The effect of skew angle on the mechanical behaviour of masonry arches

V. Sarhosis*, D.V. Oliveira+, J.V. Lemos#. P.B. Lourenco+
* School of Engineering, Cardiff University, Cardiff, UK, SarhosisV@cardiff.ac.uk
+◊ISISE, Department of Civil Engineering, University of Minho, Portugal, danvco@civil.uminho.pt, pbl@civil.uminho.pt
National Laboratory of Civil Engineering, Lisbon, Portugal, vlemos@lnec.pt

ABSTRACT

This paper presents the development of a three dimensional computational model, based on the Discrete Element Method (DEM), which was used to investigate the effect of the angle of skew on the load carrying capacity of twenty-eight different in geometry single span stone masonry arches. Each stone of the arch was represented as a distinct block. Mortar joints were modelled as zero thickness interfaces which can open and close depending on the magnitude and direction of the stresses applied to them. The variables investigated were the arch span, the span : rise ratio and the skew angle. At each arch, a full width vertical line load was applied incrementally to the extrados at quarter span until collapse. At each load increment, the crack development and vertical deflection profile was recorded. The results compared with similar "square" (or regular) arches. From the results analysis, it was found that an increase in the angle of skew will increase the twisting behaviour of the arch and will eventually cause failure to occur at a lower load. Also, the effect of the angle of skew on the ultimate load that the masonry arch can carry is more significant for segmental arches than circular one.

Keywords: Masonry, arches, discrete element modelling, cracking, in-plane loading.

1 INTRODUCTION

A skew arch is a method of construction that enables masonry arch bridges to span obstacles at an angle (Fig. 1). Bridges with a small amount of skew (i.e. less than 30º) can be constructed using bedding planes parallel to the abutments (Melbourne & Hodgson, 1995). However, bridges with large amount of skew present significant construction difficulties. Fig. 2 shows three well-known methods of construction for an arch spanning at 45 degrees skew (Page 1993). Fig. 2a shows the simplest form of construction where units are laid parallel to abutments. Fig. 2b shows the English (or helicoidal) method which is constructed such that the bed at the crown is perpendicular to the longitudinal axis of the bridge. For geometrical reasons and for the beds to remain parallel, the orientation of the block units causes the beds to "roll over" and thus rest on the springings at an angle (Fig. 1b). This is a cheap method of construction since every voussoir is cut similar to each other. Fig. 2c shows the French (or orthogonal) method which keeps the bed orthogonal with the local edge of the arch. This is the most expensive method of construction since it requires varying sized masonry blocks and availability of high skilled masons, since almost every block in the arch barrel to be of unique shape. The procedure used for the construction of such bridges and their mathematical curves are described in full detail by Rankine (1862).

There are many thousands of stone masonry arch bridges in Europe, many of which have spans with a varying amount of skew (Brencich & Morbiducci 2007). Most of these bridges are well over 100 years old and are supporting traffic loads many times above those originally envisaged. Different materials and methods of construction used in these bridges will influence their strength and stiffness. There is an increasing demand for a better understanding of the life expectancy of such bridges in order to inform maintenance, repair and strengthening strategies. Although a great deal of work has been carried out to assess the strength of square span masonry arch bridges using mainly two dimensional methods of analysis (Heyman 1966; Gilbert 1993; Page 1993; Melbourne & Hodgson 1995), comparatively little work has been undertaken to understand the three dimensional behaviour of skew arches (Hodgson 1996; Wang 2004). The analysis of skew arch bridges has many difficulties and there is no universally accepted method of analysis yet. Today, in many countries, including UK, skew arches are routinely assessed on the basis that the skew span is straight (e.g. DB 21/01; DB16/17). However, experience from previous studies has clearly shown that depending on the method of construction and geometry, the stiffness and strength of skew arches might be quite different (Hodgson 1996). In addition, such method is not suitable for non-standard geometries or for arches which suffered damage and deterioration.

(a) (b)

Fig. 1. Typical skew masonry arch constructed using the English method: (a) front view; (b) detail of the intrados

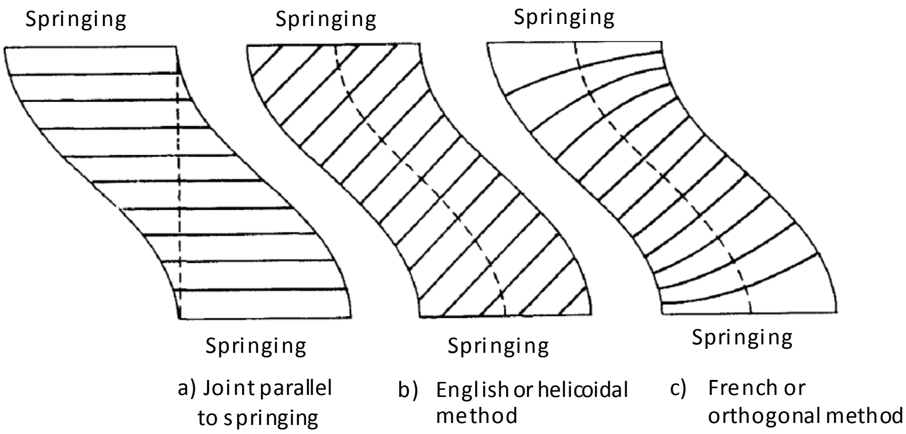


Fig. 2. Intrados of an arch spanning at 45^0 skew (Page 1993).

