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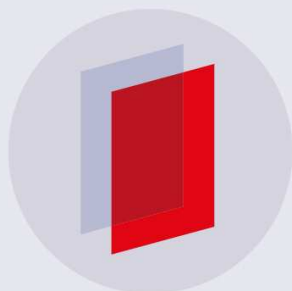
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Novelty Detection in a Cantilever Beam using Extreme Function Theory

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Abstract. Damage detection and localisation in beam-like structures using mode shape features is well-established in the research community. It is known that by inserting a localised anomaly in a cantilever beam, such as a crack, its mode shapes diverge from the usual deflection path. These novelties can hence be detected by a machine-learner trained exclusively on the modal data taken from the pristine beam. Nevertheless, a major issue in current practices regards discerning between damage-related outliers and simple noise in observations, avoiding false alarms. Extreme functions are here introduced as a viable mean of comparison. By combining Extreme Value Theory (EVT) and Gaussian Process (GP) Regression, one can investigate functions as a whole rather than focusing on their constituent data points. Indeed, n discrete observations of a mode shape sampled at D points can be assumed as 1-dimensional sets of n randomly distributed observations. From any given point it is then possible to define its *Probability Density Function* (PDF) and the *Cumulative Density Function* (CDF), whose minima, according to the EVT, belong to one of three feasible extreme distributions - *Weibull, Frechet or Gumbel*. Thus, these functions - intended as vectors of sampled data - can be compared and classified. Anomalous displacement values that could indicate the presence of a crack are therefore identified and related to damage. In this paper, the effectiveness of the proposed methodology is verified on numerically-simulated noisy data, considering several crack locations, levels of damage severity (i.e., depths of the crack) and signal-to-noise ratios.

1. Introduction

A key factor in all probabilistic and semi-probabilistic approaches used in sciences and technology fields regards the assumptions on the statistical distributions of the available experimental data. On the basis of the well-known Central Limit Theorem (CLT), Gaussian or Gaussian-like probability distributions are mostly used. Nevertheless, when dealing with extreme deviations from the mean, hypothesising Gaussianity may misguide. On the other hand, Extreme Value Statistics (EVS), based on EVT [1], has been established specifically to handle the values of the independent random variables failing in the tails of the distribution of interest [2, 3, 4]. This is particularly interesting in the field of Structural Health Monitoring (SHM).

Indeed, most widely used damage detection techniques revolve around outlier detection (see, e.g., [5] and [6]), which in turn needs, in some sense, a sort of "thresholding" between data taken from damaged and undamaged conditions. This is due to the assumption that some features in the output

recorded from the structure in an abnormal situation will be above the so-determined threshold. This can be intended as a statistical test performed on the experimental observations, which is regularly carried out considering some scalar values as the damage-sensitive features. Unfortunately, false positives are not uncommon in outlier analysis when performed on variables assumed as independent and identically distributed (*i.i.d.*) and/or Gaussian distributed. In fact, this assumption does not always hold true in practice.

EVT is an ideal statistical framework for evaluating the significance of extreme values departing from the normality model and the concept has already been exploited in this sense [7]. It must be remarked that *normality*, or *normal condition*, refers here to the pristine state of the structure; *Gaussian* is used instead to indicate the normal probability distribution function (PDF), to avoid any confusion.

In this work, the Generalised Extreme Value (GEV) distribution, which combines Gumbel, Frechet and Weibull ones in a unique model [9], has been used and its hyperparameters have been optimised through the Self-Adaptive Differential Evolution algorithm (SADE), [10]. Thanks to the adaptive nature of the adopted optimisation algorithm, the parameters of the genetic process have been deduced by the data themselves rather than being arbitrarily imposed. The novelty of this research, if compared to the previously mentioned papers and other works, resides in the application of GEV to the mode shapes as a whole. This represents a quite different approach from the more common pointwise comparison between models: the feature selected for the statistical test is not a univariate scalar, nor a multivariate collection of them, assembled in an array, but a function. This study relies on the EFT path initiated by [11] and [8]. In order to assess the mode shapes as continuous functions starting from a discrete amount of observations, GP regression has been applied. In fact, even if existing since more than a decade [12], GP Regression has become highly valued in the field of SHM only in recent years [13, 14, 15, 16]. Gaussian Processes return a Gaussian probability distribution, and the confidence intervals of the prediction; the principled Bayesian foundations make them exceptionally apt for statistical investigations. As will be explained in further detail later, combining GP Regression and the GEV distribution on modal data produces very promising results.

