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# Implication of heterogeneities on core porosity measurements



**Major inclusions**

$$H = (<\phi_m> - \phi_o)/\phi_o = F\,(R\text{-}1)$$

measured, $m$, $V_{Bm}$

outer, $o$, $V_{Bo}$

inner, $i$, $V_{Bi}$

Outer Porosity, $\phi_o = 0.20$
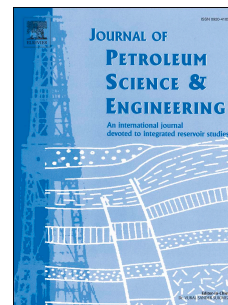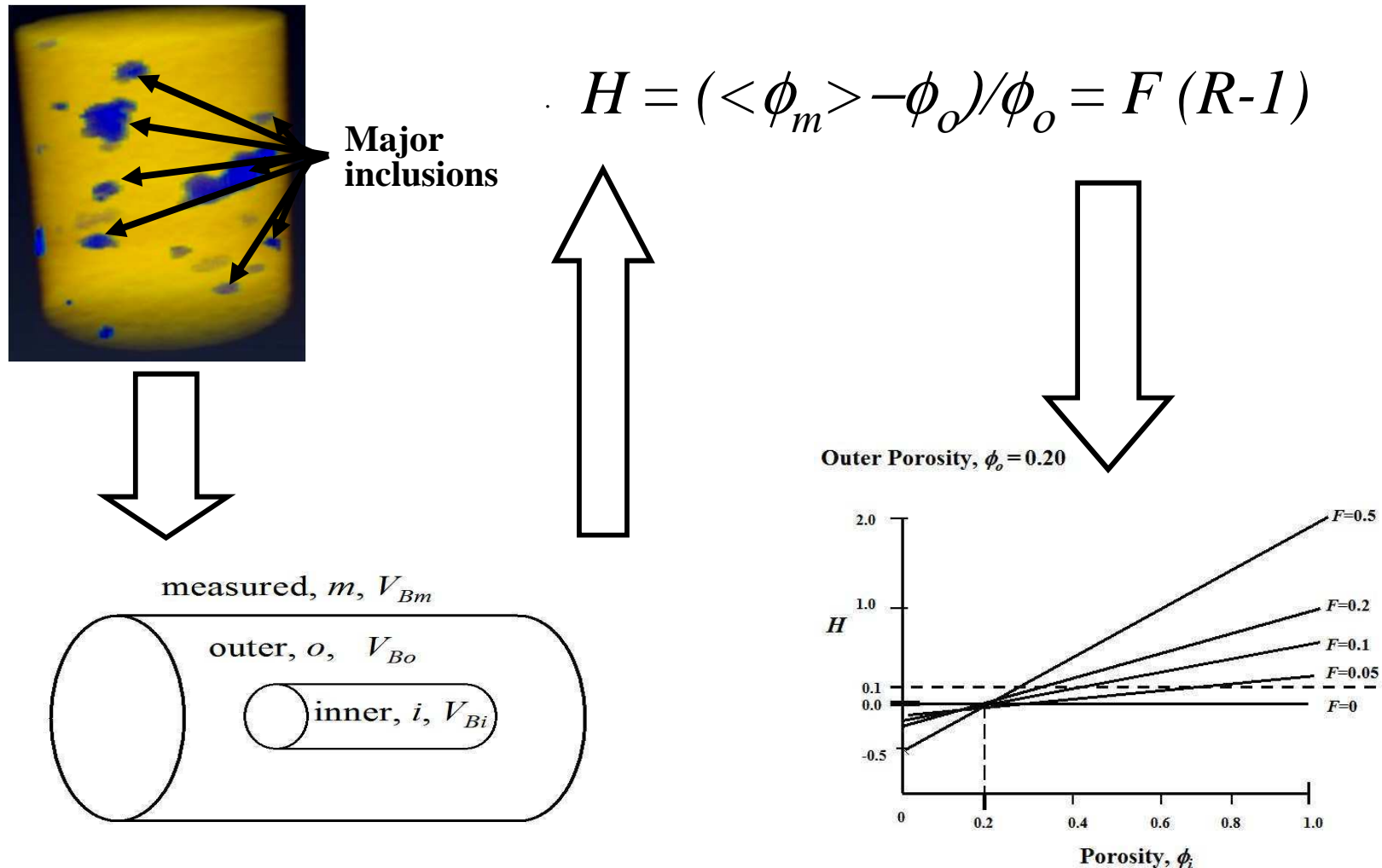


*H – Heterogeneity Index –when H>±0.1 significant effect*

$\phi$ - porosity; $i$ – inner; $o$ – outer; $m$ – measured; $<\phi_m>$ volume averaged porosity;

$V_B$ – bulk volume; $F = V_{Bi}/V_{Bm}$ ; $R = \phi_i/\phi_o$

**Implication of heterogeneities on core porosity measurements**

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**Key words**

Porosity, heterogeneity, core plugs, inclusions, vugs, clay, CT scanning.

**Abstract**

Heterogeneities within core samples affect the accuracy of the laboratory measured core plug interconnected porosity. The laboratory measures the volume averaged porosity of the interconnected pores, $<\phi_m>$. For homogeneous cores this is the correct porosity ± any experimental error. However for heterogeneous cores when any embedded material (unknown or ignored) has a differing porosity, $\phi_i$, to that of the containing outer shell, $\phi_o$, there will be increased uncertainty. We show that the difference between the measured volume averaged porosity, $<\phi_m>$, and the porosity of the outer shell of the core, $\phi_o$  can be quantified by our *Heterogeneity Factor*, *H*. *H* is defined as  $(<\phi_m> - \phi_o)/\phi_o$ and given by $H = F(R-1)$, where *F* is the ratio of the bulk volume of the embedded material to the total core bulk volume (measured), $V_{Bi}/V_{Bm}$, and *R* is the ratio of the embedded and the host outer porosity, $\phi_i/\phi_o$.

When *H* is zero there is no error in the porosity measurement due to heterogeneity, but when $H \neq 0$ then the differences can be significant and increases the experimental error bound. We present graphs for relevant industry scenarios to demonstrate the effect of any inclusions in the measured porosity. We find that when *F* is ~ 0.1 i.e., inner included porosity is 10% of bulk volume, the relative error between $<\phi_m>$ and $\phi_o$ can reach ~30% and even larger differences when *F*>0.1. We then give a real example from a faulted vuggy outcrop carbonate which demonstrates extra porosity uncertainties, even for a very small vug. Finally we discuss the possible effect of embedded clay intrusions emitting adsorbed gas on grain/pore volume determinations of porosity using gas expansion, a common laboratory method of porosity determination.

Appreciation of core heterogeneity on the precision of the laboratory porosity measurements is essential to improve the confidence in the error bounds for the quality control of laboratory core porosity measurements, and of the porosity distribution frequency for inputs to statistical methods such as Monte Carlo analysis for oil-in-place estimations, *STOIIP*.

**1.0 Introduction**

*1.1 STOIIP*

During the oil winning process, economic decisions are critically based on oil/gas in place calculations. Stock-Tank-Oil-Initially-In-Place, *STOIIP* (Dake, 1994; Dawe, 2000), is

$$STOIIP = \frac{V\phi(1-S_{wi})}{B_{oi}}$$

(1)

where $V$ is the bulk volume of rock containing reservoir oil, $S_{wi}$ is the initial average water saturation within the reservoir, $B_{oi}$ is the initial oil formation volume factor (volume of oil in the reservoir to that at surface) and $\phi$, the averaged reservoir porosity, defined in section 1.2.

Thus a field of say 100 million barrels (~16 million m$^3$) of recoverable crude oil, might have a value of ~\$5billion (taking 1 bbl oil at \$50). If the reported porosity is 0.20, an error of $\pm 0.01$ (i.e. 0.19 to 0.21) would give a ~$\pm 5\%$ uncertainty in the oil estimation, which could create an uncertainty of perhaps $\pm$\$250 million in budget estimations, which represents a considerable amount of money. Quantification of this uncertainty is gained by statistical methods. Monte Carlo analysis is the most common method and uses some form of frequency distribution for the variables (Dawe, 2000). All errors and their bounds in the porosity determination must be taken into consideration in the determination of this frequency distribution.