In recent years, sophisticated methods of analysis like Finite Element Method (FEM) have been applied to understand the three dimensional behaviour of arches (Choo & Gong 1995). A nice overview of the different arch models performed in the 1990's can be found in Boothby (2001). However, in such models, the description of the discontinuity is limited since they tend to focus on the continuity of the arch. Sophisticated FEM approaches (e.g. contact element techniques) are able to reflect the discrete nature of masonry. Examples of such models have been undertaken by Fanning and Boothby (2001), Gago et al. (2002), Ford et al. (2003) and Drosopoulos et al. (2006). The disadvantages of these methods are mainly associated with: a) high computational cost; b) crack development cannot be obtained; and c) convergence difficulties if blocks fall or slide excessively. An alternative and appealing approach is represented by the Distinct Element Method (DEM), where the discrete nature of the masonry arch is truly incorporated. The advantage of the DEM is that considers the arch as a collection of separate voussoirs able to move and rotate to each other. The DEM was initially developed by Cundall (1971) to model blocky-rock systems and sliding along rock mass. The approach was later used to model masonry structures including arches (Lemos 1995;

Lemos 2007; Mirabella & Calvetti 1998; Toth 2009; Sarhosis 2014), where failure occurs along mortar joints. These studies demonstrated that DEM is a suitable method to perform analysis of masonry arches and to describe realistically the ultimate load and failure mechanism. However, the above studies were mainly focused on the two dimensional behaviour of arches.

The aim of this paper is to study the three dimensional behaviour of single span skew masonry arches and provide useful guidance for the design engineer. Using the three dimensional DEM software 3DEC (Itasca 2004), computational models were developed to predict the serviceability and ultimate state behaviour of twenty-eight stone masonry arches with different geometries and skew angles. DEM is well suited for collapse analysis of stone masonry structures since: a) large displacements and rotations between blocks, including their complete detachment, can be simulated; b) contacts between blocks are automatically detected and updated as block motion occurs; c) progressive failure associated with crack propagation can be simulated; and d) interlocking can be overcome by rounding the corners.

At this study, arches were constructed with joints parallel to abutments (Fig. 2a). Since the intention of the authors was to investigate the effect of the arch ring geometry, the effect of fill has not been included at this stage. The variables investigated were the arch span, the span : rise ratio and the skew angle. Results are compared against the load to cause first cracking, the magnitude of collapse load, the mode of failure and the area of joints opened. The suitability of the DEM to model the three dimensional behaviour of skew arches is also outlined. It is anticipated that results of this study will provide insight into the structural performance of skew masonry arches as well as will provide useful guidance for the design engineers.

2 OVERVIEW OF 3DEC FOR MODELLING MASONRY

3DEC is an advanced numerical modelling code based on DEM for discontinuous modelling and can simulate the response of discontinuous media, such as masonry, subjected to either static or dynamic loading. When used to model masonry, the units (i.e. stones) are represented as an assemblage of rigid or deformable blocks which may take any arbitrary geometry. Typically, rigid blocks are adequate for structures with stiff, strong units, in which deformational behaviour takes place at the joints. For explicit dynamic analysis, rigid block models run significantly faster. For static problems, this computational advantage is less important, so deformable blocks are preferable, as they provide a more elaborate representation of structural behaviour. Deformable blocks, with an internal tetrahedral FE mesh, were used in the analyses reported herein. Joints are represented as interfaces between blocks. These interfaces can be viewed as interactions between the blocks and are governed by appropriate stress-displacement constitutive laws. These interactions can be linear (e.g. spring stiffness) or non-linear functions. Interaction between blocks is represented by set of point contacts, of either vertex to face or edge to edge type (Fig. 3). In 3DEC, finite displacements and rotations of the discrete bodies are allowed. These include complete detachment between blocks and new contact generation as the calculation proceeds. Contacts can open and close depending on the stresses acting on them from the application of the external load. Contact forces in both the shear and normal direction are considered to be linear functions of the actual penetration in shear and normal directions respectively (Itasca 2004). In the normal direction, the mechanical behaviour of joints is governed by the following equation:

$$\Delta\sigma_n = - JKn \cdot \Delta u_n \tag{1}$$

where JKn is the normal stiffness of the contact, $\Delta\sigma_n$ is the change in normal stress and Δu_n is the change in normal displacement. Similarly, in the shear direction the mechanical behaviour of mortar joints is controlled by a constant shear stiffness JKs using the following expression:

$$\Delta\tau_s = - JKs \cdot \Delta u_s \tag{2}$$

where $\Delta\tau_s$ is the change in shear stress and Δu_s is the change in shear displacement. These stress increments are added to the previous stresses, and then the total normal and shear stresses are updated to meet the selected non-elastic failure criteria, such as the Mohr-Coulomb model.

