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Evaluating the applicability of the radial approximation for pile heat exchangers

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This paper appraises the efficacy of using an analytical radial approximation for different thermal pile heat exchanger geometries. Unsteady radial heat-flow from fluid in a pipe set within a grouted borehole into the external ground is well-documented and can be solved analytically very rapidly using Laplace Transforms (Javed & Claesson, 2011). By comparing the radial model with finite-element simulations including explicit pile geometries, this paper provides a provisional analysis of the accuracy of this approach. Initial findings suggest that the radial model may provide an appropriate approximation to pile behaviour for certain pipe configurations, albeit with small ‘mid-time’ error.

Introduction

Energy piles are an alternative to classical borehole exchangers to provide energy-saving for newly constructed buildings. The primary heat-exchange circuit is embedded within the concrete building piles. This saves on embodied energy and has the ability to place the ground-source heat exchanger close to the building where it is needed. The contrast between the thermal properties of the concrete and the surrounding ground means that the typical assumptions used for designing borehole heat exchangers may not necessarily apply; in particular the transient heat transport due to heat storage within the pile itself may cause pile heat exchangers to perform significantly differently to ordinary boreholes.

Various arrangements of exchanger pipes are engineered, typically relating to the underlying construction method of the pile. Where piles are constructed using a Continuous Flight Auger (CFA), the fluid pipes tend to be located towards the centre of the pile, arranged around reinforcement bar for stability. Where there is rotary pile construction, the fluid pipes are located closer to the outside edge of the

pile. Both arrangements are summarised in Fig 1. These differences give rise to different thermal response characteristics.

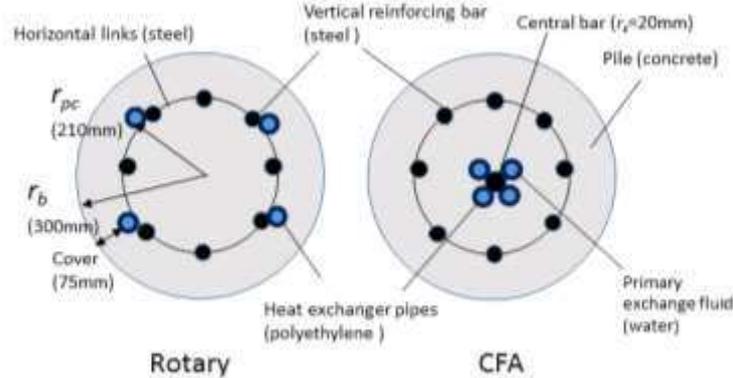


Fig. 1: Cartoon of typical thermal pile construction arrangements assumed here

There have been numerous studies on how best to capture different arrangements in terms of a step response (or ‘G-function’), without the computational cost of explicitly modelling the detailed geometry of the borehole (Loveridge & Powrie, 2013). Here we build on this general approach, making use of an elegant semi-analytical solution which exploits radial symmetry. The specific models used here we call the Claesson-Javed Radial model (CJRM) and the Claesson-Javed Radial model with Storage (CJRMS). These are detailed in Claesson (2011) and Claesson (2017, Personal communication) and are implemented here in Matlab R2017a.

The radial models and their equivalence to typical geometries are summarised in Figure 2. The radial models are consistent with the explicit models in solving unsteady-state Fourier conduction in the ground and concrete borehole, and in treating the fluid as an isothermal unit. The models differ in geometry, but also in that the pipes in the radial model are modelled as steady-state resistances. For the CJRMS model, the inner store is considered to be a further isothermal unit, which is connected to the fluid via a pipe resistance. The total pipe resistance is divided by an empirical weighting factor between the transfer to the store and the transfer to the concrete pile.

To scope the issues we keep to two different arrangements, here short-handed by the construction method (i.e. ‘CFA’ or ‘Rotary’), both times with four pipes ($N_p=4$). The steel is modelled for the CFA, since it is surrounded by the pipes, but we neglect it in the Rotary case.

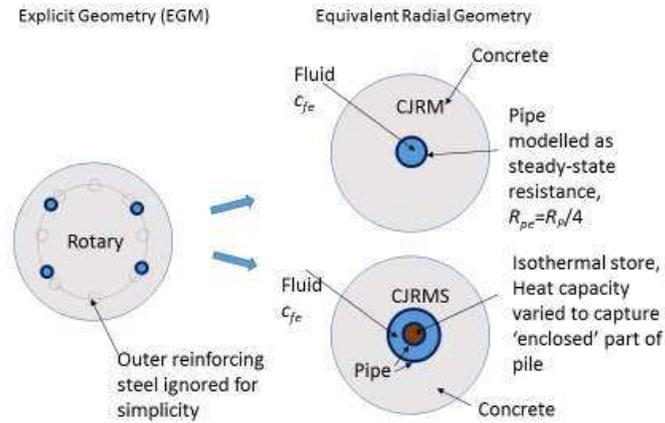


Fig 2: Radial models to represent explicit geometry (rotary case given as example)

The CJRM and CJRSM models are a geometric simplification of the true underlying geometry of a typical borehole pile arrangement, so it is prudent to examine the potential inaccuracies. Before making this analysis we conjectured that the CJRMS may improve the CJRM, which would be anticipated to be less accurate in the instance where the borehole pipes are significantly offset from the centre of the pile (including where they are set around concrete reinforcement bar).

