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A 3-parameter analytical model for the acoustical properties of porous media

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Many models for the prediction of the acoustical properties of porous media require non-acoustical parameters few of which are directly measurable. One popular prediction model by Johnson, Champoux, Allard and Lafarge [Champoux *et al*, J. Appl. Phys, 70(4), 1975 – 1979 (1991)] (334 citations, Scopus, December 2018) requires six non-acoustical parameters. This paper proves that the use of more than three parameters in the Johnson-Champoux-Allard-Lafarge model is not necessary at all. Here we present theoretical and experimental evidence that the acoustical impedance of a range of porous media with pore size distribution close to log-normal (granular, fibrous and foams) can be predicted through the knowledge of the porosity, median pore size and standard deviation in the pore size only. A novelty of this paper is that it effectively halves the number of parameters required to predict the acoustical properties of porous media very accurately. The significance of this paper is that it proposes an unambiguous relationship between the pore microstructure and key acoustical properties of porous media with log-normal pore size distribution. This new model is well suited for using acoustical data for measuring and inverting key non-acoustical properties of a wider range of porous media used in a range of applications which are not necessarily acoustic.

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I. INTRODUCTION

The model for the acoustical properties of porous media proposed by Champoux, Allard and Lafarge[1,2] relies on six non-acoustical parameters which are: (i) porosity (ϕ), (ii) tortuosity (α_∞); (iii) viscous characteristic length (Λ); (iv) thermal characteristic length (Λ'); (v) viscous permeability (κ_0); and (vi) thermal permeability (κ_0'). The improved model proposed by Pride[3] adds one more parameter, the Pride parameter (β). Some people include it in their models, but rarely. These phenomenological parameters are notoriously difficult to relate to the microstructure of real-life porous materials such as granular media, fibrous media or complicated foam structures. This fact makes the application of this kind of models for the inversion of microstructural properties of porous media from acoustical data rather difficult to understand by non-acousticians, e.g. by material scientists, process or chemical engineers.

It has recently been shown that the popular 6-parameter model[1] can be reduced to a 4-parameter model because the two characteristic lengths and two permeabilities can be expressed via the median pore size ($\bar{s}$) and standard deviation in the pore size (σ_s) provided that the size of pores in the porous medium is log-normally distributed[4], i.e.:

$$\Lambda = \bar{s} e^{-5/2(\sigma_s \log 2)^2}, \tag{1}$$

$$\Lambda^{'} = \bar{s} e^{3/2(\sigma_s \log 2)^2}, \tag{2}$$

$$\kappa_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} e^{-6(\sigma_s \log 2)^2}, \tag{3}$$

and

$$\kappa_0^{'} = \frac{\bar{s}^2 \phi}{8\alpha_\infty} e^{6(\sigma_s \log 2)^2}. \tag{4}$$

It has also been shown in[4] that the Pride parameter[3] for this type of porous media is:

$$\beta = 4/3 e^{4(\sigma_s \log 2)^2}. \tag{5}$$

A theoretical model which has a reduced number of parameters can be attractive to scientists and engineers who work beyond acoustics because: (i) the number of parameters in the model is greatly reduced; (ii) the porosity, median pore size and standard deviation in pore size become directly measurable; (iii) the physical meaning of the median pore size and standard deviation in the pore size is easier to understand than the two characteristic lengths and thermal permeability. Effectively, the model by Horoshenkov *et al*[4] suggests that the acoustical properties of a porous medium with log-normal pore size distribution can be predicted from the knowledge of ϕ, α_∞, $\bar{s}$ and σ_s. This reduction in the number of parameters in a model also paves the way for an easier inversion of key morphological characteristics of porous media from acoustical data which are relatively easy to obtain through a standard impedance tube experiment[5].

However, one question which has been overlooked by the authors of[4] is the relation between the tortuosity and the pore size distribution. Is the tortuosity of this kind of media a parameter independent from the pore size distribution statistics? This paper is an attempt to answer that question. It shows that the tortuosity of a porous medium with log-normally distributed pore size is controlled by the standard deviation in the pore size distribution. It is also shown how the acoustical impedance of a range of porous media (granular, fibrous and foams) can be predicted very accurately through the knowledge of the porosity and pore size distribution parameters only. The novelty of this paper is that it proposes a robust analytical model for the acoustical properties which relies on three directly measurable parameters of porous media. It also shows that the use of more than three parameters in the Johnson-Champoux-Allard-Lafarge model[2] is not necessary for many types of porous media. The significance of this paper is that it proposes an unambiguous relationship between the pore microstructure and key acoustical properties of porous media with log-normal pore size distribution. This new model is well suited for using acoustical data for measuring key non-acoustical properties of a wider range of porous media.

