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# A GPU-accelerated three-dimensional SPH solver for geotechnical applications

Chong Peng, Wei Wu, Hai-sui Yu

**Abstract** A three-dimensional GPU-accelerated SPH solver is developed and ready to be released as an open-source tool. In this paper, the SPH formulation, geomechanical aspects of SPH modeling and the GPU parallelization are presented. A numerical case of granular flow passing a cylindrical obstacle is simulated to show the performance of the solver.

## 1 Introduction

Smoothed particle hydrodynamics (SPH) is a Lagrangian particle-based meshless methods with wide applications in astrophysics and computational fluid dynamics (CFD). It has several advantages over conventional grid-based methods: (1) Its Lagrangian and meshless nature makes it convenient in solving problems involving free-surface flow and large deformation; (2) The generation of computational model is easier compared with the mesh preparation in grid-based methods; (3) SPH has a structure very suitable for parallel computing.

In recent years, SPH attracts many attentions from the geotechnics community. Examples of its application include large deformation analysis, granular flow modeling, fluid-soil mixture coupling and debris flow simulation, among others. However, there are several factors which discourage researchers and engineers to apply SPH in

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Chong Peng  
ESS Engineering Software Steyr GmbH, Berggasse 35, 4400 Steyr, Austria, e-mail: peng-chong@boku.ac.at

Wei Wu  
Institute of Geotechnical Engineering, University of Natural Resources and Life Sciences, Vienna, Feistmantelstraße 4, 1180 Vienna, Austria, e-mail: wei.wu@boku.ac.at

Hai-sui Yu  
School of Civil Engineering, Faculty of Engineering, University of Leeds, LS2 9JT Leeds, UK e-mail: h.yu@leeds.uk.ac

their work. First, in single-cpu mode SPH is slow, mainly due to the large amount of particle interactions and the small time step. Second, although SPH has a long history, only in recent years it is applied to geotechnical problems. Available resources are inadequate. Particularly, there is no commercial software or open-source code allowing geotechnical simulations with SPH. These two factors make the development of an efficient open-source geotechnical SPH solver essential and urgent.

In this work, we developed a three-dimensional SPH solver for geotechnical simulations. The GPU computing is chosen as the acceleration technique, because it is widely available, and more efficient and much cheaper than CPU-based parallelization. The SPH fundamentals and the SPH adoption in geotechnical simulation are introduced. The efficient GPU implementation of SPH is then detailed. A numerical example is given to demonstrate the validity, accuracy and efficiency of the solver.

## 2 SPH for geotechnical simulation

In SPH, a set of scattered particles are used to discretize the problem domain. These particles carry field variables such as mass, velocity, density and stress, and move with the material. A field function  $f(\mathbf{x})$  and its gradients can be approximately by

$$f(\mathbf{x}_i) = \sum f(\mathbf{x}_j)W_{ij}m_j/\rho_j, \quad \nabla f(\mathbf{x}_i) = \sum f(\mathbf{x}_j)\nabla_i W_{ij}m_j/\rho_j \quad (1)$$

where  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are the coordinates of particle  $i$  and  $j$ , respectively;  $W_{ij}$  is a smoothing function dependent on  $r_{ij}$ , the distance between the particles;  $m_j$  and  $\rho_j$  denotes the mass and density at particle  $j$ ,  $\nabla_i W_{ij}$  indicates the gradient of the kernel  $W_{ij}$  evaluated at particle  $i$ .

Using the above SPH approximations, the conventional continuity equation and momentum equation can be written in the following form

$$\frac{d\rho_i}{dt} = \sum m_j(\mathbf{u}_i - \mathbf{u}_j) \cdot \nabla_i W_{ij} \quad (2)$$

$$\frac{d\mathbf{u}_i}{dt} = \sum m_j \left( \frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} - \Pi_{ij}\mathbf{I} + \mathbf{S}_{ij} \right) \nabla_i W_{ij} + \mathbf{g}_i \quad (3)$$

where  $\mathbf{u}$  is the velocity,  $\boldsymbol{\sigma}$  is the Cauchy stress tensor,  $\mathbf{I}$  is a  $3 \times 3$  identity matrix,  $\mathbf{g}$  is the body forces.  $\Pi_{ij}$  and  $\mathbf{S}_{ij}$  denotes the contribution from artificial viscosity and artificial stress, used to enhance the stability of SPH computation. For details of the derivation of formulations, please refer to [? ? ].

