For the sake of brevity, only the experiments relating to a 0.1mm depth of cut are presented and discussed from herein. The results of the other experiments were observed to be broadly the same as those reported here.

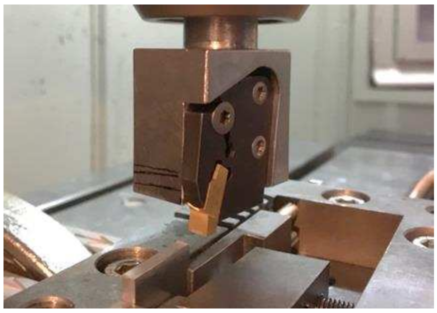

Fig. 2. Cutting rig used to perform orthogonal cutting experiments at low cutting speeds

Table 1. Matrix of experimental cutting tests undertaken as part of the study.

Cutting Speed (m/min)	Cut Type	Cut Width (mm)	Tool Inclination (°)	Tool Rake (°)	Tool Clearance (°)
2, 4, 6	Orth.	3.0	0	7.0	7.0
2, 4, 6	Obl.	Varies	10,20	7.0 (Norm.)	7.0 (Norm.)
30, 60, 120	Orth.	4.0	0	23.0	7.0
30, 60, 120	Obl.	Varies	10	23.0 (Norm.)	7.0 (Norm.)

4. Modelling orthogonal cutting using DLO

4.1 Comparison with Merchant single shear plane model

In order to first confirm that the DLO technique is able to predict the critical mechanism and associated forces for relatively simple cases, DLO results were compared with Merchant's single shear-plane model [2].

Four different cutting conditions were considered. A constant cut depth (t_1) of 0.1mm was retained throughout, while tool rake angles (γ) were varied between 0° and 11.31° and interface friction (θ) was varied between 0° and 31° (equivalent friction coefficient of $\mu = \tan\theta = 0.6$).

For all DLO validation, the yield stress (σ) of the workpiece material was converted to a cohesion (c), assuming a Von Mises material model:

$$c = \frac{\sigma}{\sqrt{3}} \quad (6)$$

As the region of primary interest in the cutting problems lies on or around the shear plane, each model was created with a concentration of nodes in this area, as shown in Fig. 3. In the current study 500 nodes were assigned to the region, as this allowed for the potential identification of complex mechanisms whilst also keeping computational effort and solution times low. However, for this specific comparison with Merchant solutions, it was deemed prudent to artificially constrain the solution to one in which the slip-line mechanism had to exit the material at the external corner of the workpiece. This was achieved by deliberately restricting nodes along the exposed boundaries so as to lie only at the corner points of the solid. (Though note that the mechanism inside the workpiece was still free to take any form.)

To simulate the cutting action, the workpiece material was pushed into the fixed cutting tool (arrows in Fig. 3). The cutting (F_c) and thrust (F_t) forces were then predicted and compared with the results of calculations using the classical Merchant's force circle assumption (Table 2).

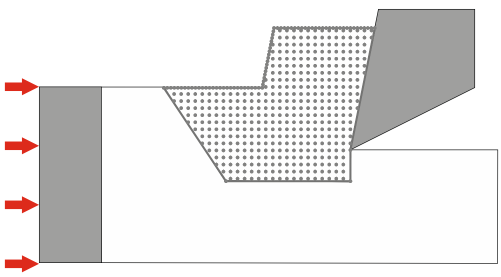

Fig. 3. Typical nodal distribution in the DLO approach for modelling chip formation (500 node discretization)

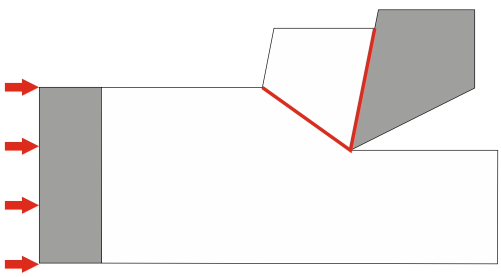

Fig. 4. DLO chip formation mechanism (slip-lines in red) for the single shear plane (Merchant [2]) mode

It should be noted that to avoid complexities associated with transient chip formation at the onset of cutting edge engagement in this study a chip of given thickness was assumed to have already been formed.

Fig. 4 shows a typical solution for the problems considered, where the set of red lines in the deformation zone indicates the predicted slip-lines for the studied case.

A comparison of the ratio of cutting to thrust forces for each case, calculated using the Merchant's circle and the DLO approaches, shows that there is perfect agreement.

Table 2. Comparison of cutting to thrust ratio values for the feasibility study cases

γ (deg.)	θ (deg.)	F_c / F_t (Merch.)	F_c / F_t (DLO)	% Error
0.00	0.00	1.00	1.00	0.00
11.31	0.00	1.22	1.22	0.00
0.00	31.00	0.56	0.56	0.00
11.31	31.00	0.70	0.70	0.00

4.2 Low speed tests

The results of the experiments, showing cutting, thrust and out-of-plane forces are presented in Fig. 5. A representative image of the cutting operations (Al6061, 2m/min feed rate, 0.1mm cut depth) is presented in Fig. 6.