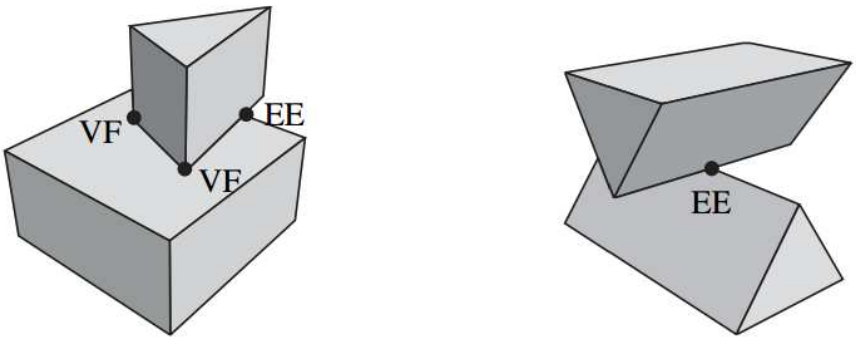


Fig. 3. Representation of block interaction by elementary vertex-face (VF) and edge-edge (EE) point contacts in 3DEC (Lemos 2007).

The calculations are made using the force-displacement law at all contacts and the Newton's second law of motion at all blocks. The force-displacement law is used to find contact forces from known displacements, while the Newton's second law governs the motion of the blocks resulting from the known forces acting on them. Convergence to static solutions is obtained by means of adaptive damping, as in the classical dynamic relaxation methods. Fig. 4 shows the schematic representations of the calculations taking place in 3DEC analysis.

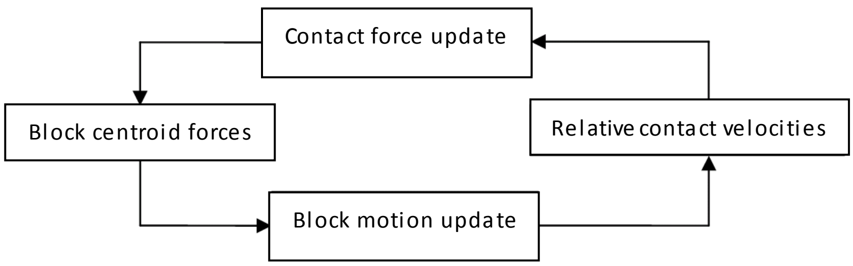


Fig. 4. Calculation cycle in 3DEC (Itasca 2004).

3 COMPUTATIONAL MODELLING OF MASONRY ARCHES WITH 3DEC

3.1 Geometry

Initially, geometric models of four square arches have been created using 3DEC. Arches A and C had a deep semi-circular shape and Arches B and D had a semi-shallow segmental shape (Fig. 5). For the semi-circular arches the rise to span ratio was 1:2, while for the segmental arches 1:4. According to Jennings (2004), segmental arches are constructed were larger spans are required and gives fewer supports and lower roadway level for a given clearance under the bridge. The width of the arches kept constant and equal to 4.8 m, which according to Oliveira et al. (2010) it is typical for stone masonry arches. Geometric data of the arches under investigation are shown in Table 1.

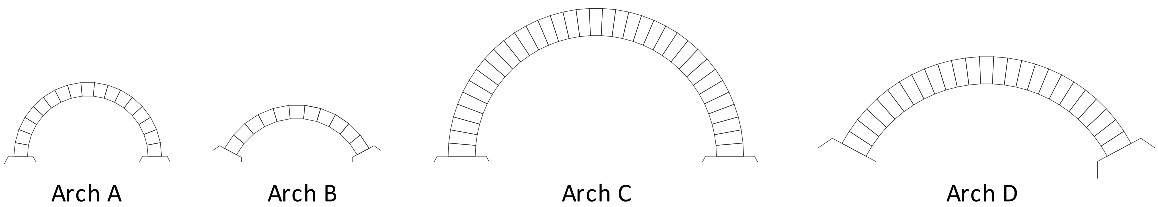


Fig. 5. Geometry of the arches studied (elevation view).

Table 1. Arch dimensions used in the analysis.

Arch	Arch shape	Skew span [m]	Rise to span ratio	Barrel thickness [m]	Width [m]
Arch A	Deep semi-circular	4.0	1:2	0.45	4.8
Arch B	Semi-shallow segmental	4.0	1:4	0.45	4.8
Arch C	Deep semi-circular	8.0	1:2	0.9	4.8
Arch D	Semi-shallow segmental	8.0	1:4	0.9	4.8

3.2 Block and interface details

Each stone of the arch was represented by a deformable block separated by zero thickness interfaces at each mortar joint. The deformable blocks were internally discretised into finite difference zone elements, each assumed to behave in a linear elastic manner. As failure in low strength masonry arches is predominantly at the brick/mortar joint interfaces (Melbourne & Hodgson 1995), the stresses in the stone blocks will be well below their strength limit and so no significant deformation would be expected to occur to them. The zero thickness interfaces between adjacent blocks were modelled using the elastic perfectly plastic coulomb slip failure criterion with a tension cut-off. So, if in any of the numerical calculations the value of tensile bond strength or shear strength is reached at a certain location, then the tensile strength and cohesion are reduced to zero at that location (Itasca 2004). Material parameters for the stone blocks and the mortar joints have been obtained from the literature (Lemos 2007; Toth 2009) and presented in Table 2 and Table 3.

Table 2. Properties of the masonry units.

