



UNIVERSITY OF LEEDS

This is a repository copy of *Optimisation procedure for material parameter identification for masonry constitutive models*.

White Rose Research Online URL for this paper:
<http://eprints.whiterose.ac.uk/146299/>

Version: Accepted Version

Article:

Sarhosis, V orcid.org/0000-0002-8604-8659 (2016) Optimisation procedure for material parameter identification for masonry constitutive models. *International Journal of Masonry Research and Innovation*, 1 (1). pp. 48-58. ISSN 2056-9459

<https://doi.org/10.1504/IJMRI.2016.074735>

This is an author produced version of an article, published in *International Journal of Masonry Research and Innovation (IJMRI)*. Uploaded in accordance with the publisher's self-archiving policy.

Reuse

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



eprints@whiterose.ac.uk
<https://eprints.whiterose.ac.uk/>

An optimization procedure for material parameter identification for masonry constitutive models

Vasilis Sarhosis¹

¹School of Engineering, Cardiff University, CF24 3AA, Cardiff, UK, sarhosisv@cardiff.ac.uk

Abstract

Constitutive models for masonry require a number of parameters to define material behaviour with sufficient accuracy. It is common practice to determine such material parameters from the results of various, relatively simple, small-scale laboratory experiments. However, the effectiveness of determining material parameters that are representative of masonry from small-scale experiments have found to be problematic. This paper investigates the material parameter identification problem for masonry constitutive models. The methodology is based on an inverse analysis containing an optimization procedure and surrogate modelling. The general framework of the non-linear estimate methodology and the parameter identification problems are discussed.

Keywords: Numerical modelling, material parameter identification, masonry, non-linear analysis

1 Introduction

Masonry is the oldest material used in construction and has proven to be both simple to build and durable. Although its simplicity of construction, the analysis of masonry is a challenging task. Masonry is an anisotropic, heterogeneous and composite material where mortar joints act as plane of weakens. The need to predict the in-service behaviour and load carrying capacity of masonry structures has led researchers to develop several numerical methods and computational tools which are characterized by their different levels of complexity. For a numerical model to adequately represent the behaviour of a real structure, both the constitutive model and the input material properties must be selected carefully by the

29 modeller to take into account the variation of masonry properties and the range of stress state
30 types that exist in masonry structures (Hendry 1998.). It is often the case that material
31 parameters are very sensitive to the mechanical behaviour of the structure and if not selected
32 accurately can lead to over or under estimations (Sarhosis 2015). A broad range of numerical
33 methods is available today ranging from the classical limit analysis methods (Heyman, 1998)
34 to the most advanced non-linear computational formulations (e.g. finite element and discrete
35 element methods of analysis). The selection of the most appropriate method to use depends
36 on, among other factors, the structure under analysis; the level of accuracy and simplicity
37 desired; the knowledge of the input properties in the model and the experimental data
38 available; the amount of financial resources; time requirements and the experience of the
39 modeller (Lourenço, 2002). It should also be expected that different methods should lead to
40 different results depending on the adequacy of the approach and the information available.
41 Preferably, the approach selected to model masonry should provide the desired information in
42 a reliable manner within an acceptable degree of accuracy and with least cost. This paper
43 investigates the material parameter identification problem for masonry and proposes an
44 alternative methodology for obtaining material parameters for non-linear constitutive laws.

45

46 **2 Conventional methods for material parameter identification**

47 Conventionally, material parameters for masonry constitutive models are determined directly
48 from the results of compressive, tensile and shear strength tests on small masonry prisms.
49 These usually consist of assemblages of masonry consisting of a small number of bricks and
50 mortar joints. It is usually assumed that the stress and strain fields in the specimen are
51 uniform. In some other cases, separate tests are carried out on material samples, such as
52 masonry units and/or mortar specimens (Rots, 1997; Van der Pluijm, 1999). The testing of
53 small specimens is simple, relatively inexpensive and involves little specialist equipment.
54 However, the conventional approach is considered to be problematic and may not produce
55 material parameters that are representative of masonry. As identified by Hendry (1998), brick
56 and mortar properties are highly variable and depend primarily on the local supply of raw
57 materials and manufacturing methods. Also, the assumption that the stress and strain in the
58 specimen are uniform is not applicable for masonry which is an intrinsically inhomogeneous
59 material. Moreover, the simple conditions under which the small specimens are tested in the