The proposed methodology has been here applied to a numerical case, based on a Finite Element (FE) model of a beam in various boundary conditions, where an edge crack has been simulated by deriving the stiffness matrix through the definition of the *stress intensity factor*, holding the hypotheses of invariance of crack in time, invariance of damaged element mass distribution and neglecting the *opening-closing* fracture mechanism [16, 17]. Several combinations of damage severity and noise levels (artificially included in simulations) have been investigated.

The paper is organised as follows: in Section two, some basic concepts of GP Regression, the EVT and the EFT, as well as some hints about the genetic algorithm used for this task, are briefly recalled; in Section three, results are reported and discussed; finally, a discussion is presented in Section four.

2. Gaussian Process and Extreme Function Theory

The theoretical FE mode shapes simulated using the Euler-Bernoulli dynamic elastic theory are noise free, hence to give them the aleatory behaviour that they would have in reality, Gaussian white noise has been added (at different levels) through the specification of the *Signal to Noise Ratio* (SNR), expressed in Decibels [Db] as,

$$SNR = 20\log_{10}\left(\frac{A_{signal}}{A_{noise}}\right) \tag{1}$$

where A_{signal} is the noise-free signal amplitude and A_{noise} is the amplitude of the added noise. First, two large sets of data have been simulated with the FE Method, the first one made up from noisy undamaged transverse mode shapes, while the second one collects the noisy damaged ones. This step is operational to obtain a robust statistical pool of data from which different subsets will be randomly sampled later, avoiding intersections among them. Then three subsets of data are randomly sampled for each analysed mode shape from the parent sets as follows:

- *Training set (TR)*: composed only of the undamaged data.
- *Validation set (VA)*: composed only of the undamaged data.
- *Test set (TS)*: composed of the undamaged and the damaged data.

The training set TR is used to train the Gaussian Process, fitting the parameters of the variable distribution through the optimisation of the predictive joint conditional *Marginal Likelihood* by minimising its negative logarithmic form [12]. Combining the current parameter estimation with the validation set VA leads to the generation of the PDF standardised logarithmic values. As explained in [12], the noisy observations $\mathbf{y} = f(\mathbf{x}) + \varepsilon$ of a 1-dimensional (D) problem, that in this case are the beam transverse displacements at the node locations $\mathbf{x}$ disturbed by the i.i.d. noise ε, have a Gaussian joint distribution of the target values $\mathbf{f}^*$ and the function values at the test locations $\mathbf{x}^*$ defined as follows,

$$p\left(\mathbf{f}^*|\mathbf{x},\mathbf{y},\mathbf{x}^*\right) \sim \mathcal{N}\left(m^*,K^*\right) \tag{2}$$

where m^* is the predictive mean and K^* is the predictive covariance matrix, expressed as,

$$m^* = k\left(\mathbf{x}^*,\mathbf{x}\right)\left[k\left(\mathbf{x},\mathbf{x}\right)+\sigma_n^2\right]^{-1}\mathbf{y} \tag{3}$$

$$K^* = cov(\mathbf{f}^*) = k\left(\mathbf{x}^*,\mathbf{x}^*\right) - k\left(\mathbf{x}^*,\mathbf{x}\right)\left(k\left(\mathbf{x},\mathbf{x}\right)+\sigma_n^2\right)^{-1}k\left(\mathbf{x},\mathbf{x}^*\right) \tag{4}$$

The resulting multivariate Gaussian probability distributions take the form,

$$p = \frac{1}{\sqrt{2\pi^D|K^*|}}\exp\left[-\frac{1}{2}(\mathbf{f}^*-m^*)^T K^{*-1}(\mathbf{f}^*-m^*)\right] \tag{5}$$

