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Brief Contributions

Cycle-Time Properties of the Timed Token Medium Access Control Protocol

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Abstract—We investigate the timing properties of the timed token protocol that are necessary to guarantee synchronous message deadlines. A tighter upper bound on the elapse time between the token's $l\mathrm{th}$ arrival at any node i and its $(l+v)\mathrm{th}$ arrival at any node k is found. A formal proof to this generalized bound is presented.

Index Terms—Protocol timing properties, timed token medium access control (MAC) protocol, timed token networks, FDDI networks, real-time communications.

1 Introduction

GUARANTEEING message deadlines is a key issue in distributed real-time applications. The timed token medium access control (MAC) protocol [1], [2] is suitable for real-time applications due to its special timing property of bounded token rotation time. With this protocol [2], messages are distinguished into two types: *synchronous* and *asynchronous*. Synchronous messages are periodic with delivery time constraints. Asynchronous messages are nonperiodic with no delivery time constraints. During network initialization time, all nodes negotiate a common value for the Target Token Rotation Time ($TTRT$) which should be small enough to meet responsiveness requirements of all nodes. Each node i is assigned a fraction of the $TTRT$, denoted as H_i, as its *synchronous bandwidth*, which is the maximum time the node is allowed to transmit its synchronous messages every time it receives the token [3], [4]. Whenever node i receives the token, it transmits its synchronous messages (if any) first for a time up to H_i. If the time elapsed between the previous and the current token arrivals at the same node i is less than $TTRT$ (i.e., the token arrived *earlier* than expected), node i can then send asynchronous messages to make up the difference.

Johnson [5] first proved that the token rotation time cannot exceed $2 \cdot TTRT$. Chen et al. [3], [4] generalized the bound derived by Johnson on the maximum token rotation time, extending it from between any two to between any v (integer $v \geq 2$) successive token's arrivals at a node. This result was widely used in studying various synchronous bandwidth allocation (SBA) schemes [3], [4], [6], [7], [8], [9], [10], [11]. Unfortunately, their generalized bound may not keep tight when v grows large and, consequently, the worst-case performance of SBA schemes, which is derived based on this bound, could be too pessimistic and, hence, much poorer than actually achievable. Han et al. [12] derived a more generalized bound between any two token visits to any two nodes, which makes the previous results by Johnson and by Chen et al. a special case. However, for the bound on successive token arrivals to a particular node, the bound by Han et al. is no tighter than that derived by Chen et al.. Zhang and Burns [13], [14] improved the bound of Chen et al. by deriving another generalized upper bound which may become tighter when the number of successive token rotations grows large.

This paper derives a tighter upper bound on the elapsed time between the token's $l\mathrm{th}$ arrival at any node i and the token's $(l+v)\mathrm{th}$ arrival at any node k (where integer $v \geq 0$). This result generalizes all the previous findings on the cycle-time properties and is better than any of them in the sense that it is more general and/or tighter. For the rest of this paper, Section 2 describes the network model, Section 3 presents a formal proof to our generalized bound, and Section 4 shows how our result generalizes and why it is better than any published result on cycle-time properties. An example, given in Section 5, shows the superiority of our derived result for guaranteeing hard real-time messages with the timed token protocol. Finally, the paper concludes with Section 6.

2 Network Model

The network is assumed to consist of n nodes forming a logical ring. The message transmission is controlled by the timed token MAC protocol [2]. Let τ_i be the maximum amount of overhead incurred from the token's arrival at node i till its immediately subsequent arrival at node $(i+1)$, then the maximum time unavailable for message transmission during one complete token rotation, denoted as τ, can be expressed by $\tau = \sum_{i=1}^{n} \tau_i$. Since τ forms part of the token rotation time and synchronous transmission with guaranteed bandwidth precedes asynchronous transmission, clearly, as a protocol constraint on the allocation of synchronous bandwidth, $\sum_{j=1}^{n} H_j \leq TTRT - \tau$ must be met. The protocol constraint is assumed to hold for the rest of this paper.

3 The Formal Proof

Before formally proving a better result on the protocol cycle-time property, we need to define some terms and to show a lemma.

Let "c, i" (a pair of integers) denote the token's $c\mathrm{th}$ visit to node i. Visit c, i is followed by $c, i+1$ if $1 \leq i < n$ or by $c+1, 1$ if $i = n$. Similarly, if $i = 1$, then $c, i-1$ (the visit immediately before c, i) should be taken to be $c-1, n$. The sum total of some quantity, say, q, for all the visits from j, k to w, z inclusive can be expressed as $\sum_{x,y=j,k}^{w,z} q_{x,y} = \sum_{y=k}^{n} q_{j,y} + \sum_{x=j+1}^{w-1} \sum_{y=1}^{n} q_{x,y} + \sum_{y=1}^{z} q_{w,y}$.

Signs "$=$," "$<$," "$\leq$," "$>$," and "$\geq$" can be used to link two visits such that "$x, y = c, i$," "$x, y < c, i$," "$x, y \leq c, i$," "$x, y > c, i$," and "$x, y \geq c, i$" mean, respectively, visit x, y being the same as, before (earlier than), no later than, after (later than), and no earlier than c, i.