60 laboratory do not usually reflect the more complex boundary conditions, the combinations of
61 stress-state types and load spreading effects that exist in a large scale masonry structure. In
62 addition, some of the parameters obtained from small scale tests are variable and sensitive to
63 the method of testing. This is likely to be due to the combined effects of eccentric loading,
64 stress concentrations and variations in the resistance to applied stress that are likely to exist in
65 the test specimens (Hendry, 1998). According to Vermeltoort (1997), the effects of boundary
66 conditions such as platen restraint and the shape and size of the test specimen can have a
67 significant influence on the magnitude of the measured parameter. For example, a mortar
68 joint between porous and absorbent masonry units will set, harden and cure in a different way
69 to the same mortar used to form a cube in a steel mould. Also, the restraint conditions on the
70 mortar in the cube test will be different to those existing in the mortar joint between masonry
71 units. Thus, the compressive strength of mortar obtained from a mortar cube test is unlikely
72 to represent the compressive strength of the mortar in between adjacent masonry units. The
73 situation is made more complex when workmanship is considered. Usually a much higher
74 standard and consistency of workmanship will be achieved by constructing small scale test
75 specimens in the laboratory compared with the construction of larger scale masonry
76 structures. Such variations in workmanship will not be captured if the material parameters are
77 based on the results from the testing of small scale specimens. In addition, the use of field test
78 results presents another set of difficulties. The stress and strain levels that are found in
79 structures in the field are likely to be very low and affected by effects such as moisture
80 movements, shrinkage and creep. Any material parameters determined from field
81 measurements are unlikely to represent the behaviour of masonry in the post-cracking and
82 near-collapse conditions. Other factors such as load spreading effects, residual thermal
83 stresses in bricks, large inclusions sometimes found in bricks, etc all contribute to the
84 uncertainty of material parameters obtained from small scale experiments. As a result of these
85 difficulties it is often necessary to adjust the material parameter values obtained from small
86 scale experiments before they can be used in the numerical model.

87 **3 Proposed method for material parameter identification**

88 From the above discussion it is evident that an alternative method of determining material
89 parameters that better reflects the complex nature of masonry and the range of stress state
90 types that exist in practice is worthy of further investigation. According to the proposed

91 method, a numerical analysis for each large scale “non-trivial” experiment is carried out
92 using an initial estimate of the material parameters. These initial values are “tuned” to
93 minimise the difference between the responses measured from the large scale laboratory
94 experiments and those obtained from the numerical simulation. It was envisaged that such
95 tests would be carried out in the laboratory and the large scale structures selected for this
96 purpose would be subjected to loading that would create a variety of different stress states.
97 The responses measured in the laboratory would normally be deflections or distortions. An
98 assumed range of material parameters is initially used in the model for the simulation of the
99 large scale experiments. These initial material parameters could be based on the results
100 obtained from conventional small-scale experiments, on values provided in codes of practice
101 or from experience and engineering judgement. It should also be mentioned that the range of
102 the selected material parameters should produce similar mechanical behaviour to that
103 obtained from the large scale experiment. The selection of the range of material parameters is
104 very important and will depend on the experience of the modeller. The material parameter
105 identification problem can then be considered as an optimization problem in which the
106 function to be minimized is an error function that expresses the difference between the
107 responses measured from the large scale experiments and those obtained from the numerical
108 analysis. Responses are based on the mechanical response of the masonry to be analyzed and
109 can include: failure load, load at initial cracking, load-deflection characteristics, etc. The use
110 of optimization software is essential for the evaluation of the approximation of responses as
111 well as for the implementation of the optimization process. One should be aware that the
112 optimization procedure should provide a single set of material parameters (e.g. global
113 minimum) that are representative for the case under investigation. The use of graphical
114 illustrations of the solution in the form of response surface analysis is highly recommended.

115

116 The proposed method of material parameter identification is illustrated in Figure 1. The
117 method was initially proposed by Toropov and Garrity (1998) and later expanded and
118 validated for low strength masonry by Sarhosis & Sheng (2014).