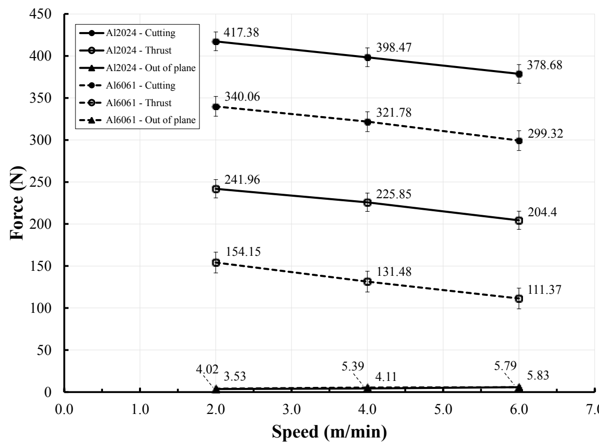

Fig. 5. Experimentally determined cutting (solid circle), thrust (hollow circle) and out of plane (triangle) forces for 0.1mm depth of cuts on Al2024 and Al6061 samples at low cutting speeds where the error bars show the standard error of the repeated cuts

Fig. 6. Image of machining 0.1mm deep orthogonal cut in Al6061 sample at low speed (2m/min)

A number of DLO analyses were undertaken and compared with the experimental data. The coefficient of friction between a tool and workpiece material during cutting has an inherent degree of uncertainty, affecting the reliability of the correlation with any modelling approach. The friction coefficient was selected based on the values from literature and interpreted from the experimental results.

In order to replicate the inherent small variabilities in the cutting operations (and also to account for parameters that could not be otherwise well quantified) the chip thickness (t_2 = 0.01mm to 0.6mm) and tool-chip interface friction values

(θ= 25° to 30°, corresponding to an assumed friction coefficient, μ, of around 0.55, which is consistent with reported values given in literature, e.g. [30]) were varied in the DLO analyses. The results were normalized so that results for different cutting depths could be directly compared to each other and the experimental data. Fig. 7 shows the results of this, with best-fit curves from DLO analyses of six different chip thickness values plotted against the experimental data.

4.3 High speed tests

The results of the high speed experiments are presented in Fig. 8. A number of DLO analyses were undertaken and compared to the experimental data. Chip thicknesses were varied from 0.04mm to 0.5mm and tool-chip interface friction values of θ = 36° to 50° (corresponding to an assumed μ of around 0.9) were investigated. Normalized results are presented in (Fig. 9).

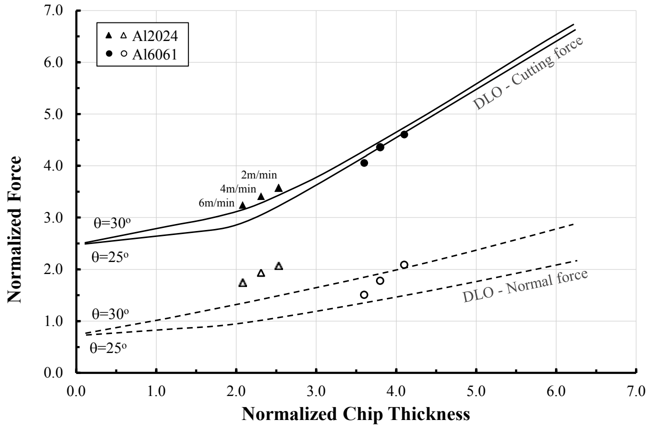

Fig. 7. Comparison of DLO calculated and experimentally measured cutting (solid lines) and normal (broken lines) forces for 0.1 mm cuts on Al2024 and Al6061 samples at low speeds (2 – 6m/min)

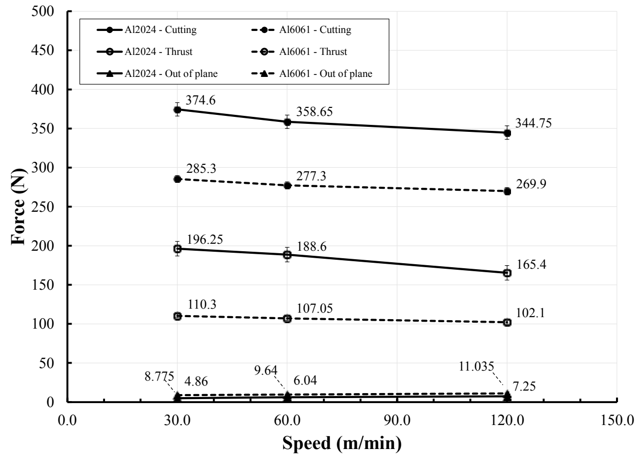

Fig. 8. Experimentally determined cutting (solid circle), thrust (hollow circle) and out of plane (triangle) forces for 0.1mm deep cuts on Al2024 and Al6061 samples at high cutting speeds

