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1 Geological discontinuity persistence: Implications and quantification

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9 **Abstract**

10 Persistence of geological discontinuities is of great importance for many rock-related
11 applications in earth sciences, both in terms of mechanical and hydraulic properties
12 of individual discontinuities and fractured rock masses. Although the importance of
13 persistence has been identified by academics and practitioners over the past
14 decades, quantification of areal persistence remains extremely difficult; in practice,
15 trace length from finite outcrop is still often used as an approximation for persistence.
16 This paper reviews the mechanical behaviour of individual discontinuities that are not
17 fully persistent, and the implications of persistence on the strength and stability of
18 rock masses. Current techniques to quantify discontinuity persistence are then
19 examined. This review will facilitate application of the most applicable methods to
20 measure or predict persistence in rock engineering projects, and recommended
21 approaches for the quantification of discontinuity persistence. Furthermore, it
22 demonstrates that further research should focus on the development of persistence

23 quantification standards to promote our understanding of rock mass behaviours
24 including strength, stability and permeability.

25 **Keywords:** Discontinuity persistence; incipient discontinuity; rock bridges;
26 geophysics; rock mass strength

27 **1. Introduction**

28 Geological discontinuities are of great importance for strength, deformability and
29 permeability of rock masses. Characterisation of discontinuity geometry (i.e.
30 aperture, persistence, length and spatial connectivity) is the first step to
31 understanding the overall behaviour of rock masses. Early references to
32 discontinuity persistence include those of Jennings (1970) and Einstein et al. (1983),
33 and the summary publication by the International Society for Rock Mechanics and
34 Rock Engineering (ISRM, 1978).

35 It is difficult to quantify true persistence due to the intrinsic three-dimensional nature
36 of discontinuities within rock masses and the number of studies that have attempted
37 to quantify this parameter has been relatively small. Some techniques have been
38 developed in recent years, for example, geophysical detection (e.g. Heike et al.,
39 2008; Deparis et al., 2011), surface terrestrial laser scanning (e.g. Sturzenegger and
40 Stead, 2009a; Tuckey and Stead, 2016) and the forensic excavation of rock masses
41 (e.g. Shang et al., 2017a). Modelling the inevitable uncertainty in the fracture
42 network is addressed in FracMan by Diershowitz and colleagues at Golder
43 Associates and by Monte-Carlo simulation (e.g. Wang et al., 2016)

44 The purpose of this paper is to consider the implications of discontinuity persistence
45 on the mechanical properties of individual discontinuities, strength and stability of

46 rock masses and to review the available techniques to quantify this parameter.
47 Several recommendations for future research are included in this paper.

48 **2. Definition**

49 **2.1 Incipient and mechanical geological discontinuities**

50 Geological discontinuity is normally recognised as a general term to describe any
51 mechanical break (lacking significant tensile strength) within rock masses, including
52 most joints, weak bedding planes, weakness zones and faults (ISRM, 1978). This
53 definition however does not apply to incipient traces, regardless of strength, although
54 such traces are often recorded during discontinuity logging in the outcrop (Hencher,
55 2014 and 2015). This common practice leads to underestimation of strength of rock
56 masses, and overestimation of permeability. It can considerably increase
57 expenditure on rock support systems and also influence reliable prediction of water,
58 oil and gas extraction. As a first step, it is therefore practically and theoretically
59 important to differentiate the degree of incipency of discontinuities in terms of their
60 tensile strength (Hencher, 2014; Shang et al., 2016).

61 Incipient discontinuities may have considerable tensile strength as a result of their
62 partial development, secondary mineralization or cementation. This concept is
63 illustrated by Fig. 1, in which a sub-vertical incipient rock joint terminates in rock.
64 Characterising the horizontal traces, would generally be disregarded in rock mass
65 characterisation, but these clearly represent a weakness.

66 Incipient rock discontinuities often develop over geological time into full mechanical
67 discontinuities (Hencher, 2014) with zero tensile strength as defined by ISRM (1978).

68 Fig. 2 shows rock cores with strong incipient traces and zero-tensile strength
69 mechanical joints; these discontinuities can be differentiated on the basis of relative

70 tensile strength of the parent rock (Hencher, 2014; Shang et al., 2015, 2016). Fig. 3
71 shows different development stages of incipient joints on a face cut by a diamond
72 wire saw. Joints can be seen as linear traces stained with iron oxides. These joints
73 were evidently formed from brittle fracture propagation at a late stage during
74 cooling/emplacement of this granite, as can be interpreted from cross-cutting
75 relationships and the geometrical association of some joints with mineral
76 differentiation (as indicated by 1, in an area washed clean with water). Note that
77 some of the joint traces terminate as visible features, as indicated at 2. Note that one
78 of the shallowly dipping joints, has an open aperture locally (indicated by 3) allowing
79 seepage of groundwater, indicating partial development to a full mechanical
80 discontinuity. Hence, it is proposed that the incipient joint pattern represents a
81 'blueprint' that, given time and appropriate conditions, will develop as interconnecting
82 true, mechanical discontinuities in the sense defined by ISRM (1978).

83 **2.2 Rock bridge and discontinuity persistence**

84 The term 'rock bridge' is defined as an area of intact/strong rock material separating
85 coplanar or non-coplanar discontinuities in rock masses (Kim et al., 2007b; Zheng et
86 al., 2015). Rock bridges usually occupy a part of the planar joint plane (Dershowitz
87 and Einstein 1988); such rock bridges in coplanar joints are the focus of this review.

88 True discontinuity persistence is the areal extent of a rock discontinuity. Fig. 4a
89 illustrates the areal discontinuity persistence (K), which is defined as the fraction of
90 continuous discontinuity area (Einstein et al., 1983) whereby:

$$91 \quad K = \sum[(A_D - A_{Bi}) / A_D] \quad (1)$$

92 where $\sum A_{Bi}$ is the total area of scattered rock bridges and A_D is reference gross
93 area including rock bridges and continuous joint segments.

94 The above definition implies that a planar discontinuity follows a predefined
95 weakness plane. For this type of geometry, the effects of the incipient parts of the
96 discontinuity represented by rock bridges have been investigated in stability analysis.
97 For example in a recent work reported by Viviana et al. (2015), effects of the spatially
98 distributed rock bridges along a preferential sliding plane was investigated. In reality,
99 however, linear persistence (K_L), see Fig. 4b, is often used as an approximation of
100 areal persistence (Einstein et al., 1983); this is defined as a linear ratio of sum of
101 joint segments ($\sum J_i$) and the total length of coplanar given line $\sum(J_i + B_i)$:

$$102 \quad K_L = \sum [J_i / (J_i + B_i)] \quad (2)$$

103 This definition has been widely used in experimental, analytical and numerical
104 studies (e.g. Lajtai, 1969a,b; Jennings, 1970; Zhang et al., 2006; Prudencio and Van
105 Sint Jan, 2007; Ghazvinian et al., 2012; Bahaaddini et al., 2013; Shang et al., 2013;
106 Jiang et al., 2015).

107 ISRM (1978) suggested a classification scheme for persistence by measuring length
108 (L) of joint trace formed by the intersection of a joint within an exposure. In that
109 scheme, five categories comprising very low persistence ($L < 1$ m), low persistence
110 ($1 \text{ m} < L < 3$ m), medium persistence ($3 \text{ m} < L < 10$ m), high persistence ($10 \text{ m} < L <$
111 20 m) and very high persistence ($20 \text{ m} < L$) were provided. That scheme however
112 only provides a description of discontinuities on a finite rock exposure (Norbury,
113 2010) and ignores the problem of joint sections that maintain strength.

114 The above definitions (based on coplanar discontinuities) mainly focus on the
115 geometrical properties of single discontinuities without consideration of stress
116 concentration around fracture tips (Kevin, 1980; Wasantha et al., 2014). Some
117 studies considered the stress influence on degree of discontinuity persistence:

118 Wasantha et al. (2014) is an example in which a new parameter was developed to
119 define persistence, considering stress distributions, however it is still difficult for the
120 practical application in rock engineering. It is noted that there is also a difference (in
121 definitions of persistence) between industries and universities (for example, nearly
122 four decades ago, the term “joint continuity”, rather than “persistence”, was used in
123 the joint survey in the Feitsui Reservoir Project, Taiwan, which is probably due to its
124 simplicity). In this review, the term “areal persistence” (Eq. 1), reflecting the three
125 dimensional nature of discontinuities, is recommended to be used to describe
126 discontinuity which is the best measure of persistence.

127 **3. Mechanical properties of individual discontinuities**

128 Tensile or shear failure of incipient discontinuities is often the ‘final straw’ leading to
129 instability of rock masses, which usually occurs in response to a number of triggers
130 including temperature and insolation (Brian and Greg, 2016), precipitation
131 (Wieczorek and Jager, 1996), weathering (Borrelli et al., 2007; Tating et al., 2013;
132 Goudie, 2016) and seismic loading (Cravero and Labichino, 2004). In exposures and
133 tunnel roofs, many overhanging and threatening rock blocks or slabs (Fig. 5) only
134 remain in place because of the strength of incipient discontinuities mainly arising
135 from rock bridges (Paolo et al., 2016). The area of rock bridge can only be viewed
136 after collapse (see for example in Fig. 6) when strength of revealed bridges can be
137 back analysed (Paronuzzi and Serafini, 2009).

138 Shang et al. (2016 and 2017c) investigated the tensile strength of incipient rock
139 discontinuities in the laboratory. They demonstrated that incipient traces can have
140 considerable tensile strength, and can be differentiated using relative tensile strength
141 to that of parent rock, as originally proposed by Hencher (2014). Based on the

142 laboratory findings by Shang et al. (2015 and 2016), a further numerical investigation
143 of the direct tensile behaviour of laminated and transversely isotropic rocks was
144 recently presented by Shang et al. 2017b, in which the incipency of bedding planes
145 (relative tensile strength to that of parent rock) was considered.

146 Many investigations have been undertaken to measure the shear strength of
147 discontinuities, mostly focusing on mechanical discontinuities with zero true cohesion
148 (Barton, 1976). For non-filled and non-persistent rock joints, shear strength is
149 however controlled by four components including fundamental shear strength of rock
150 bridges (Shang and Zhao, 2017), internal friction in solid bridges (after rock bridges
151 are mobilized), friction from the persistent joint segments (Lajtai, 1969b) as well as
152 geometry and location of bridges (Ghazvinian et al., 2007). An equivalent shear
153 strength calculation method was developed based on the Mohr-Coulomb failure
154 criterion, in which strength contributions from rock bridges and persistent joint areas
155 are linearly combined (Lajtai, 1969a; Hudson and Harrison, 2000) as expressed by
156 the following equation:

$$157 \quad \tau = c_i + \sigma \cdot \tan\varphi_i = [K_L \cdot c_p + (1 - K_L) \cdot c_B] + \sigma[K_L \cdot \tan\varphi_p + (1 - K_L)\tan\varphi_B] \quad (3)$$

158 where τ and σ are shear strength of incipient rock joints and normal stress; c_i and
159 φ_i are the equivalent cohesion and internal friction angle of incipient rock joints; c_p
160 and φ_p are the cohesion and internal friction angle of persistent joint; c_B and φ_B are
161 the cohesion and internal friction angle of intact rock bridges; K_L is the linear
162 persistence in the direction of shearing.