The prior information of the process is specified in the covariance matrix K^*, in this case assumed as a *Squared-Exponential*. In the one dimensional case it can be expressed in the form,

$$k_y(x_p,x_q) = \sigma_f^2\exp\left[-\frac{1}{2l^2}(x_p-x_q)^2\right]+\sigma_n^2\delta_{p,q} \tag{6}$$

where $\delta_{p,q}$ is the Kronecker delta and l,σ_f,σ_n are the *hyperparameters*, whose optimisation is based on the *marginalisation property* of the *marginal Likelihood* (ML). The variables x_p and x_q are the nodal locations of the training data data set corresponding to different position indices p,q. The ML is taken in the negative logarithmic form (NLML), since it is easier to perform a minimisation, resulting in,

$$NLML = \log p\left(\mathbf{y}|\mathbf{x},l,\sigma_f,\sigma_n\right) = \frac{1}{2}\mathbf{y}^T L_{xx}^T\backslash\left(L_{xx}\backslash\mathbf{y}\right) + \sum\log\left(diag(L_{xx})\right) + \frac{n}{2}\log 2\pi \tag{7}$$

where L_{xx} is the lower Cholesky decomposition of the covariance matrix $k(\mathbf{x},\mathbf{x})$ and the operator "\" indicates the left matrix division, required for a faster computation of the matrix inversion. At the end of the iterative minimisation process, when the difference in likelihood values are smaller than a given tolerance, the obtained hyperparameters are the best ones fitting the covariance function on the training data considered.

EVT is by definition a point-wise approach, generally applied to univariate data or extended to other low-dimensional spaces. It can be adapted for functional applications (EFT) [11] to identify extreme functions from a given n-dimensional multivariate Gaussian distribution; in the case proposed here, the functions are the mode shapes of the structure, interpolated from n discrete observations. Hence, defining a single value of PDF for each tested function as $\mathbf{z} = f_n(\mathbf{f}^*)$ allows a better distinction between the more extreme *normal* function (undamaged mode shapes) from the *abnormal* ones (damaged mode shapes) resulting in a reduction of wrong identifications.

The feasible extreme distributions for a sufficiently wide number of samples (tending to infinity), taken from an arbitrary parent distribution, are limited to three types: Gumbel, Frechèt and Weibull [1]. Alternatively, all the three mentioned cases can be collectively expressed using the generalised expression of the GEV, where the correspondence of their parameters is reported in equations (9).

A first posterior probability is calculated from equation (5) conditioning the validation data set VA on the training data set TR, in order to make statistically relevant the further results coming form this data combination and independent from the test data TS. The Gaussian probability z is taken in its logarithmic form ($lz = log(z)$) and processed through a genetic algorithm of the type *Differential Evolution* (DE) [18] to estimate the related CDF of the minima extreme distributions, since one could expect few occurrences of the anomalous values with respect to the normal ones. The best parameters are selected by minimising a cost function, which is the normalised mean-squared error (NMSE) in a fit of a parametric model to a given cumulative distribution function (CDF). Thanks to its heuristic approach of trying different random possible solutions and several restarts of the whole process, DE algorithms enhance robustness in the estimation while avoiding local minima, without requiring any *a-priori* knowledge on the defined range of parameters. In this study, the self-adaptive (SADE) modified version of the genetic differential algorithm proposed by [10] has been applied to fit the GEV cumulative function, with two relevant advantages: the adaptive feature of the optimisation does not require the specification of the initial parameters, such as the scaling factor or the crossover ratio, while the choice to use the generalised distribution bypasses the issue of the estimation of the domain of attraction. Regarding the GEV minima distribution (L), this has equation,

$$L(lz, \mu, \sigma, \gamma) = 1 - exp\left\{ -\left[1 + \gamma\left(\frac{\mu - lz}{\sigma}\right)\right]^{-\frac{1}{\gamma}} \right\} \tag{8}$$

where μ, σ, and γ denote, respectively location, scale, and shape of the GEV distribution. A direct correlation with the parameters of the 3 feasible limit minima distributions is reported in [9],