119

120 The aim of the identification problem is to obtain the optimum estimate of the unknown
121 model parameters taking into account uncertainties which may exist in the problem, such as
122 the inherent variation of material properties, experimental errors and errors in the model

123 estimation method. The estimates of the material parameters obtained from this approach
124 could be referred to as the “maximum likelihood estimates” and can be used to “*inform*” the
125 computational model. Sarhosis (2014) suggested that in order to account for the inherent
126 variations in the materials and unavoidable variations in workmanship, for each of the large
127 scale experiments at least three specimens should be tested. Also, it is important to note that
128 the above method can be used for any constitutive model describing masonry as long as the
129 constitutive model describes the mechanical behaviour of masonry with sufficient accuracy.
130 It is anticipated that after undertaking a series of studies, an extensive library of material
131 parameters can be obtained where one can download and use for the numerical simulation.

132

133 Examples showing studies for material parameter identification for large deformation
134 plasticity models include: a) test data of a solid bar in torsion (Toropov et al., 1993) and b)
135 test data for the cyclic bending of thin sheets (Yoshida et al., 1998). Later, Morbiducci (2003)
136 applied the method to two different masonry problems in order to: a) identify the parameters
137 of a non-linear interface model (Gambarotta et al., 1997a) to describe the shear behaviour of
138 masonry joints under monotonic loading, where shear tests were chosen as the experimental
139 tests; b) to evaluate the parameters of a continuum model for brick masonry walls under
140 cyclic loading (Gambarotta et al., 1997b); and c) to evaluate the parameters of low bond
141 strength masonry (Sarhosis 2014; Giamoundo et al. 2014). From the above studies, the
142 following points have been observed and should be taken into consideration when using such
143 method:

- 144 a) When modelling masonry, different material parameters influence different stages of
145 mechanical behaviour;
- 146 b) large number of full scale experiments may be required; and
- 147 c) a significant amount of computational time is required to carry out parameter
148 sensitivity studies.

149

150 4 Formulation of the material parameter identification problem

151 4.1 Formulation of the optimization problem

152 Consider an experimental test performed on $\mathcal{M} = 1, 2, \dots, m$ specimens. Also, the design
153 variables or unknown parameters to be estimated are $\mathcal{P} = 1, 2, \dots, p$ which form part of the
154 constitutive model for the masonry material. Let's assume that $\mathcal{N} = 1, 2, \dots, n$ represents the
155 number of responses that are recorded from the experimental data and are going to be
156 compared with the numerical simulation. Also, let's consider the variable R_n^{exp} to be the value
157 of the n^{th} measured response which corresponds to the large scale experiment carried out in
158 the laboratory. Consider R_n^{comp} as the value of the n^{th} measured response quantity
159 corresponding to the computational simulation. The model takes the general function form
160 $x = \mathcal{R}(\mathcal{P})$. To calculate this function for the specific set of parameters, x , once has to use a
161 non-linear numerical simulation, usually based on a discrete or finite element method of
162 analysis. The intention is to simulate the mechanical behaviour of the experimental test under
163 consideration. In this way, the difference between the experimental and the numerical
164 responses can be obtained. This form an error function that can be expressed by the
165 difference $D = \mathcal{R}_{M,N}^{\text{exp}} - \mathcal{R}_{M,N}^{\text{comp}}$.

166

167 The optimization problem can then be formulated as follows:-

168

$$169 \quad F_{(x)}^1 = \sum \left[(\mathcal{R}_{1,1}^{\text{exp}} - \mathcal{R}_{1,1}^{\text{comp}})^2 + (\mathcal{R}_{1,2}^{\text{exp}} - \mathcal{R}_{1,2}^{\text{comp}})^2 \dots \dots + (\mathcal{R}_{1,n}^{\text{exp}} - \mathcal{R}_{1,n}^{\text{comp}})^2 \right] \quad (1)$$

$$170 \quad F_{(x)}^2 = \sum \left[(\mathcal{R}_{2,1}^{\text{exp}} - \mathcal{R}_{2,1}^{\text{comp}})^2 + (\mathcal{R}_{2,2}^{\text{exp}} - \mathcal{R}_{2,2}^{\text{comp}})^2 \dots \dots + (\mathcal{R}_{2,n}^{\text{exp}} - \mathcal{R}_{2,n}^{\text{comp}})^2 \right] \quad (2)$$

171 \vdots

$$172 \quad F_{(x)}^m = \sum \left[(\mathcal{R}_{m,1}^{\text{exp}} - \mathcal{R}_{m,1}^{\text{comp}})^2 + (\mathcal{R}_{m,2}^{\text{exp}} - \mathcal{R}_{m,2}^{\text{comp}})^2 \dots \dots + (\mathcal{R}_{m,n}^{\text{exp}} - \mathcal{R}_{m,n}^{\text{comp}})^2 \right] \quad (3)$$