163 This equation tends to overestimate the shear strength as it assumes that rock
164 bridges and friction of persistent joint areas are mobilized simultaneously, that is at
165 the same deformation (Lajtai 1969a). In addition, the Mohr-Coulomb criterion is only

166 applicable to smooth joint surfaces; it only describes rough joints under relatively low
167 normal stress level; Eq. (3) thus has limited usefulness in practice.

168 Rock bridges significantly increase the shear strength of individual incipient rock
169 discontinuities (Shang and Zhao. 2017), especially under constant normal stiffness
170 boundary conditions (Shang et al., 2018). They effectively produce a strength
171 reserve and that is mobilised prior to failure occurring along the incipient joint plane
172 (Jennings, 1970; Stimpson, 1978; Gehle and Kutter, 2003; Paolo et al., 2016).
173 Hencher (1984) by undertaking a direct shear test on an incipient tuff joint at the core
174 scale (54 mm in diameter) with an areal persistence of around 86% found that the
175 rock bridge on the incipient joint plane produced a cohesion of 750 kPa. At a larger
176 scale, a rock bridge having a size of about 150 mm X 300 mm was identified by
177 Paolo et al. (2016) after collapse of a limestone wedge (tetrahedral block with a
178 volume of around 28 m³) at the Rosandra valley, north-eastern Italy. Cohesion of the
179 bridge was back-calculated to be around 2.4 MPa (cohesion of the intact rock is 25
180 MPa). It is however rare to see laboratory shear testing on natural incipient rock
181 discontinuity as it is not straightforward to secure and prepare groups of natural rock
182 samples containing incipient discontinuities.

183 Numerical analysis has been used as an alternative to examine the shear strength of
184 non-persistent rock joints, for example, using Itasca Particle Flow Code (e.g.
185 Cundall, 1999; Park and Song, 2009; Ghazvinian et al., 2012; Shang et al., 2018)
186 and Rock Failure Process Analysis code (e.g. Zhang et al., 2006). In numerical
187 analysis, non-persistent rock joints containing rock bridges with different geometrical
188 parameters are readily analysed (Shang and Zhao, 2017); the brittle failure of rock
189 bridges often lead to a dramatic drop in shear strength (Fig. 7). Shear strength of
190 incipient rock joints generally increases when persistence value decreases, and it

191 also varies with spatial scale of rock bridges, as illustrated by Fig. 8 in which
192 numerically simulated shear strength of three incipient rock joints with the same
193 areal persistence ($K=0.5$) varied. Such scale dependent of strength arises from
194 variations in the stress distribution (Rao et al., 2006) and therefore mode of fracture
195 initiation and propagation.

196 **4. Implications for the strength and stability of rock masses**

197 **4.1 Block size and volume for rock masses with non-persistent joint**

198 The intersections of discontinuities in rock masses leads to discrete blocks with
199 variable geometries (Mauldon, 1994; Kalenchuk et al., 2006), especially when
200 discontinuities are not fully persistent. Publications accounting for discontinuity
201 persistence and its influence on the rock block size and volume are discussed below.

202 Assessing rock block size and volume can be roughly categorized into three groups
203 such as index evaluation (e.g. ISRM, 1978; Sen and Eissa, 1992), image-based
204 measurement (e.g. Panek, 1981; Maerz, 1996), and model dissection (e.g.
205 Goodman and Shi, 1985).

206 For rock masses containing several sets of persistent rock joints, rock block volume
207 (V) within a representative rock mass can be empirically calculated by:

$$208 \quad V = \frac{S_1 \cdot S_2 \cdot S_3 \cdots S_i}{\sin\alpha_1 \cdot \sin\alpha_2 \cdot \sin\alpha_3 \cdots \sin\alpha_i} \quad (4)$$

209 where S_i and α_i are joint spacing and angle of inclination for each joint set,
210 respectively (Cai et al., 2004; Palmström, 2005).

211 Block volume calculated by Eq. (4) is an estimation of real rock block volume on the
212 assumption that discontinuities are fully persistent. This approximation tends to be

213 more problematic when the scale of rock mass increases (Lu and Latham, 1999).
 214 Rock bridges in fractured rock masses lead to irregular rock block shapes and larger
 215 rock block size (Longoni et al., 2012). An equivalent spacing S_i' for incipient rock
 216 joints can be defined as (Cai and Horri, 1992):

$$217 \quad S_i' = \frac{S_i}{\sqrt[3]{K_i}} \quad (5)$$

218 where K_i is joint persistence for each joint set i .

219 Thus the equivalent rock block volume can be expressed by the following equation:

$$220 \quad V = \frac{S_1 \cdot S_2 \cdot S_3 \cdots S_i}{\sqrt[3]{K_1 \cdot K_2 \cdot K_3 \cdots K_i} \cdot \sin\alpha_1 \cdot \sin\alpha_2 \cdot \sin\alpha_3 \cdots \sin\alpha_i} \quad (6)$$

221 It has been accepted that block size and volume are sensitive to discontinuity
 222 persistence (Rogers et al., 2007; Elmouttie and Poropat, 2012) and block volume
 223 increases when persistence decreases (Kalenchuk et al., 2006). Numerical
 224 modelling allows the sensitivity of block volume to persistence to be investigated
 225 quantitatively (Kim, 2007b; Palleske et al., 2014). Fig. 9 shows an reciprocal
 226 relationship between discontinuity persistence and rock block size (including 294
 227 cases analysed by UDEC), and volume (including 144 cases analysed by 3DEC)
 228 with parametric analysis using the discrete element method (Kim et al., 2007b and
 229 2007c). In Fig.9a, Groups 1-3 represent simulation cases that the standard deviation
 230 (SD) of joint angle between each joint set is 5°, and the SDs of spacing and trace
 231 length are 10, 20 and 30% of the mean values, respectively. Groups 4-6 (Fig. 9a)
 232 represent simulation cases that the standard deviation (SD) of joint angle between
 233 each joint set is 10°, and the SDs of spacing and trace length are 10, 20 and 30% of
 234 the mean values, respectively. In Fig. 9b, S represents simulation cases that the SDs
 235 of joint spacing and angle are within 30% of the mean value.

236 Normalised rock block size (Fig. 9a) and volume (Fig. 9b) decreased when
237 discontinuity persistence increased, asymptotically approaching unity for fully
238 persistent discontinuities. However it should be noted that the reciprocal
239 relationships shown in Fig. 9 depended on the specific discontinuity orientations and
240 number of joint sets (two sets) used in the simulation. In real projects, lithology and
241 geological conditions should also be considered in the assessment of rock mass
242 properties.

243 **4.2 Mechanical properties and deformability of non-persistently jointed rock** 244 **masses**

245 **4.2.1 Influence of persistence on rock mass behaviour**

246 Many factors control the overall mechanical properties of a rock mass which include
247 intact rock matrix strength (Hu et al., 2012a), geometrical and mechanical properties
248 of discontinuities, discontinuity intersections (stress distribution varies with the
249 number and arrangement of discontinuities, Mughieda, 1997) and the interactions
250 between discontinuities and rock matrix (such as block interlocking). There have
251 been several classic rock mass classification schemes, for example, RMR
252 (Bieniawski 1973, 1989), Q system (Barton et al., 1975) and GSI (Hoek et al., 1995),
253 to assess the strength of rock masses. Generally, these classification schemes are
254 empirically developed to provide a guidance for engineering support (except for GSI,
255 which was semi-empirically designed for rock mass strength estimation) based on
256 engineering projects and laboratory data (Hu et al., 2012b). A specific value
257 considering different influential factors is assessed and calculated to reflect the
258 quality of rock masses. Nevertheless, these schemes fail to explicitly consider the
259 influence of persistence in the mass strength determination. For example, in GSI
260 system, discontinuity persistence is only indirectly considered by the interlocking

261 descriptor (Cai et al., 2004); essentially discontinuities are assumed fully persistent.
262 GSI therefore tends to underestimate the overall strength of a rock mass, especially
263 at high confinement where interlocking effects are strong (Bharani and Kaiser 2013).
264 Rock quality designation (RQD), originally introduced by Deere (1963) for the use in
265 core logging, is one of the key parameters used in RMR and Q system. Sound core
266 pieces greater than 100 mm in length are summed and expressed as a percentage
267 of total core run. RQD however was devised to include only fully development
268 discontinuities with zero tensile strength, so when incipient joint traces (which have
269 considerable tensile strength) are also included in the assessment, rock mass
270 strength is underestimated (Hencher 2014, 2015; Pells et al., 2017).

271 Prudencio and Van Sint Jan (2007) conducted laboratory tests on physical models of
272 non-persistently jointed rock mass under biaxial loading condition. A set of non-
273 persistent rock joints was made by inserting steel sheets into the mortar mixture
274 during sample preparation. One of the key findings is that rock mass failure modes
275 and compressive strength depended on the geometry of the discontinuity, loading
276 stresses, and ratios of principle and intermediate stresses. Three basic failure modes
277 were identified (i.e. failure through incipient joint plane, stepped failure and rotational
278 failure of rock blocks).

279 Numerical modelling has been used to investigate the influence of persistence on
280 overall mechanical properties of jointed rock masses. Kim et al. (2007a, b and c)
281 examined how the incipient discontinuities with varying persistence values affect the
282 mechanical properties of jointed rock mass. UDEC and 3DEC codes combined with
283 experimental approaches were used in their study. Shear and compressive strengths
284 of a jointed rock mass with and without considering persistence (represented as t , t_0 ,
285 σ_c and σ_{c0} respectively, with the zero subscript indicating fully persistent case) were

286 studied, while GSI values with and without considering persistence were calculated
287 using the quantitative approach proposed by Cai et al. (2004). Normalised ratios
288 found from Kim et al. (2007a, b) including t / t_0 , σ_c / σ_{c0} and GSI / GSI_0 are plotted
289 against discontinuity persistence (see Fig. 10). It can be seen that normalised
290 shear strength (red curve) of jointed rock masses dramatically decreases when
291 persistence increases. The analysis shows that the shear strength of rocks can be
292 underestimated dramatically if persistence is ignored in the rock mass strength
293 assessment. The normalised compressive strength (blue curve) and normalised GSI
294 value (green curve) against persistence also show that the assumption of full
295 persistence leads to strength underestimation but by a smaller extent, i.e., by about
296 up to 1.5 times for each case.