The paper is organised in the following manner. Section II proposes the new relation between the tortuosity and standard deviation in the pore size. Section III presents a revision of the model proposed in[4]. Section IV presents a comparison between the predicted and measured acoustical and non-acoustical properties of several types of porous media. Section V is the conclusions section.

II. THE TORTUOSITY OF POROUS MEDIA WITH LOG-NORMAL PORE SIZE DISTRIBUTION

Let us assume that a sound wave propagates in a non-uniform pore which circular cross-section varies with depth as shown in Figure 1. In this figure Δx is the thickness of the porous layer, Δl is the length of the section within which the change in the pore cross-section (A_n) is considered negligible. According to the work by Champoux and Stinson (see Eq. (20) in[6]) the electrically measured tortuosity of a medium with the total surface area (A) covered by M non-uniform pores is:

$$\alpha_\infty = \frac{\phi A}{M \Delta x} \sum_{n=1}^{N} \frac{\Delta l}{A_n}, \tag{6}$$

where N is the total number of cross-sectional changes which may occur in the pore area measured along the thickness of the porous layer, Δx (for more details see Figure 1 in ref.[6]). Since $\phi = \dfrac{M}{A\Delta x} \sum_n A_n \Delta l$, and $\Delta x = \sum_n \Delta l$ for a constant value of Δl equation (6) becomes:

$$\alpha_\infty = \frac{\sum_{n=1}^{N} A_n \Delta l}{\left(\sum_{n=1}^{N} \Delta l\right)^2} \sum_{n=1}^{N} \frac{\Delta l}{A_n}. \tag{7}$$

Setting the total number of sections with a constant cross-section required to represent the complexity of the non-uniform pore shown in Figure 1 to $N \to \infty$, $\Delta l \to 0$ and swapping the sums in (7) for integrals yields (see eqs. (38)-(44) and eqs. (50), (51) in ref.[4]):

$$\alpha_\infty = \frac{I_A}{I_0^2} I_{1/A}, \tag{8}$$

where $I_A = \int_{-\infty}^{\infty} s^2 e(s) ds$, $I_{1/A} = \int_{-\infty}^{\infty} s^{-2} e(s) ds$ and $I_0 = \int_{-\infty}^{\infty} e(s) ds$ and $e(s)$ is the probability density function for the pore radius s.

It is common in the areas of geotechnics and granular porous media research to use a logarithm with base 2 to express pore size on a log-normal scale, i.e. so that $s = 2^{-\varphi}$, $e(s) = f(\varphi)\dfrac{d\varphi}{ds}$, $f(\varphi) = \dfrac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{(\varphi-\bar{\varphi})^2}{2\sigma_s^2}}$ and $\varphi = -\log_2 s$. For this choice of pore size distribution it is possible to demonstrate that the above three integrals reduce to:

$$I_{1/A} = \int_{-\infty}^{+\infty} 2^{2\varphi} e^{-\dfrac{(\varphi-\bar{\varphi})^2}{2\sigma_s^2}} d\varphi = 2^{2\bar{\varphi}} e^{2\sigma_s^2 \log^2 2}, \tag{9}$$

$$I_A = \int_{-\infty}^{+\infty} 2^{-2\varphi} e^{-\dfrac{(\varphi-\bar{\varphi})^2}{2\sigma_s^2}} d\varphi = 2^{-2\bar{\varphi}} e^{2\sigma_s^2 \log^2 2}. \tag{10}$$

and

$$I_0 = \int_{-\infty}^{+\infty} e^{-\dfrac{(\varphi-\bar{\varphi})^2}{2\sigma_s^2}} d\varphi = 1. \tag{11}$$