$$
\begin{aligned}
GUMBEL: \quad & if \quad \gamma \to 0, \quad L_G(lz, \lambda, \delta, \beta) \\
& \mu = \lambda, \quad \sigma = \delta \\
WEIBULL: \quad & if \quad \gamma < 0, \quad L_W(lz, \lambda, \delta, \beta) \\
& \mu = \lambda + \delta, \quad \sigma = \frac{\delta}{\beta}, \quad \gamma = -\frac{1}{\beta} \\
FRECHET: \quad & if \quad \gamma > 0, \quad L_F(lz, \lambda, \delta, \beta) \\
& \mu = \lambda - \delta, \quad \sigma = \frac{\delta}{\beta}, \quad \gamma = \frac{1}{\beta}
\end{aligned}
\tag{9}
$$

To individuate the anomalous mode shapes that could indicate the presence of a damage in the beam structure, it is necessary to discern them from the mode shapes that fall in a normal range. After reconstructing the CDF it is possible to define a threshold lz_{lim} in correspondence of a given quantile α, in this case set to the lower 1%. For the test data set TS, a new logarithmic posterior probability lz_{test} is now calculated from equation (5) for each tested mode shapes. When a logarithmic probability falls in a quantile α' beyond the defined limit α, the value is recorded as an outlier and the entire mode shape identified as taken from a damaged structure. Since the TS data set contains undamaged and damaged mode shapes, when an undamaged mode shape is detected as damaged it is recorded as "false positive" and, on the other way, when a damaged mode is not recognised, a "false negative" misclassification occurs. The overall rate of success in correct classification is registered since it is of primary interest to determine the effectiveness of the used damage features, in this particular case the transverse displacement of the beam. Figure 1 shows an overview of the whole algorithm.
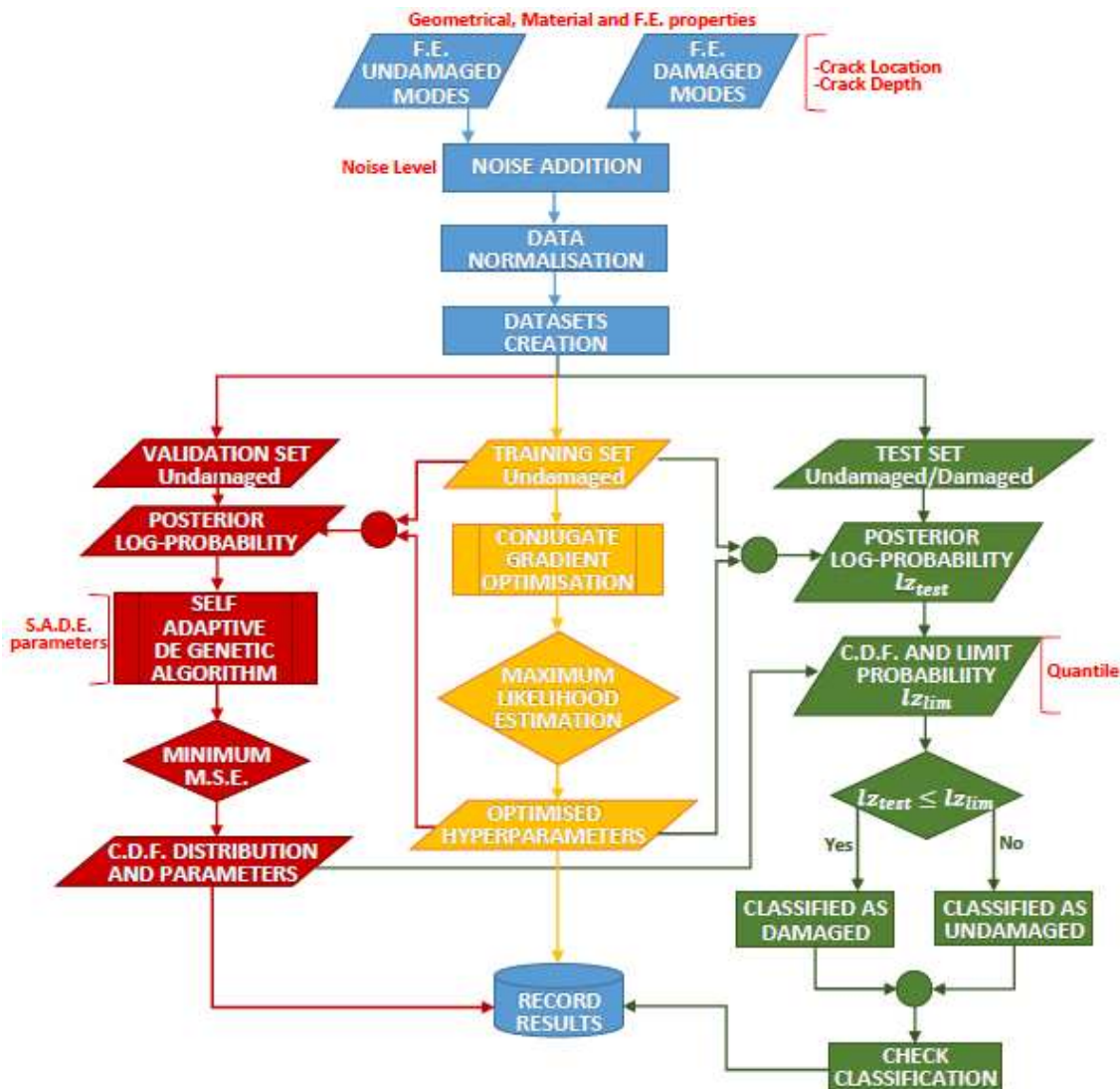
Figure 1: Flow chart showing with different colours data creation, training, validation and detection steps.