173

174 $F^M(\mathbf{x}) = F_{(x)}^1 + F_{(x)}^2 + \dots + F_{(x)}^m$ is a dimensionless function. The problem is then to find the
175 vector $\mathbf{x} = [x_1, x_2, x_3 \dots x_p]$ that minimizes the objective function:

176

$$177 \quad F_{(x)}^{\text{total}} = \sum \theta^{\mathcal{M}} (F^{\mathcal{M}}(\mathbf{x})), \quad A_i \leq X_i \leq B_i \quad (i = 1 \dots \dots N) \quad (4)$$

178 where $F_{(x)}^{\text{total}}$ is a function of the unknown parameters $(x_1, x_2, x_3 \dots x_p)$, $\theta^{\mathcal{M}}$ is the weight
179 coefficient which determines the relative contribution of information yielded by the M-th set
180 of experimental data, and A_i, B_i are the lower and upper limits on the values of material
181 parameters identified by physical considerations. The objective function is an implicit
182 function of parameters x , where $x \in \mathbb{R}$. Also, once should expect that since a series of
183 numerical simulations will be required, a considerable amount of computational time will
184 result. Also, the optimization procedure may present some level of numerical noise. Since the
185 computational simulations would involve an excessive amount of computational time to
186 execute and convergence of the above method cannot be guaranteed due to the presence of
187 noise in the objective function values, routine task analysis such as design optimization,
188 design space exploration, sensitivity analysis and what-if analysis become impossible since
189 they require thousands of simulation evaluations. One way to mitigate against such a burden
190 is by constructing surrogate models (also referred to by some researchers as response surface
191 models or metamodels). These mimic the behaviour of the model as closely as possible while
192 at the same time they are time effective to evaluate (Queipo et al., 2005). Surrogate models
193 are constructed based on modelling the response predicted from the computational model to a
194 limited number of intelligently chosen data points. In the case that a single variable is
195 involved, the process is known as curve fitting, see Figure 2. New combinations of parameter
196 settings, not used in the original design, can be plugged into the approximate model to
197 quickly estimate the response of that model without actually running it through the entire
198 analysis. This approach can result in less computational iterations leading to substantial
199 saving of computational resources and time.

200

201 Using this approach, the initial optimization problem, equation (4), is replaced with the
202 succession of simpler mathematical programming sub-problems as follows:

203

204 Find the vector \mathbf{x}_k^* that minimizes the objective function:

205

$$206 \quad \tilde{F}_k(x) = \sum \theta^{\mathcal{M}} \tilde{F}_k^{\mathcal{M}}(x), \quad A_i^k \leq X_i \leq B_i^k, \quad A_i^k \geq A_i, B_i^k \leq B_i \quad (i = 1 \dots \dots N) \quad (5)$$

207

208 where k is the iteration number. The limits A_i^k and B_i^k define a sub-region of the
209 optimization parameter space where the simplified functions $\tilde{F}_k^M(x)$ are considered as current
210 approximations of the original implicit functions $F^M(x)$. To estimate their accuracy, the error
211 parameter $r_k = |[F(x_k^*) - \tilde{F}_k(x_k^*)]/F(x_k^*)|$ is evaluated. The value of the error parameter
212 gives a measure of discrepancy between the values of the initial functions and the simplified
213 ones. Any conventional optimization technique can be used to solve a sub-problem, equation
214 (5), because the functions involved in its formulation are simple and noiseless.

215

216 **4.2 Choice of the surrogate model**

217 To construct the simplified noiseless expression for the function $\tilde{F}_k^M(x)$ in equation (5),
218 different methods of regression analysis can be used including the Least Squares Regression
219 (LSR) method, the Moving Least Squares (MLS) method and the Hyper Kriging approach for
220 building approximation models. The LSR and the MLS methods will be described for
221 approximating noisy experimental results such as those obtained from the testing of masonry
222 structures. Hyper Kriging is not considered further as it is suitable for modelling highly non-
223 linear response data that does not contain numerical noise.