This paper explores the quantification of the possible discrepancy that arises due to the presence of secondary 'embedded materials' in the real core between the laboratory measured interconnected porosity, which is normally carried out assuming that the core is homogeneous, and the porosities of the components in the real heterogeneous material.

*1.2 Porosity definition*

Porosity, $\phi$, is the void fraction of a rock volume containing pores, and in reservoirs these pores contain the oil, gas and water. It is calculated by:

$$\phi = \frac{V_P}{V_B} = \frac{V_B - V_G}{V_B} \tag{2}$$

where $V_P$ is the pore volume, the volume of the 'empty' space (the pores), $V_G$ the grain volume (the volume of the solid grains) and $V_B$ the bulk volume of the rock (pores plus grains) and

$$V_B = V_G + V_P . \tag{2a}$$

Porosity is a fraction ranging from zero to 1.0, but is often discussed in the oil industry in percentages which range from 0 to 100% and are sometimes called 'porosity units', PU. To avoid confusion, in this paper we discuss the porosity as the fraction. For petroleum recovery the pores have to be connected so that fluids can flow through them. This is termed 'effective porosity' and is the porosity considered in this paper.

Porosity can be measured in the laboratory and by logs in the field as discussed by the specialist texts, (McPhee et al., 2015; Schön, 2015; Tiab and Donaldson, 2015) and for many reservoirs is the primary source of physical property data. Industrial practice requires logs for all reservoirs e.g. electrical, sonic, neutron and often more sophisticated methods such as NMR, (Hook, 2003; Kennedy, 2015). However, some of these logs give total porosity, all the pores including pores which are isolated within the core body. Thus the data must be carefully reconciled to the required interconnected porosity (Boyle et al., 2000; McPhee et al., 2015). Core disintegration determines total porosity. As we only examine effective porosity, total porosity is not considered further.

*1.3 Rock density*

Knowledge of rock densities is sometimes needed in porosity determination to help identify heterogeneity, as will be mentioned in section 2.4. The grains have a mass $M$, so grain density is $\rho_G = \frac{M}{V_G}$ and the bulk density $\rho_B = \frac{M}{V_B}$ , so from Eq. (2),

$$\phi = \frac{V_B - V_G}{V_B} = 1 - \frac{\rho_B}{\rho_G} . \tag{3}$$

*1.4 Measurement of porosity*

A reservoir needs to have its porosity determined so it is sampled by taking cores (Baker et al., 2015; Dake, 1994). These cylindrical cores, perhaps 15cm in diameter and maybe 10's of metres in

length, are slabbed (cut in half) and then subsampled to produce core plugs usually by careful selection of 'suitable' positions along the core material, and which, after cleaning and drying, have their porosity measured in the laboratory. Core plugs are usually cylindrical but cuboids (rectangular parallelepipeds) are used when more convenient. It must always be remembered that the core plugs will be much less than a billionth and more likely a trillionth of the reservoir volume (Baker et al., 2015). Measurements can also be made on off-cuts from the core plugs and cuttings from drilling, so long as they can be accurately reconciled with the core depths and are representative of the reservoir. Critically, the core samples are usually assumed to be of a uniform and homogeneous porosity and representative of a larger volume. If the core plugs are not fully representative of the reservoir, or are heterogeneous but have been assumed to be homogeneous, then even if the laboratory measurements are perfect, there will be increased uncertainties in the output estimation of reservoir petrophysical properties, and consequently uncertainty in calculations using them.

The experimental determination of interconnected porosity is often conducted in dedicated core laboratories using well-defined best practice procedures. Methods are based on Boyle's Law and Archimedes principle and obtain pore or grain volume and the bulk volume by some immersion or direct measurement process, (Amyx et al.,1960; Boyle et al.,2000; Dawe, 1992; Hook, 2003; Kennedy, 2015; McPhee et al., 2015; Schön, 2015; Tiab and Donaldson, 2015). Such procedures determine interconnected porosity.

### 1.5 Reservoir porosity and heterogeneity difficulties

For reservoir assessments, the individual measured connected porosities along a wellbore are averaged using some procedural workflow, and these values used to upscale to reservoir scale models for *STOIIP* calculations, and as a calibrating standard for other measuring techniques. Of course all values have to be reconciled (e.g. McPhee et al., 2015) with the reservoir, depth, other well bores, geology etc. However, if there is uncertainty in the original measured laboratory porosity due to non-accounting of core heterogeneity, there will be additional uncertainty in subsequent reservoir evaluation.

### 1.6 Effect of inclusions

If a core sample has (known or unknown) inclusions embedded within it and which have a different porosity to the host, then, no matter how well the measurements have been made, because the core plug has been assumed homogeneous, there may be a difference in porosity values between the volume averaged measured value and the actual value of the major portion of the core. The significance of the effects of any embedded porosity will depend on the volume of the inner inclusions and their property variation compared to the outer medium. But by how much?

Such inclusions could range from a pebble of zero porosity (e.g. a conglomerate rock) to a cavity of porosity 1.0 (e.g. a vuggy limestone): these are clearly the extreme cases. Unless one can 'see' the heterogeneity it is often difficult to know about these inclusions and so be able to account for them. The heterogeneity may be visible on the surface of the core plug or slabbed core, or detected using special tests such as CT scanning, as shown in Fig. 1 and discussed in section 3.1, but such equipment may not be available during routine core analysis. Inclusions may also be inferred through pressure transient tests, where a change in slope of the pressure-time plot is observed (Kennedy, 2015).

The implication of core plug non-uniformity/heterogeneity on the measured connected porosity, and, in particular, the possible magnitude of any (unknown) unintended error due to non-knowledge of inclusions in the porous matrix is now examined.
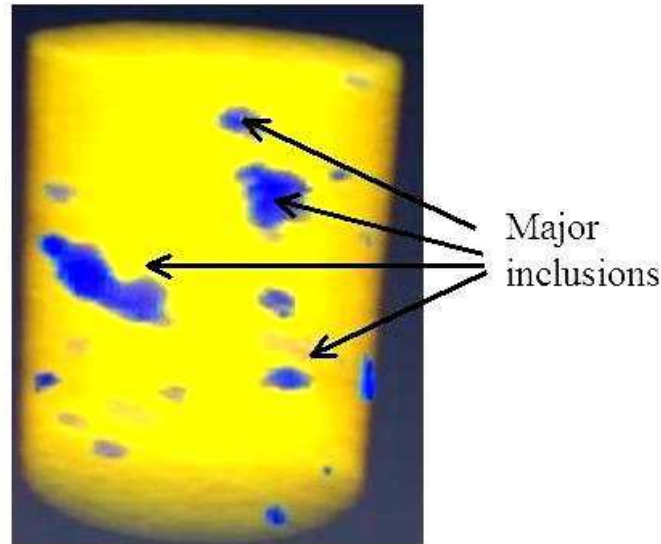
Fig. 1, Example of a CT image of a core plug of rock with inclusions inside. The diameter of the core is 3cm. The CT signals have been analysed using Avizo image analysis programme (FEI, 2007). Clearly there are major inclusions. These may be interconnected or isolated, which must be inferred from other analysis.

