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UE-Centric Clustering and Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks

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Abstract: Heterogeneous cellular Network (HetNet) has emerged as a promising technology for the 5th generation mobile networks (5G) that can be used to meet the high demand of data rate and better quality of service (QoS) performance. However, the performance of HetNet will depend on how scarce resources such as frequency, time, power and spatial resource are shared among user equipments (UEs) in the system and also how interference is controlled. In this work, we utilize UE-centric clustering as a tool to effectively determine the interfering BSs that cause significant interference to each UE in the network. These interfering BSs together with the serving BSs of these interfered UEs will coordinate and make resource allocation decision together to allocate spatial directions to each UE in the network in order to manage interference in the network. We formulate the resource allocation problem as maximizing the weighted sum-rate of HetNet while fulfilling some power, QoS and interference constraints. This optimization problem is non-convex. We readily split the RA problem into two sub-problems: the spatial direction allocation problem and the power allocation problem respectively. We are able to solve these problems efficiently using SeDumi, which provides a general purpose implementation of interior point methods.
Simulation results of our proposed method, when compared with the other existing methods, show significant improvement.

1 Introduction

As the demand for mobile data services increases by end-users, operators seek ways to enhance capacity of their networks. Unfortunately, single-tier networks (macro-cellular networks) could not provide adequate solutions to the problem of capacity and coverage in cellular networks. This prompts ideas like cell splitting which evolves into HetNets [1]. HetNet is a network that consist of planned macro base stations (MBSs) deployments which transmit signals at higher powers with overlaid smaller cells nodes such as pico base station (PBS), micro base station (mBS), femto-cell acess points (FAPs), relay nodes (RNs) and remote radio heads (RRHs). HetNet is one of the key technologies in 5G which can tackle the ever increasing demand for data rate and coverage. However the performance of HetNets depends on resource allocation (RA) which is how frequency, time, power and spatial resources are shared among UEs in order to maximize the system spectral efficiency (SE).

Interference is a limiting factor to the performance of HetNets and if not properly managed will deteriorate the achievable system wide throughput [2, 3], therefore, RA is very important. There have been different methods proposed in literature to solve the interference problem. Multi-cell processing (MCP) has emerged as an efficient way to suppress interference as well as enhancing the spectral efficiency of the system [4, 5]. In MCP, BSs cooperate together in different levels to manage interference and at the same time improve the individual BSs that forms the cluster. Clustering is very important in multi-cell processing because it can help to group specific BSs together with the goal of mitigating interference and/or improving the received signal quality for UEs at the cell edges. Different clustering schemes have been investigated in literatures and they can be categorized as UE-centric clustering [6–8], network-centric clustering and hybrid clustering [9]. In UE-centric clustering scheme, the UE selects the coordinating BSs based on its point of view, these BSs either serve or reduce interference from it. In contrast, network-centric clustering is performed by the operators on a static or semi-static basis and have been castigated for not fully utilizing the channel variations of UEs present in the network. While hybrid clustering will achieve the trade-off between the performance and complexity of the aforementoned clustering schemes.

Coordinated beamforming (CB) [10] is a type of MCP described in 3rd generation partnership project (3GPP) LTE-Advanced which require partial cooperation between the cooperating BSs. In CB, each BS serves its UEs with data while control information is exchanged between BSs with which RA decisions can be made collectively. Compared with joint transmission (JT) [11], CB has been shown to be a practical and feasible approach for mitigating interference in downlink of single-tier cellular networks [12–15]. JT has limitation from a practical perspective because it requires full phase coherence among signals received from different BSs, which is usually impossible due to difference in propagation delay. Tight synchronization [16], is a very important factor JT needs, to become practically feasible. Some new ideas have emerged on implementing JT using cloud RAN technology [17], and using tools from stochastic geometry [18, 19]. Though the theories behind it make sense but the practical implementation is where the problem lies. Even if unlimited capacity fibre optical link is utilized for data sharing, it will only increase operational expenditures (OPEX). If the net gain between OPEX and increased spectral efficiency is small, then the motivation behind increased expenditure for implementing JT cannot be justified. Although the effectiveness of CB has been well studied in single-tier homogeneous cellular networks where the multi-cell characteristics and the accompanying inter-cell interference are usually limited to at most three cooperating MBSs, its application in a dense deployed HetNet scenario requires detailed investigation. Therefore in this paper, we develop a UE-centric clustering scheme that determines the optimal interfering BSs that will coordinate with the serving BS of each interfered UE to allocate resources such as spatial directions and powers to UEs in HetNet.

1.1 Prior works

Previous works on coordinated beamforming either use the wyner model [20–22] which is a simplified model where interference only comes from the immediate neighbouring cells, or network centric model [23–25], which is network with static clusters, these clustering method limits the cooperating area in several fixed BSs thereby cannot flexible adapt to the changing topology. Furthermore, in [26, 27], BSs are divided into static disjoint cooperation clusters. Each cluster is operated as a single-cell system. However, networks with this kind of clusters usually provide poor spectral efficiency when UE distribution is heterogeneous, also these clusters suffer from out-of-cluster interference and thereby affecting the performance of the system. In [28], UE-centric based clustering is utilized for inter-cell interference nulling. However, this is done for a single-tier small cell networks. Furthermore, in [29] UE-centric based clustering and beamforming is utilized for energy efficiency optimization, however, this is targeted for cloud radio access network (RAN).

RA has attracted a lot of research in the past, however it is mainly for single-tier networks such as in [10] and references therein. The contributions made in these papers do not address the significant interference problem posed when multi-tier networks are deployed, hence cannot be used in practical realistic multi-tier networks such as HetNet. Which have more significant inter-cell interference (ICI) situations, different propagation characteristics, different cell selection procedures and different BSs power classes. We affirm that the major difficulty in RA facing HetNet is the issue of co-channel interference which degrades the performance of HetNets when UEs are served in parallel, for HetNet systems using space division multiple access (SDMA) in each cell and cooperation among coordinating BSs. Recently, RA has been investigated for different networks. In [30] and [31] RA, were investigated for the uplink of orthogonal frequency-divison multiple access (OFDMA) networks and two-cell networks respectively. However, in our work, we are interested in achieving the RA for downlink HetNet that utilizes SDMA. Furthermore, in [32–34] the RA utility function is geared towards achieving energy efficiency in HetNet. However, in this work, we differ from the aforementioned reviewed papers in the sense that our RA optimization problem is geared towards achieving spectral efficiency but also constrained the total power at each transmitter to different given values to enable energy efficiency. Furthermore, their resource allocation is done by fixed BSs without considering clustering, which in practice will reduce the improvements they claimed are achievable by their work because of the regular change of the HetNet topology. In contrast, we determine the optimal number of interfering BSs that causes significant interference to each UE based on its point of view. These interfering BSs together with the serving BS of the interfered UE will coordinate and make RA decisions together to mitigate interference and thereby improving the achievable throughput in HetNet.