224

225 4.2.1 Least Squares Regression (LSR)

226 LSR is an approximation method which finds application in data fitting (Toropov et al.,
227 2005). The best fit in the least squares sense minimizes the sum of the squared residuals i.e.
228 the difference between an observed value and the fitted value provided by the model. Let N
229 points located at positions x_i in \mathbb{R} where $i \in [1 \dots N]$. We wish to obtain a globally defined
230 function $f(x)$ that approximates the given scalar values f_i at points x_i in the least squares
231 sense with the error function $r_{LS} = \sum_i \|f(x_i) - f_i\|^2$. The following optimization problem can
232 be obtained:

$$233 \quad \min \sum_i \|f(x_i) - f_i\|^2 \quad (6)$$

234 , where f is taken from the polynomial basis vector and the vector of unknown coefficients to
235 be minimized in equation (6).

236

237 4.2.2 Moving Least Squares (MLS)

238 MLS is an approximation building technique that is proposed for smoothing and interpolating
239 data (Toropov et al., 2005). MLS is a generalisation of a conventional weighted least squares
240 model building method. The main difference between MLS and LSR is that with MLS the
241 weights associated with the individual experimental sampling points do not remain constant
242 but are functions of the normalized distance from an experimental sampling point to a point x
243 where the approximation model is evaluated. In the weighted least squares formulation, we
244 use the error function $r_{WLS} = \sum_i W_i \|f(x_i) - f_i\|^2$ for a fixed point $\tilde{x} \in \mathbb{R}$, which we
245 minimize:

$$246 \quad \min \sum_i W_i \|f(x_i) - f_i\|^2 \quad (7)$$

247

248 The function is similar to equation (6) only that, now, the error is weighted by W_i . Many
249 choices for the weighting function W_i have been proposed in the literature (Alexa et al.,
250 2003). Equation 8 shows the Gaussian formulation:

251

$$252 \quad W_i = e^{-\theta r_i^2} \quad (8)$$

253

254 , where r_i are the Euclidian normalized distances from the $i - th$ sampling point to a current
255 point. Also, the parameter θ refers to the “closeness of fit” and by varying its value we can
256 directly influence the approximating/interpolating nature of the MLS fit function. A low
257 value of θ leads to least squares smoothing (e.g. in the case where $\theta = 0$, then equation (7) is
258 equivalent to the traditional least squares regression). Alternatively, when the parameter θ is
259 large, it is possible to obtain a very close fit through the sampling points (i.e. interpolating), if
260 desired. When the MLS method is used to approximate results obtained from experiments
261 carried out on masonry structures, interpolation (i.e. a high value of θ) would not be
262 appropriate, as there is a considerable amount of variation in the masonry material properties
263 resulting in experimental noise.

264

265 **4.3 Choice of the optimization method**

266 In order to solve the sub-problem in equation (5), there are a number of available
 267 optimization methods to be used. Currently, a gradient-based method (known as Sequential
 268 Quadratic Programming) and a global search algorithm method (known as the Genetic
 269 Algorithm approach) are the two representative methods that can be used for the comparison
 270 of results (Toropov and Yoshida, 2005).

271

272 The Sequential Quadratic Programming (SQP) method is used for solving constrained
 273 optimization problems by creating linear approximations to the constraints (Toropov et al.,
 274 2010). The fundamental principle behind this method is to create a quadratic approximation
 275 of the Lagrangian function that combines the objective function with active constraints. The
 276 quadratic problem is then solved for the search direction avoiding any constraint violations.
 277 On the other hand, a Genetic Algorithm (GA) is a machine learning technique modelled after
 278 the evolutionary process theory. Genetic algorithms differ from conventional optimization
 279 techniques in that the work is based on a whole population of individual objects of finite
 280 length, typically binary strings (chromosomes), which encode candidate solutions
 281 $(x_1, x_2, x_3, \dots, x_n)$ using a problem-specific representation scheme (Toropov et al., 2010).
 282 These strings are decoded and evaluated for their fitness, which is a measure of how good a
 283 particular solution is. Following Darwin’s principle of “survival of the fittest” (or natural
 284 evolution), strings with higher fitness values have a higher probability of being selected for
 285 mating purposes to produce the next generation (i.e. new population created from current
 286 population) of candidate solutions (Toropov et al., 2010). Evolution is performed by breeding
 287 the population of individual designs over a number of generations. The advantages and the
 288 limitations of SQP and GA methods for solving optimization problems are shown in Tables 1
 289 & 2.