3. Results

As introduced previously, different boundary conditions were simulated; due to space limitations, only the results of the cantilever beam structure are here reported as reference. The results are displayed as comprehensive graphics where the x-axis and y-axis represent the crack location and the crack depth, respectively, while the cell colour in grey scale is the percentage of successful identifications of damaged modes. Figures from 2 to 4 below on the left side show resulting damage assessment using an additive noise with SNR = 80 dB for the first 3 modes. Instead, Figures from 2 to 4 on the right side show the theoretical (noise-free) inverse Euclidean distance in transverse displacements between undamaged and damaged modes, normalised to 1, in order to check the correctness of the obtained results. Initially, the different simulations were elaborated taking into account all the FE nodes as sensor locations in order to assess the correctness of the method. Successively, the number of input was reduced to verify the performance of the algorithm when dealing with fewer measurement points. In this case, among the 51 nodes, only 11 of them were considered as equally spaced sensor locations and used to train and validate the process (30 and 200 data copies, respectively). As one would expect, peaks of error occur near the free end, since the
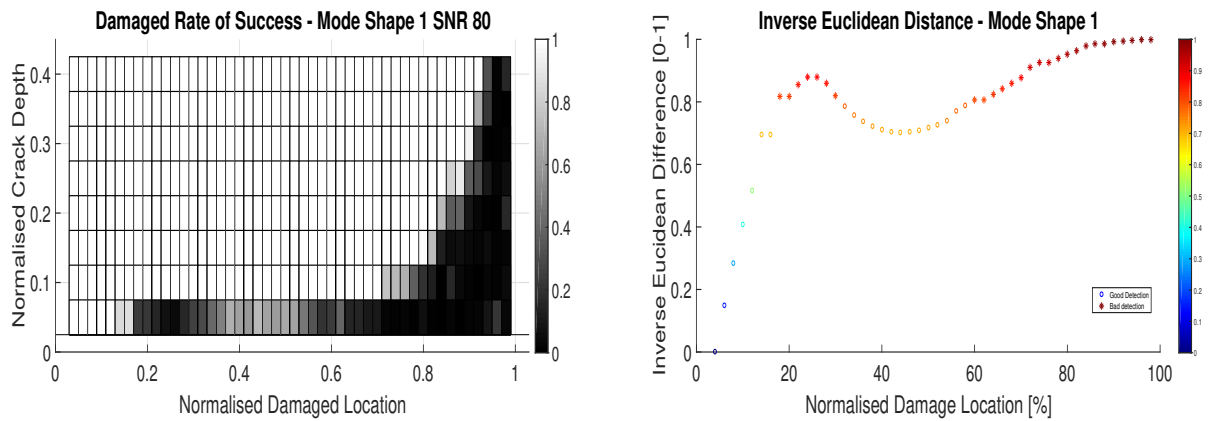
Figure 2: Results Cantilever Beam (CB) - Mode Shape (MS) 1, considering 11 sensors and SNR=80 dB (left) and its inverse Euclidean distance (right).
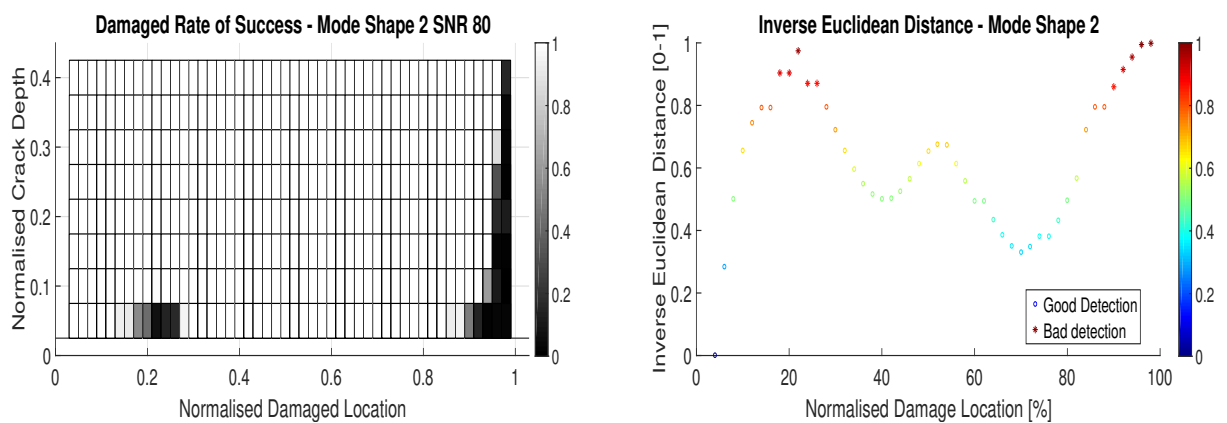