Let $t_{c,i}$ be the time when the token makes its $c\mathrm{th}$ arrival at node i. Let $h_{c,i}$, $a_{c,i}$, and $\tau_{c,i}$, respectively, represent the times spent transmitting synchronous and asynchronous traffic and the various overheads involved in visit c, i. Then, the duration of visit c, i, denoted as $v_{c,i}$, can be expressed as $v_{c,i} = h_{c,i} + a_{c,i} + \tau_{c,i}$. Further, let $B_{c,i}$ be the length of a complete token rotation ending with visit c, i, i.e., $B_{c,i} = \sum_{x,y=c-1,i+1}^{c,i} v_{x,y}$.

According to the timed token protocol [2], each node i can transmit synchronous messages for a time up to H_i and can then send asynchronous messages (if the token arrived early) up to the amount of time by which the token arrived early. So, for $c \geq 1$ and $1 \leq i \leq n$, $h_{c,i} \leq H_i$, $a_{c,i} \leq \max(0, TTRT - B_{c,i-1})$. Also, with this protocol, no nodes are allowed to hold the token for a time over

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$TTRT$, i.e., $a_{c,i} \le TTRT - h_{c,i}$. Combining these two constraints on $a_{c,i}$, we get

$$a_{c,i} \le \min\{ TTRT - h_{c,i}\,,\ \max(0, TTRT - B_{c,i-1})\}. \qquad (1)$$

Because $B_{c,i-1} = \tau$, when no nodes have messages, either synchronous or asynchronous, to send during the preceding token rotation ending with visit $c, i-1$, the bandwidth available for transmitting asynchronous messages is bounded by $TTRT - \tau$ (since $a_{c,i} \le \max[0, TTRT - B_{c,i-1}]$).

Let $\sum_e^f Q_j$ be defined as follows (where e and f are positive integers and "*mod n*" represents "**modulo_n**" operation):

$$\sum_e^f Q_j = \begin{cases} \sum_{j=e}^f Q_j & \text{if } 1 \le e \le f \le n \text{ and } e \ne (f \bmod n)+1 \\ \sum_{j=e}^n Q_j + \sum_{j=1}^f Q_j & \text{if } 1 \le f < e \le n \text{ and } e \ne f+1 \\ 0 & \text{if } e = (f \bmod n)+1. \end{cases}$$

Then, the sum of overheads incurred during less than n successive token visits (from node e to node f inclusive) can be expressed as $\sum_e^f \tau_j$.

We also need the following lemma for the proof of our generalized result.

Lemma 1 [14]. *If the token is early on visit c, i (i.e., early on its cth arrival at node i), then*

$$t_{c,i+1} - t_{c-1,i} = \sum_{x,y=c-1,i}^{c,i} v_{x,y} \le TTRT + h_{c,i} + \tau_{c,i} \le TTRT + H_i + \tau_i.$$

Before formally proving our generalized upper bound given in Theorem 1, we briefly describe the simple idea behind the proof: For visits between l,i and $l+v,k-1$, examine each visit backward starting from $l+v,k-1$. If the current visit is a late visit (where only synchronous transmission is allowed), replace it by the allocated amount of synchronous bandwidth (allocated to the node which the late visit corresponds to) plus an upper-bounded amount of overheads involved in the visit and then move onto the *next* visit immediately before this late visit. Otherwise, if the current visit is an early visit, replace $(n+1)$ successive visits ending with this early visit by the bound specified in Lemma 1 and then move onto the *next* visit immediately before these $(n+1)$ visits. Check the new current visit in exactly the same way as above and continue the backward examining process until visit l,i has been replaced by a bound formed either individually or jointly with other visits. Finally, by smartly concatenating and formulating all produced component bounds (each for one or $(n+1)$ replaced visits), one can easily reach the proposed upper bound. Following this proof route, the upper bound is easy to derive though it looks quite complex.

Theorem 1 (Generalized Johnson's Theorem). *For any integers l and v ($l \ge 1$, $v \ge 0$) and any nodes i and k ($1 \le i \le n$, $1 \le k \le n$), if $t_{l+v,k} > t_{l,i}$, then, under the protocol constraint (i.e., $\sum_{j=1}^n H_j + \tau \le TTRT$),*

$$t_{l+v,k} - t_{l,i} \le \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil \cdot TTRT + \sum_{i+1}^{k-1} H + \tau + \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor - \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil + 1 \right) \cdot \left(\sum_{j=1}^n H_j + \tau \right).$$

Proof. The time interval of $t_{l+v,k} - t_{l,i}$ exactly corresponds to visits from l,i to $l+v,k-1$, inclusive. There are two cases to consider:

CASE 1: *The token is late on all visits from l,i to $l+v,k-1$ inclusive.*

Since $a_{x,y} = 0$ $(l,i \le x,y \le l+v,k-1)$, the time elapsed during any complete token rotation, if any, is bounded by $\sum_{j=1}^n H_j + \tau$. As there are, in total, $(v \cdot n + k - i)$ visits between l,i and $l+v,k-1$, inclusive, and each token rotation consists of n successive visits, the number of complete token rotations is given by $b = \lfloor \frac{v \cdot n + k - i}{n} \rfloor$. The elapsed time in the remaining visits from node i to node $(k-1)$ inclusive, if any, is bounded by $\sum_i^{k-1} H_j + \sum_i^{k-1} \tau_j$. Based on the above analysis, we have,

$$\begin{aligned}
t_{l+v,k} - t_{l,i} &= \sum_{x,y=l,i}^{l+v,k-1} v_{x,y} = \sum_{j=0}^{b-1} \left(\sum_{x,y=l+j,i}^{l+j+1,i-1} v_{x,y} \right) + \sum_{x,y=l+b,i}^{l+v,k-1} v_{x,y} \\
&= \sum_{j=0}^{b-1} \left(\sum_{x,y=l+j,i}^{l+j+1,i-1} (h_{x,y} + \tau_{x,y}) \right) + \sum_{x,y=l+b,i}^{l+v,k-1} (h_{x,y} + \tau_{x,y}) \\
&\le b \cdot \left(\sum_{j=1}^n H_j + \tau \right) + \sum_i^{k-1} H_j + \sum_i^{k-1} \tau_j \\
&\left(\text{since } \tau_{x,y} \le \tau_y, \tau = \sum_{j=1}^n \tau_i, h_{x,y} \le H_y\right) \\
&= \left\lfloor \frac{v \cdot n + k - i}{n} \right\rfloor \cdot \left(\sum_{j=1}^n H_j + \tau \right) + \sum_i^{k-1} H_j + \tau \\
&\left(\text{since } \sum_i^{k-1} \tau_j \le \tau \right) \\
&\le \sum_{i+1}^{k-1} H_j + \tau + \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor + 1 \right) \cdot \left(\sum_{j=1}^n H_j + \tau \right) \\
&\le \sum_{i+1}^{k-1} H_j + \tau + \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor + 1 \right) \cdot \left(\sum_{j=1}^n H_j + \tau \right) \\
&+ \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil \cdot \left(TTRT - \sum_{j=1}^n H_j - \tau \right) \\
&\left(\text{since } \sum_{j=1}^n H_j + \tau \le TTRT \right) \\
&= \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil \cdot TTRT + \sum_{j=i+1}^{k-1} H_j + \tau \\
&+ \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor - \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil + 1 \right) \cdot \left(\sum_{j=1}^n H_j + \tau \right).
\end{aligned}$$

CASE 2: *There is at least one early visit from l,i to $l+v,k-1$ inclusive.*

Let p_1, q_1 ; ; ... ; p_m, q_m, where

$$l, i \le p_1, q_1 < p_2, q_2 < \cdots < p_m, q_m < l+v, k,$$

be all m early visits between l,i and $l+v,k-1$ inclusive such that, for $1 \le s \le m$ (assuming $l+v,k = p_{(m+1)} - 1, q_{(m+1)}$), $p_s, q_s < p_{(s+1)} - 1, q_{(s+1)}$, where p_s, q_s is the last early visit before $p_{(s+1)} - 1, q_{(s+1)}$. The following observations can be made from the above definitions:

1. For $1 \le s < m$, between p_s, q_s exclusive and $p_{(s+1)}, q_{(s+1)}$ inclusive, there are at least $(n+1)$ successive visits. Since there are, in total, $v \cdot n + (k-i)$ visits between l,i and $l+v,k-1$ inclusive, we have $m \le \lceil \frac{v \cdot n + k - i}{n+1} \rceil$.
2. Any visit between p_s, q_s and $p_{(s+1)} - 1, q_{(s+1)}$ exclusive (where $1 \le s \le m$ and, as assumed, $p_{(m+1)} - 1, q_{(m+1)} = l+v, k$), if it exists, is a late visit.

Thus, $\sum_{x,y=p_s,q_s+1}^{p_{(s+1)}-1,q_{(s+1)}-1} v_{x,y} = \sum_{x,y=p_s,q_s+1}^{p_{(s+1)}-1,q_{(s+1)}-1} (h_{x,y} + \tau_{x,y})$.

3. If $p_1 - 1, q_1 > l, i$, there are no early visits (thus no asynchronous transmission) between l, i and $p_1 - 1, q_1 - 1$ inclusive. Hence,

$$\sum_{x,y=l,i}^{p_1-1,q_1-1} v_{x,y} = \sum_{x,y=l,i}^{p_1-1,q_1-1} (h_{x,y} + \tau_{x,y}).$$

4. According to Lemma 1, the time elapsed in any $(n+1)$ successive visits ending with an early visit is bounded by $TTRT$ plus the synchronous bandwidth used and the amount of overheads incurred in this early visit. For simplicity of proof, suppose that this bound is formed by two imaginary parts: *the first n successive visits* (that form one complete token rotation) being bounded by $TTRT$ and *the* $(n+1)$th *visit* of only synchronous transmission. Note that this imaginary (equivalent) situation, though not what happens in reality, leads to the same theoretically derived upper bound and, at the same time, simplifies the derivation making the proof easy to follow.

   Note that removing any n successive visits (say, replaced by the imaginary upper bound of $TTRT$) does not break the neighboring relationship between nodes because any n successive visits make up one complete token rotation. That is, the node corresponding to the visit immediately before these n visits and the node corresponding to the visit immediately after these n visits neighbor each other (i.e., the latter is the immediately subsequent node of the former), although these two corresponding visits are one-token-rotation apart. For example, the node corresponding to visit $p_s - 1, q_s - 1$ (i.e., node $(q_s - 1)$) and the node corresponding to visit p_s, q_s (i.e., node q_s) neighbor each other, with the removed n visits (replaced by $TTRT$) between $p_s - 1, q_s$ and $p_s, q_s - 1$ inclusive.