290 **Table 1** Sequential Quadratic Programming: Advantages and limitations

Advantages	Limitations
<ul style="list-style-type: none"> - Converges fast to a highly accurate solution when gradients are accurate; - There is no dramatic increase in 	<ul style="list-style-type: none"> - As with any other gradient-based technique, SQP falls into the nearest local optimum so might need restarts from different points; - Converges poorly when gradients are

<p>the number of iterations when the number of design variables grows.</p>	<p>inaccurate;</p> <ul style="list-style-type: none"> - Deals with continuous problems. In the case of a discrete problem, the solution has to be discretised (e.g. rounding off); - As a sequential technique, parallelisation is only possible for getting gradients.
--	---

291

292

293 **Table 2** Genetic Algorithm: Advantages and limitations

Advantages	Limitations
<ul style="list-style-type: none"> - More likely to find a non-local solution as it works with a population of sets of variables rather than a single set; - Can handle noise and occasional failure to compute responses; - As GA is a non-deterministic search method (it exhibits different behaviours on different runs), it makes the search highly robust; - Simplicity; - Can be easily parallelised; - Only requires the objective function and not the derivatives; - Allows both discrete and continuous (discretized) variables as it codes the variables rather than taking the variables themselves. 	<ul style="list-style-type: none"> - High number of design iterations; - Lower accuracy compared with gradient based techniques for continuous problems; - Lack of indication as to how close the solution is to the optimum; - A few parameters need to be defined that affect the solution process.

294

295 **5 Conclusion**

296 A methodology for material parameter identification for nonlinear masonry constitutive laws
297 has been proposed. Usually, the material parameters used for modelling masonry within
298 computational models are based on the results of simple tests that do not reflect the more
299 complex boundary conditions and combinations of stress-state types that exist in a real
300 masonry structure. A method which is considered likely to determine more representative
301 material parameters for masonry constitutive models has been proposed. This involves the
302 computational analysis of large scale experimental tests on masonry structures. The initially
303 assumed material parameters are tuned to minimize the difference between the responses
304 measured from the large scale tests and those obtained from the computational simulations.
305 The procedure has been successfully validated by (Sarhosis, 2014) when used to determine
306 the material parameters for low bond strength masonry for a microscopic discrete element
307 model. Both computational and experimental test data from a number of low bond strength
308 brick masonry wall panels, each containing an opening to represent a large window, loaded at
309 mid-span are used. Such wall panels were chosen as they contain regions of different types of
310 stress when subjected to an externally applied load. In addition, the panels were considered to
311 be sufficiently large to include inherent variations in the masonry materials and variations in
312 workmanship. In the future, the effectiveness of the methodology is going to be applied to
313 identify material parameters for macro-models.

314

315 **Reference**

316 Alexa, M., Behr, J., Cohen-Or, D., Fleishman, S., Levin, D. and T. Silva, C., 2003.
317 Computing and rendering point set surfaces. *IEEE Transactions on visualization and*
318 *Computer Graphics*, (9)1, pp.3-15.

319

320 Gambarotta, L. and Lagomarsino, S., 1997a. Damage models for the seismic response of
321 brick masonry shear walls. Part I: The mortar joint model and its applications. *Journal of*
322 *Earthquake Engineering and Structural Dynamics*, 26(4), pp.423-439.

323

324 Gambarotta, L. and Lagomarsino, S., 1997b. Damage models for seismic response of brick
325 masonry shear walls. Part II: The continuum model and its applications. *Journal of*
326 *Earthquake Engineering and Structural Dynamics*, 26(4), pp.441-462.

327

328 Giamundo V., Sarhosis V., Lignola G.P., Sheng Y., Manfredi G. 2014. Evaluation of
329 different computational modelling strategies for modelling low strength masonry,
330 Engineering Structures, 73, pp.160-169. DOI: 10.1016/j.engstruct.2014.05.007
331

332 Hendry, A.W., 1998. Structural masonry. 2nd Edition. Palgrave Macmillan, London, UK.
333

334 Heyman, J., 1998. Structural Analysis: A historical approach. University Press, Cambridge,
335 UK
336

337 Lourenço, P.B., 2002. Computations on historic masonry structures. Progress in Structural
338 Engineering and Materials, 4(3), pp.301-319.
339

340 Morbiducci, R., 2003. Non-linear parameter identification of models for masonry.
341 International Journal of Solids and Structures, 40(15), pp.4071-4090.
342

343 Queipo, N.V., Haftka, R.T., Shyy, W., Goel, T., Vaidyanathan, R., Tucker, P.K., 2005.
344 Surrogate-based analysis and optimization. Progress in Aerospace Sciences, 41(1), pp. 1-28.
345