297 Following their laboratory investigation of discontinuity geometry (Prudencio and Van
298 Sint Jan 2007), the PFC3D code was used to investigate the effect of discontinuity
299 persistence on the failure mechanism of jointed rock masses (Bahaaddini et al.,
300 2013). Compressive strength and elastic modulus of rock masses with multiple
301 layers of coplanar non-persistent discontinuities were examined (Fig. 11). In their
302 study, persistence varied from 0.5 to 0.8 while other geometrical parameters were
303 set to be constant except for the dip angle β , which varied from 0° to 90° . Their
304 numerical results are reproduced in Fig. 12, with corresponding failure modes of
305 samples when $K=0.5$ and $\beta=90^\circ$. Compressive strength and elastic modulus of the
306 rock mass decreased when persistence increased, for the same dip angle relative to
307 the loading axis. Tensile cracks dominated at low persistence but decreased
308 dramatically when persistence increased from 0.5 to 0.8 (see the insert diagrams of
309 $\beta=90^\circ$, Fig. 12a.). This phenomenon can be attributed to the reduction of the number

310 of joint tips. A further investigation was reported by Bahaaddini et al. (2016); a similar
311 methodology was used and similar results were arrived at to those plotted in Fig. 12.

312 **4.2.2 Rock slope stability considering non-persistent discontinuities**

313
314 Non-persistent rock discontinuities have significant influence on the mechanical
315 properties and deformability of rock masses and therefore on the stability of rock
316 engineering projects such as engineered rock slopes. Large rock volumes
317 (compared with joint spacing) can contain many discontinuities and therefore
318 complex stress distributions, especially where discontinuities are randomly
319 distributed. A challenging difficulty confronting practitioners is how to consider the
320 incipency of discontinuities in large-scale stability analysis. In addition, the gradual
321 development and coalescence of discontinuities over engineering time may have
322 profound effects on stability. An illustrative example was presented by Hencher
323 (2006), in which progressive development of sheeting joints over a period of many
324 years was observed prior to the detachment of a large landslide in Hong Kong.

325 Einstein et al. (1983) proposed a probabilistic criterion for failure that was related to
326 discontinuity data, to examine the effect of discontinuity persistence on rock slope
327 stability. Only one set of parallel discontinuities with varying persistence was
328 examined in their study. The “critical path” for a given discontinuity geometry
329 (including coplanar and non-coplanar joint planes, such as an en echelon) was defined
330 to consider strength contributions from discontinuities and intervening rock bridges
331 as well as the spatial variability of discontinuity geometry. For this “critical path” they
332 defined a minimum safety margin, SM, as the ratio of available resisting force to
333 driving force. The SLOPESIM code was utilized to find the paths of minimum SM and

334 achieve probabilistic failure analysis of a jointed rock slope. In addition, the effect of
335 probabilistic distribution of persistence was investigated using a parametric method.

336 The notion of representative volume element (RVE) of jointed rock masses was
337 proposed by Pariseau et al. (2008) aiming to simultaneously enhance the reliability
338 of large-scale rock mass stability analysis and dramatically reduce computer run
339 time, from hundreds of hours to several hours. The RVE of a non-persistently jointed
340 rock mass represents the smallest volume over which a measurement can be made
341 that will yield a value representative of the whole. In this study, the stability of
342 engineered open pit slopes was investigated by utilizing a finite element modelling
343 technique in which RVE were recognised for a given discontinuity geometry, rather
344 than modelling individual discontinuities. Equivalent discontinuity properties
345 (Pariseau et al., 2008) were calculated for a given persistence for each set of
346 discontinuities within the RVE, and then employed in the slope stability analysis. The
347 main contribution of RVE approach is that numerous non-persistent discontinuities
348 within a rock mass at project scale can be effectively dealt with.

349 In another study of the effect of incipency on rock mass strength behaviour, Viviana
350 et al. (2015) proposed a method combining a probabilistic approach (assuming the
351 distribution of the rock bridges along the sliding plane follows a fractal distribution
352 law) using the discrete element method (DEM), to investigate translational sliding
353 failure along a single incipient discontinuity within rock slopes. Three different sliding
354 block geometries were investigated, that is, with block centres of gravity located in
355 the upper part (Fig.13a), lower part (Fig.13b) and middle of sliding block (Fig.13c),
356 respectively. For each situation, three different dip angles (30° , 50° , and 70°) were
357 used. The dominant slope failure mode (indicated by extent of shear versus tensile
358 crack development) was found to be dependent on the slope geometry (dip of slope

359 and centre of gravity) and discontinuity persistence (Fig.13). For all situations, tensile
360 and shear cracking increased dramatically when persistence decreased which
361 confirms the finding by Bahaaddini et al. (2013) that higher tensile cracking arises
362 from lower discontinuity persistence. For configurations where centres of gravity
363 were located in the upper (see the schematic diagram in Fig.13a) and middle section
364 of the sliding block (see the schematic diagram Fig.13c), shear cracks predominate,
365 especially for a small dip angles i.e., 30° where pure shear failure occurred. For
366 higher dip angles, rock slopes often fail in by both tensile and shear cracking.

367 **5. Quantification of rock discontinuity persistence**

368 As discussed earlier, the influence of persistence on rock mass mechanical
369 behaviour has long been known but generally has been dealt with crudely. Currently,
370 there are no recommended methods to measure or predict discontinuity persistence.
371 An approximation to real discontinuity size can be derived from measured trace
372 length from rock exposures after correcting the sampling bias (e.g. Baecher et al.,
373 1977; Priest and Hudson 1981; Mauldon 1998; Zhang et al., 2002; Latham et al.,
374 2006) but with inherent limitations.

375 **5.1 Discontinuity data collection and size estimation**

376 Data acquisition of discontinuities from exposed rock faces, can be grouped into two
377 categories: manual methods (i.e. scanline sampling and window sampling) and
378 computer-aided methods.

379 **5.1.1 Scanline and window sampling methods**

380 At planar or nearly planar rock exposures, statistical sampling methods including
381 scanline and window approaches have been widely used to measure the extent of
382 discontinuities intersected.

383 In straight scanlines, a tape is laid along rock face, and the joint traces intersecting
384 the line in a scanline survey are recorded. In practice, surveys including between
385 150 and 350 discontinuities are suggested and colour photos of exposed rock faces
386 and scale makers are useful (Hudson and Priest, 1979). Scanline surveys may be
387 grouped into two categories: quick scanline and detailed scanline. For a quick
388 scanline survey, only the location of the scanline, the chainage of each intersection,
389 plunge and azimuth of joint traces are recorded. Detailed scanline surveys normally
390 also include, discontinuity types (e.g. joints, bedding, foliation, lamination and
391 cleavage), trace length, aperture and infilling condition, planarity, waviness,
392 termination and water condition (any evidence of seepage). A good example
393 template of detailed scanline survey is produced by Hencher (2015), in which relative
394 strength to parent rock was additionally suggested to be considered.

395 Fig. 14 diagrammatically shows a scanline survey on a planar rock face of limited
396 extent. This survey is subject to some drawbacks, for example, sampling biases,
397 orientation bias and censoring bias, which have been noted by many researchers
398 (e.g. Cruden 1977). These biases are summarised as follows:

399 (1) Size bias. Scanlines will preferentially identify those discontinuities with a
400 longer trace length, and small traces on exposures are missed (Priest and
401 Hudson 1981)

402 (2) Orientation bias. Discontinuities striking roughly parallel to the scanline will be
403 under-represented and excluded from the sampling results. This will lead to a
404 serious misinterpretation of discontinuity extent as some critical information is
405 omitted. Park and West (2002) verified and emphasised the orientation bias
406 based on the examination of the differences in results from vertical borehole
407 fracture mapping method and horizontal scanline sampling. Selection of

408 several scanline directions in the measurement of trace length can, to some
409 extent, eliminate the orientation bias and it is recommended that scanlines
410 should be measured in each orthogonal direction (Priest 1993; Hencher 2015)

411 (3) Censoring bias. Rock exposures are limited and relatively small compared
412 with major joints. Inevitably for large discontinuities, one end or both ends,
413 may extend beyond the visible exposure, therefore they are censored to some
414 degree depending on discontinuity size (Cruden 1977). The censoring bias
415 should be considered in the inference of discontinuity size (Baecher 1980).

416 Window sampling, another manual data acquisition technique, has also been used
417 for sampling the discontinuities exposed at a given rock face. The preliminaries and
418 measurement techniques are similar to scanline survey except that all discontinuities
419 are measured in a finite area, rather than the intersection of the scanline. For setting
420 up window sampling, a rectangle or circular area is defined on the outcrop. The
421 window should be sufficiently large to reduce the sampling bias, with each side
422 intersecting between 30 and 100 discontinuities. Discontinuities are counted and
423 classified into three classes (Pahl 1981; Zhang and Einstein 2000):

424 (1) Discontinuities contained in the window: both ends of discontinuities are
425 visible in the sampling domain.

426 (2) Discontinuities that transect the window: both ends of discontinuities are
427 invisible in the sampling domain, this is, ends beyond the limits of window.

428 (3) Discontinuities that intersect the window: only one end is visible in the window
429 and another one beyond the limits of sampling area.

430 Although window sampling still suffers from the censoring issue, this method
431 normally is able to eliminate size and orientation biases (Mauldon et al., 2001). In

432 addition, discontinuity termination characteristics can also be logged by using
433 window sampling (Dershowitz and Einstein 1988), but it does not provide any
434 information about discontinuity orientation or surface geometry (Priest 1993).

435 Manual data acquisition methods suffer from some limitations. The first is that they
436 are labour and time consuming. In order to minimise the sampling bias, sampling
437 should be conducted at many different locations. The operator's safety during
438 sampling is another issue. The second is that unbiased discontinuity characterisation
439 requires a skilled interpretation (rock engineer or geologist). The third limitation is
440 that manual methods cannot collect data from rock exposures that are not
441 accessible. So researchers have paid a lot attention to producing alternative ways to
442 obtain discontinuity data from outcrop.

443 **5.1.2 Computer aided sampling**

444 Computer aided sampling methods for discontinuity characterisation have made
445 significant advances over the last 25 years. An image analysis technique, perhaps
446 the pioneer work towards this topic, was proposed by Ord and Cheung (1991) to
447 describe discontinuities in outcrop automatically. Since then, computer-aided
448 techniques have been developed. Roberts and Poropat (2000) proposed a digital
449 photogrammetric technique to investigate three dimensional models of rock faces.
450 Feng et al. (2001) proposed a portal system, in which a laser range finder was used,
451 to identify discontinuities in outcrop. Several computer aided techniques including
452 digital photogrammetry (e.g. Tuckey and Stead 2016), ground-based LiDAR (e.g.
453 Mattew and Malte 2012), and digital trace mapping (Tuckey et al., 2012) have been
454 applied to develop a standardized and adaptable methodology for assessing
455 discontinuity persistence. An example among these techniques is shown in Fig. 15

456 (Tuckey et al., 2012), in which the image processing code Image-J was used to trace
457 discontinuities and infer rock bridges. The results of the study were used to
458 supplement field window sampling. Umili et al. (2013) developed an automatic
459 method to map and identify discontinuity traces based on a digital surface model
460 (DSM), which consists of a triangulated point cloud that approximates the true
461 surface. Terrestrial Laser Scanner (TLS) and Terrestrial digital photogrammetry
462 (TDP) have also been widely used in characterising discontinuities and rock face
463 morphology (e.g. Rosser et al., 2005; Sturzenegger and Stead 2009 a, b; Slob 2010;
464 Sturzenegger et al., 2011; Brideau et al., 2012). Abellan et al., (2014)
465 comprehensively reviewed the application of TLS technique to rock exposure
466 characterization. These methods are generally based on the segmentation of the
467 rock exposures, and discontinuity characteristics are obtained from the boundaries
468 and orientations of the identified planes (Umili et al., 2013). Data collected is
469 statistically examined and is used for the rock mass characterisation.