Figure 3: Results CB - MS 2 considering 11 sensors and SNR=80 dB (left) and its inverse Euclidean distance (right).
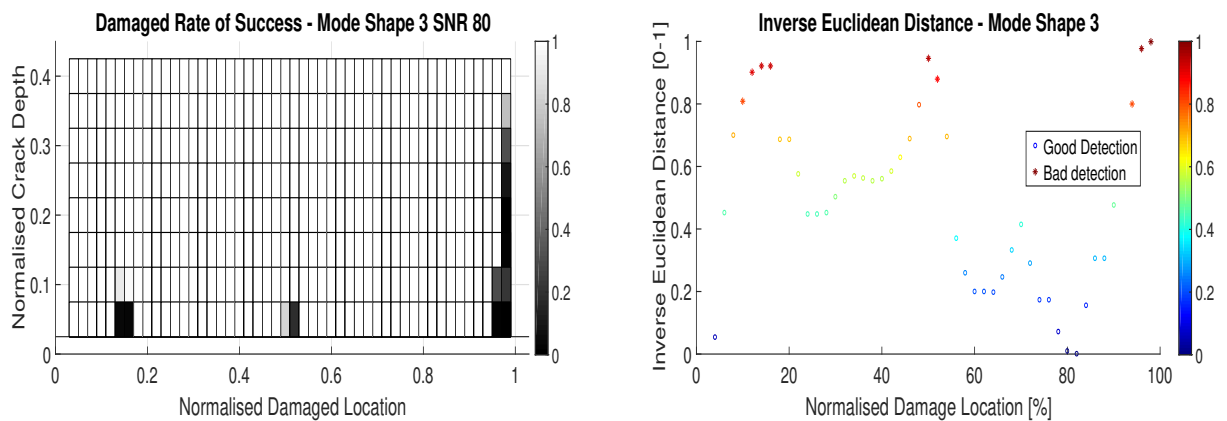