5. Based upon how far the visit p_1, q_1 is from l, i, several cases are considered below:

   a. If $p_1 - 1, q_1 \geq l, i$ (i.e., $p_1, q_1 \geq l + 1, i$).

      All m early visits, each corresponding to $(n+1)$ successive visits, fall within the visits from l, i to $l + v, k - 1$, inclusive. By 4, we can replace the first n successive visits of each $(n+1)$ successive visits by the imaginary upper bound of $TTRT$. So, the final derived upper bound (on the elapse time from l, i to $l + v, k - 1$ inclusive), for this case, should include "$m \cdot TTRT$."

      After removing m sets of token rotations (replaced by $m \cdot TTRT$), the number of remaining visits will be the total number of all visits minus the number of removed visits, i.e., $(v.n + k - i) - m \cdot n$. Note that we should only consider transmission of synchronous messages in each of these remaining visits because it is either a late visit x, y (if $x, y \neq p_s, q_s, 1 \leq s \leq m$) or has been supposed so (if $x, y = p_s, q_s$) in our imaginary equivalent scenario stated in 4. Due to the unbroken feature of neighboring relationships between nodes (whenever the removal of any n successive nodes happens), the number of the imaginary equivalent token rotations within these remaining visits is given by $q = \lfloor\frac{(v \cdot n + k - i) - m \cdot n}{n}\rfloor = \lfloor\frac{v \cdot n + k - i}{n}\rfloor - m$. The elapsed time in the q equivalent token rotations is bounded by

$$q \cdot \left(\sum_{j=1}^{n} H_j + \tau\right) = \left(\left\lfloor\frac{v \cdot n + k - i}{n}\right\rfloor - m\right) \cdot \left(\sum_{j=1}^{n} H_j + \tau\right),$$

      which should also be a component of the final derived upper-bound expression.

      Also, with the unbroken neighboring feature between nodes, it is easy to check that the elapsed time in the leftover visits (after taking out the q rotations) is bounded by $\sum_{i}^{k-1} H_j + \sum_{i}^{k-1} \tau_j$, which should also appear in the final expression.

   b. If $l - 1, i \leq p_1 - 1, q_1 < l, i$, i.e.,

$$l, i \leq p_1, q_1 < l + 1, i.$$

      Divide all $(n \cdot v + k - i)$ visits (from l, i to $l + v, k - 1$ inclusive) into two groups:

      Group $1 = \{x, y \mid p_1, q_1 + 1 \leq x, y \leq l + v, k - 1\}$
      Group $2 = \{x, y \mid l, i \leq x, y \leq p_1, q_1\}$.

      We now discuss the upper bounds for visits in these two groups, respectively.

      For visits in Group 1, we can do exactly the same analysis as that adopted in (5.a) for $(m - 1)$ early visits (i.e., "p_2, q_2";"p_3, q_3"; ... ; "p_m, q_m"). So, the final upper-bound expression (for Group 1) should include "$(m-1) \cdot TTRT$" and

$$q = \left\lfloor\frac{(l + v - p_1) \cdot n + (k - q_1) - 1}{n}\right\rfloor - (m - 1).$$

      Similarly, the time elapsed in the q rotations is upper bounded by

$$q \cdot \left(\sum_{j=1}^{n} H_j + \tau\right) = \left(\left\lfloor\frac{(l + v - p_1) \cdot n + (k - q_1) - 1}{n}\right\rfloor - (m - 1)\right) \cdot \left(\sum_{j=1}^{n} H_j + \tau\right)$$

      and the leftover visits can never exceed $\sum_{q_1+1}^{k-1}(H_j + \tau_j) = \sum_{q_1+1}^{k-1} H_j + \sum_{q_1+1}^{k-1} \tau_j$. Also, both of these two bounds, together with "$(m-1) \cdot TTRT$," form an upper bound for Group 1.

      For Group 2, by Lemma 1, we have

$$\sum_{x,y=l,i}^{p_1,q_1} v_{x,y} < \sum_{x,y=p_1-1,q_1}^{p_1,q_1} v_{x,y} \leq TTRT + h_{p_1,q_1} + \tau_{p_1,q_1} \leq TTRT + H_{q_1} + \tau_{q_1}.$$

      Note that l, i becomes the only visit in Group 2 when $p_1, q_1 = l, i$. By (1), when $p_1, q_1 = l, i$, we have $v_{l,i} \leq TTRT + \tau_{l,i} \leq TTRT + \tau_i$.