Density [kg/m^3]	Young Modulus [N/m^2]	Poisson's ratio [-]	Bulk Modulus [N/m^2]	Shear Modulus [N/m^2]
2700	50E9	0.2	27.7E9	20.8E9

Table 3. Properties of the interfaces.

Joint Normal Stiffness [N/m^3]	Joint Shear Stiffness [N/m^3]	Joint Friction Angle [Degrees]	Joint Tensile Strength [N/m^2]	Joint Cohesive Strength [N/m^2]	Joint Dilation Angle [Degrees]
7.64E9	1.79E9	35	0.1E6	0.1E6	0

3.3 Boundary conditions and loading

Since the intention of the authors was to investigate the effect of the arch ring geometry, the abutments of the arch were modelled as rigid supports in the vertical and horizontal directions. The local damping option was selected for the static analysis algorithm.

Self-weight effects were assigned as a gravitational load. Gravitational forces cause the raise of compressive forces within the blocks of the arch and result in the stabilisation of the arch. Initially, the model was brought into equilibrium under its own self weight. An external full width descending linear load was applied incrementally on the arch at one quarter of the span parallel to the abutments until the arch collapsed. The loading history was imposed by applying a velocity at the loading block. In order to determine the applied load at each time-step, a subroutine has been written using FISH (an embedded language in 3DEC) which was able to trace the reaction forces from the fixed velocity grid points acting on the loading block. Evolution of the displacement of the block below the loading point was recorded. This was later used to obtain load-displacement relationships.

3.4 Validation of the computational model

The reliability of the numerical model evaluated by comparing the ultimate load obtained from 3DEC against those obtained by imposing the limit equilibrium of the arch at collapse using limit analysis software RING (LimitState 2009). Since RING is a two dimensional software, comparisons were made with respect to the four square span arches (i.e. zero skew). Also, for this comparative study and with the assumption that the limit analysis theorem applies (Heyman 1966), the joint tensile and cohesive strength in 3DEC model have been assumed to be equal to zero. The comparisons between the numerical and analytical results, for the four square arches, are shown in Fig. 6. The little peaks in the curves shown in Fig. 6 represent relaxation of the loading and moment redistribution in the arch due to the formation of a new crack. When a crack propagates there is an abrupt loss in stiffness in the arch. Good correlation was obtained between the results from the limit analysis and the 3DEC model.

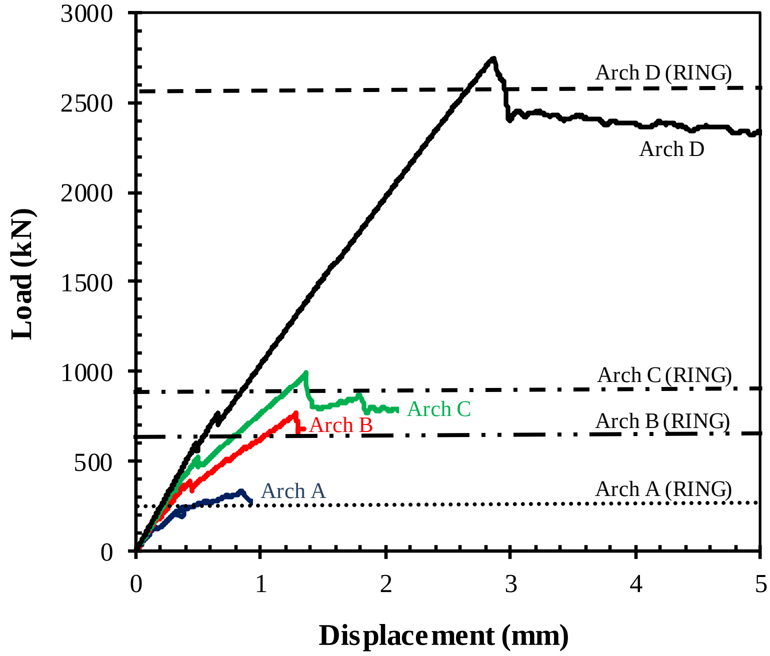


Fig. 6. Load against displacement relationship for the four square (i.e. zero skew) arches studied.

4. PARAMETRIC ANALYSIS

4.1 Influence of the angle of skew

The influence of the angle of skew is investigated by comparing square arches against those with different angles of skew with respect to the load at first crack, mode of failure, load carrying capacity and area of joint opened. All arches were constructed with joints parallel to springing. According to Melbourne & Hodgson (1995), this type of construction is found to arches with small angles of skew. For this reason, the angle of skew (φ) was varied from 0 to 30 degrees with 5 degrees interval. Also, the span (S) parallel to the axis of the arch has been kept constant for all arches. As a result, the square span (s) of the arches decreased as the angle of skew (φ) increased (Fig. 7). The square span of each arch was equal to $s = S \times \cos(\varphi)$.