Figure 4: Results CB - MS 3 considering 11 sensors and SNR=80 dB (left) and its inverse Euclidean distance (right).
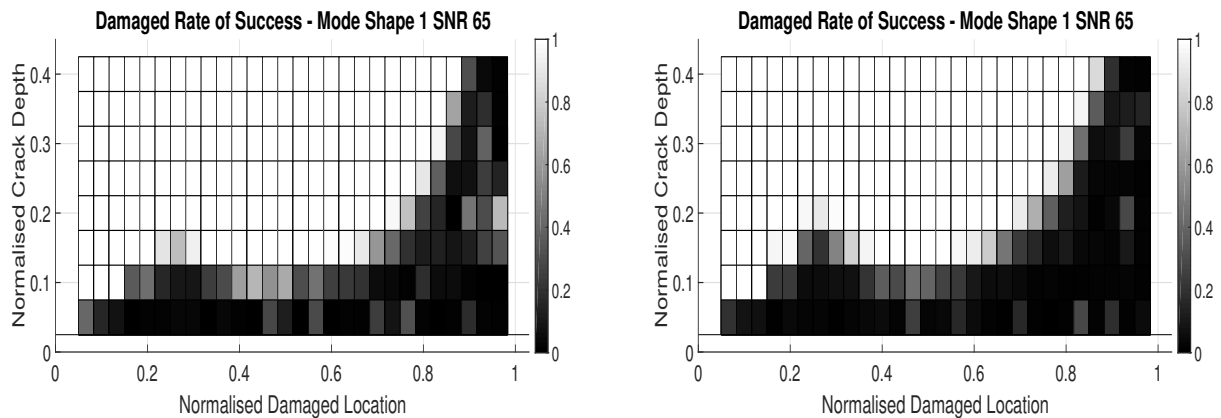
Figure 5: Results CB - MS 1 SNR=65 dB, considering 30 sensors (left) and 11 sensors (right).
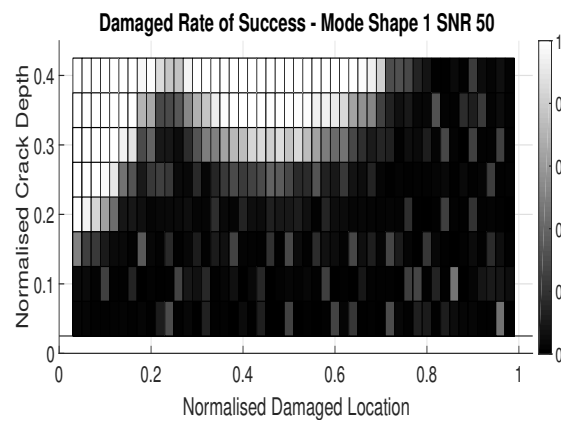

Figure 6: Results CB - MS 1 considering 11 sensors and SNR=50 dB.

crack influence decreases moving away from the clamped end, and in correspondence of the null-displacement points (i.e. modal nodes). Less intuitive is the peak of error located around the 20% of beam length for the particular boundary conditions of the cantilever beam: when a crack occurs in this location, the Euclidean distance between damaged and undamaged mode shapes is very small thus a correct detection result is difficult. The correspondence between peaks of the error in detection and peaks in the curve representing the inverse Euclidean distance is evident from the Figures 2 to 4. A comparison between results obtained with a different number of used sensors is shown for the first mode Figure 5: the left figure is obtained considering a noise level of 65 dB and it is evaluated with a coarser discretisation of 30 elements (instead of 50) and taking into account all of them as sensor locations. The right figure, instead, considers 11 sensors among the 31 nodes. As can be noted, the overall performance of the algorithm remains almost unvaried, while the computational time is definitely lower. Finally, Figure 6 shows the obtained results for a noise level of SNR = 50 dB, that can be assumed as limit for damage assessment beyond which the noisy mode shapes are too disturbed to be analysed.