This paper presents examines the nature of this simplification, focusing on a 600 mm diameter concrete pile. A wider analysis is to follow.

Explicit geometry simulations

We generate synthetic data in our 'Explicit Geometry Model' (EGM), which is a 2D solid conductive heat-flow pile model in COMSOL. The assumptions are summarised in Table 1. A key assumption is that there is no heat-flow out of plane (i.e. we simulate a unit depth with no effects from the end of the borehole, the ground surface or due to different fluid temperatures in the pipes). Each thermal unit (i.e. ground, concrete, pipe and fluid) is considered to be a homogeneous uniform continuum each characterised by thermal parameters constant in time and temperature.

Four fluid pipes per borehole are modelled. The thermal properties of the fluid in these pipes are of water, which is assumed to be of equal temperature throughout the pipe (i.e. well-mixed). For simplicity we neglect to include a wall heat-transfer coefficient, which could be added to the pipe resistance if necessary.

The assumed initial condition is for uniform, equilibrated temperatures. At the start of modelling period, there is a step-change in input power to the fluid. The model is run for 1×10^7 s (i.e. ~ 116 days), outputting ten times per logarithmic interval starting at for 1×10^{-5} s. The early end of the range is of course substantially shorter than the timescales of practical interest and are included to enable validation of early-time asymptotic behaviour.

A typical duty might crudely be summarised as a square-wave (on-off) cycle of period one day, being sustained for months or years at a time. Therefore, in practice the daily timescale (i.e. from $\sim 1 \times 10^3$ s to $\sim 1 \times 10^5$ s) is of particular interest for optimisation purposes.

Table 1: input assumptions. Two generic pile types are simulated: ‘CFA’ and ‘Rotary. The thermal conductivities of the concrete and ground (λ_c, λ_g) are permuted as (1,1); (1,2); (2,1) and (2,2).

Item	Geometry	Properties	Comments
Fluid	$N_p=4, r_{pi}=0.0123$ m.	$\rho_f=1000$ kg/m ³ $c_f=4217$ J/kgK $C_f=\rho_f c_f. \pi r_{pi}^2=8017.2$ J/mK	Isothermal. Constant heating of 50 W/m applied to the fluid (i.e. 12.5 W/m/pipe)
Pipes	$r_{po}=0.015$ m $N_p=4$, at 90° separation. CFA: $r_{pc}=0.035$ m Rotary: $r_{pc}=0.21$ m	$\rho_p=950$ kg/m ³ $c_p=1900$ J/kgK $\lambda_p=0.45$ W/mK	High density polyethylene (Cecinato & Loveridge, 2015)
Con-crete pile	Circular. $r_b=0.3$ m	$\rho_c=2000$ kg/m ³ $c_c=800$ J/kgK $\lambda_c=1$ or 2 W/mK	
Ground	Circular outer model domain ($r_o=25$ m)	$\rho_g=2000$ kg/m ³ $c_g=2000$ J/kgK $\lambda_g=1$ or 2 W/mK	
Steel	$r_s=0.02$ m at the centre of the pile.	$\rho_s=7801$ kg/m ³ $c_s=473$ J/kgK $\lambda_s=43$ W/mK	Only for CFA Loveridge & Cecinato (2016)

Matching to radially-symmetric model

The method for matching the radially-symmetric models (CJRM and CJRMS) to the synthetic explicit model simulations (EGM) are summarised in Figure 2. The philosophy of the matching is to preserve the true geometry and physics whenever possible. Therefore, the concrete and ground thermal properties and geometry (i.e. $r_b=0.3$ m) are kept unchanged from the EGM. In theory these could be varied to give additional degrees of freedom thereby potentially improving the fit.