1.2 Contributions

In this paper, we propose a UE-centric clustering scheme that can determine the optimal interfering BSs that cause significant interference to each UE in HetNet. Afterwards, these interfering BSs coordinate with the serving BSs of the interfered UEs to make resource allocation decisions such as allocating spatial directions and powers to UEs in HetNet to mitigate interference and improve UE performance. The specific methodology for selecting these interfering BSs among all other BSs in the system is as follows. Foremost, each UE measures the interfering signal power from a subset of the interfering BSs, if the interfering signal power sensed by it is less than or equal to the noise power it will not be considered as significant, hence will be regarded as negligible and modeled as noise. However, if the sensed interfering signal power is greater than the noise power then it informs its serving BS. The serving BS will now select the n-tuple interfering BSs that will cause the aggregate highest interference to this UE based on the information it receives. The serving BS for each of the UEs will now make resource allocation decisions with these interfering BSs to mitigate interference by allocating spatial directions and powers to UEs in the system.

The aim of our RA is to allocate powers and spatial directions to UEs in the system in order to maximize the system sum-rate while satisfying powers, QoS and interference constraints.

The rest of this paper is organized as follows. In Section 2 we present the system model while a new UE-clustering scheme is presented in Section 3. Section 4 presents the RA problem formulation, which is readily split into spatial direction and power resource allocation optimization problems respectively and how they are solved. In Section 5 we summarise the branch and bound method which gives global optimum solutions for the NP-hard non-convex weighted sum-rate maximization problem. Simulation results are provided in Section 6, and the conclusions are given in the last section. Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $||\cdot||_2$ denotes the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the statistical expectation over a random variable. We use upper-case boldface letters for matrices and lower-case boldface for (column) vectors and either upper-case or lower-case letters without boldface for scalars.

2 System Model

We consider the downlink of a two-tier HetNet as depicted in Fig. 1*, which consists of K_p pico cells and K_m macro cells making it a total of K_t cells in the system. We assume that all cells in the HetNet use the same carrier frequency, note that this is not the case in orthogonal frequency-division multiplexing (OFDM) systems. The jth BS is denoted BS$_j$ which can be any of the BSs (PBS or MBS) and is assumed to have N antennas with which it communicates with at least one active UE per cell which is assumed to have a single antenna†. The set of UEs served by BS$_j$ is denoted by $\mathcal{S}_j \subset \{1,\ldots,K_r\}$, where K_r denotes the total number of UEs in HetNet, also the kth UE is denoted UE k. While the selected n-tuple BSs that interferes UE k is denoted by $\mathcal{C}_n^k$. The main system parameters are listed in Table 1. Note that macro-pico HetNet scenario is preferred in this work to macro-femto HetNet scenario. Because coordination among BSs will be much easier due to the connecting backhaul link, which uses fibre optical link whereas the macro-femto utilizes internet connection.

The complex-baseband received signal at UE k is $y_k \in \mathbb{C}$ and given by

$$y_k = \sum_{j=1}^{K_t} \sqrt{g_{j,k}}(\mathbf{h}_{j,k}^s)^H \mathbf{x}_j + z_k, \tag{1}$$

where $\sqrt{g_{j,k}}$ is the large-scale path-loss from BS$_j$ to UE k. Also $\mathbf{h}_{j,k}^s \in \mathbb{C}^N$ is the small-scale frequency-flat fading channel vector from BS$_j$ to UE k, while $\mathbf{x}_j \in \mathbb{C}^N$ is the data signal vector transmitted at BS$_j$ and intended for it served UEs. Furthermore, $z_k \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex Gaussian, distributed as $z_k \sim \mathcal{CN}(0,\sigma^2)$, where σ^2 is the noise power. Note that the above model seems to assume perfect symbol-to-symbol synchronization, this assumption will not be valid and can be removed in OFDM systems. Assuming BS$_l$ is the serving BS of UE k, the received signal at UE k in (1) can be rewritten as

$$y_k = \mathbf{h}_{l,k}^H \mathbf{w}_k s_k + \mathbf{h}_{l,k}^H \sum_{p \in \mathcal{S}_l, p \neq k} \mathbf{w}_p s_p + \sum_{\substack{j \in \mathcal{C}_n^k \\ j \neq l}} \mathbf{h}_{j,k}^H \sum_{\substack{m \in \mathcal{S}_j \\ m \neq k}} \mathbf{w}_m s_m + z_k, \tag{2}$$

*Note that the number of pico cells considered for each macro-cell is not limited to one, as suggested by Fig. 1 but for clarity we just showed a simplified schematic representation of our considered model. In our simulation, the total number of pico cells considered will be stated.

†We limit each UE to have a single antenna for practical reasons, such as, reducing the UE hardware complexity and also preserving of battery life.

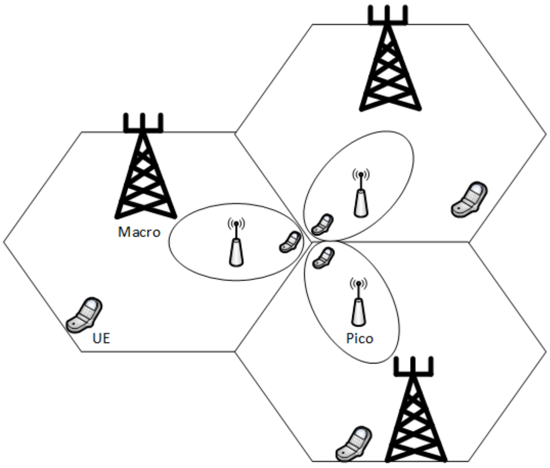

Fig. 1: Downlink two-tier HetNet model with overlaid pico cells in the coverage area of MBS,

where $\mathbf{h}_{j,k} \triangleq \sqrt{g_{j,k}}\mathbf{h}^s_{j,k}$, also the transmitted data signal vector is a linear function of the symbols, i.e., $\mathbf{x}_j = \sum_{p\in\mathcal{S}_j} \mathbf{w}_p s_p$, where $\mathbf{w}_p$ denotes the transmit beamformers for each symbol s_p. The first summand of (2) is the desired signal transmitted to UE k while the second and third summands represent the intra-cell interference caused by co-channel UEs within the same BS and the inter-cell interference caused by co-channel UEs in neighbouring BSs respectively.