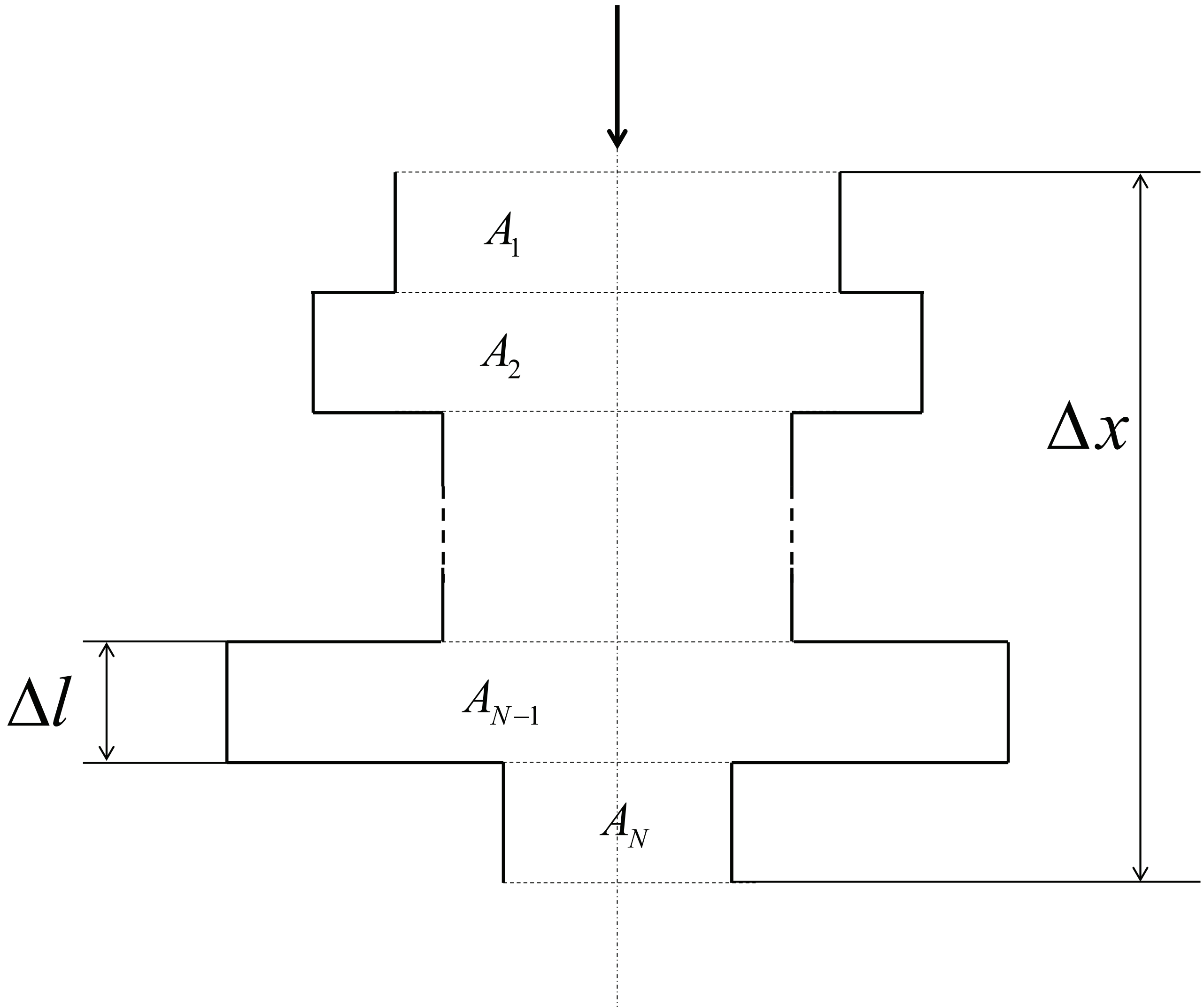

FIG. 1. Sound propagation in a non-uniform pore with a circular cross-section.

In these expressions $\bar{\varphi} = -\log_2 \bar{s}$ is the mean pore size on the logarithmic scale and $\bar{s}$ is the median pore size on the linear pore size scale. The substitution of Eqs. (9)-(11) in Eq. (7) yields the new equation for the tortuosity:

$$\alpha_\infty = e^{4(\sigma_s \log 2)^2}. \tag{12}$$

Eq. (12) suggests that the tortuosity of a porous medium with well-interconnected non-uniform pores which size is log-normally distributed depends only on the standard deviation in the pore size (σ_s). In the limit case, when the pores are uniform and their size is identical, $\sigma_s = 0$ and $\alpha_\infty = 1$, which makes physical sense for materials with a relatively high porosity.

The main implication of Eq. (12) and previously derived expressions for the characteristic lengths and permeabilities (Eqs. (1)-(4)) is that it enables us to predict the acoustical properties of porous medium with three directly measurable parameters only rather than six[1,2]. These parameters are: (i) the porosity (ϕ); (ii) the median pore size ($\bar{s}$); and (iii) the standard deviation in pore size distribution (σ_s).

