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A principled multiresolution approach for signal decomposition

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Abstract. One of the common issues with the Structural Health Monitoring (SHM) of civil infrastructure, is that signals measured from structures are nonstationary. The nonstationarity is the result of uncontrolled variations in the environmental and operational variations (EOVs) in the structure, which result from the fact that monitoring can only be carried out *in situ*. Unfortunately, the EOVs prove to be a problem for damage detection, as they can mask the onset of damage, itself a nonstationary process. The aim of the current paper is to demonstrate a principled, but simple, decomposition method, which can separate signals into stationary and nonstationary components; the stationary component can then be used as a feature for SHM. The method is based on a wavelet/multiresolution analysis of the signal of interest, followed by a sequence of wavelet level-by-level tests for stationary using the ADF (Augmented Dickey-Fuller) test statistic. The decomposition is illustrated using data from the famous SHM campaign on the Z24 Bridge. The paper should be of interest, not just to engineers, but to econometricians, or anyone concerned with nonstationary signal processing.

1. Introduction

In the context of Structural Health Monitoring (SHM), signals obtained by continuous monitoring exhibit variability, both in the shorter and longer terms, which can be associated with the impact of Environmental and Operational Variations (EOVs). This is often manifested in the form of signal nonstationarity and has been observed to mask the nonstationarity of that signals that can be a signature of damage; something that makes damage detection difficult [1]. Within SHM, it is common practice to employ methods such as Statistical Process Control (SPC) charts [2] for damage detection purposes, which require the signal to be stationary. Thus, it is desirable to quantify and project out any nonstationarity introduced into SHM signals by confounding influences such as EOVs.

An important conclusion from the SHM literature is that the impact of EOVs on SHM signals can occur on widely disparate time scales; their quantification and elimination is not a straightforward matter. For such purposes, refined means originated from the field of signal processing have developed throughout the years and are used within SHM. Of particular interest here is the concept of Multiresolution Analysis (MRA) [3], which has been employed in SHM in order to decompose a given SHM signal into band-limited frequency components and evaluate the damage sensitivity of each one [4]. The latter reference also considered the method of *cointegration* [5, 6], as a means of combining multiple nonstationary signals into a stationary residual which could employed as an SHM feature. An important precursor to cointegration is to

establish which of the signals or interest are actually nonstationary; this is usually accomplished by using a statistical hypothesis test. The most common test for stationarity of this sort is the Augmented Dickey-Fuller test, which is a type of t-test; the ADF t-statistic is computed for a signal and then compared to a critical value.

In [7], the current authors proposed a simple interpretation of the ADF statistic, based on dimensional analysis. A byproduct of the analysis was to show that there existed a critical frequency f_c for given sample parameters, such that any signal components containing only frequencies below f_c would be judged nonstationary by the ADF test. This result is used here to motivate the definition of a simple, but principled, decomposition method for signals, which can resolve them into their stationary and nonstationary components.

The layout of this paper is as follows; firstly, a brief description of the analysis methods including the MRA, the ADF test and autocorrelation functions is given. The second section provides an illustration of the method on the Z24 bridge data. Finally, brief conclusions are presented.

2. A Signal Decomposition Method based on Wavelet Analysis and the ADF Test

Based on the analysis presented in [7], it can be assumed that one can decompose a signal into its stationary and nonstationary components by separating out the components with frequency content above and below a critical frequency f_c, established using the ADF test criterion. However, in the previous analysis the existence of the critical frequency was only demonstrated for the case of a harmonic signal corrupted by noise; in the general case, the critical frequency would require a more complicated analysis. It will be shown in this section that the decomposition idea can be implemented in the context of wavelet analysis, without an *a priori* calculation of f_c. This will prove important in the context of SHM.

The issue in SHM, is that there will generally be nonstationary (periodic) components in time series at more than one frequency. If one considers the example the Z24 Bridge, Figure 1 shows the fluctuations in natural frequency associated with seasonal variations in temperature over a period of one year. However, other environmental and operational variations (EOVs) generate nonstationarity at other time scales e.g. day-night variations in temperature and traffic loading will create nonstationarity with a daily period; patterns of traffic loading will induce weekly variations as a result of the different loads during the working week. SHM signals will thus require the more general decomposition method alluded to earlier. As mentioned in the introduction, it is necessary to remove all the confounding influences caused by benign EOVs, so that any potential signatures of damage can be detected.