470 **5.1.3 Discontinuity size estimation from censored measurements**

471 Discontinuity size is often estimated based on censored sampling measurements
472 using the aforementioned techniques. As visible trace length does not equal to true
473 persistence, probability distributions of trace lengths need to be corrected for
474 sampling biases to provide an estimate of the true discontinuity size (or trace length)
475 distribution. Well formulated probability sampling planes should be used otherwise
476 errors will occur (Baecher and Lanney 1977). Table 1 presents a selection of key
477 publications advancing these approaches highlighting the methods used, sampling
478 techniques they are applicable, and major assumptions.

479 **5.2 Discontinuity persistence in the subsurface**

480 Geophysical techniques have been used to investigate discontinuities in the
481 subsurface (e.g. Grandjean and Gourry 1996; Willenberg et al., 2008; Kana et al.,
482 2013). The paper by Longoni et al. (2012) provided illustrating insights into the
483 application of radar in the investigation of subsurface discontinuity persistence. In
484 their work, ground penetrating radar surveys were conducted to image the 3D
485 discontinuity planes inside rock mass, thereafter discontinuity persistence was
486 calculated. Geological discontinuities in the subsurface are usually complex thus
487 sometimes will frustrate geophysical sampling, geophysical approaches requires a
488 high resolution to be able to sample discontinuities as these are relatively thin, and
489 an experienced operator is also needed to process and interpret discontinuities
490 within radar datasets.

491 **5.3 Forensic excavation of rock masses**

492 In a recent work reported by Shang et al. (2017a), a new technique, termed forensic
493 excavation of rock masses (FERM), was introduced as an approach for investigating
494 discontinuity areal persistence. Fig. 16 shows the FERM testing procedures. This
495 technique involves non-explosive excavation of rock masses by injecting an
496 expansive grout along incipient discontinuities. The agent causes the incipient rock
497 discontinuity traces to open as open joints, thus allows the observation of areal joint
498 surfaces and determination of areal persistence. Laboratory and field tests has been
499 conducted on two lithologies (Midgley Grit Sandstone and Horton Formation
500 Siltstone) by the authors, which demonstrated that FERM allows measurement of
501 areal persistence at laboratory scale and field scale over the range of a few meters.
502 Project scale tests will hopefully to be conducted to verify the capability of FERM at
503 larger scales.

504 **6. Summary, conclusion and recommendations for future research**

505 **6.1 Summary and conclusion**

506 It has been nearly four decades since awareness of the importance of discontinuity
507 persistence in earth science applications (Cruden 1977; ISRM 1978). Some
508 endeavours have been made to consider persistence during the measurement of
509 discontinuities (e.g. ISRM 1978; Priest and Hudson 1981; Latham et al., 2006) and in
510 the assessment of rock mass stability (e.g. Einstein 1983; Pariseau et al., 2008).
511 These endeavours however have not led to standard methods to quantify real
512 persistence. This review has described the fundamentals of this topic e.g. definitions
513 (incipient, mechanical discontinuities and persistence), mechanical properties of
514 individual rock discontinuities, and those of rock masses containing non-persistent
515 joints. State-of-the-art methodologies in the description and quantification of
516 discontinuity persistence were summarised and reviewed.

517 Areal persistence, reflecting the three dimensional nature of geological
518 discontinuities, is the best measure of persistence. Studies aiming at quantification of
519 discontinuity persistence have been relatively few in number. In rock engineering
520 practice, “geological judgements” are often used, but these can fail to represent the
521 three dimensional nature of discontinuities, for example where linear persistence is
522 used to represent areal persistence.

523 The size and volume of rock blocks within rock masses are sensitive to discontinuity
524 persistence and will be underestimated if 100% persistence is assumed. Geometrical
525 considerations based on uniform joint spacing imply a reciprocal cube-root
526 relationship between discontinuity persistence and block size / block volume (Eq. 6),
527 whereas previous studies using more realistic spacing distributions suggest a

528 reciprocal relationship i.e. $V_b/V_0 \sim K^{-1}$. However, the specific lithology and geological
529 conditions should be considered in the assessment of rock mass properties based
530 on persistence values.

531 Failure modes of a rock mass are generally controlled by the discontinuities. Studies
532 show that discontinuity persistence, orientation and number of discontinuities
533 overshadow the efficacy of other factors. Potential for sliding failure of rock slopes
534 along planar discontinuities is mainly controlled by the persistence and orientation of
535 discontinuities. In addition, the spatial distributions and geometries of intact rock
536 bridges as well as mineral infills influence the mechanical properties of incipient
537 discontinuities (Shang et al., 2016).

538 **6.2 Recommendations for future research**

539 The authors recommend some topics that might be taken up for future research.

540 These are as follows:

541 (1) Current definitions of persistence (i.e., Eqs. 1 and 2) only apply to planar
542 discontinuities. Engineering applications based on the definitions will unavoidably
543 have some limitations, as some discontinuities are not planar in shape (e.g., 'zig-zag'
544 and 'en-echelon' fractures). Thus, there is a need to define persistence for non-
545 planar discontinuities; thereafter a full spectrum of discontinuity persistence is able to
546 be quantified and implemented into engineering applications such as discrete
547 fracture network modelling.

548 (2) Up to date, rock engineering practise lacks standard methods to deal with the
549 incipency of some discontinuities, i.e. those that are not fully developed mechanical
550 break with some tensile strength. The degree of incipency of discontinuities can be
551 described by their tensile strengths relative to that of parent rock. Tensile strength is

552 suggested because incipient discontinuity shear strength is complicated by other
553 factors, including roughness and asperities of the persistent sections. A classification
554 scheme differentiating incipient discontinuities has been conceptually proposed
555 by Hencher (2014) with different bands including open fracture, weak, moderate and
556 high. Direct tensile tests on incipient rock discontinuities have been conducted by
557 Shang (2016) in the laboratory to follow up that topic. However, limited tests were
558 involved due to the difficulty of the natural sample collection and preparation. It is
559 therefore suggested that more tests need to be performed to facilitate the production
560 of the classification scheme of discontinuity incipient.

561 (3) In a recent study by Shang et al. (2017a), the quantification of areal persistence
562 was attempted by “forensic excavation of rock masses (at block sale)”; this technique
563 needs proof of concept at larger scales.

564 (4) Non-invasive quantification of persistence might also be achieved using
565 geophysics, which if successful will improve the ability to predict rock mass
566 properties.

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569 **References**

- 570 Abellan, A., Oppikofer, T., Jahoyedoff, M., Rosser, N.J., Lim, M., Lato, M.J., 2014.
571 Terrestrial laser scanning of rock slope instabilities. *Earth Surf. Process.*
572 *Landf.* 39, 80-97.
- 573 Baecher, G.B., 1980. Progressively censored sampling of rock joints traces. *Math.*
574 *Geo.* 12(1), 33-40.

575 Baecher, G.B., Lanney, N.A., Einstein, H.H., 1978. Statistical description of rock
576 properties and sampling. In: Proceeding of the 18th U.S. Symposium on Rock
577 Mechanics, Colorado, pp. 22-24.

578 Bahaaddini, M., Sharrock, G., Hebblewhite, B.K., 2013. Numerical investigation of
579 the effect of joint geometrical parameters on the mechanical properties of a
580 non-persistent jointed rock mass under uniaxial compression. *Comput.
581 Geotech.* 49, 206-225.

582 Bahaaddini, M., Hagan, P., Mitra, R., Hebblewhite, B.K., 2016. Numerical study of
583 the mechanical behaviour of nonpersistent jointed rock masses. *Int. J.
584 Geomech.* 16(1),1-10.

585 Barton, N., 1976. The shear strength of rock and rock joints. *Int. J. Rock Mech. Min.
586 Sci.* 13, 255-279.

587 Bharani, N., and Kaiser, P.K., 2013. Strength degradation of non-persistently jointed
588 rock mass. *Int. J. Rock Mech. Min. Sci.* 62, 28-33.

589 Bieniawski, Z.T., 1973. Engineering classification of jointed rock masses. *Trans. S.
590 Afr. Inst. Civ. Eng.* 15, 335-344.

591 Bieniawski, Z.T., 1989. *Engineering Rock Mass Classifications*. Wiley, New York, pp.
592 249.

593 Borrelli, L., Greco, R., Gulla, G., 2007. Weathering grade of rock masses as a
594 predisposing factor to slope instabilities: reconnaissance and control
595 procedures. *Geomorphology.* 87, 158-175.

596 Brian, D.C., Greg, M.S., 2016. Rockfall triggering by cyclic thermal stressing of
597 exfoliation fractures. *Nat. Geosci.* 9, 395-400.

598 Brideau, M.A., Sturzenegger, M., Stead, D., Jaboyedoff, M., Lawrence, M., Roberts,
599 N., Ward, B., Millard, T., Clague, J., 2012. Stability analysis of the 2007

600 Chehalis lake landslide based on long-range terrestrial photogrammetry and
601 airborne LiDAR data. *Landslides*. 9, 75-91.

602 Bridges, M.C., 1976. Presentation of fracture data for rock mechanics. In:
603 Proceeding of 2nd Australian – New Zealand Conference on Geomechanics,
604 pp. 144-148.

605 Cai, M., Horri, H., 1992. A constitutive model of highly jointed rock masses. *Mech.*
606 *Mater.* 13(3), 217-246.

607 Cai, M., Kaiser, P.K., Uno, H., Tasaka, Y., Minami, M., 2004. Estimation of rock
608 mass deformation modulus and strength of jointed hard rock masses using
609 the GSI system. *Int. J. Rock Mech. Min. Sci.* 41, 3-19.

610 Cravero, M., Labichino, G., 2004. Analysis of the flexural failure of an overhanging
611 rock slab. *Int. J. Rock Mech. Min. Sci.* 41(S1), 605-610.

612 Cruden, D.M., 1977. Describing the size of discontinuities. *Int. J. Rock Mech. Min.*
613 *Sci.* 144, 133-137.

614 Cundall, P.A., 1999. Numerical experiments on rough joints in shear using a bonded
615 particle model. In: *Aspects of tectonic faulting*. Berlin, Springer, pp. 1-9.

616 Deere, D.U., 1963. Technical description of rock cores for engineering purposes. In:
617 13th Colloquium, International Society for Rock Mechanics, Salzburg, Austria,
618 pp. 16-22.