III. MODELLING THE ACOUSTICAL PROPERTIES OF POROUS MEDIA WITH THE 3-PARAMETER MODEL

Following[4] we will propose Padé approximations for the equations for the frequency dependent bulk dynamic density ($\tilde{\rho}(\omega)$) and bulk complex compressibility ($\tilde{C}(\omega)$) in the equivalent fluid model to predict the acoustical properties of porous media with log normal distribution. Here ω stands for the circular frequency.

The bulk dynamic density can be approximated with the following expression (Eqs. (64) and (65) in[4]):

$$\tilde{\rho}(\omega)/\rho_0 \simeq \frac{\alpha_\infty}{\phi}\left(1+\epsilon_\rho^{-2}\tilde{F}_\rho(\epsilon_\rho)\right), \tag{13}$$

where

$$\tilde{F}_\rho(\omega) = \frac{1+\theta_{\rho,3}\epsilon_\rho+\theta_{\rho,1}\epsilon_\rho}{1+\theta_{\rho,3}\epsilon_\rho} \tag{14}$$

is the Padé approximant to the viscosity correction function with $\epsilon_\rho = \sqrt{\frac{-i\omega\rho_0\alpha_\infty}{\phi\sigma}}$ (we note, there is a typo in the expression for ϵ_ρ in ref.[4]. The flow resistivity for a single pore (σ_x) must appear in the denominator. Also, the term (ρ_0) is missing from the denominator in the left hand side of Eq. (64) in[4]), $\theta_{\rho,1} = 1/3$, $\theta_{\rho,2} = e^{-1/2(\sigma_s \log 2)^2}/\sqrt{2}$ and $\theta_{\rho,3} = \theta_{\rho,1}/\theta_{\rho,2}$ (see Section III in[4]). In these equations the bulk flow resistivity of the porous medium is (see also Eq. (3)):

$$\sigma = \frac{\eta}{\kappa_0} = \frac{8\eta\alpha_\infty}{\bar{s}^2\phi}e^{6(\sigma_s \log 2)^2}, \tag{15}$$

where η is the dynamic viscosity of air and ρ_0 is the ambient density of air.

Similarly, the bulk complex compressibility of the fluid in the material pores can be given in the following form (see Eqs. (68) and (69) in[4]):

$$\tilde{C}(\omega) = \frac{\phi}{\gamma P_0}\left(\gamma - \frac{\gamma-1}{1+\epsilon_c^{-2}\tilde{F}_c(\epsilon_c)}\right), \tag{16}$$

where

$$\tilde{F}_c(\epsilon_c) = \frac{1 + \theta_{c,3}\epsilon_c + \theta_{c,1}\epsilon_c}{1 + \theta_{c,3}\epsilon_c}. \tag{17}$$

In the above two equations $\epsilon_c = \sqrt{\dfrac{-i\omega\rho_0 N_{Pr}\alpha_\infty}{\phi\sigma'}}$, $\theta_{c,1} = \theta_{\rho,1} = 1/3$, $\theta_{c,2} = e^{3/2(\sigma_s \log 2)^2}$, $\theta_{c,3} = \theta_{c,1}/\theta_{c,2}$, γ is the ratio of specific heats, N_{Pr} is the Prandtl number and P_0 is the ambient atmospheric pressure. The thermal flow resistivity here is defined as as the inverse of the thermal permeability (see also Eq. (4)):

$$\sigma' = \frac{\eta}{\kappa_0'} = \frac{8\eta\alpha_\infty}{\bar{s}^2\phi} e^{-6(\sigma_s \log 2)^2}. \tag{18}$$

Eqs. (13) and (16) can be used to predict the characteristic acoustic impedance:

$$z_b(\omega) = \sqrt{\tilde{\rho}(\omega)/\tilde{C}(\omega)} \tag{19}$$

and complex wavenumber:

$$k_b(\omega) = \omega\sqrt{\tilde{\rho}(\omega)\tilde{C}(\omega)} \tag{20}$$

in a porous medium with log-normal pore size distribution.

IV. RESULTS

Eqs. (19) and (20) were used to predict the acoustical surface impedance of hard-backed layers of three types of porous media: (i) loose granular media; (ii) fibrous media; and (iii) foams. The details of these three types of media are given in Table I. The values of the intrinsic air properties (e.g. ρ_0, η, γ and N_{Pr}) were calculated from the standard equations for the temperature of 20^{o}C and ambient air pressure of $P_0 = 101320$ Pa. The equations for these parameters come from several textbooks. These equations were carefully compiled and provided kindly by Matelys[8].