For a HetNet that uses universal frequency reuse one deployment, the important issues that needs to be addressed are:

- **issue 1**: how to identify the dominant inter-cell interference from BSs in HetNet to UE k. In other words, which BSs should be selected among the possible n-tuple BSs that interferes UE k the most. Any BS whose interference power towards UE k is less than or equal to the noise power is regarded as negligible interference, hence will not be considered for coordination.
- **issue 2**: How to jointly design the transmit beamformers that will spatially separate the transmitted signal vector from the interfering BSs in order to avoid interference towards UE k. Note that this interfering BSs are not fixed but selected for UE k by solving issue 1.

3 UE-Centric Clustering

In this section we try to resolve issue 1. We provide solution to it by finding an optimal BS subset that will give the aggregate largest interference to UE k at a given time slot. We now write an abridged expression of (2) to show only the summation of inter-cell interference signals.

$$int_{sig} = \sum_{\substack{j\in\mathcal{C}^k_n \\ j\neq l}} \mathbf{h}^H_{j,k}\mathbf{x}_j. \tag{3}$$

The inter-cell interference power corresponding to (3) can be represented by

$$int = \sum_{\substack{j\in\mathcal{C}^k_n \\ j\neq l}} |\mathbf{h}^H_{j,k}\mathbf{x}_j|^2. \tag{4}$$

Let $\{int^k_n\}_{k\in\mathcal{S}_l}$ denote the set of all aggregate inter-cell interference power calculated from n-tuple BSs interfering UE k with $n \leq K_t$.

TABLE 1 KEY PARAMETERS

K_p	Total number of PBS in HetNet.
K_m	Total number of MBS in HetNet.
K_t	Total number of BSs in HetNet, ($n \leq K_t$).
BS_j	The jth BS.
$\mathcal{S}_j$	The set of UEs served by BS_j.
N	Total number of transmit antenna at PBS or MBS.
K	Total number of active served UEs in each cell.
$\sqrt{g_{j,k}}$	The large-scale pathloss from BS_j to UE k.
$\mathbf{h}^s_{j,k}$	The small scale (fading) channel vector from BS_j to UE k.
$\mathbf{x}_j$	The data signal vector transmitted at BS_j and intended for its served UEs.
$\mathcal{C}^k_n$	The selected n-tuple BSs that interferes UE k.
$\mathcal{U}_n$	The collection of all possible n-tuple BS subsets.
$\mathbf{R}_{j,k} \succeq \mathbf{0}$	Means $\mathbf{R}_{j,k}$ is a positive semi-definite matrix.
K_r	Total number of UEs in HetNet.
σ^2	Noise Power .
τ_p	Limit of interference power at UE p.
q_j	Power limit at BS_j.

It is important to note that for a system that comprises of K_t BSs as shown in Fig. 1, there are altogether 2^{K_t} possible BS subsets. Let $\mathcal{U}_n$ represent the collection of all possible n-tuple BS subsets in HetNet. The optimal BS subset that will maximize the interference suffered by UE k can be expressed as

$$\mathcal{C}^{k*}_n = \arg\max_{\mathcal{C}_n\in\mathcal{U}_n} int^k_n \;\; \forall k. \tag{5}$$

To be able to find the optimal number of BSs in the optimal BS subsets that will cause the highest interference to UE k, we determine that through the following expression:

$$l_n = \max_{\mathcal{C}_n\in\mathcal{U}_n} int^k_n, \tag{6}$$

where l_n denote the maximum value of the interference generated to UE k by n-tuple BSs. Accordingly, the serving BS to UE k can choose the optimal number of interfering BSs that it will coordinate with based on l_n. This can be expressed as

$$n_{opt} = \arg\max_{n=1,\ldots,K_t} l_n. \tag{7}$$

However, it involve finding $\mathcal{C}^{k*}_n$ using (5) and l_n using (6) for each n before selecting the optimal one using (7).

The optimal interfering BS set for UE k is easily found as $\mathcal{C}^{k*}_{n_{opt}}$ and the optimal number of interfering BSs that needed to coordinate interference with the serving BS of UE k is n_{opt}. Consequently, the signal received by UE k after identifying its dominant inter-cell interferers is given by

$$y_k = \mathbf{h}^H_{l,k}\mathbf{x}_l + \sum_{j=1, j\in\mathcal{C}^{k*}_{n_{opt}}}^{n_{opt}} \mathbf{h}^H_{j,k}\mathbf{x}_j + z_k, \tag{8}$$

furthermore, the achievable data rate for UE k in beamforming terms, with s_k normalized to unit power, can also be expressed as

$$r_k = log_2\left(1 + \frac{|\mathbf{h}^H_{l,k}\mathbf{w}_k|^2}{\sigma^2 + \sum_{p\in\mathcal{S}_l}|\mathbf{h}^H_{l,k}\mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}}\sum_{m\in\mathcal{S}_j}|\mathbf{h}^H_{j,k}\mathbf{w}_m|^2}\right). \tag{9}$$

For a particular selected BS subset, the received signal y_k in (8) suffers from the highest significant inter-cell interference that exist in the system and peculiar to UE k. The corresponding achievable data rate r_k will diminish if these interference sources are not mitigated. Note that if a significant interference source to UE k is not identify and dealt with, it will hinder the performance of UE k.

Next Section presents how we resolve issue 2 through RA to make sure that these interference sources are dealt with effectively.

4 Resource Allocation

In this section, the serving BS of UE k will make RA decisions together with the selected BS subset that causes interference to UE k. The implementation of this RA needs to be done centrally. Note, RA problems can be formulated in many different ways to suit the desires or objectives of the system designer. For example, if the objective of the system designer or operator is to maximize the throughput for the worst served UE, then max-min based RA optimization will be the right way to tackle that. Furthermore, if the system designer wants to achieve a maximal throughput, while ensuring that none of the UEs are starving, proportionality based RA could be good for it. Also, if the aim is to achieve the maximal aggregate throughput of the system, then some of the system resource parameters such as high transmit powers will be allocated to those UEs whose channels have high signal to noise ratios (SNRs), while little or no powers will be allocated to UEs with attenuated channel gain. All the aforementioned RA optimization procedures have some advantages and disadvantages in terms of improving system utility and/or individual UE performance. Depending on the RA procedure adopted, there are two major consequences. Firstly, it will define the balance between performance of the system utility and that of each UE in the system. Secondly, it will also determine the extent of computational complexity involve in solving the RA problem. In this paper, we seek to achieve the fundamental trade-off between maximizing the spectral efficiency of HetNet and achieving a minimum performance level for all UEs in the system. This decision is motivated by the poor individual performance of UEs located at the cell range expansion (CRE)[38] area of pico cells in a macro-pico HetNet scenario.