Previous work on confounding influences and cointegration has exploited multiresolution analysis (MRA) and wavelet analysis [3], as a means of decomposing time series into frequency bands so that the EOVs at different timescales were exposed [4]. In particular, a scale-by-scale analysis performed in [4] considered the effects of using cointegration at different time scales. It was shown that the most nonstationary time scales, associated with lower-frequency signal components, manifested greater damage sensitivity than the higher-frequency ones, due to the fact that the onset of damage leads to a stronger reassertion of nonstationarity than in the original signal and correspondingly to a greater effect on the cointegration residual.

The *level decomposition* used in [4], will be used here to motivate the required decomposition into stationary and nonstationary components. In fact, by making use of autocorrelation functions, it will prove quite simple to decompose signals – in a principled manner – into four main components: (*i*) a mean, (*ii*) a stationary part, (*iii*) a non-stationary part and (*iv*) a noise component. In essence, the main idea will be to obtain the MRA signal levels and then assess them using the ADF test to identify those that can be judged as nonstationary and those as stationary. In this way, a *critical level* can be found and used to decompose the original signal into a nonstationary and stationary part. This critical level (denoted L^*) is based on the
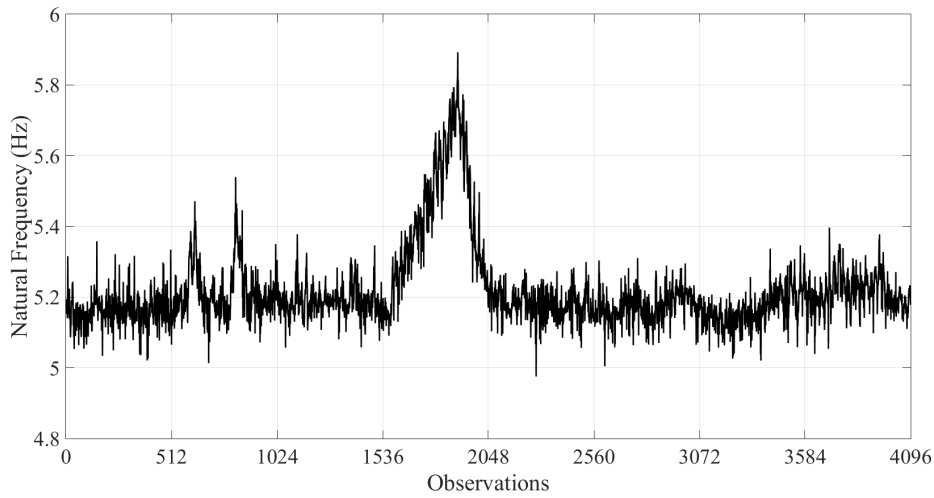
Figure 1. Second natural frequency of Z24 Bridge over a single year of monitoring.

ADF statistic associated with the 99.73% confidence interval, which corresponds to a critical t-statistic of -3.045. The second step in the method proposed is use *autocorrelation functions* (ACFs) in order to test the MRA levels and identify those that are *delta-correlated*, and can therefore be considered to be noise. In order to explain the decomposition, it will be necessary to summarise a little wavelet analysis.

2.1. The Orthogonal Wavelet Transform and MRA

As is now well known, the wavelet transform is a linear transform of the function $x(t)$,

$$x_k^m = \int_{-\infty}^{\infty} x(t)\psi_{m,k}(t)dt \qquad (1)$$

into a translation (k) and scale (m) domain [3]. Unlike the Fourier transform, the expansion basis is in terms of functions localised in time, and this allows the wavelet transform to represent nonstationary signals, among its other benefits. If the set of scale and translation parameters are reduced to a discrete set, one obtains the *discrete wavelet transform*. A common choice for the discrete parameters is the dyadic framework in which, $a_j = 2^j$ and $b_{j,k} = k/2^j$. In this framework, the expansion basis functions are defined by,