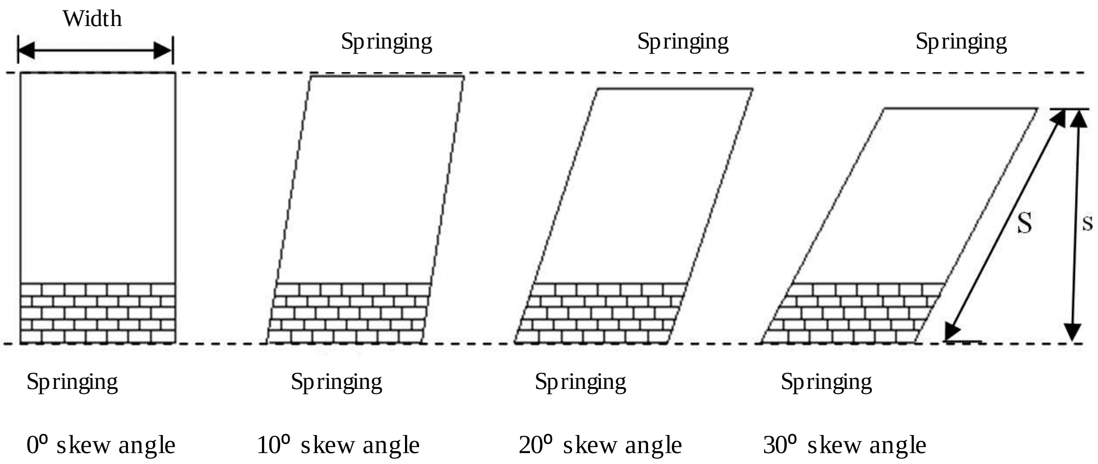


Fig. 7. Geometry of the arches studied: Plan view of a typical arch.

4.2 Load at first crack

Cracks in masonry may not open uniformly but may open and close according to the variation of the stress field over a period of time. In 3DEC, a contact point is defined as "open" if there is currently on the contact a zero normal force. For the purpose of this study, a FISH function has been written that was able to trace contact opening greater than 0.2 mm. Usually, cracks of 0.2 mm and wider are visible to the naked eye. The load required to cause crack opening greater or equal to 0.2 mm for each of the arches modelled with 3DEC is shown in Fig. 8. From Fig. 8, for all of the arches studied, the load at which first cracking occurs linearly decreases as the angle of skew increases.

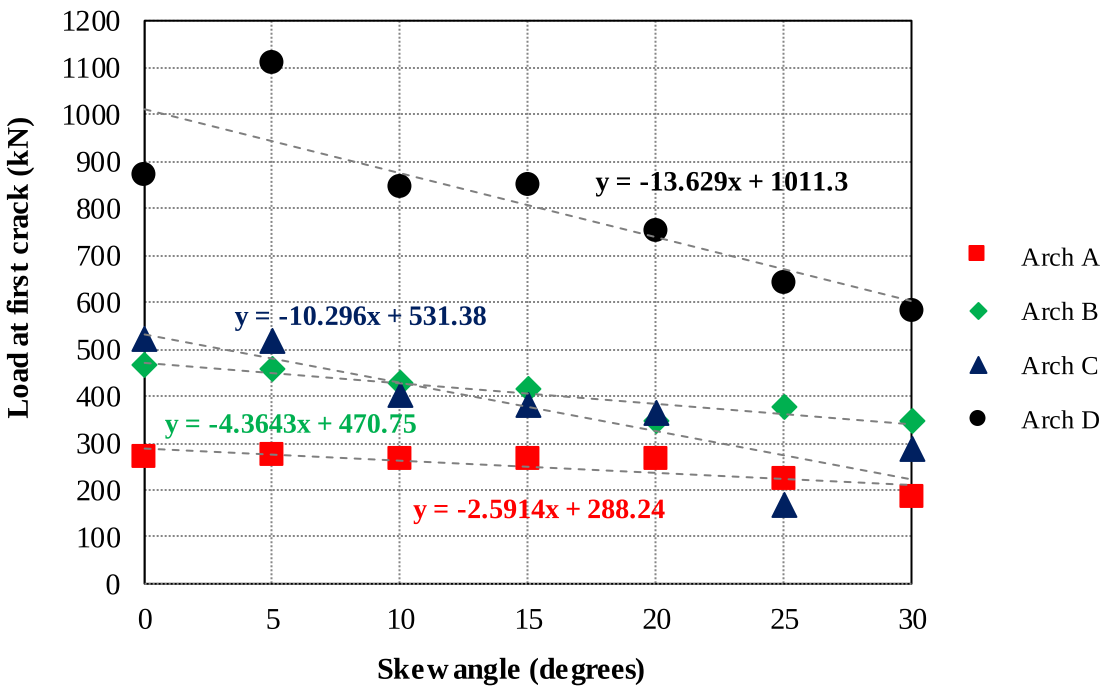
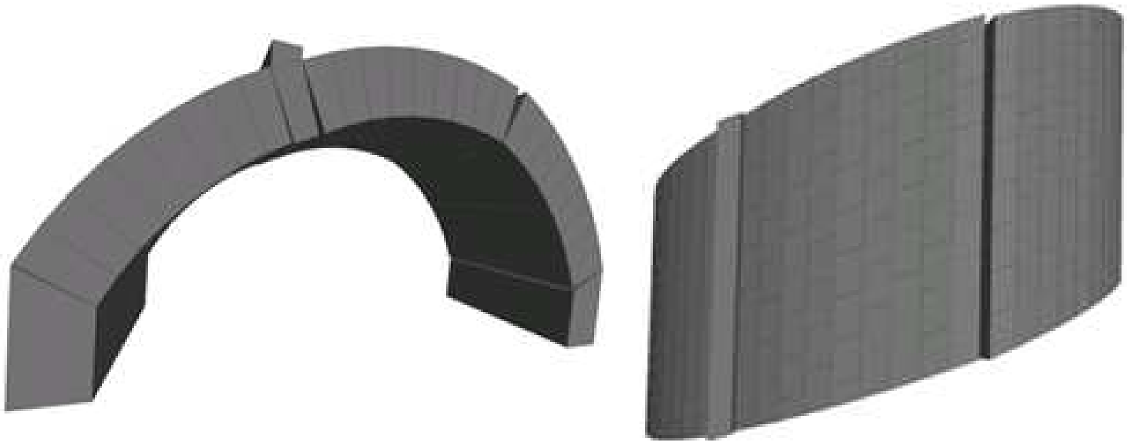