4.1 Problem formulation

Our target is to select $\{\mathbf{w}_k\}_{k=1}^{K_r}$ to maximize the weighted sum-rate, while fulfilling some power, quality of service (QoS) and interference constraints (IC) [36, 37] respectively. It is important to note that the individual rate r_k is a function of the signal-to-interference-and-noise-ratio ($SINR_k$). And the optimal interfering BS set $\mathcal{C}_{n_{opt}}^{k^*}$ that affects r_k has been used to determine $SINR_k$ as expressed in (9). We therefore, formulate the optimization problem as

$$\begin{aligned}
\underset{\{\mathbf{w}_k\}}{\text{maximize}} \quad & \sum_{k=1}^{K_r} u_k r_k(\{\mathbf{w}_k\}) \\
\text{subject to} \quad & C1: SINR_k \geq \gamma_k \quad k = 1, \ldots, K_r, \\
& C2: \sum_{k \in \mathcal{S}_s} ||\mathbf{w}_k||_2^2 \leq q_s \quad s = 1, \ldots, K_p, \\
& C3: \sum_{k \in \mathcal{S}_m} ||\mathbf{w}_k||_2^2 \leq q_m \quad m = 1, \ldots, K_m, \\
& C4: \sum_{k \in \mathcal{S}_m} \mathbf{w}_k^H \mathbf{R}_{m,p} \mathbf{w}_k \leq \tau_p \quad \forall p \in \mathcal{S}_s.
\end{aligned} \tag{10}$$

Where the utility function represents the weighted sum-rate of the system with the non negative factor u_k denoting the individual weight assigned to each UE, chosen to reflect the different level of concern about the individual channel gains. Also constraints $(C1 \sim C4)$ represent the desired quality of service constraint, with γ_k denoting the QoS threshold for UE k; PBS power constraint, MBS power constraint and interference power constraint (i.e., interference generated from MBS to UE p) respectively. $\mathbf{R}_{m,p} \triangleq \mathbf{h}_{m,p}\mathbf{h}_{m,p}^H$ is a positive semidefinite (PSD) matrix ($\mathbf{R}_{m,p} \geq \mathbf{0}$), where $\mathbf{h}_{m,p}$ is the channel vector from the MBS to UE p and τ_p is the non negative threshold which controls the allowable level of interference at UE p. Note, that by adding the IC constraint in (10), we aim to shape the transmission from the MBS in order to control the significant interference to UEs served by PBS.

Maximizing the weighted sum-rate of HetNet under some given constraints, as expressed in $(C1 \sim C4)$ is generally regarded as a non-convex non-deterministic polynomial-time hard (NP-hard) problem because there are no known efficient algorithms that can solve it in polynomial time. However, this intractable problem can be solved by computer algorithms that run in exponential-time such as branch and bound (B&B) algorithms [39], which can give global optimal solutions. B&B algorithms can only be considered for small scale problems, i.e. problems with very small problem size, because their running times are exponential functions of their problem sizes. Note, the problem size in this paper is regarded to be the number of variables and constraints involved in the optimization problem. To pinpoint the actual cause of non-convexity of the resource allocation optimization problem of (10), let's analyse each function that make up the resource allocation problem: firstly, the utility function in (10) is a concave function which can be maximized, though it depends on the SINRs of UEs in the system. The power constraint functions in $C2 \sim C3$ together with the MBS interference power constraint function in $C4$ are all convex functions. The SINR constraint function in $C1$ is a non convex function of beamforming vectors $\{\mathbf{w}_k\}_{k=1}^{K_r}$, which cannot be classified as a semidefinite constraint or second-order cone constraint. In order to make the constraint convex, $SINR_k \geq \gamma_k$ can be expressed as [41]

$$\frac{1}{\gamma_k}|\mathbf{h}_{l,k}^H\mathbf{w}_k|^2 \geq \sum_{p \in \mathcal{S}_l, p \neq k} |\mathbf{h}_{l,k}^H\mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m \in \mathcal{S}_j} |\mathbf{h}_{j,k}^H\mathbf{w}_m|^2 + \sigma^2, \tag{11}$$

we note that the absolute values in (11) make $\mathbf{w}_k$ and $e^{j\theta_k}\mathbf{w}_k$ equivalent for any common phase rotation $\theta_k \in \mathbb{R}$, hence we exploit this phase ambiguity to rotate the phase such that $\mathbf{h}_{l,k}^H\mathbf{w}_k$ is real-valued and positive. This insinuate that $\sqrt{|\mathbf{h}_{l,k}^H\mathbf{w}_k|^2} = \mathbf{h}_{l,k}^H\mathbf{w}_k \geq 0$. Therefore, $SINR_k \geq \gamma_k$ can now be rewritten as

$$\frac{1}{\sqrt{\gamma_k}}\Re(\mathbf{h}_{l,k}^H\mathbf{w}_k) \geq \sqrt{\sum_{\substack{p \in \mathcal{S}_l \\ p \neq k}} |\mathbf{h}_{l,k}^H\mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m \in \mathcal{S}_j} |\mathbf{h}_{j,k}^H\mathbf{w}_m|^2 + \sigma^2}, \tag{12}$$

where $\Re(\cdot)$ denotes the real part. The γ_k value at each UE needs to be fixed and we assume these values to be known a $priori$ but can be computed as $\gamma_k \triangleq 2^{r_k} - 1$ obtainable from (9). Therefore, the SINR constraint in (10) can now be classified as a second-order cone constraint, which is a convex type constraint [42].

We are interested in producing approximate solutions, that are feasible in practice for large scale problems, consequently, we seek to solve the non-convex problem using convex heuristics approach.