## 2. Theory - derivation of an equation comparing measured porosity with the actual values

*2.1 Derivation of a heterogeneity factor for a measured (assumed homogeneous) core but the real (heterogeneous) core has components of differing porosity*

Let us consider a heterogeneous core having an outer host and inner inclusion, Fig. 2. We denote the core outer part *'o'*, the inner part as *'i'*, (multiple inclusions are discussed in section 2.3), and the laboratory measured values as '*m*'.
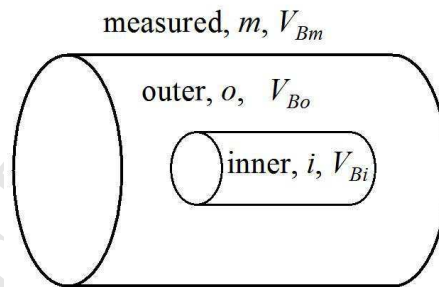


Fig.2. Cartoon of a Laboratory Core with an inclusion: $V_{Bm} = V_{Bi} + V_{Bo}$

For the porosity measurement of this non-homogeneous core sample in the laboratory, let the laboratory measured volume averaged porosity be $<\phi_m>$ (i.e. if it were a homogeneous core), and if we assume, for the moment, $\phi_o$ and $\phi_i$ are known, then using Eq. (2) and the fact that $V_{Bm} = V_{Bi} + V_{Bo}$, Eq (2a),

$$<\phi_m> = V_p/V_{Bm} = (V_{Bi} \phi_i + V_{Bo} \phi_o) / (V_{Bi} + V_{Bo}). \tag{4}$$

If the ratio of inner bulk volume compared to the measured core bulk volume is defined as *F*, then
$$F = V_{Bi}/V_{Bm} \text{ and } (1-F) = V_{Bo}/V_{Bm}. \tag{5}$$
*F* can range from 0, (no inner inclusion) to 1.0, (i.e. only a fine shell of outer media).
Substituting Eq. (5) into Eq. (4)
$$<\phi_m> = \phi_o + F (\phi_i - \phi_o) \tag{6}$$
Eq. (6) shows that the volume averaged measured core porosity that is reported by the laboratory is $<\phi_m>$, but if the core is heterogeneous the porosity will be complicated because it has an outer

(possibly) observable fabric core porosity, $\phi_o$, and an inner (unknown) core porosity $\phi_i$. When $F > {\sim}0.001 <\phi_m>$ will be within an acceptable error of $\phi_o$ as experimental errors are often quoted as $\pm.01$ ($\pm$5% porosity), but when $F>0.01$ differences in porosity between $<\phi_m>$ and $\phi_o$ could be $>\pm0.01$ which is greater than the measured experimental error. When $F > 0.10$ i.e., 10% of the bulk volume is inner material, the difference in the porosities $(<\phi_m> - \phi_o)$ can be much greater than 0.01, which maybe concerning. Further examination of the structure of the core may then be required.

A quantitative measure of this error/uncertainty in the porosity measurement can be a *Heterogeneity Factor H,* defined as $\{(<\phi_m> - \phi_o)/\phi_o\}$, and if we define the ratio of the inner porosity to the outer porosity as $R = \phi_i/\phi_o$, then from Eq. (6),

$$H = \{(<\phi_m> - \phi_o)/\phi_o\} = F(R - 1). \tag{7}$$

Eq. (7) gives a measure of an additional error, $H$, caused by assuming that a heterogeneous core is homogeneous. This error is there even if all the experimental measurements are correct and error free.

*2.2 Examples of implications of the Heterogeneity Factor on porosity*
It must be emphasised that any additional error for heterogeneous core does not arise from measurements; it arises in the volume averaging procedures and because the core is not, as assumed, homogeneous, but is heterogeneous with the differing porosities, $\phi_i$ and $\phi_o$. The appropriate porosity that needs to be finally reported for *STOIIP* estimations is not $<\phi_m>$ with its laboratory related error bounds, but a value having increased error bounds determined through Eqs. (6 or 7). To evaluate $H$, the measured sample-averaged porosity and the porosity of the heterogeneous materials (particularly the dominant material) must be known, as well as the value of $F$. This is not often the case, although prudent estimates can be made.

The porosity of any inner inclusion, $\phi_i$, can range from 0.0 (pebble) to 1.0 (hole). The outer porosity, $\phi_o$, will practically only range from 0.01 to 0.5, because if $\phi_o$ is less than 0.01 there is little significant porosity and if $>0.5$ the specimen is rather holey. Note that the maximum porosity for a pack of uniform spheres is 0.48 and is independent of their grain size (Amyx et al., 1960; Dawe, 1992; Grattoni and Dawe, 1995).

If $\phi_i = \phi_o$ then $R = 1.0$ so $H = 0$, and the measured porosity, $<\phi_m>$, and the outer porosity, $\phi_o$, will be the same, and there is no heterogeneity error in porosity. Otherwise the value of $H$ depends through Eqs. (6 and 7) on the value of $F$, the volume fraction of inner matrix in the core, and $R$, the ratio between the porosities of the inner inclusions and the outer shell. Clearly, if the sample has a pebble included $(\phi_i = 0)$ then $H < 0$, and the measured porosity $<\phi_m>$ will be lower than $\phi_o$, the outer portion of the heterogeneous rock. If the core has a vug (hole) $(\phi_i =1)$ then $H > 0$, and $<\phi_m>$ will be higher than $\phi_o$.

However, even if the values of $\phi_i$ and $\phi_o$ are unknown, we can gain some appreciation of the magnitude of any error caused by the heterogeneity through thought experiments using Eqs. (6 and 7) and assuming realistic (reservoir) values. Table 1 summarises the heterogeneity error for some typical examples, and Figs. 3 and 4 show values of $H$ computed for functions of $F$ and $\phi_i$ at fixed values of $\phi_o$. In Fig.3, $\phi_o$ is constant at 0.20 (a porosity of 20%) and in Fig. 4 $\phi_o$ is kept constant at porosities of either 0.05 (5%) or 0.50 (50%) the normal practical limits of the porosity for oilfield cores.

Figs. 3 and 4 show that there can be significant differences of porosity between the real heterogeneous core (with inner and outer porosities) and $<\phi_m>$, when $H$ is not close to 0.0. If for example the inner inclusion was a pebble, then $\phi_i= 0$ and $R= 0$. If there is a 10% inclusion of material, then $F = 0.1$ and if $\phi_o = 0.20$, by Eq. (6) $<\phi_m> = 0.18$, a 10% difference, and by Eq. (7) $H$

= -0.10. On the other hand, if the inclusion was a vug (maybe an oolite), $\phi_i = 1.00$ and if $\phi_o = 0.20$, then $R = 5$, and the reported $<\phi_m> = 0.28$, rather than the 0.20 if the core was all of the outer shell, a 40% difference, and $H = 0.4$. These are significant differences (additional error) above the reported measurement experimental error. Such differences, particularly when $F > 0.01$ could create significant additional uncertainty in reservoir economic decisions.

Table 1 shows through Eqs. (6) and (7) some typical values of porosity differences and $H$ due to heterogeneity

| $F$ | $\phi_o$ | $\phi_i$ | $<\phi_m>$ | difference $<\phi_m>-\phi_o$ | Error, %, diff*100/<φm> | $R$ $\phi_i/\phi_o$ | $H$ Eq. (7) |
|---|---|---|---|---|---|---|---|
| 0.100 | 0.200 | 0.000 | 0.180 | -0.020 | -11 | 0.00 | -0.100 |
| 0.100 | 0.200 | 1.000 | 0.280 | 0.080 | 29 | 5.00 | 0.400 |
| 0.010 | 0.200 | 0.000 | 0.198 | -0.002 | -1 | 0.00 | -0.010 |
| 0.010 | 0.200 | 1.000 | 0.208 | 0.008 | 4 | 5.00 | 0.040 |
| 0.001 | 0.200 | 0.000 | 0.200 | 0.000 | 0 | 0.00 | -0.001 |
| 0.001 | 0.200 | 1.000 | 0.201 | 0.001 | 0 | 5.00 | 0.004 |
| 0.050 | #0.158 | 0.000 | 0.150 | -0.008 | -5 | 0.00 | -0.051 |
| 0.050 | #0.105 | 1.000 | 0.150 | 0.045 | 30 | 9.52 | 0.429 |
| 0.022* | 0.100 | 0.000 | 0.098 | -0.002 | 2.0 | 0.00 | -0.022 |
| 0.022* | 0.100 | 1.000 | 0.120 | 0.020 | 20 | 10.00 | 0.198 |
| 0.010** | 0.100 | 0.000 | 0.099 | -0.001 | -1.0 | 0.00 | -0.010 |
| 0.010** | 0.100 | 1.000 | 0.109 | 0.009 | 9.0 | 10.00 | 0.090 |
| 0.500 | 0.200 | 0.000 | 0.100 | -0.100 | -100 | 0.00 | -0.500 |
| 0.500 | 0.200 | 1.000 | 0.400 | 0.200 | 50 | 5.00 | 2.000 |

# obtained by iteration via Eq. (6) from the assumed values of $\phi_I$ and $\phi_o$

*Consider a plug 4.0cm length and 3.8cm with a vug or pebble of 1cc then $F = 0.022$ (section 3.1).