Constitutive models should be included in SPH to correctly capture the mechanical behaviors of geomaterials. In the SPH solver, three constitutive models are implemented: the advanced  $\mu(I)$  rheological model for granular media, and the Drucker-Prager elastoplastic (D-P) model and hypoplastic model for general simulation. The hypoelastic approach is used to extend the infinitesimal strain-based D-P model and hypoplastic model to large deformation analysis. The SPH solution

are obtained by numerical integrating Eq. (3), (4) and the constitutive equation using explicit integration schemes. In the solver, the second-order Predictor-Corrector scheme is employed.

### 3 SPH implementation on GPU

In three-dimensional simulations, large number of SPH particles are needed. Parallel computing technique is necessary for fast SPH simulation. The GPU based parallel computing becomes increasingly popular in recent years in large-scale computing for both research and application. Compared with CPU-based parallelization, GPU is much cheaper and easily available, and is extremely fast because usually a main stream graphic card has more than one thousand computing cores.

The solver is implemented using the CUDA programming language, an extension to the standard C. All the time-expensive parts of SPH computation, including neighbor search, particle interaction and numerical integration, are performed using graphic card. CPU-based code is used to control the whole computation flow. The efficiency of SPH solver is highly dependent on the neighbor search algorithm. In the solver, the efficient cell based searching scheme is adopted. First, the computational domain is partitioned into regular three-dimensional cells and each cell has a cell number according to its location, the size of the cell is  $2h$ . All particles are assigned a cell index in which cell it locates. Then all the particles are sorted according to their cell index, and the beginning particle and ending particle of each cell are computed. With the constructed neighbor list, a particle only has to search interacting particles in its own and the neighboring cells. This neighbor search scheme fully makes use of the GPU parallel architecture and is highly efficient. In our simulations, usually less than 5% of the total simulation time is spent on neighbor search.

### 4 Numerical example

A simple yet representative example is numerically simulated using the GPU-based SPH solver. The example is based on a test performed by Cui and Gray [?] where granular flow passes a cylinder obstacle. This test case is of practical interest in industry and engineering. A body of granular material of diameter 0.002 m is at rest in a container initially at the top of a inclined chute. Once the gate of the container opens, the granular material flows down the inclined chute and passes a cylinder. A three dimensional simulation is carried out using the presented SPH solver. The chute is set as 1 m wide and 1.5 m long. The cylinder has a diameter of 0.2 m and is placed 0.3 m downside of the gate. The open of the gate is 8 cm. Two simulations with different resolutions, 0.004 m and 0.002 m, are performed. In the two simulations, the particle numbers are 3.3 million and 22.8 million, respectively. The simulated physical time is 3 seconds. With GTX 1080Ti graphic card, the simula-

tion times for the two cases are approximately 7 hours and 47 hours, respectively. Although a huge number of particles are simulated, the solver can deliver results in reasonable time. The results at several time instances are shown in Figure 1. Salient features of the granular flow are captured by the SPH simulation.

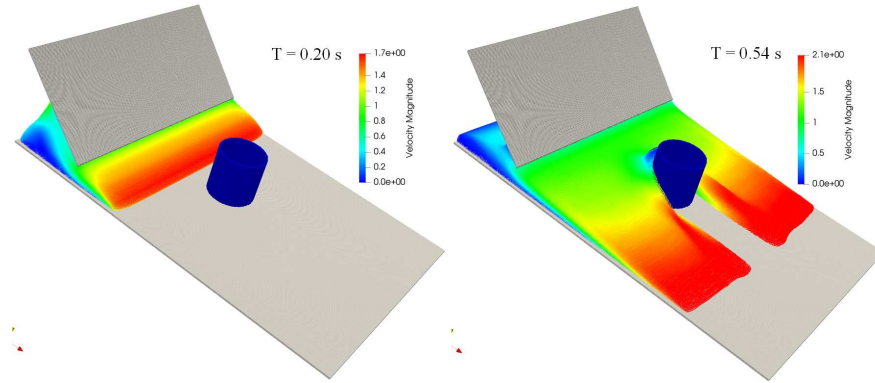


Fig. 1: Snapshots of the granular flow simulation with resolution 0.004 m.

## 5 Summary

We developed a three-dimensional GPU-accelerated SPH solver for geotechnical applications. State-of-the-art SPH formulations for geotechnical problems are implemented. The GPU parallelization is efficient and the solver is capable of performing large-scale simulation using conventional desktops and laptops. Preliminary validation are performed and it is shown that the solver is applicable to geotechnical problems. We hope the solver can promote the application of SPH in geotechnical engineering once it is mature and released to public.

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