4. Conclusions
The method proposed here shows some advantages, but also some weak points. A first point of interest surely is the capability to work without differentiating transverse displacement data, which always involves a manipulation of values that leads to an amplification of noise. Moreover, from a computational point of view, the algorithm elaborates sufficiently accurate results starting from

a reduced number of input data or without a dense discretisation of the structure, remaining able to collect all the available information in the posterior probability in a few seconds per whole cycle with a non-optimised Matlab ® code. When working with a reduced number of sensors, the "false positive" occurrences decrease significantly because of the diminution of possible false outliers, which is a characteristic of primary importance in the SHM field. In spite of this, the presence of less sensitive points such as the modal nodes or crack position corresponding to a minimum Euclidean distance between damaged and undamaged mode shapes locally reduces the effectiveness of the method. Another issue regards the accuracy of the sensors used: the current available measurement technology, state-of-art devices able to consider the vast number of output channels considered in the case here reported, like laser sensors or high resolution cameras, allow to measure transverse displacements with a noise around 3% of the maximum displacement involved, which corresponds to an additive noise level of SNR = 40 dB.

Acknowledgements

References

[1] Fisher R.A., Tippett L.H.C., (1928) Limiting forms of the frequency distributions of the largest or smallest members of a sample, *Proceedings of the Cambridge Phil. Soc.*, **24**:180-190.

[2] Gumbel E.J. (1958), Statistics of Extremes, *Columbia University Press*, New York.

[3] Castillo E. (1988), Extreme Value Theory in Engineering. *Academic Press Series in Statistical Modelling and Decision Science*, San Diego, CA.

[4] Kotz S., Nadarajah S. (2000), Extreme Value Distributions, Theory and Applications. *Imperial College Press.*

[5] Worden K., Manson G., Fieller N.R.J. (2000), Damage detection using outlier analysis. *Journal Of Sound And Vibration*, **229(3)**:647-667.

[6] Worden K., Manson G., Sohn H., Farrar C. (2005), Extreme value statistics from differential evolution for damage detection, *Proc. 23rd Int. Modal Analysis Conf.*

[7] Roberts S., Novelty detection using extreme value statistics (1998), *IEE Proc.in Vision, Image and Signal Processing* **146**:124-129.

[8] Papatheou E., Dervilis N., Maguire A.E., Campos C., Antoniadou I., Worden K. (2017), Performance monitoring of a wind turbine using extreme function theory, *Renewable Energy*, **113**:1490-1502.

[9] Park H.W., Sohn H., (2006), Parameter estimation of the generalized extreme value distribution for structural health monitoring, *Probabilistic Engineering Mechanics*, **21(4)**:366-376.

[10] Qin A.K., Suganthan P.N. (2005), Self-adaptive differential evolution algorithm for numerical optimization, *Proc. IEEE Congr. Evol. Comput.*, **2**:1785–1791.

[11] Clifton D.A., Clifton L., Hugueny S., Wong D., Tarassenko L. (2013), An extreme function theory for novelty detection. *IEEE Journal of Selected Topics in Signal Processing*, **7(1)**:28-37.

[12] Rasmussen C.E., Williams C. (2006) Gaussian Processes for Machine Learning, *The MIT Press*, Cambridge, MA, USA **38**:715-719.

[13] Worden K., Farrar C.R. (2013), Structural Health Monitoring: a Machine Learning Perspective, *Wiley and Sons.*

[14] Corrado N., Surace C., Montanari L., Spagnoli A. (2015), Comparing three derivative discontinuities detection methods for the localisation of cracks in beam-like structures. *Proc. Int. Workshop on Structural Health Monitoring*, Palo Alto, CA, USA.

[15] Civera M., Surace C., Worden K. (2017), Detection of cracks in beams using treed Gaussian processes, *Structural Health Monitoring and Damage Detection*, Springer, **7**:85-97.

[16] Corrado N., Durrande N. Gherlone M, Hensman J., Mattone M., Surace C. (2017), Single and multiple crack localization in beam-like structures using a Gaussian process regression approach, *Journal of Vibration and Control*, Sage, **1**:16.

[17] Qian G.L., Gu S.N., Jiang J.S. (1990), The dynamic behaviour and crack detection of a beam with crack, *Journal of Vibration and Control*, **138(2)**:233-243.

[18] Storn R., Price R. (1997), Differential evolution a simple and efficient heuristic for global optimisation over continuous spaces, *Journal of Global Optimization*, **11**:341-359.