We chose glass beads because it has been shown that the pore size distribution in glass beads is very close to log-normal[7]. We chose melamine foam and rebound felt as the other two examples because these materials are commonly used as acoustic absorbers in various noise control applications.

The normalised surface impedance of finite, hard-backed layers of these porous media was calculated by:

$$z_s(\omega) = z_b(\omega)\coth(-ik_b(\omega)h)/(\rho_0 c_0), \tag{21}$$

where h is the layer thickness and c_0 is the sound speed in air. The measurements of the surface impedance of glass beads were carried out in a 45 mm diameter impedance tube. The measurements of the melamine foam and felt samples were made in a 100 mm diameter impedance tube. The both tubes were supplied by Materiacustica[9]. Each material sample was measured three times and the averaged data were used for the comparison with the model. The repeatability of the measurements was within ±5%.

Figure 2 presents a comparison between the measured (dotted lines) and predicted (solid and dashed lines) normalized surface impedance of a 40 mm thick, hard-backed stack of 2 mm diameter glass beads. These beads were deposited in the sample holder in a random manner and were left uncompacted. The solid lines correspond to the prediction made with the proposed Padé approximation model (equations (13) and (16)). The dashed lines correspond to the prediction made with the Johnson-Allard-Champoux-Lafarge model[1,2]. The predictions correspond to the following values of the key non-acoustical parameters: $\phi = 0.376$, $\bar{s} = 305\ \mu$m; and $\sigma_s = 0.388$. These non-acoustical parameters were estimated by fitting the Padé approximation model (Eqs. (13) and (16)) to the measured data for the surface impedance through the Nelder-Mead function minimization technique[10] as described in[11]. In this process the tortuosity and flow resistivity required for the Padé approximation model were predicted using Eqs. (12) and (3), respectively. The thermal permeability and characteristic lengths required for the subsequent comparison with the Johnson-Champoux-Allard model were predicted using Eqs. (1), (2) and (4), respectively. The values of all these parameters for glass beads and for other materials studied in this work are summarized in Table I.

The results obtained for glass beads suggest that the viscous characteristic length we recovered from the acoustical data is consistent with that reported by Glover *et al* for 2 mm glass beads[12] (this work: $\Lambda = 255\ \mu$m; Table 1 in Glover *et al*[12]: $\Lambda = 252\mu$m). The compaction states for these two materials were comparable so that the porosities were similar (in this work: $\phi = 0.376$; Glover *et al*[12]: $\phi = 0.386$). The porosity value estimated from our experiment is also consistent with that measured by Leclaire *et al* for uncompacted glass beads. It was shown to be in the range of $0.362 \le \phi \le 0.385$ (see Table I in[7]). The estimated value of the median pore size for our glass beads seems sensible because the work by Glover *et al* (page E20 in[12]) suggests that the ratio of the sphere diameter (2 mm) to the effective pore diameter (611 μm) in a stack of identical beads should be 3.44, which makes our estimate of $\bar{s}$ accurate within 5%. Our estimate of the thermal characteristic