$$\psi_{m,k}(t) = 2^{m/2}\psi(2^m t - k), \qquad (m, k \in Z) \qquad (2)$$

where ϕ is termed a *mother wavelet*. The mother wavelet must satisfy certain properties e.g. it must be localised in the variable t, and this can be ensured by using functions ψ with compact support for example. The conditions on the mother wavelet allow many families of wavelet functions, but the most important families are those which have additional orthogonality properties,

$$< \psi_{m,k}, \psi_{n,l} >= \delta_{mn}\delta_{kl} \qquad (3)$$

where $<,>$ is the standard inner product,

$$< h, g >= \int_{-\infty}^{\infty} h^*(t)g(t)dt \qquad (4)$$

where the $*$ denotes complex conjugation. The term δ_{mn} represents the *Kronecker Delta function*, which is equal to unity when $m = n$, while zero for $m \neq n$.

With the extra condition on the wavelet functions, the transform in equation (1) becomes the *orthogonal wavelet transform* (OWT). The great advantage of the OWT is that it allows the definition of extremely efficient algorithms for computation, for example Mallat's pyramidal algorithm, which can calculate the expansion coefficients in $O(N)$ time, where N is the number of sample points in the time series of interest. There are various families of orthogonal wavelets that can be used; however, by far the most commonly used are the *Daubechies* families [3]. There do not exist closed forms for the Daubechies mother wavelets; instead, they are computed in terms of scaling functions ϕ defined by recursion relations [8],

$$\phi(t) = \sqrt{2} \sum_{k=0}^{2r-1} c_k \phi(2t - k) \tag{5}$$

where different sets of coefficients c_k yield different families. The order of the family is controlled by the integer r; mother wavelets of higher order are smoother than those of lower order (i.e. more differentiable), but the low-order families have more compact support. The lowest-order mother wavelet in the Daubechies family is the Haar wavelet which essentially yields a square-wave expansion basis.

Once the wavelet coefficients have been computed, one can construct a decomposition in terms of *wavelet levels*,

$$x_m(t) = \sum_k x_k^m \psi_{m,k}(t) \tag{6}$$

where the sum is over all translation parameters at a common scale (which is roughly 1/frequency), and thus provides a decomposition in terms of frequency band-limited time series. The different levels potentially capture different low-frequency components corresponding to EOVs, so the level decomposition is used here as the basis of a new decomposition into stationary and nonstationary parts.

The idea behind the decomposition is quite simple. Having established the level decomposition, one simply applies the ADF test at each level in order to establish if a given level time series is stationary or not. It is clear that one level will capture the critical frequency f_c, so that level – denoted by the index L^* – has a frequency band covering the transition from stationary to nonstationary. Summing the levels up to, and including level L^* will determine the nonstationary component of the original signal (as dictated by the ADF test) and the sum over the remaining levels will yield the stationary component. By applying the ADF test level-wise, one does not need to pre-determine the critical frequency f_c.

As before, for the assessments in this paper, the 99.73% confidence interval on the ADF test is used, which corresponds to a t-critical value of -3.045.

The decomposition proposed here can be made a little more informative, with a little more effort. First of all, the mean of the signal can be extracted before the wavelet decomposition; secondly, one can make a further separation within the stationary component in order to identify a 'noise' component.

2.2. Autocorrelation Functions (ACFs)
According to [9], for a stochastic process X_t for a finite sample, with mean μ and sample variance σ_x^2, which are both time-independent, the autocorrelation function (ACF) ϕ_{xx}, can be defined which determines the degree of dependence between any two samples in the process (signal). For stationary signals, the ACF depends only on the time interval τ between the pair of values considered and not their individual positions in time,

$$\phi_{xx}(\tau) = \frac{E\left[(X_t - \mu)(X_{t+\tau} - \mu)\right]}{\sigma_x^2} \tag{7}$$

where E again denotes the expectation operator.

Now, if one defines 'noise' as a component within the signal with *no* temporal correlations (i.e. one cannot predict any future values from past values), one can test for this property using the ACF. A signal will be termed noise, if it is *delta-correlated* i.e.

$$\phi_{xx}(k) = \delta_{k0} \tag{8}$$

with the index k now representing the sample index in terms of discrete time. With the normalisation given in equation (7), one can compute confidence intervals for a zero result; the intervals corresponding to 99.73% confidence are $\pm 3/\sqrt{N}$.