Our RA problem in (10) is centralized and the optimization variable is the transmit beamformers. Note that the properties of this transmit beamformers include both the spatial characteristic and the corresponding transmission powers. Recall that the aim of our RA is to allocate powers and spatial directions to UEs in the system in order to maximize the system sum-rate while satisfying power, QoS and interference constraints. Having said that, we therefore readily split (10) into two sub-problems. The first problem is formulated as a spatial direction allocation problem, while the second is formulated as a power allocation problem. The former needs to be solved centrally while the latter will be solved in a decentralized manner because HetNet is naturally distributed. This technically means that the RA problem in (10) is decomposed into two sub problems, giving more freedom to each BS to determine the performance power level for each served UE.

4.2 Spatial Direction Allocation Problem

The spatial direction allocation problem is expressed as

$$\begin{aligned} \tilde{\mathbf{w}}_k = \underset{\{\mathbf{w}_k\}_{k=1}^{K_r}}{\text{argmax}} \quad & \sum_{k=1}^{K_r} u_k r_k(\{\mathbf{w}_k\}) \\ \text{subject to} \quad & C1: \frac{1}{\sqrt{\gamma_k}} \Re(\mathbf{h}_{l,k}^H \mathbf{w}_k) \geq \Gamma_k, \\ & C2 \sim C4 \; in \; (10), \\ & C5: ||\mathbf{w}_k||^2 = 1 \;\; k = 1, \ldots, K_r, \end{aligned} \tag{13}$$

where $\Gamma_k = \sqrt{\sum_{p \in \mathcal{S}_l, p \neq k} |\mathbf{h}_{l,k}^H \mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m \in \mathcal{S}_j} |\mathbf{h}_{j,k}^H \mathbf{w}_m|^2 + \sigma^2}$. To solve (13) efficiently we use SeDumi [43], which is a general purpose implementation of interior point method, with CVX [44], providing a Matlab based modelling platform for it. Therefore, the unit-norm beamformers or spatial directions of the system are $\{\tilde{\mathbf{w}}_1, ..., \tilde{\mathbf{w}}_{K_r}\}$.

Next Section presents how we design the optimal transmit power allocated to each UE in each cell to improve UE performance and maximize the sum-rate of HetNet.

4.3 Power Allocation Problem

Since the major interference problem has been tackled* in the previous section by designing unit-norm beamformers $\{\tilde{\mathbf{w}}_1, ..., \tilde{\mathbf{w}}_{K_r}\}$ that will spatially separate data symbols when transmitting to UEs. Any negligible interference in the system will be modelled as part of the background noise. What is left to be done is to select the power allocation coefficient $\{p_k\} \forall k \in \mathcal{S}_j$ which will act as optimum scale factors to each spatial directions $\{\tilde{\mathbf{w}}_k\} \forall k \in \mathcal{S}_j$ in order to maximize the SE of the system as well as satisfying each UE with a minimum performance level. We proceed by formulating our power resource allocation problem as

$$\begin{aligned} \underset{\{p_k\} \forall k \in \mathcal{S}_j}{\text{maximize}} \quad & \sum_{k \in \mathcal{S}_j} log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2}\right), \\ \text{subject to} \quad & \sum_{k \in \mathcal{S}_j} p_k \leq q_j, \\ & log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2}\right) \geq R_k \; \forall k \in \mathcal{S}_j, \\ & p_k \geq 0 \;\; \forall k \in \mathcal{S}_j. \end{aligned} \tag{14}$$

Where R_k denotes the minimum required data rate for UE k to have good quality of experience (QoE). One can easily observe that the power RA problem in (14) is a convex optimization problem [35, 45], because the utility function is a concave function while the constraint functions are: convex function, concave function and concave function respectively. Hence, the global power solution can be obtained efficiently using CVX, a package for specifying and solving convex programs. For fairness in this power RA formulation to be achieved, this hard constraint $log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2}\right) \geq R_k$ needs to be active. In some cases it is not but it all depends on how large this threshold R_k is.

We summarized the resource allocation procedure in this paper using Algorithm 1.

We note that this proposed power allocation scheme will be optimal for transmit strategy utilizing zero-forcing method. However, we also found out that forcing zeros may also cause a distorted beam pattern with high side lobes which can lead to increase in the background interference level in the system.

Algorithm 1 Allocation of spatial directions and powers for each UE in two-tier HetNet

Input and variables
$\mathcal{S}_j$: set of UEs served by BS$_j$;
K : total number of UEs in each cell;
procedure
1: **for** UEs $\in \mathcal{S}_j$ i.e. k = 1 to K **do**
2: compute the unit-norm beamformers $\tilde{\mathbf{w}}_k$ using (13);
3: compute $p_k \forall k \in \mathcal{S}_j$ from using (14) and;
4: **end for**
BS$_j$ transmits $\mathbf{x}_j = \sum_{k \in \mathcal{S}_j} \sqrt{p_k} \tilde{\mathbf{w}}_k s_k$

Branch and Bound method will be introduced in our next section. This method gives global optimal solutions for NP-hard, non-convex, weighted sum-rate optimization problem in HetNet, we aim to compare our heuristic method to it.

5 Branch and Bound method

Branch and Bound (B&B) method [40] is the method through which we can get global optimal solution of an NP-hard intractable non-convex weighted sum-rate maximization problem for a two-tier HetNet. It is an iterative method that requires at least two procedures that can efficiently calculate and improve a lower bound (f_{min}) and an upper bound (f_{max}) on the optimal value of the non-convex problem over a given set or region. In our case, the set or region considered is a subset of a box (K_r-dimensional) interval, [**a b**]. This set is the feasible set that satisfies our problem formulation in (10). Also the utility function in our optimization problem is Lipschitz*continuous and monotonically increasing over this box interval. The Lipschitz constant will provide limit on how the fast the function varies. We denote the initial box as $\mathcal{B} = [\mathbf{a} \;\; \mathbf{b}] \subseteq \mathbb{R}_+^{K_r}$, this box is assumed to be compact† and normal‖ [46] and houses all kind of rates from the worst to the best rates. Furthermore, **a** denotes the worst rate vector achievable by UEs in the system thus $\mathbf{a} = \mathbf{0} \in \mathbb{R}_+^{K_r}$ while $\mathbf{b} \in \mathbb{R}_+^{K_r}$ is the best rate vector achievable by UEs in the system using egoistic beamforming [47] scheme such that $\mathbf{a} < \mathbf{b}$. Also [**a b**] is defined to be the set of all rates ($\mathbf{r} \in \mathbb{R}_+^{K_r}$) achievable in the system such that $\mathbf{a} \leq \mathbf{r} \leq \mathbf{b}$. Egoistic beamforming is a beamforming scheme where transmit beamformers are designed to maximize the array gain of a single UE in a system. Note this beamforming scheme will always be suboptimal if there are other sources of interference, hence

$$b_k = log_2 \left(1 + \frac{p_k |\mathbf{h}_{k,l}^H \tilde{\mathbf{w}}_\mathbf{k}|^2}{\sigma^2}\right) \; k = 1, \ldots, K_r, \tag{15}$$

where $\mathbf{b} = [b_1 \ldots b_{K_r}]^T$. p_k is the transmit power constraint at each transmitter. The egoistic transmit beamformers can be obtained using

$$\tilde{\mathbf{w}}_k = \arg \max_{\substack{\mathbf{w}_k \in \mathbb{C}^{N \times 1} \\ ||\mathbf{w}_k||^2 = 1}} |\mathbf{h}_k^H \mathbf{w}_k|^2. \tag{16}$$