Fig. 8. Variation of load to cause first crack with change in skew angle.

4.3 Cracking

The initiation and propagation of cracks under increasing applied load have been simulated. Arches failed by the development of a four hinge mechanism (Fig. 9). Due to the line loading which was applied in the arches, the hinge lines developed where parallel to the abutments. This was possibly facilitated by the effect of the stiff abutments. Similar findings have also been reported by Abdunur (1995). The failure mode of the Arch D constructed with a 20 degrees angle of skew shown in Fig. 9.



(a) (b)

Fig. 9. Failure mode of the Arch D with 20 degrees angle of skew: (a) front view; (b) plan view.

4.4 Ultimate load

The magnitude of the ultimate load that each of the studied arches can carry is presented in Fig. 10. From the results analysis, the ultimate load decreases linearly as the angle of skew increases from 0° to 30°. Similar trends were also reported by Melbourne (1995). The absolute decrease in ultimate load due to skew is more significant for the arches with longer span and higher load capacity. Also, from Fig. 10, segmental arches can carry almost two times more load than the circular ones. The effect of barrel thickness and span has an effect on the load carrying capacity. By doubling the barrel thickness and span, the arch can sustain approximately three times more load.

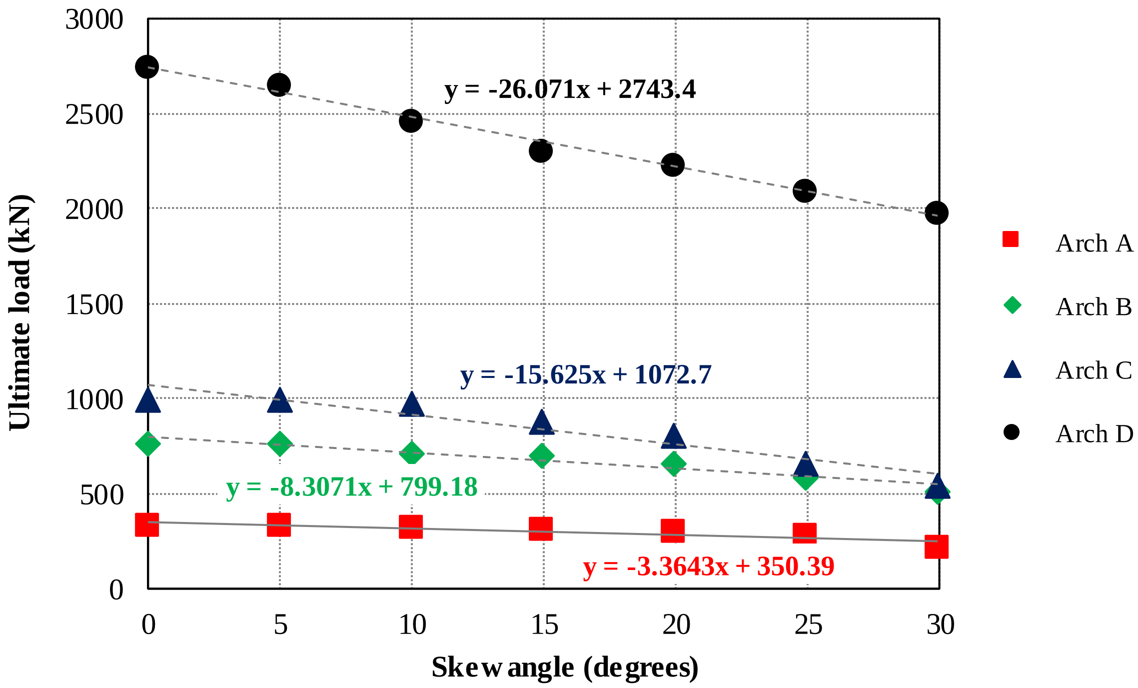


Fig. 10. Variation of ultimate load with change in skew angle.

4.5 Influence of the angle of skew on the total area of joints opened.

The increase of joint opening in the masonry arch, with the application of external load, relates to the accumulation of damage. The effect of skew on the total area of joints opened in the arch for each load increment has investigated. The cumulated area of joints opened has been calculated using a FISH function in which a joint defined as “open” when the normal force at this area is equal to zero and an opening equal or greater to 0.2 mm occurs. Fig. 11 shows the relations between the cumulative area of joints opened with the application of load for all of the arches studied. From Fig. 11, as the angle of skew increases, joint opening starts at lower loads, and for the same application of load, the cumulative area of joints opened increases.