346 Rots, J.G., 1997. Structural masonry: An experimental/numerical basis for practical design
347 rules. A.A. Balkema Publishers, Netherlands.
348

349 Sarhosis V, Sheng Y. 2014. Identification of material parameters for low bond strength
350 masonry. Engineering Structures, 60(1):100-110.
351

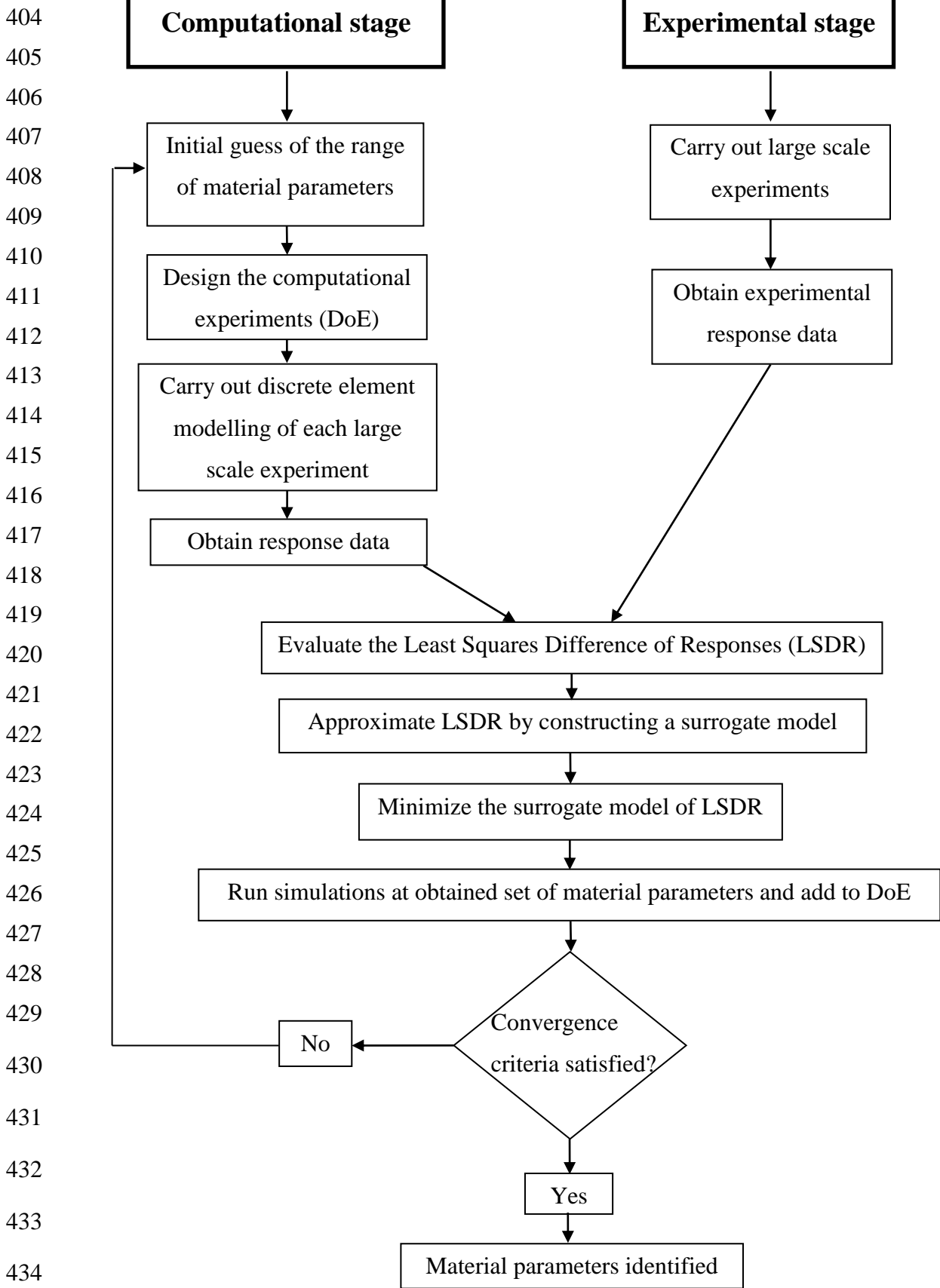
352 Sarhosis V., Garrity S.W., Sheng Y. 2015. Influence of the brick-mortar interface on the
353 mechanical response of low bond strength masonry lintels, Engineering Structures, 88, 1-11.
354 DOI: 10.1016/j.engstruct.2014.12.014
355