With the above observations 1-5, the upper bound of Theorem 1 can be derived as follows:

$$
\begin{aligned}
& t_{l+v,k} - t_{l,i} \\
&= (t_{l+v,k} - t_{p_m,q_m+1}) + \sum_{j=0}^{m-2} \left[(t_{p_{m-j},q_{m-j}+1} - t_{p_{m-j}-1,q_{m-j}}) \right. \\
&\left. + (t_{p_{m-j}-1,q_{m-j}} - t_{p_{m-j-1},q_{m-j-1}+1}) \right] \\
&+ \begin{cases} (t_{p_1,q_1+1} - t_{p_1-1,q_1}) + (t_{p_1-1,q_1} - t_{l,i}) & \text{if } p_1-1,q_1 > l,i \\ (t_{p_1,q_1+1} - t_{l,i}) & \text{if } l-1,i < p_1-1,q_1 \le l,i \\ (t_{l,i+1} - t_{l,i}) & \text{if } p_1-1,q_1 = l-1,i \end{cases} \\
&= \sum_{x,y=p_m,q_m+1}^{l+v,k-1} v_{x,y} + \sum_{j=0}^{m-2} \left[\sum_{x,y=p_{m-j}-1,q_{m-j}}^{p_{m-j},q_{m-j}} v_{x,y} + \sum_{x,y=p_{m-j-1},q_{m-j-1}+1}^{p_{m-j}-1,q_{m-j}-1} v_{x,y} \right] \\
&+ \begin{cases} \sum_{x,y=p_1-1,q_1}^{p_1,q_1} v_{x,y} + \sum_{x,y=l,i}^{p_1-1,q_1-1} v_{x,y} & \text{if } p_1-1,q_1 > l,i \\ \sum_{x,y=l,i}^{p_1,q_1} v_{x,y} & \text{if } l-1,i < p_1-1,q_1 \le l,i \\ v_{l,i} & \text{if } p_1-1,q_1 = l-1,i \end{cases} \\
&\le \sum_{x,y=p_m,q_m+1}^{l+v,k-1} (h_{x,y}+\tau_{x,y}) + \sum_{j=0}^{m-2} \left[\sum_{x,y=p_{m-j}-1,q_{m-j}}^{p_{m-j},q_{m-j}} v_{x,y} \right] \\
&+ \sum_{j=0}^{m-2} \left[\sum_{x,y=p_{m-j-1},q_{m-j-1}+1}^{p_{m-j}-1,q_{m-j}-1} v_{x,y} \right] \\
&+ \begin{cases} \sum_{x,y=p_1-1,q_1}^{p_1,q_1} v_{x,y} + \sum_{x,y=l,i}^{p_1-1,q_1-1} (h_{x,y}+\tau_{x,y}) & \text{if } p_1-1,q_1 > l,i \\ \text{(by } (C)) & \\ \sum_{x,y=p_1-1,q_1}^{p_1,q_1} v_{x,y} & \text{if } l-1,i < p_1-1, \\ \text{(since } \sum_{x,y=l,i}^{p_1,q_1} v_{x,y} < \sum_{x,y=p_1-1,q_1}^{p_1,q_1} v_{x,y}) & q_1 \le l,i \\ h_{l,i} + a_{l,i} + \tau_{l,i} & \text{if } p_1-1,q_1 = l-1,i \end{cases} \\
&\text{(by observation 2 above)} \\
&\le \sum_{x,y=p_m,q_m+1}^{l+v,k-1} (h_{x,y}+\tau_{x,y}) + \sum_{j=0}^{m-2} (TTRT + h_{p_{m-j},q_{m-j}} + \tau_{p_{m-j},q_{m-j}}) \\
&+ \sum_{j=0}^{m-2} \left[\sum_{x,y=p_{m-j-1},q_{m-j-1}+1}^{p_{m-j}-1,q_{m-j}-1} (h_{x,y}+\tau_{x,y}) \right] + \\
&\begin{cases} (TTRT + h_{p_1,q_1} + \tau_{p_1,q_1}) & \text{if } p_1-1,q_1 > l,i \\ \quad + \sum_{x,y=l,i}^{p_1-1,q_1-1} (h_{x,y}+\tau_{x,y}) & \\ TTRT + h_{p_1,q_1} + \tau_{p_1,q_1} & \text{if } l-1,i < p_1-1,q_1 \le l,i \\ h_{l,i} + \tau_{l,i} + \min\{TTRT - h_{l,i}, & \text{if } p_1-1,q_1 = l-1,i \\ \max(0, TTRT - B_{l,i-1})\} \text{(by (1))} & \end{cases} \\
&\text{(by Lemma 1)} \\
&\le \begin{cases} m\cdot TTRT + (\lfloor \frac{v\cdot n+k-i}{n} \rfloor - m)\cdot(\sum_{j=1}^{n} H_j + \tau) & \\ + \sum_i^{k-1} H_j + \sum_i^{k-1} \tau_j & \text{if } p_1-1,q_1 \ge l,i \\ \text{(by observation 5.a above)} & \\ (m-1)\cdot TTRT + & \\ (\lfloor \frac{(l+v-p_1)\cdot n+(k-q_1)-1}{n} \rfloor - (m-1))\cdot & \text{if } l-1,i < p_1-1, \\ (\sum_{j=1}^{n} H_j + \tau) + \sum_{q_1+1}^{k-1} H_j + \sum_{q_1+1}^{k-1} \tau_j & q_1 < l,i \\ +(TTRT + H_{q_1} + \tau_{q_1}) & \\ \text{(by observation 5.b above)} & \\ (m-1)\cdot TTRT + & \\ (\lfloor \frac{v\cdot n+(k-i)-1}{n} \rfloor - (m-1))\cdot(\sum_{j=1}^{n} H_j + \tau) & \text{if } p_1-1, \\ + \sum_{i+1}^{k-1} H_j + \sum_{i+1}^{k-1} \tau_j + (TTRT + \tau_i) & q_1 = l-1,i \\ \text{(since } a_{l,i} \le TTRT - h_{l,i}) & \\ \text{(by observation 5.b above)} & \end{cases} \\
&\text{(because } \tau_{x,y} \le \tau_y, \tau = \sum_{j=1}^{n} \tau_i \text{ and } h_{x,y} \le H_y) \\
&\le m\cdot TTRT + \left(\lfloor \frac{v\cdot n+(k-i)-1}{n} \rfloor - (m-1) \right) \\
&\cdot \left(\sum_{j=1}^{n} H_j + \tau \right) + \sum_{i+1}^{k-1} H_j + \tau \\
&\le \left\lceil \frac{v\cdot n+k-i}{n+1} \right\rceil \cdot TTRT + \sum_{i+1}^{k-1} H_j + \tau + \\
&\left(\left\lfloor \frac{v\cdot n+k-i-1}{n} \right\rfloor - \left\lceil \frac{v\cdot n+k-i}{n+1} \right\rceil + 1 \right) \cdot \left(\sum_{j=1}^{n} H_j + \tau \right) \\
&\text{(by observation 1 above and the fact of the above} \\
&\text{upper bound being an increasing function of } m).
\end{aligned}
$$