**Consider a plug 4.0cm length and 3.8cm with a vug or pebble of 0.454cc (1%BV) then $F = 0.01$ and $\phi_o = 0.01$ (section 3.1).

Fig. 3 which is a plot of $H$ as a function of $\phi_i$ at a constant $\phi_o$ of 0.20 shows that as $F$ increases (i.e. the volume of the inner inclusions increase) the heterogeneity error will increase until if half the rock volume has empty pore spaces (i.e. $\phi_i = 1.00$ and $F = 0.5$), the error could be as large as a factor of 2.0 (100%)! If the outer matrix has a porosity of $\phi_o=0.20$ and an inner inclusion of $\phi_i$ =0.1, then $R = 0.5$, and if $F = 0.1$, then 90% of the core has a porosity $\phi_o$, and from Eqs. (6) and (7) $<\phi_m> = 0.19$ and $H = -0.05$. If the inclusion had not been detected $<\phi_m>$ would be reported as 0.19 but there will be an unintended additional relative error of 5% in the core porosity. In Fig. 3 at the dotted line of $H = 0.1$, the difference between $<\phi_m>$ and $\phi_o$ will be ~10%. But below this value there may be little significant extra uncertainty in many field cases.

Similar observations can be made in Fig. 4 for outer core porosities as low, 0.05, and very high, 0.50. For cores with a low outer porosity and large inner porosity, $\phi_i \sim 1.00$ ($R = 20$) the differences can be very significant even for volume inclusions of less than 10%. For a high outer porosity e.g. $\phi_o=0.50$, Fig. 4 shows that for $\phi_o = 0.50$ as $F$ increases from zero and $\phi_i$ moves down from 0.50 there can be a small, but still significant, additional difference.

## Outer Porosity, $\phi_o = 0.20$
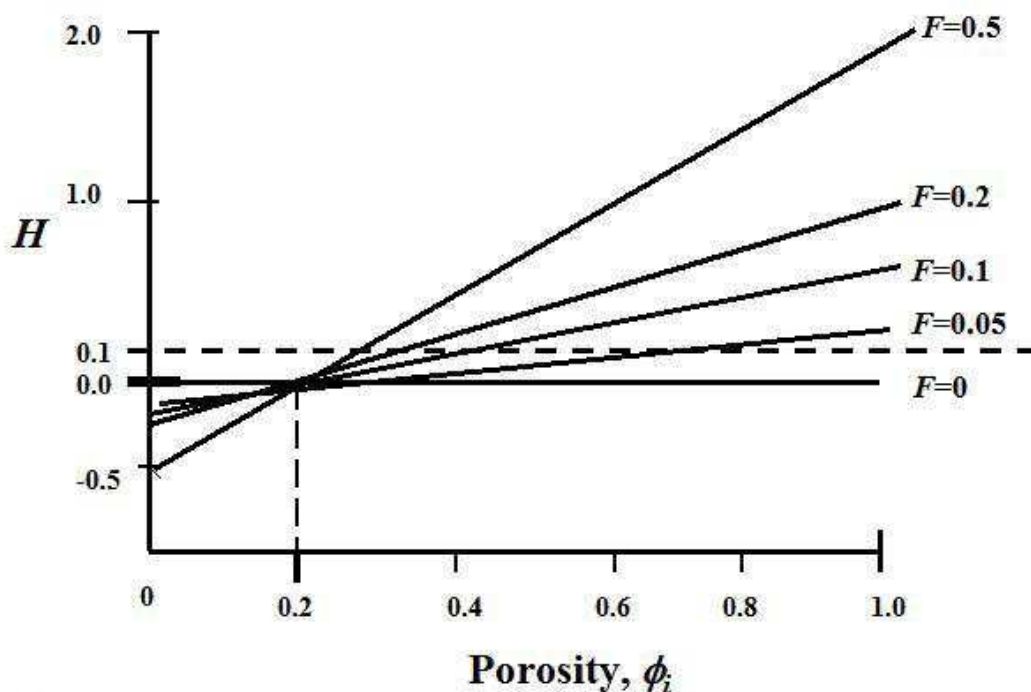


Fig. 3. Plot of $H$ against $\phi_i$ using Eq. (7), with $\phi_o = 0.20$. $H = 0$ when $\phi_i = 0.20$. When $F = 0$ there are no inclusions and thus $<\phi_m>$ and $\phi_o$ will be the same (since there is no $\phi_i$). At other values of $\phi_i$, $H \neq 0$, and there could be error. At $\phi_i = 0.20$, $H = 0$ for all the $F's$. At $H = 0.1$, the dashed line, the difference between $<\phi_m>$ and $\phi_o$ will be ~10%, but below this value the assumption of core homogeneity may probably create little extra uncertainty.
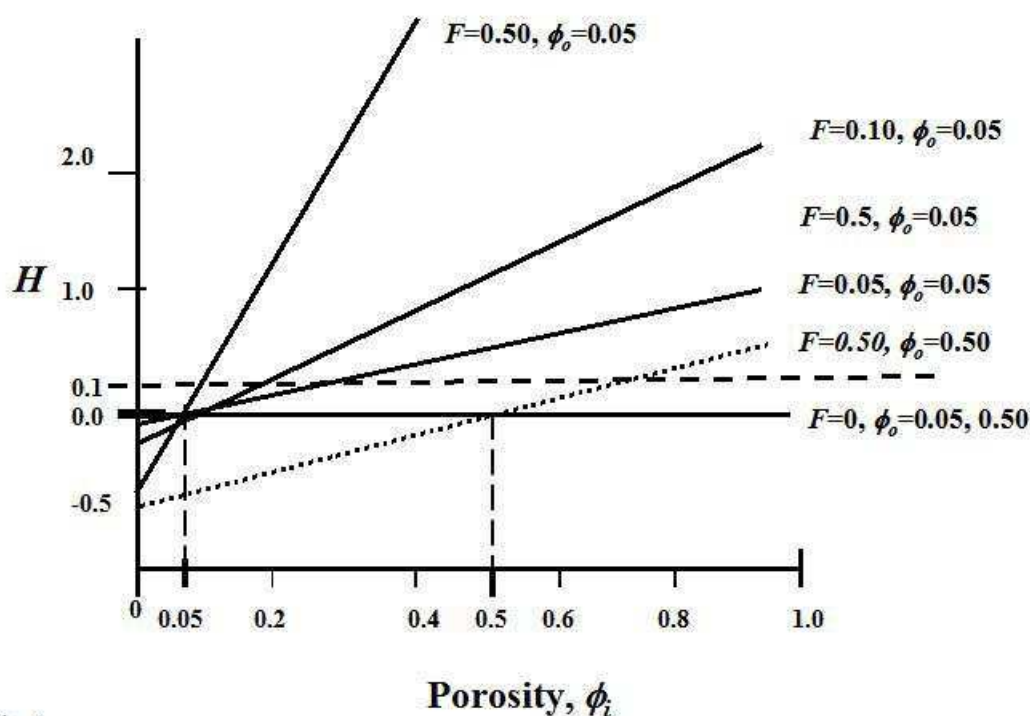
Fig. 4. Plot of $H$ against $\phi_i$ using Eqs. (6 and 7), with $\phi_o = 0.05$ and 0.50. $H = 0$ when $\phi_i = 0.05$ for lines and 0.50 for dotted line. When $F = 0$ there are no inclusions and thus $<\phi_m>$ and $\phi_o$ will be the same, (since there is no $\phi_i$). At other values of $\phi_i$, $H \neq 0$, and there could be error. At $H = 0.1$, shown by the dashed line, the difference between $<\phi_m>$ and $\phi_o$ will be ~10%, but below this value the assumption of core homogeneity may probably create little extra uncertainty.