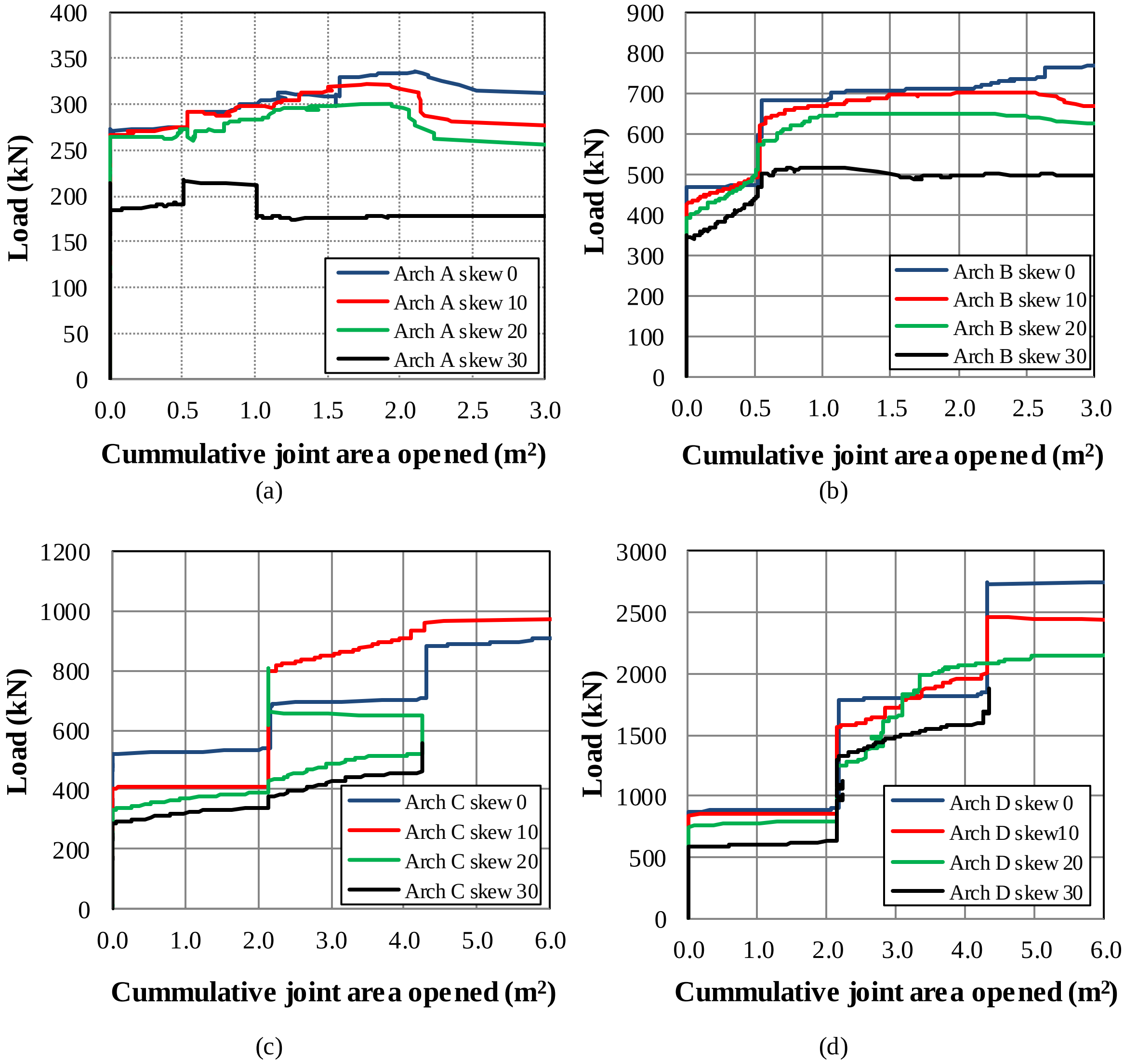


Fig. 11. Variation of the cumulative joint area opened with load for the arches studied.

6 CONCLUSIONS

The Discrete Element Method in the form of the 3DEC software has been used to investigate the effect of the angle of skew on the load carrying capacity of twenty eight single span stone masonry arches. A full width linear increasing load was applied to the extrados of the arch at quarter span until collapse. The load at first cracking, the mode of failure, the ultimate load that the arch can carry and the area of joints opened with the application of load were recorded. The main conclusions that can be made based on the above study are:

a) In order to capture the complex geometry and behaviour of skew arches, it is necessary to make use of three dimensional computational models;
b) 3DEC was able to relate the evolution of load with the progressive development of hinges;
c) Each arch barrel failed by the development of a four-hinge mechanism. In some cases, hinges developed parallel to the abutments;
d) The simulations of the ultimate load indicated that an increase in the angle of skew will increase the twisting behaviour of the arch and will eventually cause failure to occur at a lower load;
e) The ratio between the load at first cracking and the ultimate load depends on the geometry of the arch and ranges from 0.3 (Arch D) to 0.9 (Arch A);
f) The effect of the angle of skew on the ultimate load that the arch can carry is more significant for segmental arches than circular one;
g) Variations in the span and rise: span ratios have an effect on the strength of the arch bridges;
h) For the same application of load, the cumulative area of joints opened increases as the angle of skew increases.