In order to establish the noise component in the signal, one computes the ACF for any levels in the stationary component (stationarity is required in the definition of the ACF) and moves into the noise component any levels which are delta-correlated.

This completes the description of how the new decomposition is defined. To summarise, the steps are as follows:

(i) Remove the mean from the signal $\{x_i : i = 1, \ldots, N\}$. The mean μ is the first component.

(ii) Compute the wavelet transform, and determine the wavelet levels. (Note that this may require truncation or zero-padding in order to have 2^M points for the OWT, if N does not satisfy this condition already.)

(iii) Apply the ADF test level by level and determine L^*.

(iv) Resum the levels corresponding to the stationary and nonstationary parts.

(v) If desired, one can calculate the ACFs for the stationary levels and separate out the noise component.

The procedure will now be illustrated on a case study.

3. Case Study: The Z24 Bridge

As the motivation for this work came from the SHM context in the first place, the case study will be taken from data acquired in a major SHM campaign based on the monitoring of the Z24 Bridge. The second natural frequency from the bridge has already been encountered in Figure 1. The Z24 Bridge (overpass) was constructed in Switzerland in the beginning of the 1960s in order to link Koppigen and Utzenstorf by passing over the main national Highway A1 (Lausanne-Zurich). The bridge was superceded by a new construction in the late 1990s, and authorisation was given for an SHM campaign before its demolition in the autumn of 1998. The exercise, which included the introduction of realistic but controlled damage, is described in [10]. Various sensor signals were acquired, these included measurements of the ambient environmental variables like air and deck temperatures, humidity etc. The main features for SHM were modal quantities extracted automatically by stochastic subspace identification (SSI) at regular intervals [11]. The signal examined here will be the second natural frequency series.

From a first look at the signal (Figure 1) one can discern two major components. There is a high-frequency component, which is essentially the uncertainty associated with the SSI estimates; one would expect this component to be stationary. There is also a clear low-frequency trend in the time series that reflects the EOVs experienced by the bridge over the year. The most dramatic features in the signal are the peaks, which are associated with stiffening of the bridge asphalt layer during sub-zero temperature conditions in the winter months [11].

Moving now to the decomposition, Figure 2 shows the levels from the wavelet/MRA analysis. The Daubechies 4th order wavelet was used as a compromise between smoothness and support.

In total, 12 levels were extracted. Note that levels zero and one both seem to be copies of the mother wavelet; in fact the sum of the two levels carries the mean of the original signal, according to the implementation of the algorithm, which was taken from the book *Numerical Recipes* [12]. Furthermore, the algorithm is applied in such a way that the levels are numbered to increase with increasing frequencies. It is immediately obvious from the figure, that the trend behaviour – from the EOVs – is captured in the lower levels.
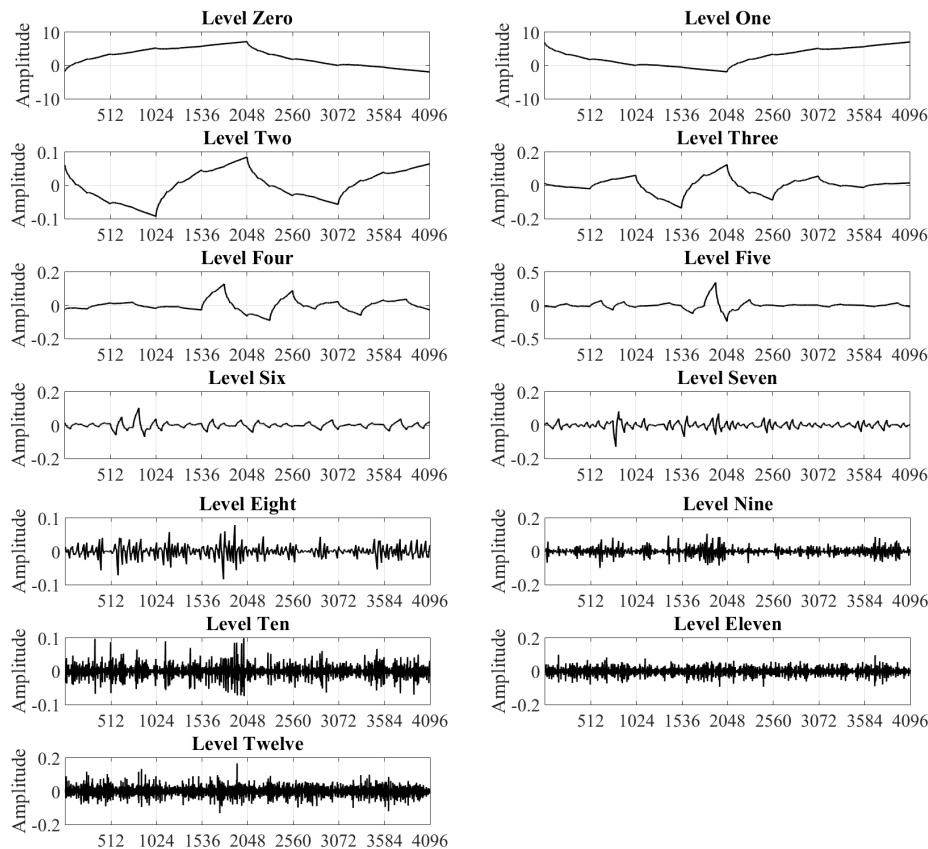