The effective radius of the single effective pipe (r_{pe}) is set so as to provide the identical fluid temperature drop over the concrete pile during quasi steady-state

conduction at the end of the simulation as occurs for the EGM (i.e. at $t=1\times 10^7$ s). This is achieved by computing the averages of T_{po} and T_b around the borehole and pipe boundaries and rearranging the steady-state heat-flow equation [1] to find r_{pe} :

$$\bar{T}_{po} - \bar{T}_b = qR_b = \frac{q}{2\pi\lambda_c} \ln\left(\frac{r_b}{r_{pe}}\right) \quad [1]$$

By doing this we remove the issue of how to estimate R_b given a particular borehole geometry; this is discussed very comprehensively by Javed and Spitler (2017). The thermal capacity of the fluid C_f is kept equal to the capacity in the EGM. Since the effective area of fluid has changed, this is achieved by using a scaled specific heat of fluid, c_{fe} .

The characteristic time for thermal diffusion within the pipe wall is $c_p\rho_p(r_{po} - r_{pi})^2/\lambda_p$, which is relatively short (~ 30 s). This is the basis for the assumption made in the radial model of steady-state heat transfer in the pipe. Thus, in a similar manner for R_b , the thermal resistance is estimated as:

$$R_p = \frac{1}{2\pi\lambda_p} \ln\left(\frac{r_{po}}{r_{pi}}\right) \quad [2]$$

The equivalent pipe resistance for the EGM is a quarter since there is only one pipe in the CRM dispersing the same total power, i.e. $R_{pe} = R_p/4$.

The above matching via R_p , R_b (or r_{pe}) and c_{fe} ensures long- and short- time asymptotic matching. However, between these times the models may diverge. For the CRM, there is no obviously sensible adjustment to the parameters that can be made to improve the mid-time. We test the conjecture that the CJRMS may instead provide some adjustment, and with a physically-logical basis. For the CJRMS different values may be appropriate for the thermal capacity of the central store. For CFA geometry the thermal capacity of the store is set equal to the capacity of the steel. For the radial geometry a reasonable upper bound for the capacity of the store C_{ST} is to lump the capacity of the concrete at $r < r_{pc}$.

Results

Figure 3 illustrates the comparison, plotted in real temperatures and times. The solutions converge to the asymptotic solutions for short and long times, albeit there is numerical error appearing at the very earliest part of the EGM simulation. At mid-time, there is a departure which reaches a maximum of 0.6 °K at 2511 s before declining. This is a small absolute error when compared to the temperature at the end of the period (18.7 °K), but is a 17% relative error at that point in time. The matching departure can be seen to occur within the approximate range 20s-20,000s.

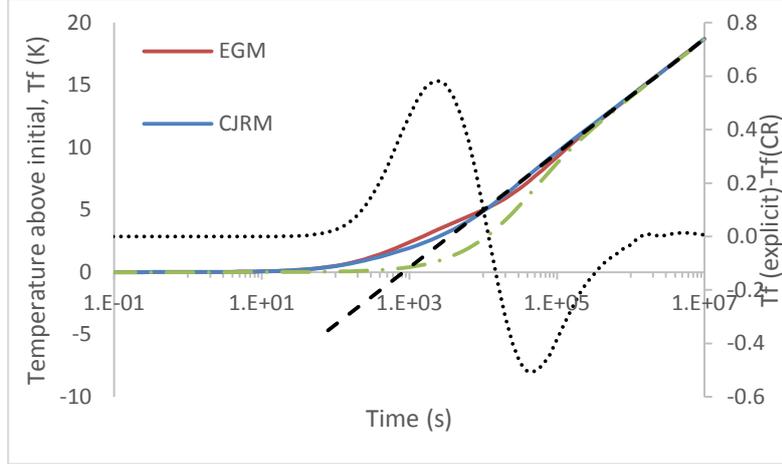


Fig 3: EGM and CJRM compared for rotary case ($r_b=300$ mm) for $(\lambda_c, \lambda_g) = (1,2)$ W/mK $T_f \sim \frac{q}{4\pi\lambda_g} \left[\ln \left(\frac{4\lambda_g t}{c_g \rho_g r_b^2} \right) - 0.5772 \right] + q[R_p + R_b]$ at long time (dashed line). $C_{ST} = 100,000$ J/(Km) in the CJRMS.

Table 2: Summary parameters. Note R_p calculated via equation [2] is 0.0702. So, $R_{pe}=0.0175$. This is confirmed in the EGM at 1×10^7 s.

Case	λ_c, λ_g (W/mK)	c_{fe} (J/kgK)	R_{be} (Km/W)	$ \text{Max}(T_f _{EGM} - T_f _{CJRM}) $
CFA1-2	1,2	1471.0	0.31	0.66
CFA2-1	2,1	1556.9	0.16	0.24
CFA2-2	2,2	1560.1	0.16	0.24
Rot1-1	1,2	141.3	0.13	0.58
Rot2-1	2,1	142.2	0.06	0.38
Rot2-2	2,2	142.1	0.06	0.47

The pattern from Figure 3 is repeated in Figure 4– a relatively small error occurs at mid-time in all the plots. CJRMS cannot remedy the discrepancy between the models; it lowers the mid-time temperatures as the store capacity C_s increases, and thereby worsening the match.