619 Deparis, J., Johgmans, D., Garambois, S., Levy, C., Baillet, L., Meric O., 2011.
620 Geophysical detection and structural characterization of discontinuities in rock
621 slopes. *Rockfall Engineering*, Wiley, ISTE, pp. 1-38.

622 Dershowitz, W.S., Einstein, H.H., 1988. Characterizing rock joint geometry with joint
623 system models. *Rock Mech. Rock Eng.* 21, 21-51.

624 Einstein, H.H., Veneziano, D., Baecher, G.B., Oreilly, K.J., 1983. The effect of
625 discontinuity persistence on rock slope stability. *Int. J. Rock Mech. Min. Sci.*
626 20(5), 227-236.

627 Elmouttie, M.K., Poropat, G.V., 2012. A method to estimate in situ block size
628 distribution. *Rock Mech. Rock Eng.* 45, 401-407.

629 Feng, Q., Sjogren, P., Stephansson, O., Jing, L., 2001. Measuring fracture
630 orientation at exposed rock faces by using a non-reflector total station. *Eng.*
631 *Geol.* 59(1-2), 133-146.

632 Gehle, C., Kutter, H.K., 2003. Breakage and shear behaviour of intermittent rock
633 joints. *Int. J. Rock Mech. Min. Sci.* 30(40), 687-700.

634 Ghazvinian, A., Nikudel, M.R., Sarfarazi, V., 2007. Effect of rock bridge continuity
635 and area on shear behaviour of joints. In: 11th Congress of the International
636 Society for Rock Mechanics, pp. 247-250.

637 Ghazvinian, A., Sarfarzai, V., Schubert, W., 2012. A study of the failure mechanism
638 of planar non-persistent open joints using PFC2D. *Rock Mech. Rock Eng.*
639 45, 677-693.

640 Goodman, R.E., Shi, G., 1985. Block theory and its application to rock engineering,
641 Prentice-Hall, New Jersey, p. 352

642 Goudie, A.S., 2016. Quantification of rock control in geomorphology. *Earth-Sci. Rev.*
643 159, 374-387.

644 Grandjean, G., Gourry, J.C., 1996. GPR data processing for 3D fracture mapping in
645 a marble quarry. *J. Appl. Geophys.* 36, 19-30.

646 Heike, W., Simon, L., Erik, E., Keith, F.E., Thomas, S., Bjorn, H., Hansruedi, M.,
647 Alan, G.G., 2008. Internal structure and deformation of an unstable crystalline

648 rock mass above Randa (Switzerland): Part I – Internal structure from
649 integrated geological and geophysical investigations. Eng. Geol. 101, 1-14.

650 Hencher, S.R., 2006. Weathering and erosion process in rock - implications for
651 geotechnical engineering. In: Proceedings symposium on Hong Kong soils
652 and rocks, Centre for research and professional development, IMMM,
653 Geological Society, HKRG, Hong Kong, pp. 29-79.

654 Hencher, S.R., 2014. Characterizing discontinuities in naturally fractured outcrop
655 analogues and rock core: the need to consider fracture development over
656 geological time. Advances in the study of fractured reservoirs. Geol. Soc.
657 London Spec. Publ. 374, 113-123.

658 Hencher, S.R., 2015. Practical rock mechanics. Taylor and Francis Group, London,
659 pp.356.

660 Hencher, S.R., 1984. Three direct shear tests on volcanic rock joints. Geotechnical
661 Control Office Information Notes, IN 15/84. Hong Kong Governments, pp. 33
662 (unpublished).

663 Hoek, E., Kaiser, P.K., Bawden, W.F., 1995. Support of underground excavations in
664 hard rock. Rotterdam, Balkema, p. 215.

665 Hu, J., Shang, J., Tao, L., 2012a. Rock mass quality evaluation of underground
666 engineering based on RS-TOPSIS method. J. Cent. South Univ. 43(11),
667 4412-4419.

668 Hu, J., Xu, H., Luo, X., Shang, J., Ruan, D., 2012b. Determination of mechanical
669 parameters of fractured rock mass based on GSI. J. Guangxi Univ. 37(1),
670 178-183.

671 Hudson, J.A., Harrison, J.P., 2000. Engineering rock mechanics: An interdiction to
672 the principles. Pergamon, p. 441.

673 Hudson, J.A., Priest, S.D., 1979. Discontinuities and rock mass geometry. *Int. J.*
674 *Rock Mech. Min. Sci.* 16, 339-362.

675 ISRM, 1978. Suggested methods for the quantitative description of discontinuities in
676 rock masses. *Int. J. Rock Mech. Min. Sci.* 15, 319-368.

677 Jennings, J.E., 1970. A mathematical theory for the calculation of the stability of
678 slopes in open cast mines: Planning Open Pit Mines. In: *Proceedings of the*
679 *Symposium on the Theoretical Background to the Planning of Open Pit Mines*
680 *with Special Reference to Slope Stability*, Johannesburg, pp. 87-102.

681 Jiang, M.J., Jiang, T., Crosta, G.B., Shi, Z.M., Chen, H., Zhang, N., 2015. Modelling
682 failure of jointed rock slope with two main joint sets using a novel DEM bond
683 contact model. *Eng. Geol.* 193, 79-96.

684 Jimenze-Rodriguez, R., Sitar, N., 2006. Inference of discontinuity trace length
685 distributions using statistical graphical models. *Int. J. Rock Mech. Min. Sci.* 43,
686 877-893.

687 Kalenchuk, K.S., Diederichs, M.S., Mckinnon, S., 2006. Characterizing block
688 geometry in jointed rock masses. *Int. J. Rock Mech. Min. Sci.* 43(8): 1212-
689 1225.

690 Kana AA; West LJ; Clark RA (2013) Fracture aperture and fill characterization in a
691 limestone quarry using GPR thin-layer AVA analysis, *Near Surface*
692 *Geophysics*, 11, 293-305.

693 Kevin, J.O., 1980. The effect of joint plane persistence on rock slope reliability. Ph.D.
694 thesis, Massachusetts Institute of Technology, United State, p. 553.

695 Kim, B.H., Cai, M., Kaiser, P.K., 2007a. Rock mass strength with non-persistent
696 joints. In: *1st Canada – U.S. Rock Mechanics Symposium*, Vancouver,
697 Canada, pp. 1-8.

698 Kim, B.H., Cai, M., Kaiser, P.K., Yang, H.S., 2007b. Estimation of block sizes for
699 rock masses with non-persistent joints. *Rock Mech. Rock Eng.* 40, 169-192.

700 Kim, B.H., Kaiser, P.K., Grasselli, G., 2007c. Influence of persistence on behaviour
701 of fractured rock masses. *Rock physics and geomechanics in the study of*
702 *reservoirs and repositories. Geol. Soc. London Spec. Publ.* 284, 161-173.

703 Kulatilake, P.H.S.W., Wu, T.H., 1984. Estimation of mean trace length of
704 discontinuities. *Rock Mech. Rock Eng.* 17, 215-232.

705 Lajtai, E.Z., 1969a. Shear strength of weakness planes in rock. *Int. J. Rock Mech.*
706 *Min. Sci.* 6, 499-515.

707 Lajtai, E.Z., 1969b. Strength of discontinuous rocks in direct shear. *Geotechnique.*
708 19(2), 218-233.

709 Latham, J.P., Meulen, J.V., Dupray, S., 2006. Prediction of in-situ block size
710 distributions with reference to armourstone for breakwaters. *Eng. Geol.* 86,
711 19-36.

712 Li, X., Zuo, Y., Zhuang, X., Zhu, H., 2014. Estimation of fracture trace length
713 distributions using probability weighted moments and L-moments. *Eng. Geol.*
714 168, 69-85.

715 Longoni, L., Arosio, D., Scaioni, M., Papini, M., Zanzi, L., Roncella, R., Brambilla, D.,
716 2012. Surface and subsurface non-invasive investigations to improve the
717 characterization of a fractured rock mass. *J. Geophys. Eng.* 9, 461-472.

718 Lu, P., Latham, J.P., 1999. Developments in the assessment of in-situ block size
719 distributions of rock masses. *Rock Mech. Rock Eng.* 32(1), 29-49.

720 Lyman, G.J., 2003. Rock fracture mean trace length estimation and confidence
721 interval calculation using maximum likelihood methods. *Int. J. Rock Mech.*
722 *Min. Sci.* 40, 825-832.

723 Maerz, H.H., 1996. Image sampling techniques and requirements for automated
724 image analysis of rock fragmentation. In: Proceedings of the FRAGBLAST 5
725 Workshop on Measurement of Blast Fragmentation, Montreal, Quebec,
726 Canada, pp. 115-120.

727 Mattew, J.L., Malte, V., 2012. Automated mapping of rock discontinuities in 3D lidar
728 and photogrammetry models. *Int. J. Rock Mech. Min. Sci.* 54, 150-158.

729 Mauldon, M., 1994. Intersection Probabilities of impersistent joints. *Int. J. Rock*
730 *Mech. Min. Sci.* 31(2), 107-115.

731 Mauldon, M., 1998. Estimating mean fracture trace length and density from
732 observations in convex windows. *Rock Mech. Rock Eng.* 31(4), 201-216.

733 Mauldon, M., Dunne, W.M., Rohrbaugh Jr, M.B., 2001. Circular scanlines and
734 circular windows: new tools for characterizing the geometry of fracture traces.
735 *J. Struct. Geol.* 23, 247-258.

736 Mughieda, O., 1997. Failure mechanisms and strength of non-persistent rock joints.
737 Ph.D. thesis, University of Illinois at Urbana-Champaign, United State, p. 512.

738 Norbury, D., 2010. Soil and rock description in engineering practice. CRC Press,
739 Taylor & Francis Group, USA, p. 288.

740 Odling, N.E., Gillespie, P., Bourgine, B., Castaing, C., Chiles, J.P., Christensen,
741 N.P., Fillion, E., Genter, A., Olsen, C., Thrane, L., Trice, R., Aarseth, E.,
742 Walsh, J.J., Watterson, J., 1999. Variations in fracture system geometry and
743 their implications for fluid flow in fractured hydrocarbon reservoirs. *Petrol.*
744 *Geosci.* 5, 373-384

745 Ord, A., Cheung, L.C., Hobbs, B.E., Blanc, D.L., 1991. Automatic mapping of rock
746 exposures for geotechnical purposes. In: The 2nd Australian Conference on

747 Computer Applications in the Mineral Industry, The University of Wollongong,
748 Australia, pp. 1-6.

749 Pahl, P.J., 1981. Estimating the mean length of discontinuity traces. *Int. J. Rock*
750 *Mech. Min. Sci.* 18(3), 221-228.

751 Palleske, C., Diederichs, M.S., Hutchinson, D.J., Elimo, D., 2014. Block size
752 distributions as a rock mass classification tool. In: 48th U.S. Rock
753 Mechanics/Geomechanics Symposium, Minneapolis, Minnesota, pp. 1-8.