In Table 1 and Section 3.1 we find for a carbonate core with 3.8 cm diameter and 5.0 cm length, for a matrix of $\phi_o$ of porosity 0.10 containing a very small vug ($\phi_i = 1.00$) of only 10 mm$^3$, $F$ is approximately 0.0002, $R$ is 10, so that from Eq. (7), $H$ is 0.0017 and by Eq.(6) $<\phi_m>$ is 0.10018. There is thus an additional error of 0.2% to be added to any experimental error. For a pebble of the same size, $R$ is 0 so $<\phi_m>$ would be 0.0998 so that the error will be pleasingly very small, ~0.02%. However for larger volumes of inclusions i.e. larger values of $F$, any error will be proportionally larger.

As a further example, if the laboratory reported a measured porosity $<\phi_m>$ of 0.15 with an experimental error ±0.01 i.e. 0.14 to 0.16 porosity (±6%,) and from examination of the slabbed core there was an estimated heterogeneity of 5% volume of embedded pebbles ($F$=0.05), then $\phi_i =$ 0.00, and the porosity of the bulk of the core $\phi_o$ from Eq. (6) will be ~0.158 (iterations needed), which is an additional +5% difference. This means that the reported porosity will have an increased total error of +5 %, which may be significant. If the core had embedded vugs, then $\phi_i =$ 1.00 and if $F$=0.05, $\phi_o$ will be 0.108 and thus an uncertainty of 0.043 in porosity from the measured value, a 30% difference. Here the porosity as measured is 0.150 but the outer fabric porosity of the core is 0.108. These increased error limits, must be accounted for in the input for the probability distribution functions for porosity in the *STOIIP* estimations, rather than just using the laboratory experimental errors. This will give a significantly wider range for the frequency distribution, and hence a wider range for the reserves for economic evaluation.

So far in our discussions we have assumed (realistic) values of $F$ and $R$, but in practice, we do not usually know them unless we can 'see' the heterogeneity, for example using CT scanning (Fig. 1), or the heterogeneity is visible on the surface of the core. A careful examination of the slabbed core from the well-bore core samples, which are sometimes available (e.g. if it is demanded by the host government license agreements or by the regulating authority), should give some indication of the heterogeneity of the core. This information can be augmented by observations of the local geology and any outcrops. However what inclusions are within the actual laboratory core sample will still be usually unknown. CT scanning (section 3) or cutting-up the core plug can give more guidance, but with cutting-up, the core plug would be destroyed.

In summary, it is clear that $H$ can assist quality control by helping to give more realistic ranges for the error bars on the porosity of the reservoir for inclusion in Monte Carlo *STOIIP* estimations of reserves (Dawe, 2000). If $F < 0.01$ or $H < ±0.005$ then the errors due to heterogeneity in the core are likely to be sufficiently small (equivalent porosity error ~0.002) that they can be neglected compared to the normal experimental error, which are often quoted as porosity ±0.01. In practice for typical reservoirs, the major area of interest will normally be the porosity range 0.050 and 0.300, $R(\phi_i/\phi_o)$ can range from 0.0 (pebble) to $\{1.0/\phi_o\}$ (vug), and the included volume, $F$, will probably not be more than 0.2, but this may not always be so.

### 2.3 The value of internal porosity $\phi_i$ for multiple internal volumes

Porosity is a scalar property, so that the porosity of a heterogeneous core can be calculated as the sum of the volume weighted parts. If there are a number of inclusions in the core plug, Fig. 5a, the core can be simplified to that shown in Fig. 5b, where all the individual occluded (inner) volumes $V_{Bj}$, $V_{Gj}$, $V_{Pj}$ with associated porosity $\phi_j$ and $j$ represents the individual inner volumes (1 to $n$), can

all be added together to give the total included volume $V_{Bi}$. Hence, $V_{Bi} = \sum_{j=1}^{n} V_{Bj}$, $V_{Gi} = \sum_{j=1}^{n} V_{Gj}$,

and $V_{Pi} = \sum_{j=1}^{n} V_{Pj}$ so that from Eq. (2), the porosity of the inclusive porosities is

$\phi_i = (\sum_{j=1}^{n} V_{Bj} - \sum_{j=1}^{n} V_{Gj}) / \sum_{j=1}^{n} V_{Bj}$, which is the value to be used in Eqs. (6) and (7).
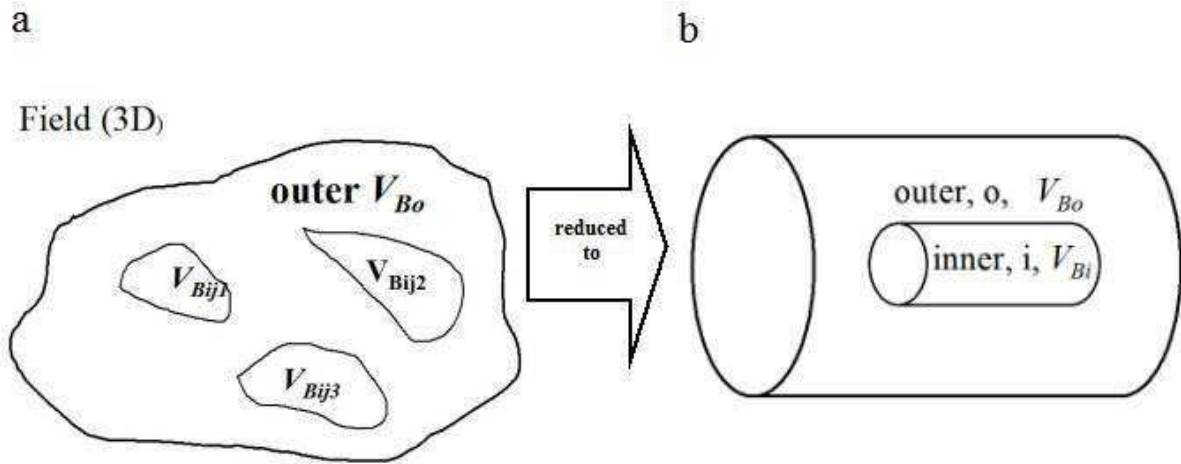


Fig. 5. Set-up of system with inclusions.
a. The core with inclusions. $V_{Bo}$ is the outer bulk volume, $V_{Bij}$ are the inner bulk volume inclusions and $j = 1$ to $n$, the number of inclusions .
b. The inclusions have been integrated into a single inner inclusion, $i$, drawn for simplicity as a cylinder.

*2.4 Rock density*

A possible check of whether the rock has inclusions can be by density analysis by back-checking with grain and bulk density, Sections 1.3 and 3.1. Grain density can be evaluated by weightings and using Eq. (3), and can be determined with separate measurements on leftover rock samples after coring for the laboratory sample plugs or with appropriate cuttings. If the measured rock density and the expected rock density for this rock do not match, then further investigation might be necessary to resolve the conflict.

**3. Example of porosity uncertainty for material with included vugs**
Figures 3 and 4 are in effect results from a thought experiment using realistic possibilities i.e., 'What if scenarios?' In order to test the worrisome significance of Eqs. (6 and 7) on porosity error, measurements were conducted on core plugs taken from an outcrop carbonate fault block from the Eden Valley, Cumbria (Fisher at al., 2017).

