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Supporting Information

As discussed in the main text, the Lorentz force provides the largest contribution to the forcing of TWs at the ICB. To determine the nature of the excitation mechanism of TWs we have identified which of the many terms that make up the Lorentz force, $F_L$, are largest in magnitude. As given by equation (14) of Teed et al. (2014), the time derivative of $F_L$ can be written

$$\dot{F}_L = \frac{P_m}{E} \frac{1}{h s^2} \frac{\partial}{\partial s} s^2 h \left\{ \langle s B_x (B \cdot \nabla) u_\phi \rangle + \langle \frac{B_\phi}{s} (B \cdot \nabla) (s u_s) \rangle \right. $$

$$\left. - \left( \frac{\mathbf{u} \cdot \nabla + 2}{s^2} \right) \langle B_x B_\rho \rangle \right\} + \langle B_x \nabla^2 B_\phi + B_\phi \nabla^2 B_x \rangle \right\},$$

(S1)

where overbars and angle brackets represent spatial averages in azimuth ($\phi$) and vertical depth ($z$), respectively. The velocity and magnetic field are split into their mean and fluctuating parts leaving numerous terms on the right-hand-side of Eq. (S1). The magnitude of each individual term was calculated, leading to the results described in the main text.

For the primary simulation (simulation 1) discussed in the main text of this paper the parameters take the values: $E = 5 \times 10^{-6}$, $Ra = 1.8 \times 10^8$, $Pr = 1$, $Pm = 0.1$ as well as imposed field strength $B_0 = 10$. To ensure the robustness of our result we performed several further simulations close to the parameter regime of simulation 1, which are summarized here. The parameters for these simulations are as follows: simulation 2 is as simulation 1 but with $Ra = 1.5 \times 10^8$; simulation 3 is as simulation 1 but with $Ra = 1.5 \times 10^8$ and $Pm = 0.2$; simulation 4 is as simulation 1 but with $Ra = 1.5 \times 10^8$ and $Pm = 0.05$.

Figs. S3-S5 show the torsional wave signals and power spectra for these three further simulations. In simulation 2 (Fig. S3), the convective driving has been slightly reduced from $Ra = 6 Ra_c$ (in simulation 1) to $Ra = 5 Ra_c$. The effects are minimal; a clear periodic signal exists with only a small modification to the period of the torsional wave ($\sim$8 yr compared with $\sim$6 yr in simulation 1). Next, in simulation 3, the ratio of the viscosity to the magnetic diffusion has been increased
from $Pm = 0.1$ (in simulation 2) to $Pm = 0.2$. Periodic waves remain but their period is now significantly increased to $\sim 25$ yr (Fig. S4a). This is also evidenced by peaks in the power spectra found at smaller values of the frequency (Fig. S4b). Larger modes of convection (i.e. smaller values of $m$) are now more important which is to be expected as the role of viscosity is increased. In simulation 4, the ratio of the viscosity to the magnetic diffusion has been adjusted once more, in this case to a lower value of $Pm = 0.05$. The results are now more dramatic; most obvious is that the wave signal has lost its clear periodicity with some oscillations notably stronger than others (Fig. S5a). Waves excited at later times are also considerably weaker suggesting that the driving mechanism is sporadic and only partially effective. Fig. S5b shows that the structure of the convective spectra is more complicated than the other cases. Power is now located at several distinct frequency ranges; significantly, the locations of the peaks in the axisymmetric mode ($m = 0$) and the TW ($U_A$) do not match as closely as in the previous cases (cf. Fig. 3b and Figs. S3b and S4b).

**Supporting Video 1**

This video demonstrates the excitation mechanism of the waves discussed in the main text. The fluctuating azimuthal velocity, represented by the quantity $u'_\phi$, is plotted at the same three heights above the equatorial plane marked by the horizontal orange lines in Fig. 4d but now also extended to the full width of the core, i.e. extended into the region OTC. The location of the TC is marked by the dotted black line in each plot. The area of the domain of the plot varies with $z$ as the extent of the core depends on height but the region ITC is of constant area. The video runs for the period from $t = 0$ yr to $t = 18.5$ yr of simulation 1 (the simulation discussed in the main text).

The convection OTC is chaotic but waves can be seen to propagate radially outward from the TC in the form of patches of highly axisymmetric positive (red) or negative (blue) velocity. The patches represent the torsional waves (TWs) travelling in the cylindrical radial direction from the TC to the core-mantle boundary. On the TC itself the $\sim 20$ patches of convection discussed in the main text can be seen operating. The region ITC displays very different behaviour; here convection is inefficient and the velocity is highly axisymmetric. This represents a resonant cavity where TWs travel from the TC to the polar axis and back setting the frequency of the waves.
Figure S1. **Thermal gradient across the TC.** Time-average of (normalized) temperature in $s\phi$-space at height 1,700 km above the equatorial plane.

As waves in the resonant cavity reach the TC (e.g. in the video at $\sim 5$ yr or again at $\sim 11$ yr) a new wave is triggered OTC through the interaction at the TC of convection and the jet (shown in the main text). The sign of the velocity ITC changes after the excitation of a wave as the mechanism restarts to trigger the next wave. The velocity profile ITC is retained across the three heights shown indicating that the resonant cavity effect operates throughout the depth of the core. At locations further from the equatorial plane (i.e. large $z$) the wave propagation OTC remains although its signal is diminished in the surrounding convection.
Figure S2. Meridional sections of the azimuthal velocity, $u_\phi$, averaged in $\phi$ at: (a) $t = 9.5$ yr; (b) $t = 21$ yr.
Figure S3. Torsional wave signal and power spectrum for simulation 2. (a) Torsional wave signal. (b) Power spectrum of the ageostrophic convection, calculated at a point on the TC (at $s = r_i$, $z = r_i$) and normalised by the largest spectral mode. These plots are the same as Figs. 2 and 3b but using data for simulation 2 (which has parameter values: $E = 5 \times 10^{-6}$, $Ra = 1.5 \times 10^8$, $Pr = 1$, $Pm = 0.2$, $B_0 = 10$).
Figure S4. Torsional wave signal and power spectrum for simulation 3. As Fig. S3, but for simulation 3 (which has parameter values: $E = 5 \times 10^{-6}$, $Ra = 1.5 \times 10^8$, $Pr = 1$, $Pm = 0.2$, $B_0 = 10$).
Figure S5. Torsional wave signal and power spectrum for simulation 4. As Fig. S3, but for simulation 4 (which has parameter values: $E = 5 \times 10^{-6}$, $Ra = 1.5 \times 10^8$, $Pr = 1$, $Pm = 0.05$, $B_0 = 10$).