754 Palmström, A., 2005. Measurements and correlations between block size and rock
755 quality designation (RQD). *Int. J. Rock Mech. Min. Sci.* 20, 362-377.

756 Panek, L.A., 1981. Design and operation of caving and sublevel stopping mines.
757 Society for Mining Metallurgy, p. 843.

758 Paolo, P., Alberto, A., and Elia R., 2016. 3D stress-strain analysis of a failed
759 limestone wedge influenced by an intact rock bridge. *Rock Mech. Rock Eng.*
760 49, 3223-3242.

761 Pariseau, W.G., Puri, S., and Schmelter, S.C., 2008. A new model for effects of
762 impersistent joint sets on rock slope stability. *Int. J. Rock Mech. Min. Sci.* 45,
763 122-131.

764 Park, H.J., West, T.R., 2002. Sampling bias of discontinuity orientation caused by
765 linear sampling technique. *Eng. Geol.* 66, 99-110.

766 Park, J.W., Song, J.J., 2009. Numerical simulation of a direct shear test on a rock
767 joint using a bounded-particle model. *Int. J. Rock Mech. Min. Sci.* 46, 1315-
768 1328.

769 Paronuzzi, P., Serafini, W., 2009. Stress state analysis of a collapsed overhanging
770 rock slab: A case study. *Eng. Geol.* 108: 65-75.

771 Pells, P., Bieniawski, Z.T.R., Hencher, S.R., Pells, S., 2017. Rock quality designation
772 (RQD): Time to rest in peace. *Can. Geotech. J.* 54: 825-834.

773 Piggott, A.R., Elsworth, D., 1992. Analytical models for flow through obstructed
774 domains. *J. Geophys. Res.* 97(B2), 2085-2092.

775 Priest, S.D., 1993. *Discontinuity analysis for rock engineering*. Chapman & Hall,
776 London, p. 473.

777 Priest, S.D., and Hudson, J.A., 1981. Estimation of discontinuity spacing and trace
778 length using scanline surveys. *Int. J. Rock Mech. Min. Sci.* 18, 183-197.

779 Prudencio, M., Van Sint Jan, M., 2007. Strength and failure modes of rock mass
780 models with non-persistent joints. *Int. J. Rock Mech. Min. Sci.* 44, 890-902.

781 Rao, Q., Sun, Z., Stephansson, O., Li, C. Stillborg, B., 2003. Shear fracture (Mode II)
782 of brittle rock. *Int. J. Rock Mech. Min. Sci.* 40, 355-375.

783 Rawnsley, K.D., 1990. *The influence of joint origin on engineering properties*. Ph.D.
784 thesis, the University of Leeds, Leeds, United Kingdom, p.388.

785 Roberts, G., Poropat, G.V., 2000. Highwall joint mapping in 3-D at the Moura mine
786 using SIROJOINT. In: *Bowen Basin Symposium, Coal and Mining The New*
787 *Millennium*, Rockhampton, pp. 371-377.

788 Robertson, A.M., 1970. The interpretation of geological factors for use in slope
789 theory. In: *Planning Open Pit Mines*. Van Rensburg, P.W.J. (Ed.). Cape Town,
790 South Africa: Balkema, pp. 55-71.

791 Rogers, S.F., Kennard, D.K., Dershowitz, W.S., Van As, 2007. Characterising the in
792 situ fragmentation of a fractured rock mass using a discrete fracture network
793 approach. In: *1st Canada – US. Rock Mechanics Symposium*, pp. 137-144.

794 Rosser, N.J., Petley, D.N., Lim, M., Dunning, S.A., Allison, R.J., 2005. Terrestrial
795 laser scanning for monitoring the process of hard rock coastal cliff erosion. Q.
796 J. Eng. Geol. Hydroge. 38(4), 363-375.

797 Sen. Z., Eissa, E.A., 1992. Rock quality charts for log-normally distributed block
798 sizes. Int. J. Rock Mech. Min. Sci. 29(1), 1-12.

799 Shang, J., Hu, J., Zhou, K., 2013. Numerical tests on heterogeneous effect in
800 damage and acoustic emission characteristic of rocks under uniaxial loading.
801 J. Cent. South Univ. 6, 2470-2475.

802 Shang, J., Hencher, S.R., West, L.J., 2015. Tensile strength of incipient rock
803 discontinuities, In: Proceedings of the ISRM regional symposium Eurock 2015
804 & 64th Geomechanics Colloquium, Salzburg, Austria, pp. 565-570.

805 Shang, J., 2016. Persistence and tensile strength of incipient rock discontinuities.
806 Ph.D. thesis, University of Leeds, United Kingdom, p. 248.

807 Shang, J., Hencher, S.R., West, L.J., 2016. Tensile strength of geological
808 discontinuities including bedding, rock joints and mineral veins. Rock Mech.
809 Rock Eng. 49(11), 4213-4225.

810 Shang, J., Hencher, S.R., West, L.J., Handley, K., 2017a. Forensic excavation of
811 rock masses: a technique to investigate discontinuity persistence. Rock Mech.
812 Rock Eng. 50(11), 2911-2928.

813 Shang, J., Duan, K., Gui, Y., Handley, K. Zhao, Z., 2017b. Numerical investigation of
814 the direct tensile behaviour of laminated and transversely isotropic rocks
815 containing incipient bedding planes with different strengths. Comput. Geotech.
816 <https://doi.org/10.1016/j.compgeo.2017.11.007>.

817 Shang, J., West, L.J., Hencher, S.R., Zhao, Z., 2017c. Tensile strength of large-scale
818 incipient rock joints: a laboratory investigation. *Acta Geotech.*
819 <https://doi.org/10.1007/s11440-017-0620-7>

820 Shang, J., Zhao, Z., 2017. 3D particle-based DEM investigation into the shear
821 behaviour of incipient rock joints with various geometries of rock bridges.
822 *Rock Mech. Rock Eng.* Under Review.

823 Shang, J., Zhao, Z., Ma, S., 2018. On the shear failure of incipient rock
824 discontinuities under CNL and CNS boundary conditions: Insights from DEM
825 modelling. *Eng. Geol.* 234(21): 153-166.

826 Slob, S., 2010. Automated rock mass characterisation using 3-D terrestrial laser
827 scanning. Ph.D. thesis, The University of Twente, Netherlands, p.301.

828 Song, J.J., Lee, C.I., 2001. Estimation of joint length distribution using window
829 sampling. *Int. J. Rock Mech. Min. Sci.* 38, 519-528.

830 Stimpson, B., 1978. Failure of slopes containing discontinuous planar joints. In:
831 *Proceedings of the 19th US Symposium on Rock Mechanics*, Stateline,
832 Nevada, pp. 296-302.

833 Sturzenegger, M., Stead, D., 2009a. Quantifying discontinuity orientation and
834 persistence on high mountain rock slopes and large landslides using
835 terrestrial remote sensing techniques. *Nat. Hazards Earth Syst. Sci.* 9, 267-
836 287.

837 Sturzenegger, M., Stead, D., 2009b. Close-range terrestrial digital photogrammetry
838 and terrestrial laser scanning for discontinuity characterization on rock cuts.
839 *Eng. Geol.* 106(3-4), 163-182.

840 Sturzenegger, M., Stead, D., Elmo, D., 2011. Terrestrial remote sensing-based
841 estimation of mean trace length, trace intensity and block size/shape. Eng.
842 Geol. 119(3-4), 96-111.

843 Tating, F., Hack, R., Jetten, V., 2013. Engineering aspects and time effects of rapid
844 deterioration of sandstone in the tropical environment of Sabah, Malaysia.
845 Eng. Geol. 159, 20-30.

846 Tuckey, Z., Stead, D., 2016. Improvements to field and remote sensing methods for
847 mapping discontinuity persistence and intact rock bridges in rock slopes. Eng.
848 Geol. 208, 136-153.

849 Tuckey, Z., Stead, D., Sturzenegger, M., Elmo, D., Terbrugge, P., 2012. Towards a
850 methodology for characterizing intact rock bridges in large open pits: In: 46th
851 US Rock Mechanics/Geomechanics Symposium held in Chicago, IL, USA, pp.
852 24-27.

853 Umili, G., Ferrero, A., Einstein, H.H., 2013. A new method for automatic discontinuity
854 traces sampling on rock mass 3D model. Comput. Geosci. 51,182-192.

855 Viviana, B.S., Luc, S., Frederic-Victor, D., Marc, E., 2015. DEM analysis of rock
856 bridges and the contribution to rock slope stability in the case of translational
857 sliding failures. Int. J. Rock Mech. Min. Sci. 80, 67-78.

858 Wang, X.G., Jia, Z.X., Chen, Z.Y., Xu, Y., 2016. Determination of discontinuity
859 persistent ratio by Monte-Carlo simulation and dynamic programming. Eng.
860 Geol. 203, 83-98.

861 Wasantha, P.L.P., Ranjith, P.G., Xu, T., Zhao, J., Yan, Y.L., 2014. A new parameter
862 to describe the persistency of non-persistent joints. Eng. Geol.181, 71-77.

863 Wieczorek, G.F., Jager, S., 1996. Triggering mechanisms and depositional rates of
864 postglacial slope-movement processes in the Yosemite Valley, California.
865 *Geomorphology*. 15, 17-31.

866 Willenberg, H., Simon, L., Eberhardt, E., Evans, K.F., Spillmann, T., Heincke, B.,
867 Maurer, H., Green, A.G., 2008. Internal structure and deformation of an
868 unstable crystalline rock mass above Randa (Switzerland): Part I – Internal
869 structure from integrated geological and geophysical investigations. *Eng.*
870 *Geol.*101, 1-14.

871 Zhang, H.Q., Zhao, Z.Y., Tang, C.A., Song, L, 2006. Numerical study of shear
872 behaviour of intermittent rock joints with different geometrical parameters. *Int.*
873 *J. Rock Mech. Min. Sci.* 43, 802-816.

874 Zhang, L., Einstein, H.H., 2000. The planar shape of rock joints. *Rock Mech. Rock*
875 *Eng.* 43(1), 55-68.

876 Zhang, L., Einstein, H.H., Dershowitz, W.S., 2002. Stereological relationship
877 between trace length and size distribution of elliptical discontinuities.
878 *Geotechnique*. 52(6), 419-433.

879 Zheng, Y., Xia, L., Yu, Q.C., 2015. Analysis of the removability and stability of rock
880 blocks by considering the rock bridge effect. *Can. Geotech. J.* 53(3), 384-395.

881 Zimmerman, R.W., Chen, D.W., Cook, N.G.W., 1992. The effect of contact area on
882 the permeability of fractures. *J. Hydrol.* 139, 79-96.

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888 **Figure Captions**

889 **Fig 1** Partially developed discontinuities that are incipient (non-persistent), Horton-in-
890 Ribblesdale, Yorkshire, England.