This best rate vector $\mathbf{b} = [b_1 \ldots b_{K_r}]^T$ is not always feasible when co-channel interference is considered in the system while designing the beamformers.

Our feasible set from the original RA problem formulation in (10)

A function $f : [\boldsymbol{a} \; \boldsymbol{b}] \to \mathbb{R}$ is said to be Lipschitz continuous with Lipschitz constant L_f, if $|f(\mathbf{r}) - f(\hat{\mathbf{r}})| \leq L_f ||\mathbf{r} - \hat{\mathbf{r}}||_1$, $\forall \, \mathbf{r}, \hat{\mathbf{r}} \in [\boldsymbol{a} \; \boldsymbol{b}]$ and $\mathbf{r} \geq \hat{\mathbf{r}}$
†*A compact set, intuitively can be described as an interval set, bounded and having the elements of the set close to each other.*
‖*A set $\mathcal{M} \subset \mathbb{R}_+^K$ is set to be normal if, for any two points $\mathbf{x}, \hat{\mathbf{x}} \in \mathbb{R}_+^K$ such that $\hat{\mathbf{x}} \leq \mathbf{x}$, if $\mathbf{x} \in \mathcal{M}$, then $\hat{\mathbf{x}}$, too.*

for the r_k that optimizes the sum-rate can be denoted as

$$\mathcal{Z} = \Big\{ \Big(r_1(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}), \ldots, r_{K_r}(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) \Big) : (\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) \in \mathcal{W} \Big\}. \quad (17)$$

Where $\mathcal{W}$ is the set of feasible transmit beamforming vectors:

$$\mathcal{W} = \Big\{ (\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) : \sum_{k \in \mathcal{S}_s} ||\mathbf{w}_k||_2^2 \le q_s, \quad \sum_{k \in \mathcal{S}_m} ||\mathbf{w}_k||_2^2 \le q_m \quad m = 1, \ldots, K_m, \sum_{k \in \mathcal{S}_m} \mathbf{w}_k^H \mathbf{R}_{m,p} \mathbf{w}_k \le \tau_p \;\; \forall p \in \mathcal{S}_s \Big\}. \quad (18)$$

Note that $\mathcal{Z}$ denotes the set of all feasible solution $(r_1, \ldots, r_{K_r})$ for which $(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r})$ are feasible and satisfy $C2 \sim C4$ in (10). Therefore, our optimization problem for maximizing the sum-rate of the system in this section is similar to searching for a feasible solution in the box that has the minimum Euclidean distance to $\mathbf{b}$, and this is formulated as

$$\begin{aligned} &\underset{\{\mathbf{r}\}}{\text{maximize}} && f(\mathbf{r}) \\ &\text{subject to} && \mathbf{r} \in \mathcal{Z}. \end{aligned} \quad (19)$$

Note that our utility function in (19) is given as

$$f(\mathbf{r}) = \sum_{k=1}^{K_r} u_k r_k(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}), \quad (20)$$

where $\mathbf{r} = [r_1 \ldots r_{K_r}]^T$ is the rate vector achievable by UEs in the system, also, that $\mathcal{Z} \subseteq [\mathbf{a}\ \mathbf{b}]$. The lower bound on the optimal value of the non-convex problem can be found from its convex reformulation, and in this paper, by removing C5 and changing the argmax term to maximize in (13), we will be able to determine the lower bound on the optimal value of (19). Let $\hat{\mathbf{r}}$ denotes the feasible solution of the box $\mathcal{B}$, that is used to obtain the lower bound on the optimal value. Hence we denote the lower bound on the optimal value of this box as $f^{\mathcal{B}}_{min} = f(\hat{\mathbf{r}})$. Similarly, since $\mathbf{b}$ represents the best rate vector in the system, though might not be feasible, we denote the upper bound on the optimal value of this box as $f^{\mathcal{B}}_{max} = f(\mathbf{b})$. Hence, $f^{\mathcal{B}}_{min} \le f_{opt} \le f^{\mathcal{B}}_{max}$, where f_{opt} represents the optimal value of the sum-rate of the system, f_{min} and f_{max} denote lower bound and upper bound on the optimal value of the weighted sum-rate of the system respectively. Similarly, $\hat{\mathbf{r}} \le \mathbf{r}_{opt} \le \mathbf{b}$, where $\mathbf{r}_{opt}$ denotes the optimal solution of the system while $\hat{\mathbf{r}}$ and $\mathbf{b}$ denote a local feasible solution and the best solution achievable in the system.

Branching involves the process of splitting the initial box $\mathcal{B}$ into more than one partitions provided that $f^{\mathcal{B}}_{max} - f^{\mathcal{B}}_{min} > \epsilon$. Where $\epsilon > 0$ is the accuracy tolerance of the sum-rate in the B&B method while bounding involves computing for the lower and upper bound on the optimal value for each of the partitions, for the purpose of improving a lower bound $f^{\mathcal{B}}_{min}$ and an upper bound $f^{\mathcal{B}}_{max}$ on the optimal value. The complete algorithm that solves the problem can be seen in one of our work [40]. It is important to note that B&B method will produce better sum-rate than the proposed convex-heuristic method because of the following reasons. Firstly, the convex heuristic method had to reduce the search space for the optimization problem by including the QoS constraint, this constraint will limit the degree of freedom (DoF) for selecting beamforming vectors. Whereas the B&B method did not consider the QoS constraint as one of its constraints, hence having a larger search space to select the optimum solution from. Furthermore, the convex heuristic solution is utilized as the starting point of the B&B algorithm. However, the trade-off for such performance will be in the complexity of the algorithm to search for the optimum solution.