Nineteen cylindrical core plugs of 38 mm diameter and 40 to 50 mm length were drilled from the block. Vugs (dissolved evaporites) of various sizes up to 7 mm diameter were observed on some of the surfaces. CT scanning was performed on the plugs using a GE Brivo 385 instrument (Brivo XR385, 2017) to discover whether there were any hidden vugs or coring induced damage (fractures) present within the core plugs. The voxel size for these scans is $0.20*0.20*0.625$ mm$^3$. The majority of the core plugs, Fig. 6, had inclusions with one or more vugs of various sizes, two had partial fractures and only one core plug (sample VE3) was homogeneous, which gave baseline information.

16 of the samples were tested following the recommendations of RP40 (McPhee et al., 2015) and the porosity of each plug was determined. Their grain volumes were measured using a Quantachrome pycnometer model SPY-2 (Quantachrome, 2017) using helium as the expansion gas, and their core plug bulk volume determined using callipers. However, bulk volume measurements of cores is always troublesome because the sample are usually not perfect cylinders due to the non-uniformity of the surface of the rock plug. The bulk volume of some of the plugs were also determined with mercury immersion, but plugs where the vugs appear on the surface would invalidate the calliper method due to possible mercury intrusion. Separately, mercury porosimetry, MICP, was performed on a small uniform piece to obtain the overall grain density (2.62 g/cc) and its individual porosity.
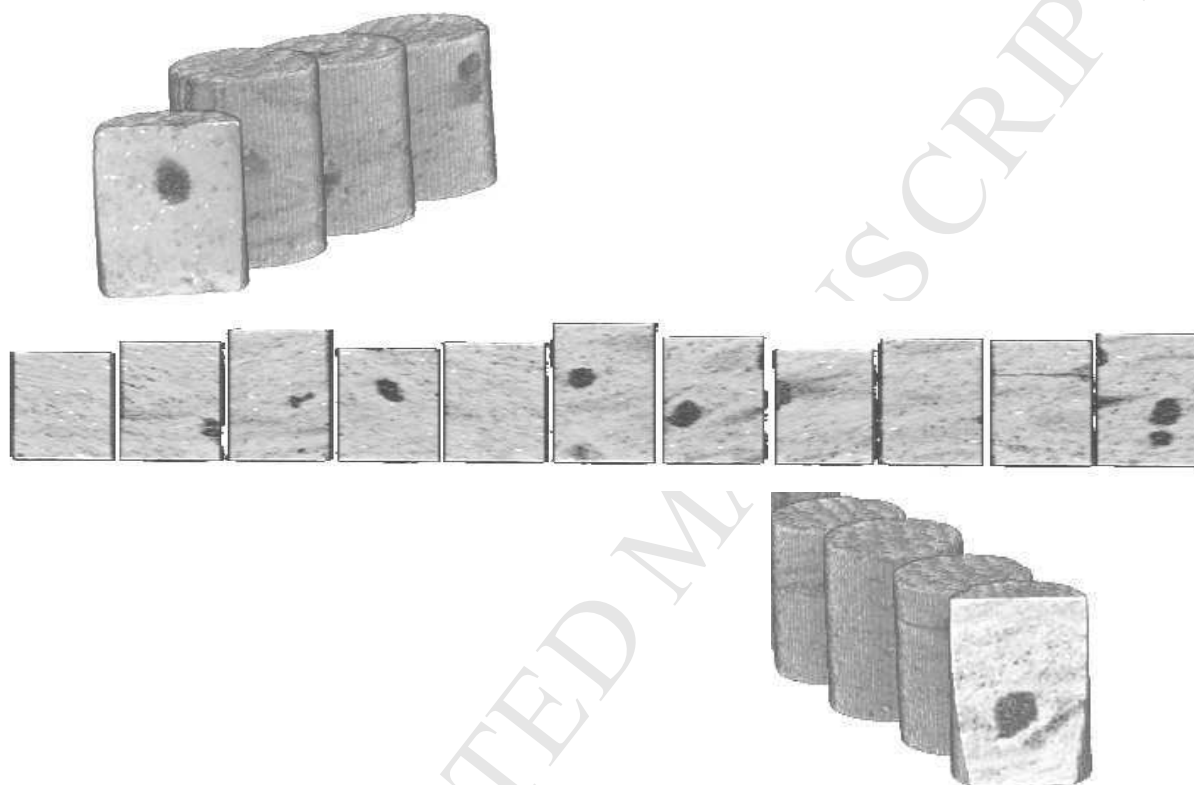


Fig. 6. CT scans of 19 plugs, in greyscale, showing internal vugs in black, deformation bands (light grey) and fractures within the core plugs. Cut-off of some plugs was from a 3-D reconstruction of the images. The cylindrical core plugs were 38 mm diameter and 40 to 50 mm length.

The range of measured porosities obtained from the helium grain volumes and calliper bulk volumes of these plugs, assuming they were homogeneous, was from 0.069 to 0.105 with an average of 0.083. The average systematic errors in bulk volume, grain volume and porosity were $\pm 0.08$ cm$^3$, $\pm 0.15$ cm$^3$ and $\pm 0.004$ respectively. The porosities were also calculated using density matrix calculations, Sections 2.4, to take vugs into consideration, using the grain density of 2.62 g/cm3 obtained by MICP. The average porosity of the matrix was 0.064 $\pm 0.005$ with agrees well with the porosity of the uniform core plug (0.065) but is larger than the porosity from MICP (0.057). The comparison between the uncorrected porosities is shown in Fig. (7) and in Table 2. Significant difference between these two methods with a large scatter is observed, due to the effect of the unknown volume of vugs on the determinations. Section 3.1 will show that CT scanning and dedicated Thermo Scientific Avizo Software calculations can give calculations of vug volumes and reconcile the measurements, Fig. (8) .
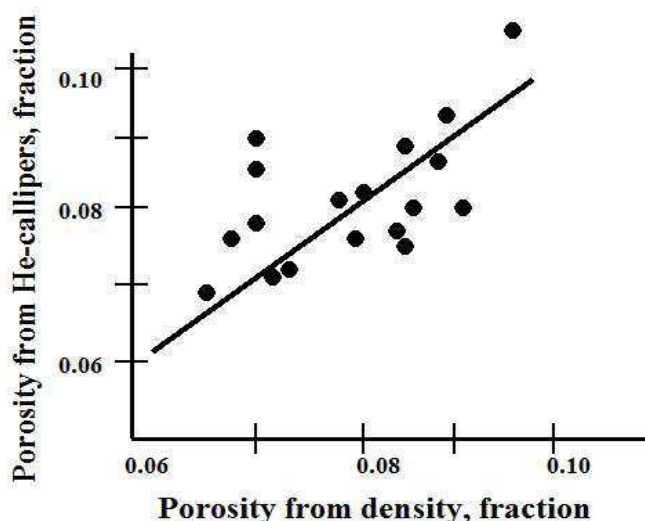
Fig. 7. Comparison of porosity calculated from core plug measurements assuming they are homogeneous. The average systematic errors for He-callipers is 0.004 and from densities 0.006 respectively. There is significant differences due to the vugs not being accounted for.