□

As shown in the above proof process, the derived upper bound is independent of $h_{x,y}$ $(l, i/leqx, y < l+v, k)$ as long as the protocol constraint holds. So, the bound still works even when $h_{x,y} = 0$ for some x, y. Realizing this fact is important for real-time communication with the timed token protocol.

4 Comparison with Previous Results

Zhang and Burns [13] demonstrate how the previous findings by Johnson [5] become a special case of their generalized upper bound and why their bound is tighter than that derived by Chen et al. when the number of consecutive token rotations grows large enough under $\sum_{j=1}^{n} H_j < TTRT - \tau$. It is easy to check that Theorem 1 becomes the generalized upper bound expression derived by Zhang and Burns [13] when $k = i$.

Let $d_i(l)$ be the time when the token makes its lth departure from node i and $\Delta_{b,i}(l,c)$ be the time difference between $d_b(l)$ and the time when the token departs from node i the cth time after $d_b(l)$ [12], i.e.,

$$\Delta_{b,i}(l,c) = \begin{cases} d_i(l+c-1) - d_b(l) & \text{if } 1 \le b < i \le n \\ d_i(l+c) - d_b(l) & \text{if } 1 \le i \le b \le n. \end{cases} \quad (2)$$

Han et al. [12] derived a generalized upper bound on $\Delta_{b,i}(l,c)$ (i.e., the elapses time between the token's lth departure from node b and the token's $(l+c)$th departure from node i) which makes the previous results by Johnson [5] and by Chen et al. [3] a special case of their result. The following theorem shows their generalized upper bound expression.

Theorem 2 (Generalized Johnson's Theorem by Han et al. [12]). *For the timed-token MAC protocol, under the protocol constraint, for any $l \ge 1$ and $c \ge 1$,*

$$\Delta_{b,i}(l,c) \le c\cdot TTRT + \sum_{j=b+1}^{i} H_j + \tau \le (c+1)\cdot TTRT,$$

where $\sum_{j=b+1}^{i} H_j$ is subject to the definition of $\sum_{j=e}^{f} H_j$ as shown below:

$$\sum_{j=e}^{f} H_j =$$

$$\begin{cases} H_e + H_{e+1} + \cdots + H_f & \text{if } 1 \le e \le f \le n \\ H_e + H_{e+1} + \cdots + H_n + H_1 + H_2 + \cdots + H_f & \text{if } 1 \le f < e \le n. \end{cases}$$

We now show our generalized bound given in Theorem 1 is tighter (thus better) than that given in Theorem 2. To show this, we relax our upper bound as follows:

$$
\begin{aligned}
& t_{l+v,k} - t_{l,i} \\
& \leq \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil \cdot TTRT + \sum_{i+1}^{k-1} H_j + \tau + \\
& \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor - \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil + 1 \right) \cdot \left(\sum_{j=1}^{n} H_j + \tau \right) \\
& \leq \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil \cdot TTRT + \sum_{i+1}^{k-1} H_j + \tau + \\
& \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor - \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil + 1 \right) \cdot TTRT \\
& \left(\text{since } \left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor - \left\lceil \frac{v \cdot n + k - i}{n+1} \right\rceil + 1 \geq 0 \right. \\
& \left. \text{and } \sum_{j=1}^{n} H_j + \tau \leq TTRT \right) \\
& = \left(\left\lfloor \frac{v \cdot n + k - i - 1}{n} \right\rfloor + 1 \right) \cdot TTRT + \sum_{i+1}^{k-1} H_j + \tau.
\end{aligned} \tag{3}
$$