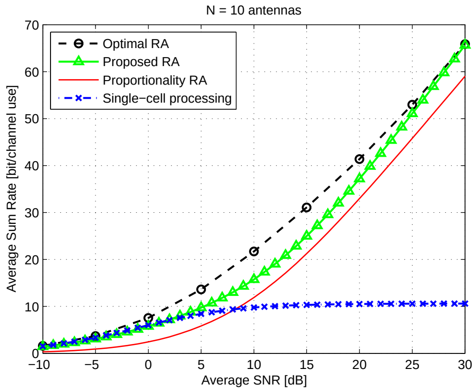

Fig. 2: Average sum-rate as a function of SNR for different RA implementation.

6 Simulation Results

In this section, we evaluate the performance of our proposed RA methods by comparing with the global optimal method and other existing RA methods based on the average achievable sum-rate, SNR, number of transmit antennas and computational complexity.

6.1 Simulation Settings

We consider a simple simulation setting with minimum of five randomly distributed PBSs deployed at hotspot locations in the coverage area of MBS. The minimum distance among pico sites is set to 40m, and we assume that all PBSs are not geometrically separated, hence interference among PBS is possible and therefore considered. The minimum distance from the macro site to the pico sites is 75m. We assume that the UEs in the HetNet are uniformly distributed and are located at the CRE such that each UE will receive significant inter-cell interference (ICI). Note we concentrate on UEs at the CRE because they suffer from both signal attenuation from their serving BS as well as inter-cell interference from neighboring cells. The UEs served by PBS are uniformly distributed between 35m and 55m from the PBS. Similarly, the UEs served by MBS are uniformly distributed between 220m and 260m from the MBS, also, the distance between the macro cell UEs and the PBS is roughly between 40m and 45m, while the distance between the pico cell UEs and the MBS is between 230m and 270m. Other system parameters are also based on the 3GPP simulation baseline parameters and can be found in [48]. The total BS transmit powers for MBS and PBS are 46dBm and 30dBm respectively, assuming a 10MHz bandwidth. The channel vector between BS$_j$and UE k is modelled as $\mathbf{h}_{j,k} \triangleq \sqrt{g_{j,k}}\mathbf{h}^s_{j,k}$, where $\sqrt{g_{j,k}}$ is the large-scale path-loss from BS$_j$ to UE k, also $\mathbf{h}^s_{j,k} \in \mathbb{C}^N$ is the small scale (fading) channel vector from BS$_j$ to UE k, and the large scale path-loss in linear scale is expressed as

$$g_{j,k} = \frac{\psi}{d^n_{j,k}}, \quad (21)$$

where ψ is a constant which accounts for system losses, n is the path-loss exponent, typically $n > 3$, while $d_{j,k}$ is the distance between BS$_j$ and UE k. The large-scale path loss model in dB for the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{j,k}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{j,k}}{10^3})$. This simulation settings will be used except otherwise indicated.

In Fig. 2, we show the average sum-rate achievable as a function of SNR. It compares the average sum-rate achieved in the system using our proposed method, the optimal RA method, proportionality RA method and the single-cell processing RA method. Note,

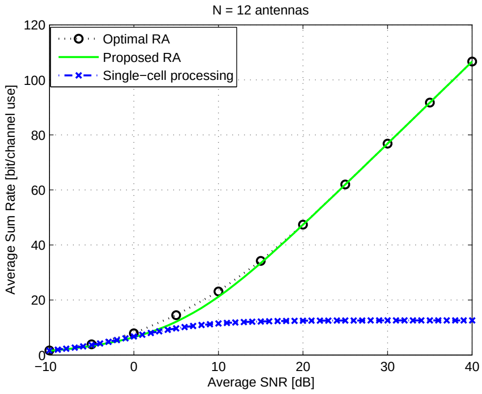

Fig. 3: Average sum-rate achievable at different SNR for $N = 12$, $K_r = 9$.

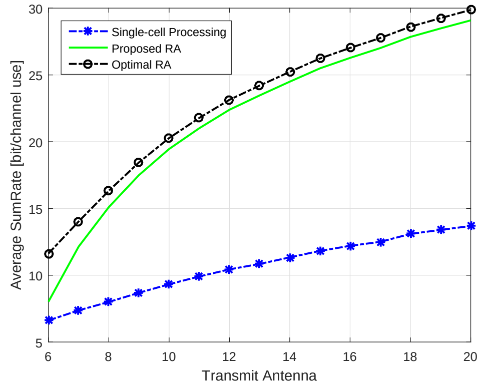

Fig. 4: Average sum-rate achievable at different transmit antennas for $SNR = 10$ dB, $K_r = 6$.

we implement both our proposed RA method and the optimal RA method using our proposed UE clustering scheme. The optimal RA method utilizes the B&B method. Our proposed method is outperformed by the B&B method whose trade off for such performance is in complexity of the B&B algorithm. The proportionality RA method performance is inferior to our proposed method because it utilized the semi-static clustering method proposed by the authors in [49] to determine the coordinating BSs that will coordinate interference to each UE. The loss in performance is due to the fact that the BSs that are selected to form cluster are semi-static hence does not always change with the changing topology of HetNet. It fails to identify the strongest inter-cell interfering BSs that affect each UE at a given time. The least performed RA method performs poorly because it only consider its served UEs while designing the beamformers without coordination with other BSs in the system. Furthermore, it model any out-of-cell interference in the system as part of the background noise.

In Fig. 3, we show that the performance of our proposed method improves as $N = 12$ transmit antennas while the B&B only slightly outperforms it at low SNR. It goes ahead to prove that our proposed method though suboptimal is asymptotically optimal as N increases. Note, that increase in the number of transmit antenna is one of the factors that improves the beamforming resolution for our proposed method. It also helps to improve the diminishing signal power due to interference cancellation. Furthermore, Fig. 4, shows that as N increases it helps in getting better spatial directions that will improve the performance of the system due to increase in degree of freedom (DoF).

In Fig. 5, we show the effect of the interference threshold $\tau \in [0, \dots, 1]$ on the average sum-rate of HetNet. The performance of our proposed RA method, the optimal RA method and the single-cell processing RA method are compared when the interference threshold τ is varied. These methods suffer rate loss as τ increases. The proposed method and B&B method perform best when the allowable interference from the MBS to UEs served by PBS in the system is $\tau = 0.1$. The single-cell processing (no-cooperation) method starts well at $\tau = 0.1$ but suffers consistent rate loss than our proposed method and the global optimal method.