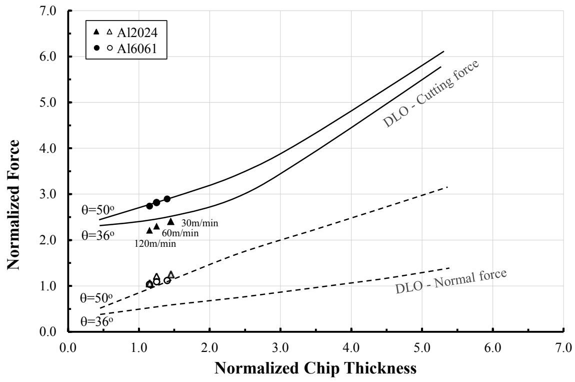

Fig. 9. Comparison of DLO calculated and experimentally measured cutting (solid lines) and normal (broken lines) forces for a depth of cut of 0.1 mm Al2024 and Al6061 samples at high speeds (30 – 120m/min)

5. Modelling oblique cutting using DLO

Oblique cutting experiments were modelled in DLO using a 2.5D approach involving a geometrical transformation of the 3D geometry on to a 2D plane.

Simulations showed only minor variation of cutting force and thrust forces with the tool inclination (oblique cut angle) for those angles tested. This mirrored the results from the experimental work. For example, see Fig. 10, where it is evident that the cutting and normal forces vary by less than 10% between an orthogonal cut and a 20° oblique one

5.1 Alternative slip-line mechanisms

An interesting finding to emerge from the DLO analyses was the potential range of slip-line mechanisms that can exist.

As would be expected, with an artificially low narrow starting chip thickness, the form of the critical mechanism was one that indicated a thicker chip would form under steady state conditions (Fig. 11a). Similarly, with an artificially high starting thickness, the form of the critical mechanism implied a narrower chip would emerge at steady state (Fig. 11d). However, between these two extremes, the slip-line mechanism was shown to take the form of a fan zone or single shear plane (as proposed by Merchant) over a range of initial starting chip thicknesses (Fig. 11b and Fig. 11c), suggesting that a range of metastable states can exist.

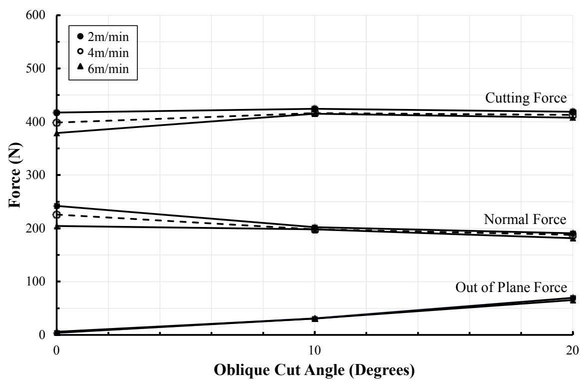

Fig. 10. Variation in cutting, normal and out of plane forces with differing oblique cut angle for 0.1mm deep cuts in Al2024

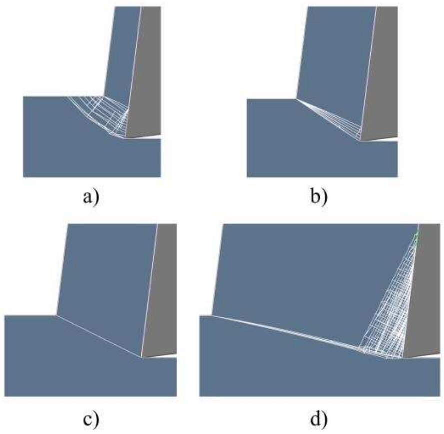

Fig. 11. Four types of critical slip line pattern in metal cutting, as identified using DLO: a) widening chip; b) steady state (with tool/chip sticking); c) steady state (Merchant type shear plane) and d) narrowing chip

6. Discussion

This investigation provides an indication of the promise of DLO. Unlike analytical methods, DLO is generally applicable and can automatically identify the critical mechanism for arbitrary input parameters. Unlike FEA, the discontinuous chip formation mechanism is modelled directly and in an entirely natural manner, with no need for tailored meshes nor adaptive mesh refinement. Solutions can be obtained rapidly using standard linear optimization algorithms that are very well developed (see [24] for further reading).

Prediction of normal force was found to be slightly less reliable than the cutting force. Further investigations into why this is and how greater uncertainty would affect predictions of tool wear are needed. Also, although not considered here, there is clearly the potential to simulate evolving chip formation geometries by coupling a DLO tool capable of modelling gross deformations and strain rate effects with a thermal model (to model material softening).

7. Conclusions

DLO has been established as a promising new means of modelling the basic physics of metal-cutting operations. Comparison with existing models has shown it provides an accurate means of determining critical slip-line mechanisms and associated forces. The generality of the technique means that a wide range of possible chip formation mechanisms can be identified.

Considering the modelling of the primary physics, DLO models were found to be generally capable of reliably predicting the magnitude of the cutting force. However, there were some exceptions that require further investigation.

Determination of the tool/chip coefficient of friction is challenging; therefore a range of values were adopted in the analysis models. Protocols for establishing this value and its associated uncertainty will need to be developed for use.

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