Figure 2. Wavelet decomposition/MRA levels for the Z24 second natural frequency.

The ADF test was then applied to each level in turn, and the statistic values obtained are summarised in Table 1. At the 99.73% confidence level applied here, the critical t-statistic is -3.045; this means that the critical level here is $L^* = 5$, and the sum of levels 0 to 5 is determined as the nonstationary component of the signal.

The final stage of the decomposition is to compute the ACFs for the stationary levels, so that one can separate out the delta-correlated levels as a noise component. The ACFs are given in Figure 3 (the ACFs for the nonstationary levels are also shown here for comparison); the red dashed lines in the figures are the 99.73% confidence levels. In this case, levels 10-12 are considered sufficiently delta-correlated and are separated out as a noise component.

The final decomposition is shown in Figure 4, where there is a clear separation into the nonstationary and stationary components. The further separation into a stationary and a noise component also appears meaningful, with the noise component resembling more the Gaussian white noise sequence one might associate with noise; however, one should recognise that the component is the result of essentially high-pass filtering the original signal, and therefore cannot

Table 1. T-statistics corresponding to different wavelet/MRA levels.

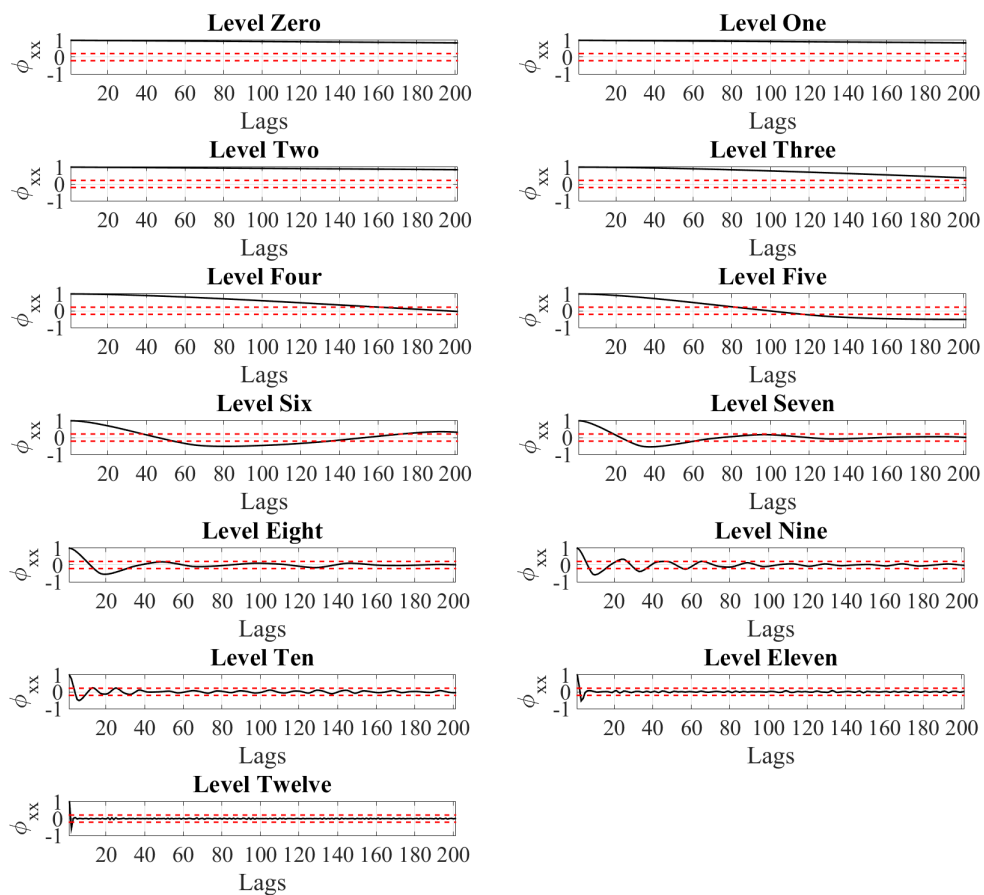| Level	t_{ADF}
0	-0.025
1	0.705
2	-0.1929
3	-1.103
4	-1.501
5	-2.724
6	-5.021
7	-9.0322
8	-15.177
9	-25.852
10	-42.085
11	-87.96
12	-93.58