356 Toropov, V.V. and Garrity, S.W., 1998. Material parameter identification for masonry
357 constitutive models. In: Proceedings of the 8th Canadian Masonry Symposium. Jasper,
358 Alberta, Canada, pp.551-562.
359

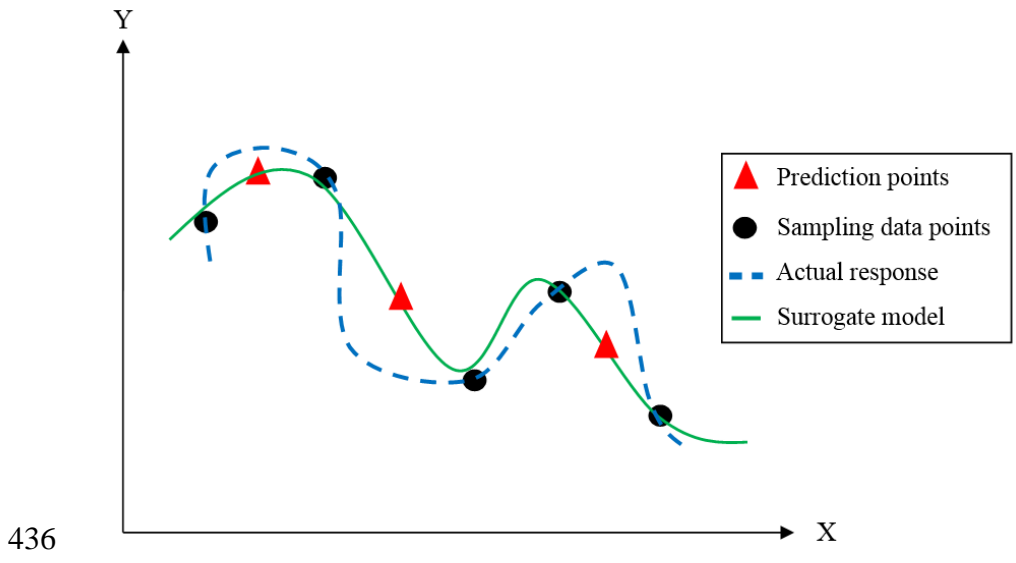
360 Toropov, V.V. and Van der Giessen, E., 1993. Parameter identification for nonlinear
361 constitutive models: Finite element simulation - optimization - nontrivial experiments. In:
362 Pedersen, P. (ed.), Proceedings of IUTAM Symposium, Optimal design with advanced
363 materials - The Frithiof Niordson volume. Lyngby, Denmark, pp.113-130.
364

365 Toropov, V.V. and Yoshida, F., 2005. Application of advanced optimization techniques to
366 parameter and damage identification problems. In: Mroz, Z. and Stavroulakis, G.E. (eds.),

367 Parameter Identification of Materials and Structures. CISM Courses and Lectures vol. 469,
368 International Centre for Mechanical Sciences, pp.177-263.
369
370 Toropov, V.V., Alvarez, L.F. and Querin, O.M., 2010. Applications of GA and GP to
371 industrial design optimization and inverse problems. In: Waszczyszyn, Z. (ed.), Advances of
372 Soft Computing in Engineering. CISM Courses and Lectures vol. 512, International Centre
373 for Mechanical Sciences, pp.133-189.
374
375 Van der Pluijm, R., 1999. Out-of-plane bending of masonry behaviour and strength, Ph.D
376 thesis, Eindhoven University of Technology, The Netherlands.
377
378 Vermeltoort, A.T., 1997. Effects of the width and boundary conditions on the mechanical
379 properties of masonry prisms under compression. In: Proceedings of the 11th International
380 Brick/Block Masonry Conference. Shanghai, 27-29 October, pp.181-190.
381
382 Yoshida, F., Urabe, M. and Toropov, V.V., 1998. Identification of material parameters in
383 constitutive model for sheet metals from cyclic bending tests. International Journal of
384 Mechanical Sciences, 40(2), pp.237-249.
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403



435 **Figure 1** Proposed methodology for the identification of material parameters (Sarhosis 2014)



436

437 **Figure 2** Curve fitting

438

439

440

441

442

443

444

445