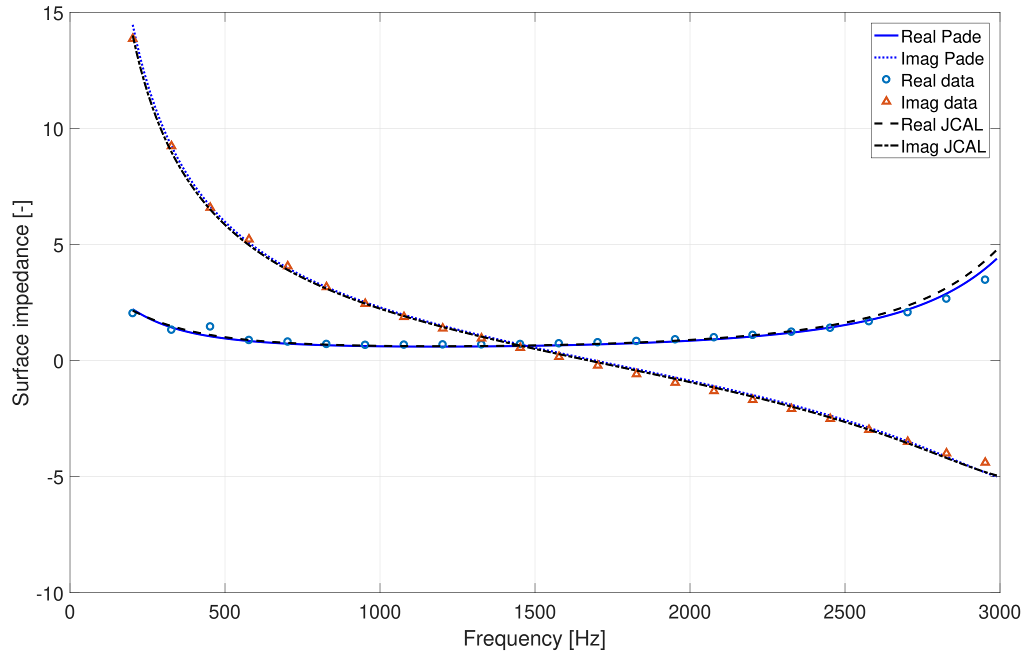

FIG. 2. The measured and predicted normalized surface impedance of a 40 mm thick, hard-backed layer of 2mm diameter glass beads.

length ($\Lambda' = 340\,\mu$m) also makes sense because it scales favourably to $\Lambda' = 321\,\mu$m from $\Lambda' = 263\,\mu$m measured by Leclaire *et al* for 1.64 mm diameter glass beads in an independent water suction experiment (see Table II in[7]).

On the other hand, the flow resistivity predicted with Eqs. (3 and (15))($\sigma = 8560$ Pa s m^{-2}) was found to be 17% higher than that we measured at the University of Sheffield with an AFD AcoustiFlow 300 device on 100 mm diameter samples. This apparatus was supplied by Akustik Forschung Dresden and used alongside their AFD 311 software package[13]. This discrepancy can be related to differences in the packing conditions and diameters of the sample holders used in the impedance tube and flow resistivity experiments. The estimated value of tortuosity for our glass beads ($\alpha_\infty = 1.33$) is consistent with that one can estimate from a typical value of the formation factor

$$F \approx \alpha_\infty/\phi, \tag{22}$$

which, according to literature (e.g.[12,14]), can be in the range of $3.0 \leq F \leq 4.0$ for a stack of uniform spherical beads. This suggests that the tortuosity of our glass beads should be in the range of $1.13 \leq \alpha_\infty \leq 1.50$. Our tortuosity estimate of $\alpha_\infty = 1.33$ is very close to the median for this range.

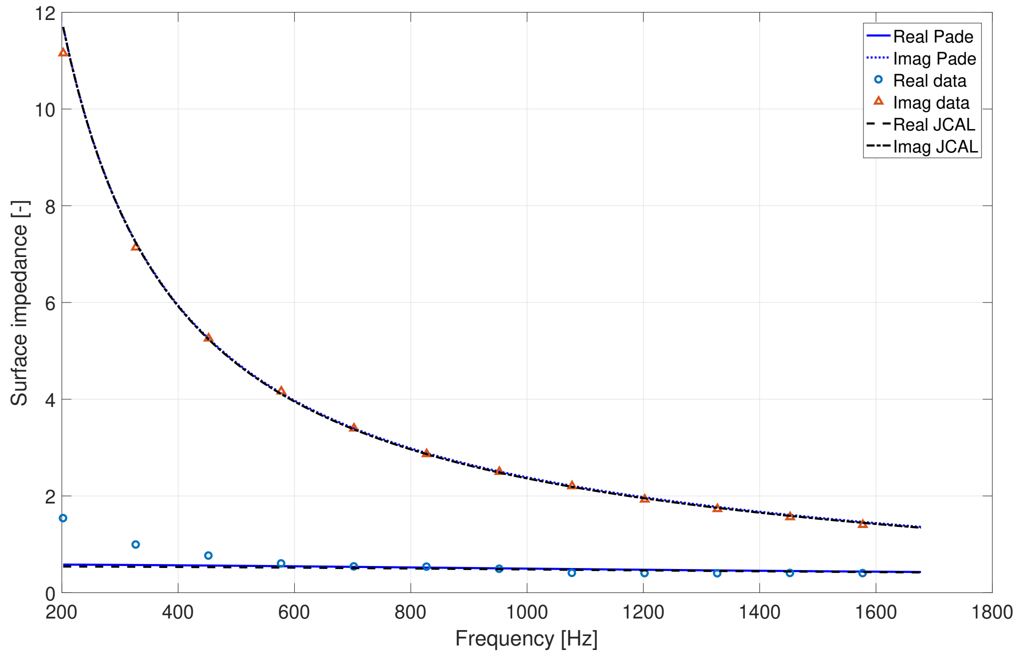

FIG. 3. The measured and predicted normalized surface impedance of a 16.5 mm thick, hard-backed layer of melamine foam used in a range of acoustic applications.