891 **Fig 2 a** Section of andesitic tuff cores (Hong Kong) with incipient and mechanical
892 discontinuities and **b** Same core (disassembled). Relative tensile strength, i.e.,
893 high, moderate and weak strength relative to the strength of the parent rock, is
894 proposed to differentiate these discontinuities. Adapted from Hencher (2014).

895 **Fig 3** A Face cut by a diamond wire saw in dimension stone quarry in granite near
896 Tui, Galicia, Spain. Joints 1 and 2 are in earlier incipient stages (which are
897 always poorly defined by current standards). Joint 3 is in later incipient stage
898 and it has a persistent area partially, allowing seepage of fluid. After Shang et
899 al. (2016)

900 **Fig 4** Definitions of rock discontinuity persistence. **a** Areal extent of a discontinuity
901 plane (true persistence) and **b** Linear extent definition (approximation of
902 persistence).

903 **Fig 5** Slope with daylighting rock slabs threatening highway in central Taiwan. The
904 incipient nature of the discontinuities contributes tensile and shear strength and
905 allows temporary stability.

906 **Fig 6** General view of a collapsed overhanging limestone slab located at northern
907 part of Cellina Valley gorge on January 26th, 1999. A rock bridge (red-hatched
908 area) was exposed after failure. The average tensile strength of this rock bridge
909 was calculated as 5.19 MPa through back-analysis. After Paronuzzi and
910 Serafini (2009).

911 **Fig 7** Relationship between shear displacement and horizontal shear force for
912 various numerical models containing non-persistent rock joint with different
913 geometrical parameters. Adapted from Zhang (2006).

914 **Fig 8** Stress and strain curves of Midgley Grit Sandstone joints with the same areal
915 persistence ($K=0.5$) in numerical direct shear tests under constant normal
916 stresses of 4 and 6 MPa. Three samples showing the spatial distribution of rock
917 bridges (Rb) and persistent joints (Pj) are shown. Particles representing rock
918 matrix (within the top and bottom shear boxes) are not shown for clarity. After
919 Shang and Zhao (2017).

920 **Fig 9** Relationship between joint persistence and normalized block size (**a**) and block
921 volume (**b**). Raw data from Kim et al. (2007b and 2007c).

922 **Fig 10** Relationship between relative rock mass strengths and persistence. Raw
923 data from Kim et al. (2007b and 2007b).

924 **Fig 11** Discontinuity geometrical parameters used in the numerical modelling by
925 Bahaaddini et al. (2013). Reproduced from Bahaaddini et al. (2013).

926 **Fig 12** Effects of discontinuity persistence on relative compressive strength of rock
927 masses (**a**) and on relative elastic modulus of rock masses (**b**). Note that yellow
928 is rock matrix in PFC model; green is non-persistent rock joint; red is tension
929 crack and blue is shear crack (rarely can be seen). K refers to linear
930 persistence. Adapted from Bahaaddini et al. (2013).

931 **Fig 13** DEM study results of the relationship between number of micro-cracks and
932 discontinuity areal persistence. Schematic diagrams of simulated slopes with
933 three different geometries are included (cracks are not shown): Centres of
934 gravity were located in the upper part (**a**), lower part (**b**) and the middle (**c**),
935 respectively. Shear cracks dominated when centres of gravity were located in
936 the upper part (**a**) and middle (**c**) of sliding block. Both tensile and shear cracks
937 occurred when centre of gravity was in the lower part of block. The dashed lines
938 correspond to tensile crack while continuous lines represent shear crack.
939 Adapted from Viviana et al. (2015)

940 **Fig 14** Diagrammatic representation of discontinuity traces intersecting a scanline
941 set up on a planar exposure of limited extent. For small size discontinuities or
942 those that are roughly parallel to scanline or concealed, bias will occur when
943 sampling. Adapted from Latham et al. (2006).

944 **Fig 15 a** Digital trace mapping of incipient discontinuities and blast-induced fractures
945 on local part of the Stawamus Chief (granite), British Columbia, Canada; **b**
946 Discontinuity traces were analysed after tracing. Irregular blast-induced
947 fractures were traced in red, bedding planes traced in green and scattered
948 joints traced in cyan and orange. After Tuckey et al. (2012).

949 **Fig 16** Schematic diagram showing the testing procedures for the forensic
950 excavation of rock masses (FERM). After Shang et al. 2017a

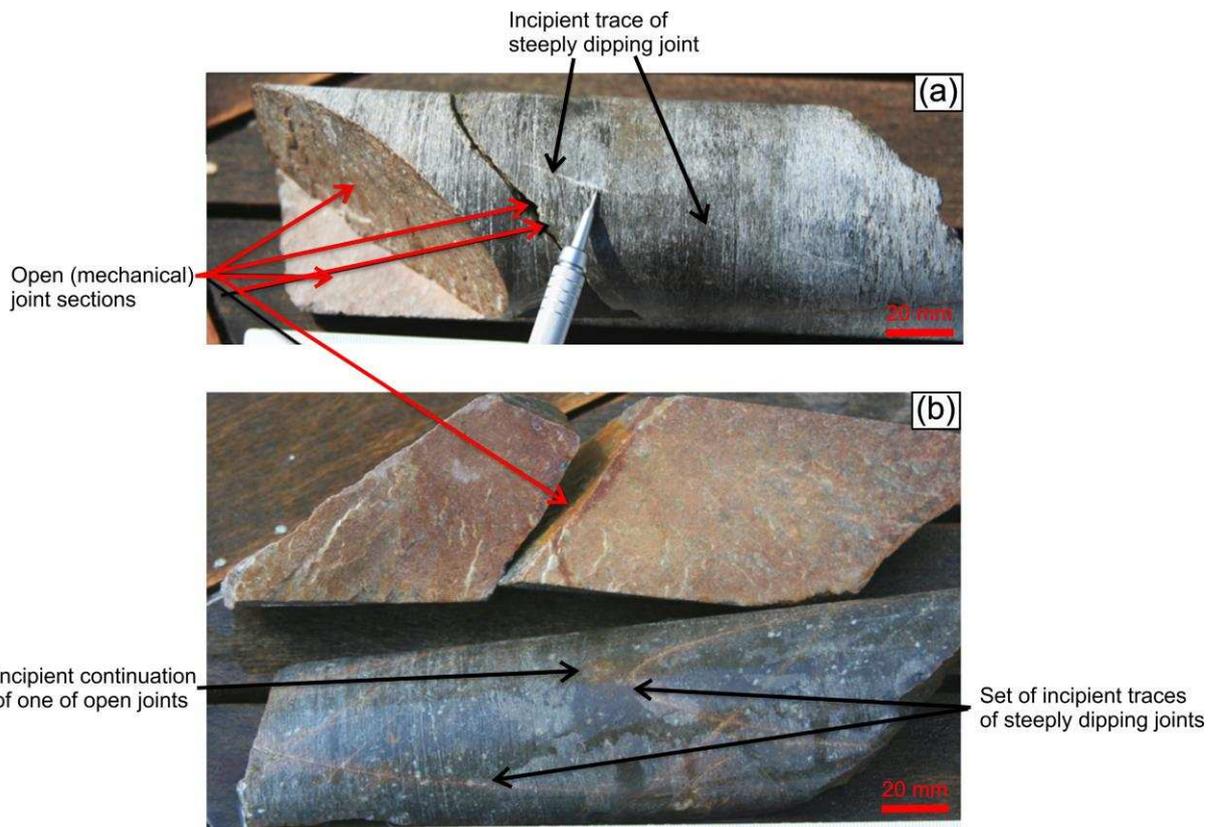


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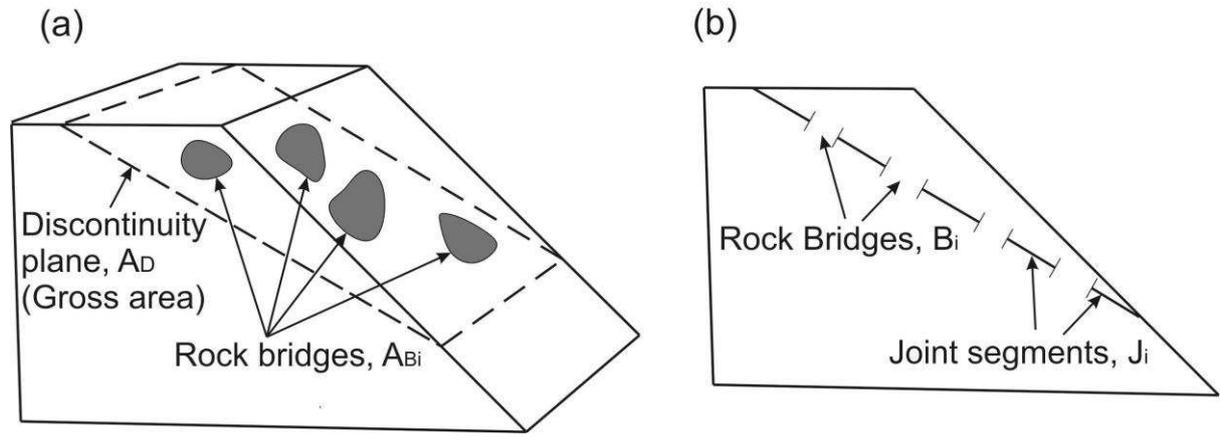
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976 Fig 4

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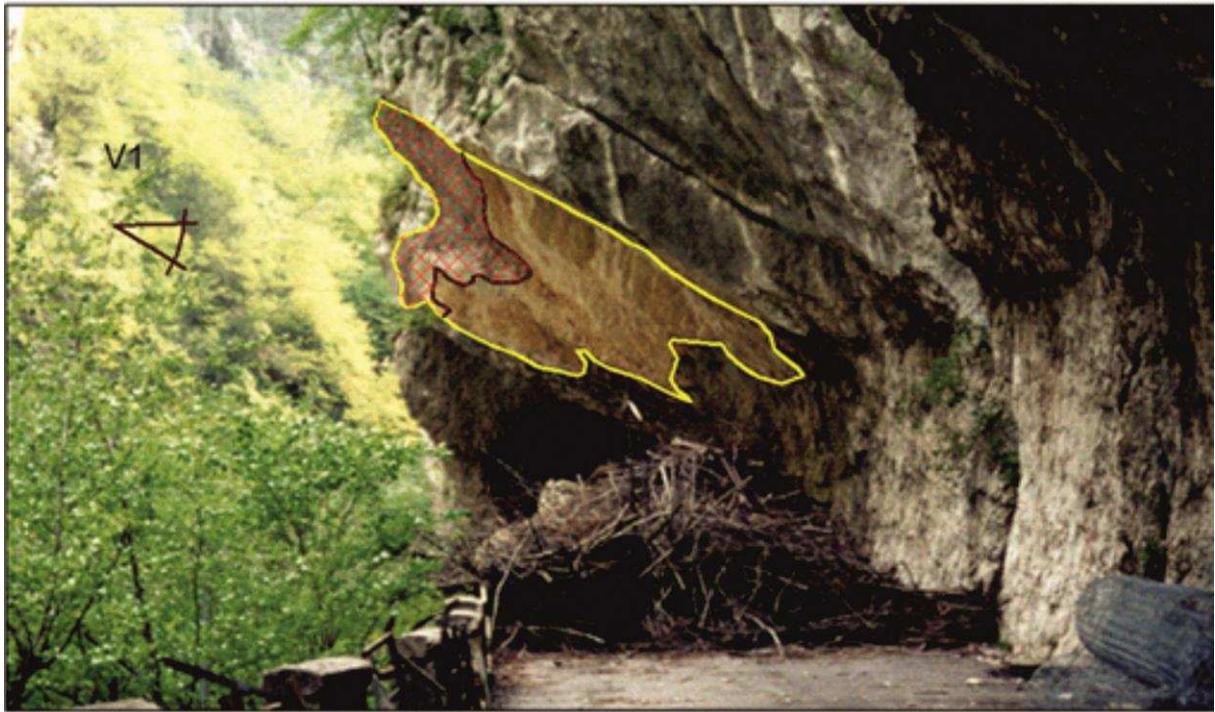
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1003 Fig 6