Table 2. The data used for 17 core plugs. The average porosity of the matrix is 0.064 ±0.005 which agrees well with the porosity of the uniform core plug (0.065) and is larger than the porosity from MICP (0.057). The porosity measured by He expansion neglects the effects of the included vugs and so is much greater. A comparison between measured and matrix porosity calculated using Avizo software is shown in Fig. 8 and this Table, where the corrections make the agreement excellent.

| Sample number | Bulk volume (cc) | | | Porosity*100 using He expansion for grain volume | | Matrix porosity *100 using rock density | Values from deconvoluted CT scan using Avizo software | | |
|---|---|---|---|---|---|---|---|---|---|
| | Calliper | Avizo | Hg immersion | calliper | density | | $F$ | $R$ | $H$, Eq.7 |
| VE1 | 56.27 | 56.48 | ---- | 8.80 | 8.35 | 6.75 | 0.017 | 14.8 | 0.24 |
| VE2 | 52.89 | 53.58 | ---- | 8.00 | 8.08 | 6.89 | 0.013 | 14.5 | 0.17 |
| VE3 * | 53.85 | 53.55 | 53.22 | 6.40 | 6.48 | 6.48 | 0.000 | 15.4 | 0.00 |
| VE4 | 53.94 | 54.93 | ---- | 9.40 | 8.77 | 6.23 | 0.027 | 16.0 | 0.41 |
| VE5 | 58.47 | 58.15 | ---- | 8.40 | 8.23 | 6.23 | 0.021 | 16.0 | 0.32 |
| VE6 | 46.70 | 46.49 | ---- | 8.90 | 8.91 | 6.34 | 0.027 | 15.7 | 0.40 |
| VE7 | 54.01 | 54.10 | ---- | 8.20 | 8.61 | 6.82 | 0.019 | 14.6 | 0.26 |
| VE8 | 59.07 | 58.52 | ---- | 8.00 | 8.43 | 6.26 | 0.014 | 16.0 | 0.35 |
| VE10 | 50.77 | 51.18 | ---- | 6.90 | 6.80 | 6.53 | 0.003 | 15.3 | 0.04 |
| VE11 | 50.40 | 49.19 | ---- | 9.20 | 7.03 | 6.48 | 0.006 | 15.4 | 0.09 |
| VE12 | 48.05 | 47.48 | ---- | 8.70 | 7.43 | 6.42 | 0.030 | 15.5 | 0.03 |
| VE13 | 57.90 | 57.28 | 57.38 | 8.80 | 7.14 | 6.54 | 0.006 | 15.3 | 0.09 |
| VE15 | 58.21 | 58.36 | ---- | 8.30 | 8.10 | 6.10 | 0.021 | 16.4 | 0.33 |
| VE17 | 55.88 | 54.72 | ---- | 8.10 | 7.18 | 6.39 | 0.008 | 15.6 | 0.12 |
| VE18 | 50.09 | 49.17 | ---- | 7.50 | 7.69 | 5.87 | 0.019 | 17.0 | 0.31 |
| VE19 | 45.73 | 44.37 | ---- | 7.60 | 7.76 | 6.06 | 0.018 | 16.5 | 0.28 |
| VE20 | 56.11 | 55.94 | ---- | 10.50 | 9.50 | 6.16 | 0.036 | 16.2 | 0.54 |

*no vugs

## 3.1 Determination of volumes of inclusions by CT scans

The volume of the inclusions is needed in order to resolve these pore volume differences. This was achieved by the deconvolution of volumes which was performed using stacks of CT scanned images and the use of Thermo Scientific Avizo Software (Thermo Fisher Scientific) (FEI, 2017). The stack of CT scans allowed us to manually identify the vugs as separate material and to segment the volumes. Pleasingly, the total bulk volume calculated with Avizo and those calculated from callipers agreed very well, as can be seen in Table 2 and Fig. 8. Here it was assumed that the porosity within the vugs (a hole) was 100% i.e. porosity 1.00. The average porosity of the matrix was 0.0643 ±0.0053 which agrees well with the porosity of the uniform core plug (0.065) sample VE3, and is larger than the porosity from MICP (0.057). A comparison between the measured and matrix porosity, Fig. 8, shows that after the matrix porosity has been calculated by the Avizo procedures there is now only small variations, giving confidence in the deconvolution methodology. However, from Table 2, as there are non-zero values of $H$, the sample-averaged porosity, $<\phi_m>$, will not be the same as the porosity of the non-vuggy volumes (i.e. outer matrix, $\phi_o$) within these carbonate samples, and must be included in the error bounds.
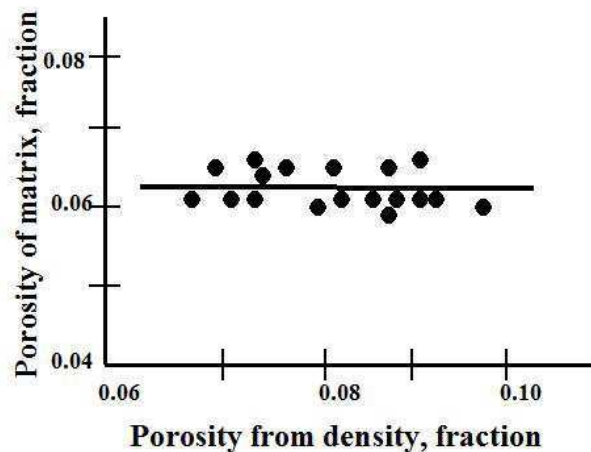


Fig. 8. Matrix porosity calculated using Avizo software against measured porosity using densities. Typical errors of porosity from density are estimated to be 0.006. Clearly there is now good agreement.

### 3.2 Cores with inclusions with porosity not zero or 100%

The carbonate example above was carried out on material with included vugs of 100% porosity. For core plugs containing inclusions that have a porosity different from 0 (pebble) or 100 % (vugs), the analysis becomes more complex as the porosity of the inclusions also need to be carefully estimated. A methodology based on multi-slice CT scans and 3-D reconstruction is possible where the CT scans can be used to obtain porosity profiles and 3-D distributions applied to slices or smaller volumes.

The porosity can be calculated by a complex method based on Beer's law of X-ray energy absorption, where the adsorption is proportional to the bulk density of the material (rock and fluids) contained within, and the linear attenuation coefficients, commonly presented in Hounsfield units. Then, two CT scans of the same pore space filled with different fluids (commonly air and water), can give a value of porosity (Akin and Kovsec, 2003) using Eq. (8),

$$\phi = \frac{CTrw - CTra}{CTw - CTa} \tag{8}$$

where $rw$ and $ra$ indicate the CT number of rock saturated with air and water, and $CTa$ and $CTw$ are the CT numbers of air (-1000) and water (0) respectively. In general the CT numbers of sandstone grains range from 1600 to 2100. Although air and water/brine are the commonly used

fluids, any two fluids with a contrasting CT numbers can be used. Volumes of 10 mm$^3$ or larger are suitable representative volumes as they would contain approximately 25000 voxels or more. However, for smaller core volumes the method has some limitations due to problems with beam hardening and shifts in X-Y-Z positioning. Such procedures to obtain the required properties require expensive equipment and dedicated staff.

## 4. Implications of included clays with large surface areas

As indicated is section 1.4, the determination of the pore volume in the laboratory is often by a gas expansion technique, (Amyx et al.,1960; Boyle et al.,2000; Dawe, 1992; Hook, 2003; Kennedy, 2015; McPhee et al., 2015; Schön, 2015; Tiab and Donaldson, 2015). However, when changing the pressure within the porosity apparatus, significant de/adsorption of gas from/to the grain surfaces can occur, particularly with clayey sandstones. Such desorption on pressure-drop changes the free gas volume in the apparatus and may cause a significant error in volume measurements. More gas will desorb on pressure drop when there is a large volume of interstitial clay and may be sufficient to distort interpretations of the experimental pore/grain volume measurements, and so give unintended errors.

The magnitude of any uncertainty in porosity measurements due to desorption depends on how much clay is present and which gas is used (Beliveau, 1993; Lowell and Shields, 1991). Desorption/ adsorption has been much discussed in the context of BET surface area measurements (Lowell and Shields, 1991) where nitrogen and krypton are normally used, but for porosity pore volume gas expansion measurements, helium is often used because it has small molecular size and lower adsorption. If the particles (grains) are large, say 1mm, then because the surface area of the clay grains will be relatively small little desorption occurs and there will be little problem. However, as the dimensions of the clay particles get smaller, the surface area of the same weight of material becomes larger as Table 3 demonstrates. If fine clay particles (smectite, chlorite, kaolinite, montmorillonite) are present in the pores of the sandstone material, they may have dimensions of ~10nm, then their surface area within the core material may become greater than for the grains of a clean sandstone, even if only present in small concentrations. The finer the grains the larger the surface area, so that an average particle size of ~50μm will have a specific surface area per gm of clay material of around 1m$^2$, and for nanoparticles will be significantly greater, perhaps more than 1000m$^3$ per gm. This has particular importance for physical property shale rock projects (Rezaee, 2015).