The results presented in Figure 2 show that the Padé approximation model used with a set of sensible values of the three key non-acoustic parameters predicts the real and imaginary part of the impedance with the normalized mean error of ±4.4%. This error was calculated as:

$$E = \frac{\sum_m \|z_s^{(m)} - z_s^{(p)}\|}{\|z_s^{(m)}\|}, \tag{23}$$

where the indices (m) and (p) stand for the measured and predicted values, respectively.

For this set of parameters the Johnson-Champoux-Allard model predicts the impedance with the normalized mean error of ±4.0%. These errors are within the experimental accuracy of the impedance tube apparatus used in this work.

Figure 3 presents the normalized surface impedance of a 16.5 mm thick, hard-backed layer of melamine foam. Similarly to the results presented in Figure 2 the parameters of the best fit were obtained through the minimization procedure described in[11]. The parameters of best fit are: $\phi = 0.998$, $\bar{s} = 115\,\mu$m; and $\sigma_s = 0.243$. These and other parameters for this material are listed in Table I. The predictions by the two models made for this material are almost identical. The normalized mean error between the two models and measured data is within ±2.4%. The two models are not very sensitive to the value of the porosity because

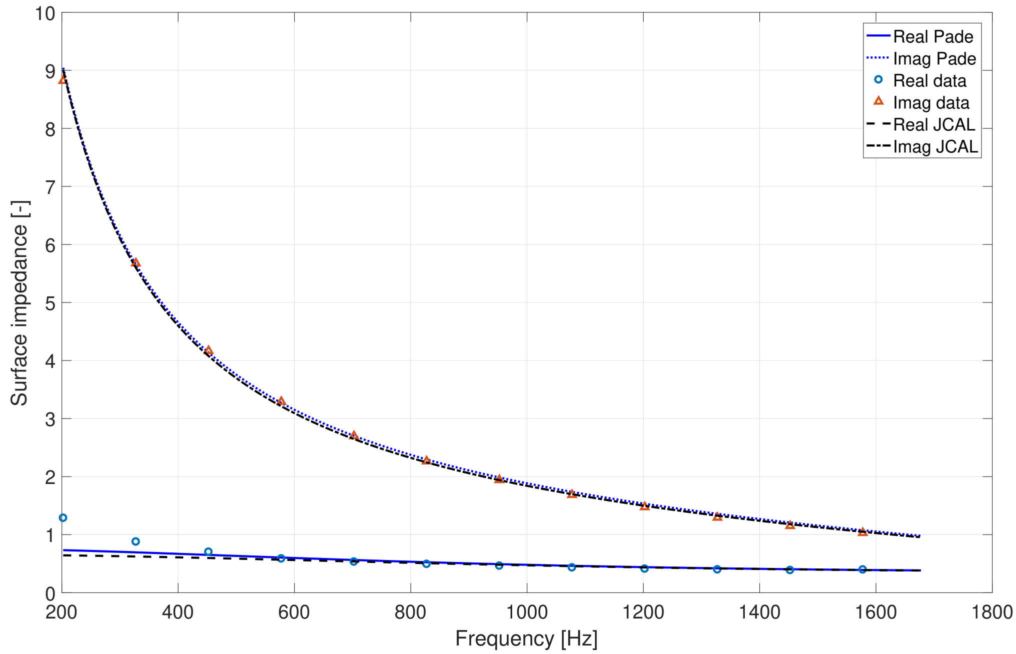

FIG. 4. The measured and predicted normalized surface impedance of a 21.5 mm thick, hard-backed layer of rebound felt used in automotive noise control applications.

its true value is close to unity ($\phi = 99.3\%$). The estimated median pore size ($\bar{s} = 115\ \mu$m) makes sense because typical pore count in melamine foams is 150-200 cells per inch[15], which suggests that the average cell size should be in the range of 127 – 169 μm. For similar melamine foam (see page 94 in[16]) it was estimated from optical images at $\bar{s} = 128 \pm 67\ \mu$m, which makes our estimate fall within the experimental error. Our estimate for the flow resistivity of melamine foam (Eqs. (3) and (15)) is $\sigma = 14600$ Pa s m^{-2}. This is 10.4% below the value of $\sigma = 16400$ Pa s m^{-2} measured with our flow resistivity apparatus[13]. The standard deviation in the pore size in this material is relatively small ($\sigma_s = 0.243$). As a result, the tortuosity estimated with Eq. (12) is close to unity ($\alpha_\infty = 1.12$) and it is not a dominant parameter in this case.