Figure 3. Autocorrelation functions corresponding to each wavelet/MRA level. The red dashed lines indicate the 99.73% confidence interval.
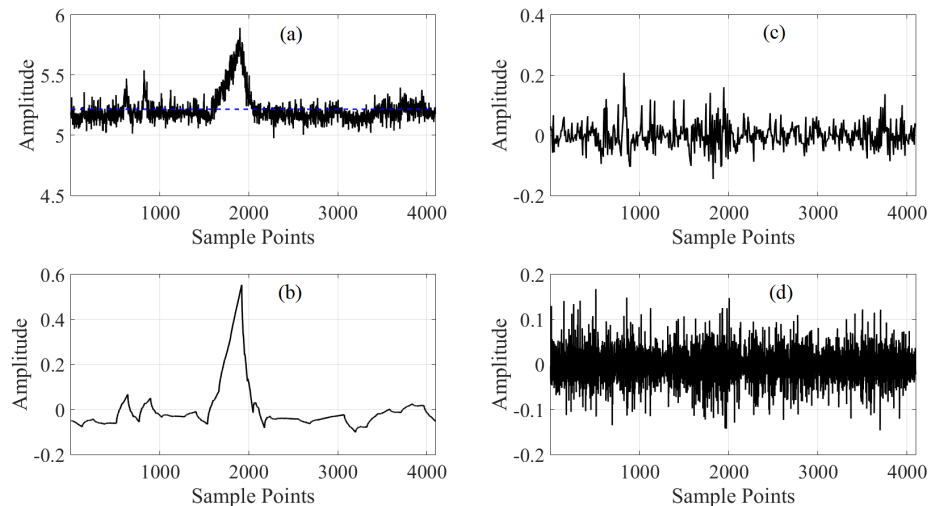
be truly white.

Figure 4. Decomposition of the Z24 second natural frequency: (a) original signal; (b) nonstationary component; (c) stationary component; (d) noise. The mean of the original signal is indicated using a blue dashed line in (a).

4. Conclusions

In this paper, a new decomposition method is presented, able to separate stationary and nonstationary components of signals. The basic idea is that a wavelet/MultiResolution Analysis is used to separate the time series of interest into a set of frequency band-limited wavelet levels. The levels are then individually tested for stationary by using the ADF statistic; the stationary levels are re-summed to give the stationary component of the original series and likewise the nonstationary levels. It is shown that one can also separate a 'noise' component from the stationary component by using autocorrelation functions (ACF). The 'noise' component is not a Gaussian white noise, but is distinguished by the fact that it is delta correlated, so that future values of the component are independent of past values. The decomposition method is demonstrated successfully on a signals from a Structural Health Monitoring (SHM) campaign on a real structure – the Z24 bridge.

Despite the SHM context selected here, the results of this paper are quite general and should be of interest, not only to engineers, but also to econometricians, or anyone seriously concerned with time series analysis where stationarity is an issue.

Acknowledgements

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