For the purpose of this study, arches were assumed to be constructed with joints parallel to the springing. Further studies will be carried out to investigate the influence of the other construction methods (Fig. 2b & 2c) to the mechanical behaviour of skew arches.

REFERENCES

Abdunur, C., 1995. Direct assessment and monitoring of stresses and mechanical properties in masonry arch bridges. Arch Bridges. Thomas Telford, London.

Boothby, T.E., 2001. Analysis of masonry arches and vaults. Progress Structural Engineering Materials. 3, 246-256.

Brencich, A., Morbiducci, R., 2007. Masonry arches: Historical rules and modern mechanics. International Journal Architectural Heritage. 1(2), 165–189.

Choo, B.S., Gong, N.G., 1995. Effect of skew on the strength of the masonry arch bridges. In Arch Brides: Proceedings of the First International Conference of Arch Bridges. 205-214. London: Thomas Telford.

Cundall, P.A., 1971. A computer model for simulating progressive large scale movements in blocky rock systems. In: International Society of Rock Mechanics; Proc. Intern. Symp., Nancy, France.

Department of Transport, 1993. The assessment of highway bridges and structures, Note, BA16/93.

Design Manual for Roads and Bridges (DMRB), 2001. Highway structures: Inspection and maintenance. Assessment of highway bridges and structures. DB 21/01. London: Highway Agency.

Design Manual for Roads and Bridges (DMRB), 2001. Highway structures: Inspection and maintenance. Assessment of highway bridges and structures. DB 16/17. London: Highway Agency.

Drosopoulos, G.A., Stavroulakis, G.E., Massalas, C.V., 2006. Limit analysis of a single span masonry bridge with unilateral frictional contact interfaces. Engineering Structures. 28, 1864–1873.

Gago, A., Alfaiate, J., Gallardo, A., 2002. Numerical analyses of the Bargower arch bridge. In: Finite Elements in Civil Engineering Applications: Proceedings of the Third Diana World Conference, Tokyo, Japan, 9–11 October.

Gilbert, M., 1993. The behaviour of masonry arch bridges containing defects. PhD thesis. UK: Manchester University.

Fanning, P.J., Boothby, T.E., 2001. Three-dimensional modelling and full-scale testing of stone arch bridges. Computer & Structures. 79, 2645–2662.

Ford, T.E., Augarde, D.E., Tuxford, S.S., 2003. Modelling masonry arch bridges using commercial finite element software. In: 9th International Conference on Civil and Structural Engineering Computing, Egmond aan Zee, The Netherlands, 2–4 September

Henry A., 1986. Test on masonry arch bridge at Bargower, Department of Transport, TRRL contractor report 7, Crowthorne, UK.

Heyman, J., 1966. The stone skeleton. International Journal of Solids and Structures. 2(2), 249-256.

Hodgson, J. 1996. The behaviour of skewed masonry arches. PhD thesis. UK: University of Salford.

Itasca, 2004. 3DEC – Universal Distinct Element Code Manual. Theory and Background. Minneapolis: Itasca Consulting Group.

LimitState, 2009. RING: Theory and modelling guide. LimitState, Sheffield.

Lemos, J.V., 1995. Assessment of the ultimate load of a masonry arch using discrete elements. Computer Methods in Structural Masonry – 3, J. Middleton & G.N. Pande Ed., Swansea, UK: Books & Journals International.

Lemos, J.V., 2007. Discrete element modelling of structures. International Journal of Architectural Heritage 1(2):190-213.

Melbourne, C., Hodgson, J.A., 1995. The behaviour of skewed brickwork arch bridges. In Arch Brides: Proceedings of the First International Conference of Arch Bridges: 309-320. London: Thomas Telford.

Mirabella, R.G. & Calvetti, E. 1998. Distinct element analysis of stone arches. In Sinopoli (ed.), Arch Bridges; Proc. intern. symp., Paris, 6-9 October 1998. Rotterdam: Balkema.

Oliveira, D.V., Lourenço, P.B. & Lemos, C. 2010. Geometric properties and ultimate load carrying capacity of masonry arch bridges from the Nortwest Iberian Peninsula. Engineering Structures 32: 3955-3965.

Page, J., 1989. Load tests to collapse on two arch bridges at Strahmashie and Barlae, TRRL research report 201, Crowthorne, UK.

Page, J., 1987. Load tests on collapse in two arch bridges at Preston, Shropshire and Prestwood, Staffordshire, Department of Transport TRRL research report 110, Crowthorne, UK.

Page, J., 1993. Masonry Arch bridges, TRL State of the Art Review. London: HMSO.

Rankine, W.J.M., 1862. A manual for Civil Engineering. London:C. Griffin and Co.

Sarhosis, V., Sheng, Y., 2014. Identification of material parameters for low bond strength masonry, Engineering Structures, 60, 100-110

Toth, A.R., Orban, Z., Bagi, K., 2009. Discrete element modelling of a stone masonry arch. Mechanics Research Communications. 36(4), 469-480.

Wang, J., 2004. The three dimensional behaviour of masonry arches. PhD thesis. UK: University of Salford.