Finally, Figure 4 presents the normalized surface impedance for a 21.5 mm layer of rebound felt used as an acoustic lining in vehicles. Here we compare the predictions with the two models against measured data. This material corresponds to Sample 3 described and characterized in detail in[17]. The parameters of best fit for this material are: $\phi = 0.998$, $\bar{s} = 147\ \mu$m; and $\sigma_s = 0.325$. These and other parameters for this material are listed in Table I. The estimated value of porosity is 3% higher than the $\phi = 0.97$ value reported in[17]. It is easy to check that this small discrepancy has only a marginal effect on the predicted

TABLE I. The values of non-acoustical parameters for the three materials studied in this work. The values in brackets correspond to those which were measured directly and non-acoustically.

Material	h, mm	$\bar{s}$, μm	ϕ	σ_s, φ-units	σ, Pa s m^{-2}	κ_0', m^2	Λ, μm	Λ', μm
2 mm glass beads	(40.0)	305	0.376	0.388	8560	5.06×10^{-9}	255	340
					(7290)		(253)	(321)
melamine foam	(16.5)	115	0.998	0.243	14600	1.75×10^{-9}	107	120
			(0.993)		(16400)			
rebound felt	(21.5)	147	0.998	0.325	11000	2.99×10^{-9}	131	160
			(0.970)		(10260)			

impedance because the acoustical behaviour of this material is dominated by the pore size and it is relatively insensitive to small variations in the porosity value. The normalized mean error between the measured data and prediction with the Padé approximation model is $\pm 1.5\%$. The normalized mean error between the measured data and prediction with the Johnson-Champoux-Allard model is $\pm 1.8\%$. There is a small difference between the predictions made with the Johnson-Champoux-Allard model and Padé approximation. This difference can be reduced by adjusting the porosity between the measured ($\phi = 0.97$) and estimated ($\phi = 0.998$) values. The flow resistivity of this material estimated using Eqs. (3) and (15) is $\sigma = 11000$ Pa s m^{-2}. This value compares very well against the measured flow resistivity of $\sigma = 10260 \pm 180$ Pa s m^{-2} and it is within the experimental error. The standard deviation in the pore size in this material ($\sigma_s = 0.325$) is higher than that estimated for melamine foam ($\sigma_s = 0.243$). As a result, the tortuosity estimated with Eq. (12) is noticeably higher than unity ($\alpha_\infty = 1.22$) and it is becoming an influential parameter in the modelling process.

V. CONCLUSIONS

This paper proposes a new equation for the tortuosity of porous media with pore size close to log-normal (Eq. (12)) suggesting that the tortuosity of this kind of media is dependent on the standard deviation in the pore size only. This new equation can be combined with equations (1) - (4) to effectively halve the number of non-acoustical parameters used in the popular Johnson-Champoux-Allard model[1,2]. It has been demonstrated through an experiment that this model can be used to predict accurately the acoustical properties of granular media, fibrous media and foams with three non-acoustical parameters only. These parameters are: (i) median pore size ($\bar{s}$); (ii) porosity (ϕ); and (iii) standard deviation in the log-normal pore size distribution (σ_s).

The paper also proposes a new Padé approximation model for the prediction of the acoustical properties of porous media with non-uniform pore size and pore size distribution close to log-normal. This model can be used as an alternative to the Johnson-Champoux-Allard model. It has been shown that the predictions with these two models are almost identical and close to our measured surface impedance data within $\pm 4\%$.

An approach to reduce the number of non-acoustical parameters in an acoustical model seems very useful because it enables us to make an accurate estimation of key parameters of pore size distribution and porosity in various types of porous media from acoustical data. It also enables us to relate these quantities to those parameters in the Johnson-Champoux-Allard model which are difficult or impossible to measure non-acoustically. This makes application of this approach more attractive for the inversion of microstructural properties of porous media from acoustical data by non-acousticians, e.g. by material scientists, chemical, process or geotechnical engineers.

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LIST OF FIGURES

Figure 1: Sound propagation in a non-uniform pore with a circular cross-section.

Figure 2. The measured and predicted normalized surface impedance of a 40 mm thick, hard-backed layer of 2 mm diameter glass beads.

Figure 3. The measured and predicted normalized surface impedance of a 16.5 mm thick, hard-backed layer of melamine foam used in a range of acoustic applications.

Figure 4. The measured and predicted normalized surface impedance of a 21.5 mm thick, hard-backed layer of rebound felt used in automotive noise control applications.