### 4.1 Mesoporosity effects

If the rocks have grains with microporosity/mesoporosity characteristics then the surface area within the rock will be significantly increased, so that large volumes of adsorbed/desorbed gas may occur. This makes determination of porosity challenging. Such rocks are found in coal-bed methane and shale gas reserves (Rezaee, 2015). These rocks will have additional porosity due to the meso/micro pores. The surface area can be huge and as a result large volumes of gas can be released due to the desorption of gas from the presence of organic material (organic carbon) within the matrix (Beliveau, 1993; Rezaee, 2015). This occurs in catalyst materials e.g. zeolites (Lowell and Shields, 1991), shale gas recovery and coal-bed methane extraction (Rezaee, 2015).

If such grains are in the test samples for porosity measurements then, when determining the grain or pore volume by gas expansion, more gas will be emitted. This would create significant uncertainty in the confidence in the determination of porosity. Increased knowledge of the composition of the core material would be needed to enhance porosity certainty. A more sophisticated method of porosity determination might be needed (Hook, 2003; Fisher at al., 2017; Lowell and Shields, 1991; McPhee et al., 2015; Tiab and Donaldson, 2015).

Table 3. Surface area as a function of particle size for 1 cm$^3$ of solid material. Here we have taken a 1cm$^3$ solid cube with the side dimension of 1cm. and then we have cut the cube into smaller cubes, of length 1/x cm, (Lowell and Shields, 1991). The number of these smaller cubes will be N $(x^3)$ and the volume of each cube is $x^{-3}$ cm$^3$ and its area is $6*x^{-2}$ cm$^2$, so although the total volume is still 1 cm$^3$ the total surface area of the material is now much greater and will be N $*6*x^{-2}$ cm$^3$.

| Length of one side of cube, x | In mm | Type of particle | Volume of a cube, mm$^3$ | No of cubes in 1cc, N | Surface area of a cube, mm$^2$ | Total surface area of 1cc of these cubes, m$^2$ | Total specific surface area of 1gm of these cubes with density 2.5 gm/cc, m$^2$ |
|---|---|---|---|---|---|---|---|
| 1cm | 10 | | $10^3$ | 1 | $6*10^2$ | $6*10^{-4}$ | $1.5*10^{-3}$ |
| 1mm | 1 | coarse sand | 1 | $10^3$ | $6*10^{-0}$ | $6*10^{-3}$ | $1.5*10^{-2}$ |
| 100μm | 0.1 | fine sand | $10^{-3}$ | $10^6$ | $6*10^{-2}$ | $6*10^{-2}$ | $1.5*10^{-1}$ |
| 10μm | 0.01 | silt | $10^{-6}$ | $10^9$ | $6*10^{-4}$ | $6*10^{-1}$ | $1.5*10^0$ |
| 1μm | 0.001 | clay | $10^{-9}$ | $10^{12}$ | $6*10^{-6}$ | $6*10^{-0}$ | $1.5*10$ |
| 100nm | 0.0001 | mud | $10^{-12}$ | $10^{15}$ | $6*10^{-8}$ | $6*10^1$ | $1.5*10^2$ |
| 10nm | 0.00001 | colloid | $10^{-15}$ | $10^{18}$ | $6*10^{-10}$ | $6*10^2$ | $1.5*10^3$ |
| 1nm | 0.000001 | nanoparticles | $10^{-18}$ | $10^{21}$ | $6*10^{-12}$ | $6*10^3$ | $1.5*10^4$ |

## 5. Additional comments

- For cores with low porosity and low permeability (e.g. shale rocks, Fisher at al., 2017), a purpose-built porosimeter is needed, although it still may not be fully certain of the effects of any inclusions.

- If large surface areas due to clays are present, these large surface areas will adsorb not only gases but also polymers and surfactants; this may be a challenge to the design of enhanced oil recovery schemes.

- Inclusions occur at all scales in the reservoir, from pore core to reservoir scale. Knowledge of the internal structure of the porous media including volumes of reservoir rock with different properties enclosed inside the matrix is necessary for porosity reporting and scaleup considerations.

- Because of the way fluids flow through heterogeneous porous rocks, the occluded volumes will create complex flow patterns so that even more caution is needed for accurate permeability determinations than for porosity measurements, as porosity measurements are normally static measurements. Also when considering reservoir hydrocarbon production, flow patterns and reservoir simulation, the absolute permeability and relative permeabilities are required to estimate the time of fluid breakthrough which is vitally important for design considerations. Such problems have been discussed elsewhere (Dake, 1994; Dawe, 2000; Dawe et al., 2011; McKean and Dawe, 1990).

- Normally the inclusions are unknown although core slabbings may give a clue. Unless one can 'see' the heterogeneity (e.g. the heterogeneity is visible on the surface of core) it will need special tests, such as 3-D CT as discussed in Section 3.1. These tests may move the unknown to a known and possibly quantifiable quantity.

## 6. Conclusions

The core plug measurement is regarded as the basic laboratory source for porosity values. Core plug measurements determine interconnected pores. In reality, often logs are more available than core data, but some logs determine total porosity so the core and log data must be reconciled to get representative values of interconnected porosity. This paper has presented the results of both thought and actual experiments for quantifying the additional errors that might occur in the measured results from a porosity determination of a core plug which is assumed to be homogeneous but when (maybe unknown) inclusions are present.

In the laboratory, cores plugs are often assumed to be homogeneous black boxes. To be certain that any output data are relevant, accurate and representative, one needs to know the structure within the core (i.e. make the black box transparent). CT scanning may help identify any inclusions so they can be accounted for in the porosity determination. The questions always to be asked is 'Is the core plug sample really representative of the reservoir?', and 'Is the assumption of homogeneity erroneous, and if so how much extra error occurs?'

We have derived a *Heterogeneity Factor*, *H*, Eq. (7), *H = F(R-1),* based on the ratios of internal and outer volumes, *F*, and porosity ratios, *R*. Values of *H* can be a guide to the laboratory personnel and the operating company engineers for the quality control of core selection and porosity measurements. Figs. 3 and 4 have given a visual guide to the value of *H* that might be needed for a core plug measurement.

In practice for typical reservoirs, the major area of interest will normally be the porosity range 0.05 to 0.30, the included volume, *F,* is probably not more than 0.2, but this may not always be so, and that *R* ($\phi_i/\phi_o$) can range from 0.0 (pebble) to $\{1.0/\phi_o\}$ (vug). If *H* is > ±0.01 then the errors due to heterogeneity in the core may be of concern, particularly for vuggy samples.

Before budget estimations for reservoir evaluation and investment using Eq. (1), appreciation of core and reservoir heterogeneity is essential. An extremely important part of this process is the initial accurate laboratory measurement of the porosity of core plugs and with correct error estimates. The assumption that the core plug is homogeneous and only has the experimental laboratory errors may be erroneous and create significant increased uncertainties in *STOIIP* and budget estimations.

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**Highlights**

**Implication of heterogeneities on core porosity measurements**
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- The core plug sample is often assumed homogeneous when determining porosity, $\phi$
- Embedded material (unknown or ignored) creates uncertainties in core porosity values
- Differences can occur between sample-averaged $<\phi_m>$ and material specific $\phi_o, \phi_i$
- Introduce a *Heterogeneity Factor* $H = (<\phi_m> - \phi_o)/\phi_o = F(R-1)$
- When $H \neq 0$, then error bounds increase caused by the ignoring of heterogeneities
- Error bounds on porosity needs to be reassessed

$F$ = ratio of inner bulk volume to total bulk volume $F = V_{Bi}/V_{Bm}$
$R$ = ratio inner to outer porosity $(\phi_i/\phi_o)$