In Fig. 6 we compare the performance of our proposed RA method with our proposed method in the conference paper [50]. We achieve this by increasing the number of pico cells in each macro cell to ten. Afterwards, our clustering scheme enables cell splitting gain by dealing with the largest aggregate interference affecting each UE in the HetNet. Whereas the proposed method in the conference paper, without the clustering scheme performs inferior to that of the journal, because the selected number of interfering BSs though the same, however do not account for the largest aggregate interference affecting each UE.

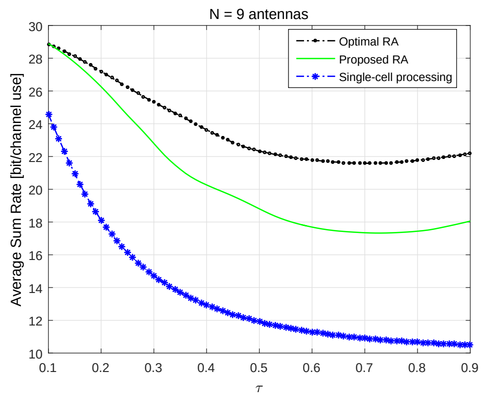

Fig. 5: Effect of the interference threshold τ on the sum-rate of HetNet for $N = 9$, and $SNR = 15$ dB.

In Fig. 7, using our proposed RA, we want to evaluate the impact of adding more pico cells in the coverage area of the MBS will have on the spectral efficiency of the considered HetNet. For each pico cell considered, we assummed that four UEs are served by the PBS while the MBS serve only two active UEs. Fig. 7 compares the downlink cell spectral efficiencies of macrocell with pico cells, where the number of pico cell is increasing. The first observation is that the deployment of the pico cells in the coverage area of the MBS does not affect the performance of the macro cell. Secondly, the second bar in Fig. 7 depicts cell-splitting gain provided by the increment in the number of pico cells. Lastly, the cell splitting gain cannot be said to be a linear function of the number of pico cells due to the effect of channel gain but can be said to close. Furthermore, this shows that our proposed RA method helps in managing interference.

In B&B method, it is well known that in practice the complexity grows exponentially in order t^n, where n is the problem size (input size) and t is just a constant [51]. In Fig. 8, we use a simple scenario to show how different input size configurations give rise to varying order of complexity for our proposed method and the B&B method. The number of variables, $v_a = NK_r$, where N and K_r have already been used to denote number of antennas and total number of UEs in the system. When $K_r = 3$ UEs, $N = 4$ transmit antennas, and $m = 4$ constraints (power and interference constraints), the order of complexity for our proposed method takes roughly 100 seconds to

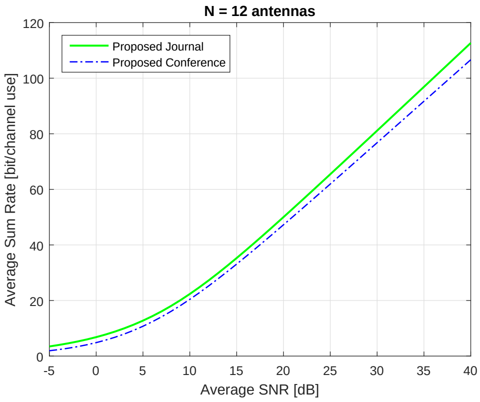

Fig. 6: Average sum-rate as a function of SNR for the Journal and Conference RA implementation

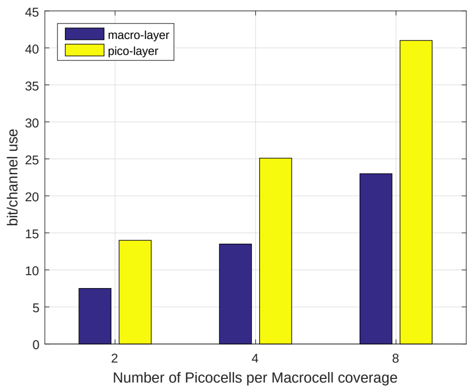

Fig. 7: Cell spectral efficiencies comprising two, four and four

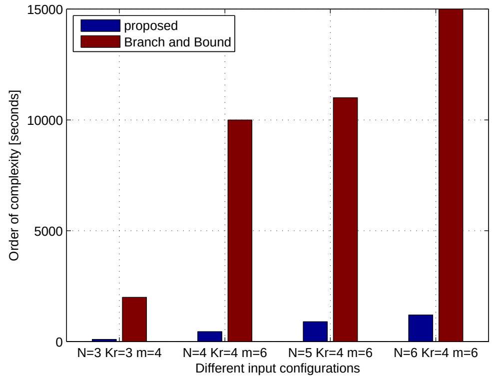

Fig. 8: Order of complexity as a function of the input size (configurations).

complete a problem size containing NK_r while that of B&B method takes 2000 seconds. Our proposed method computational complexity is polynomial in the number of UEs, transmit antennas, power and interference constraints while that of B&B method has worst case complexity that increases exponentially with the number of UEs. We cannot recommend it to be used for more than $Kr = 6$ UEs, hence should not be used for large scale real time application but can be used for small scale applications and for off-line benchmarking.

7 Conclusion

In this paper, we have developed an UE-centric clustering scheme that can effectively determine the significant interfering BSs that will cause the highest interference to each UE. Afterwards, the serving BSs for these UEs together with these selected interfering BSs will coordinate and make resource allocation decisions to allocate spatial direction to each UE in the system. Our RA strategy can be practically implemented in HetNet. The resources allocated to UEs are the spatial directions (unit-beamformers) and the power resource.

The resource allocation optimization problem for selecting spatial directions is done centrally and is formulated as an NP-hard non-convex problem, which we reformulate to a convex problem for practical implementation purposes and solved using SeDumi, which is a general purpose implementation of interior point method. While our power resource allocation scheme is decentralized and is formulated as maximizing the sum-rate of each cell while achieving a minimum performance level for each UE in the cell. The power RA problem is found to be convex and hence, can be solved efficiently using CVX (a package for specifying and solving convex programs). Results obtained show that our proposed method though suboptimal, when compared to the B&B method, which provides the global optimal solution for the non-convex NP-hard weighted sum-rate maximization problem, improves when the number of transmit antenna increases. Also, our results show that the B&B method has the worst case complexity that increases exponentially with the number of UEs, hence cannot be recommended for large-scale applications but can be used for off-line benchmarking.

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