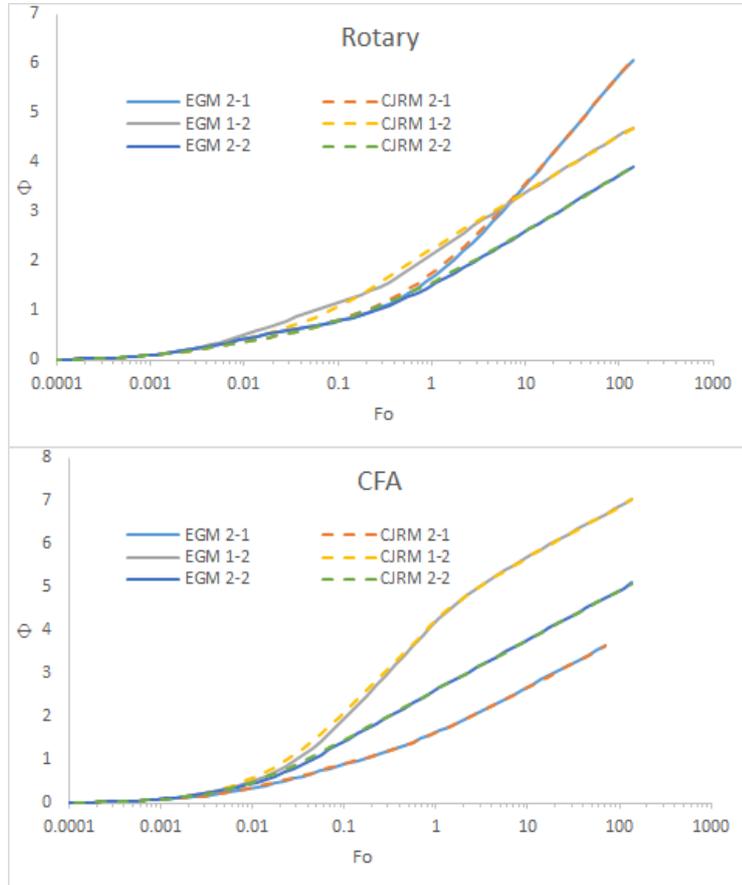


Figure 4: Comparison of step response behaviour between geometries for dimensionless temperature $\Phi = 2\pi\lambda_g\Delta T_f/q$ against dimensionless time $Fo = \alpha_g t/r_b^2$. $\lambda_c-\lambda_g$ in legend.

Discussion

Figure 3 shows that the addition of the store in CJRMS doesn't improve the fit; the effect of the store is to lower the mid-time temperature. Simulating the store as a fully diffusive cylinder also provides no improvement (numerically or analytically via block-geometry functions provided by Barker, 1985). The departure from the idealised radial solution is due to the fact that the pipes are not centred on the borehole, which can be illustrated qualitatively at the simplest level by varying the location of a single pipe. Thus, despite the intuitive nature of the CJRMS, it does not correctly capture the physics of the situation. The dominant cause of the discrepancy away from a purely radially-symmetric case is due to the asymmetry of the location of the pipes. This can be demonstrated by comparing the response for a single-pipe as it is moved out from the centre.

In terms of the CJRM simplification applied to non-centred pipes set within a large-pile situation where there is contrasting ground-concrete thermal conductivities, it is notable how close the radial approximation is. The step response is dominated by the fluid thermal capacity at early time and by the ground resistance at late-time, transitioning between these, via a period dominated by borehole resistance at mid-time.

By computing R_b in the EGM, asymptotic late-time matching was guaranteed. In practise, since the point of the radial models is to avoid building EGMs, R_b requires estimation. As is demonstrated in Javed & Spitler (2017), the multi-pole method is an accurate means of doing this.

A more generalised analysis over a wider geometry range is the logical next step to build on these provisional findings. In particular it will be useful to quantify the accuracy of the CRM over a wider range of geometries and parameters (including number of pipes).

Conclusions

For the parameter ranges and geometries assumed in the paper (which we believe to be reasonably typical) the CJRM is demonstrated to be reasonably accurate, given the geometric simplification. There are, nonetheless, differences that arise primarily due to the asymmetry. The radial (CJRM) and the radial-store model (CJRMS) although able to match the asymptotic behaviour at both early and late time, develops discrepancy during the mid-time. The CJRMS worsens CJRM fit, since it is not introducing the correct physical behaviour relating to this asymmetry.

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