As shown below, even the above relaxed upper bound (3) is still tighter than that derived by Han et al. To enable comparison, we need to represent $\Delta_{b,i}(l,c)$ using our notation $t_{c,i}$. Let θ be the delay between the departure of the token from any node i and its immediate arrival to node $(i+1)$, i.e., $d_i(l) = t_{l+\lfloor i/n \rfloor, (i \bmod n)+1} - \theta$, then $\Delta_{b,i}(l,c)$ can be converted as follows:

$$
\Delta_{b,i}(l,c) = \begin{cases} d_i(l+c-1) - d_b(l) & \text{if } 1 \leq b < i \leq n \\ d_i(l+c) - d_b(l) & \text{if } 1 \leq i \leq b \leq n \end{cases}
$$

$$
= \begin{cases} (t_{(l+c-1)+\lfloor i/n \rfloor, (i \bmod n)+1} - \theta) - (t_{l,b+1} - \theta) & \text{if } 1 \leq b < i \leq n \\ (t_{(l+c)+\lfloor i/n \rfloor, (i \bmod n)+1} - \theta) \\ \quad -(t_{l+\lfloor b/n \rfloor, (b \bmod n)+1} - \theta) & \text{if } 1 \leq i \leq b \leq n \end{cases}
$$

$$
= \begin{cases} t_{l+c,1} - t_{l,b+1} & \text{if } 1 \leq b < i = n \\ t_{l+c-1,i+1} - t_{l,b+1} & \text{if } 1 \leq b < i < n \\ t_{(l+c)+1,1} - t_{l+1,1} & \text{if } i = b = n \\ t_{l+c,i+1} - t_{l+1,1} & \text{if } 1 \leq i < b = n \\ t_{l+c,i+1} - t_{l,b+1} & \text{if } 1 \leq i \leq b < n \end{cases}
$$

$$
\leq \begin{cases} (\lfloor \frac{c \cdot n + 1 - (b+1) - 1}{n} \rfloor + 1) \cdot TTRT \\ + \sum_{[(b+1) \bmod n]+1}^{n} H_j + \tau & \text{if } 1 \leq b < i = n \\ (\lfloor \frac{(c-1) \cdot n + (i+1) - (b+1) - 1}{n} \rfloor + 1) \cdot TTRT + \\ \sum_{b+2}^{i} H_j + \tau & \text{if } 1 \leq b < i < n \\ (\lfloor \frac{c \cdot n + 1 - 1 - 1}{n} \rfloor + 1) \cdot TTRT + \sum_{2}^{n} H_j + \tau & \text{if } i = b = n \\ (\lfloor \frac{(c-1) \cdot n + (i+1) - 1 - 1}{n} \rfloor + 1) \cdot TTRT + \sum_{2}^{i} H_j + \tau & \text{if } 1 \leq i < b = n \\ (\lfloor \frac{c \cdot n + (i+1) - (b+1) - 1}{n} \rfloor + 1) \cdot TTRT + \\ \sum_{[(b+1) \bmod n]+1}^{i} H_j + \tau & \text{if } 1 \leq i \leq b < n \end{cases}
$$

(by (3))

$$
= \begin{cases} c \cdot TTRT + H_{b+2} + \cdots + H_i + \tau & \text{if } 1 \leq b \leq n-2 < i = n \\ c \cdot TTRT + \tau & \text{if } 1 \leq b = n-1 < i = n \\ c \cdot TTRT + H_{b+2} + \cdots + H_i + \tau & \text{if } 2 \leq b+1 < i < n \\ c \cdot TTRT + \tau & \text{if } 2 \leq b+1 = i < n \\ c \cdot TTRT + H_2 + \cdots + H_i + \tau & \text{if } i = b = n \\ c \cdot TTRT + H_2 + \cdots + H_i + \tau & \text{if } 1 < i < b = n \\ c \cdot TTRT + \tau & \text{if } 1 = i < b = n \\ c \cdot TTRT + H_1 + \cdots + H_i + \tau & \text{if } 1 \leq i \leq b = n-1 \\ c \cdot TTRT + H_{b+2} + \cdots + H_n \\ + H_1 + \cdots + H_i + \tau & \text{if } 1 \leq i \leq b \leq n-2. \end{cases}
$$

TABLE 1
Message and Network Parameters ($TTRT = 8ms$, $\tau = 1ms$)

S_i	SN	C_i	D_i	P_i	H_i	DN
S_1	1	$3.1\,ms$	$40\,ms$	$36\,ms$	$1\,ms$	3
S_2	2	$4.3\,ms$	$21\,ms$	$21\,ms$	$2.16\,ms$	3
S_3	3	$2.2\,ms$	$34\,ms$	$30\,ms$	$0.84\,ms$	1

On the other hand, according to Theorem 2, we have,

$$
\Delta_{b,i}(l,c) \leq \begin{cases} c \cdot TTRT + H_{b+1} + \cdots + H_i + \tau & \text{if } 1 \leq b < i \leq n \\ c \cdot TTRT + H_1 + \cdots + H_i + \tau & \text{if } 1 \leq i \leq b = n \\ c \cdot TTRT + H_{b+1} + \cdots + H_n \\ + H_1 + \cdots + H_i + \tau & \text{if } 1 \leq i \leq b < n. \end{cases}
$$

Comparing each case of the bound obtained from Theorem 2 with those of its corresponding cases obtained from (3), clearly, Theorem 1 gives a tighter upper bound (for $\Delta_{b,i}(l,c)$) than Theorem 2.

5 An Example

Consider a network with three nodes. Assume that each node i ($i = 1, 2, 3$) has a synchronous message stream S_i characterized by a period P_i, a maximum transmission time C_i, and a *relative* deadline D_i. Messages from S_i arrive regularly with period P_i and have relative deadline D_i (i.e., if a message from S_i arrives at time t, it must finish its transmission at node i by time $t + D_i$). Table 1 lists all network and message parameters, where *SN* and *DN* represent *Source Node* and *Destination Node*, respectively.

Clearly, the protocol constraint holds for the given network parameters. We now check if these parameters can ensure that all synchronous messages will arrive at their destination nodes before their deadlines by, respectively, using our proposed generalized upper bound (Theorem 1) and that derived by Han et al. (Theorem 2). Let R_i be the message response time for a message from S_i (i.e., the interval from arrival of the message till completion of its transmission at node i) and R_i^w ($\leq R_i$) be the worst-case message response time (i.e., the longest possible interval). In the worst case, a message from S_i becomes available for transmission immediately after some $t_{l,i}$, thus it misses the *first* chance of being transmitted on visit l, i [13], [14]. Because C_i units of time are needed for transmission of a whole message from S_i and node i can use at most H_i time units for transmitting synchronous messages whenever it receives the token, a total of $\lceil C_i / H_i \rceil$ times' token arrivals is required for transmitting the whole message, which is divided into $\lceil C_i / H_i \rceil$ frames (to be transmitted separately on each of the token arrivals). Since the message misses the first chance at time $t_{l,i}$ in the worst case, we have

$$
R_i = \begin{cases} (t_{l+\lceil C_i/H_i \rceil, i} - t_{l,i}) + C_i - (\lceil \frac{C_i}{H_i} \rceil - 1) \cdot H_i & \text{(for use with Theorem 1)} \\ \Delta_{i-1,i-1}(l, \lceil C_i/H_i \rceil) \\ + C_i - (\lceil \frac{C_i}{H_i} \rceil - 1) \cdot H_i & \text{(for use with Theorem 2)} \end{cases} \tag{4}
$$

With the above analysis, we can calculate R_1 based on Theorem 2 as follows:

$$
\begin{aligned}
R_1 &= \Delta_{3,3}\left(l, \left\lceil\frac{C_1}{H_1}\right\rceil\right) + C_1 - \left(\left\lceil\frac{C_1}{H_1}\right\rceil - 1\right)\cdot H_1 \qquad \text{(by (4))}\\
&\leq \left\lceil\frac{C_1}{H_1}\right\rceil \cdot TTRT + \sum_{j=1}^{3} H_j + \tau + C_1 - \left(\left\lceil\frac{C_1}{H_1}\right\rceil - 1\right)\cdot H_1\\
&\quad \text{(by Theorem2)}\\
&= \left\lceil\frac{3.1}{1}\right\rceil \cdot 8 + 1 + 2.16 + 0.84 + 1 + 3.1 - \left(\left\lceil\frac{3.1}{1}\right\rceil - 1\right)\cdot 1 = 37.1.
\end{aligned}
$$

Thus, $R_1^w = 37.1\,ms > D_1 = 36\,ms$, i.e., the message of S_1 misses its deadline when judged with Theorem 2. However, as shown below, this is not the case. Based on Theorem 1, we have

$$
\begin{aligned}
R_1 &= \left(t_{l+\lceil\frac{C_1}{H_1}\rceil,1} - t_{l,1}\right) + C_1 - \left(\left\lceil\frac{C_1}{H_1}\right\rceil - 1\right)\cdot H_1 \qquad \text{(by (4))}\\
&\leq \left\lceil\frac{\lceil\frac{C_1}{H_1}\rceil \cdot n}{n+1}\right\rceil \cdot TTRT + \sum_{j=2}^{3} H_j + \tau + \left(\left\lceil\frac{C_1}{H_1}\right\rceil - \left\lceil\frac{\lceil\frac{C_1}{H_1}\rceil \cdot n}{n+1}\right\rceil\right)\\
&\quad \cdot \left(\sum_{j=1}^{3} H_j + \tau\right) + C_1 - \left(\left\lceil\frac{C_1}{H_1}\right\rceil - 1\right)\cdot H_1 \qquad \text{(by Theorem 1)}\\
&= \left\lceil\frac{\lceil\frac{3.1}{1}\rceil \cdot 3}{3+1}\right\rceil \cdot 8 + 2.16 + 0.84 + 1 + \left(\left\lceil\frac{3.1}{1}\right\rceil - \left\lceil\frac{\lceil\frac{3.1}{1}\rceil \cdot 3}{3+1}\right\rceil\right)\\
&\quad \cdot (4+1) + 3.1 - \left(\left\lceil\frac{3.1}{1}\right\rceil - 1\right)\cdot 1 = 33.1.
\end{aligned}
$$

Thus, $R_1^w = 33.1\,ms < D_1 = 36\,ms$, i.e., the deadline of S_1 is met when judged with Theorem 1.

Similarly, calculating R_2 and R_3 based on Theorem 1, we have: $R_2 = 20.98\,ms < D_2 = 21\,ms$; $R_3 = 28.68\,ms < D_3 = 30\,ms$. So, no messages miss their deadlines when judged with Theorem 1.

6 Conclusion

We have presented a formal proof to a generalized upper bound on the elapsed time from the token's lth arrival at any node i till its $(l+v)$th arrival at any node k (where integer $v \geq 0$). Our derived upper bound expression, which is particularly important for hard real-time communications in any timed token network, is better than any of the previous findings on the protocol cycle-time properties due to the fact that it is more general and may produce a tighter upper bound.

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