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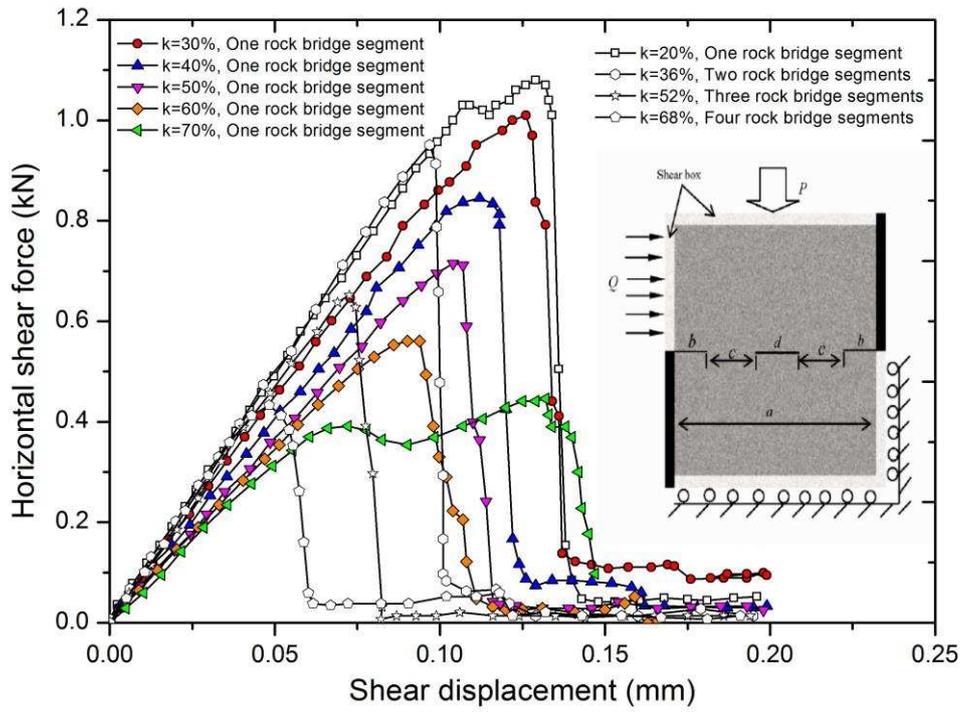
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1021 Fig 7

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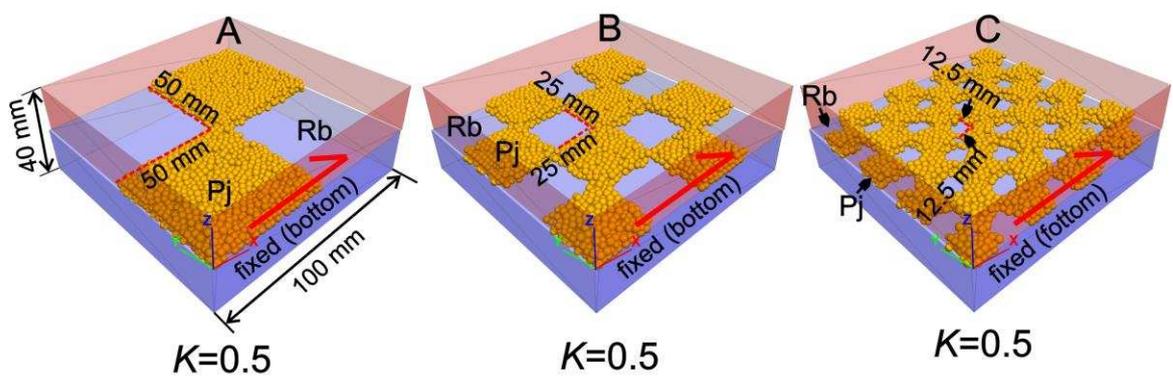
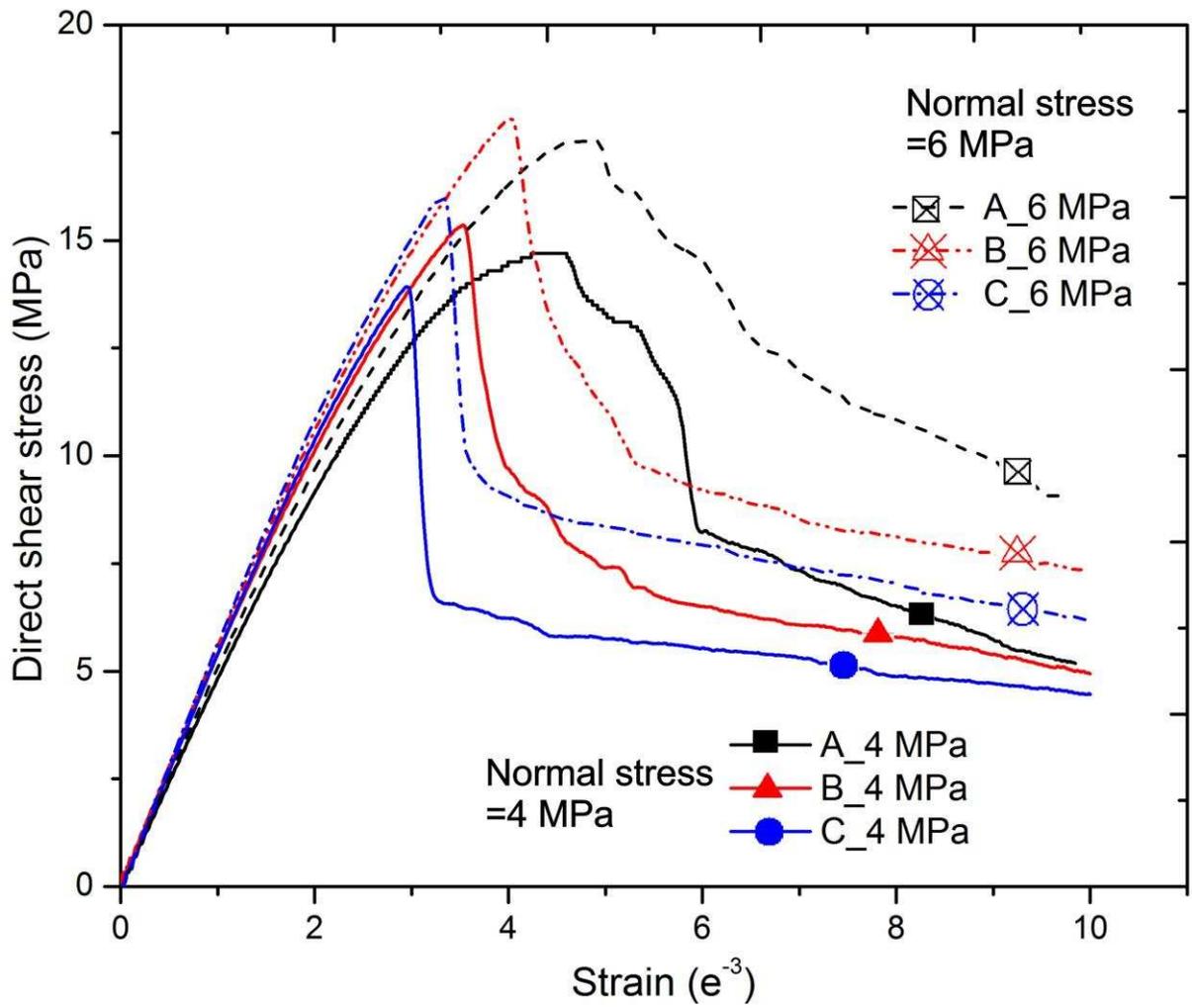
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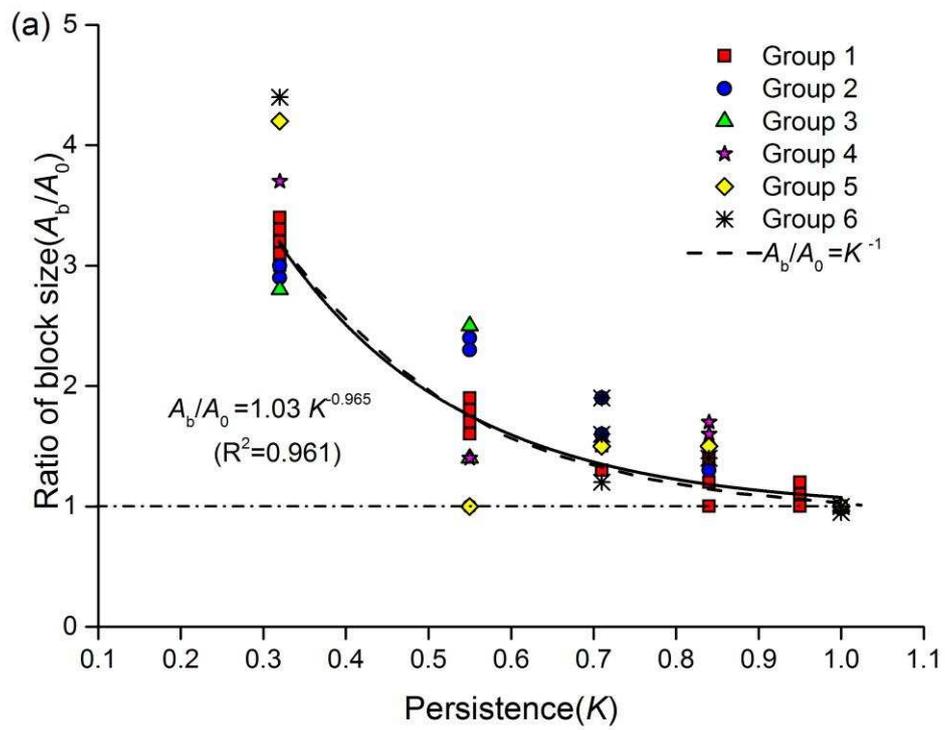
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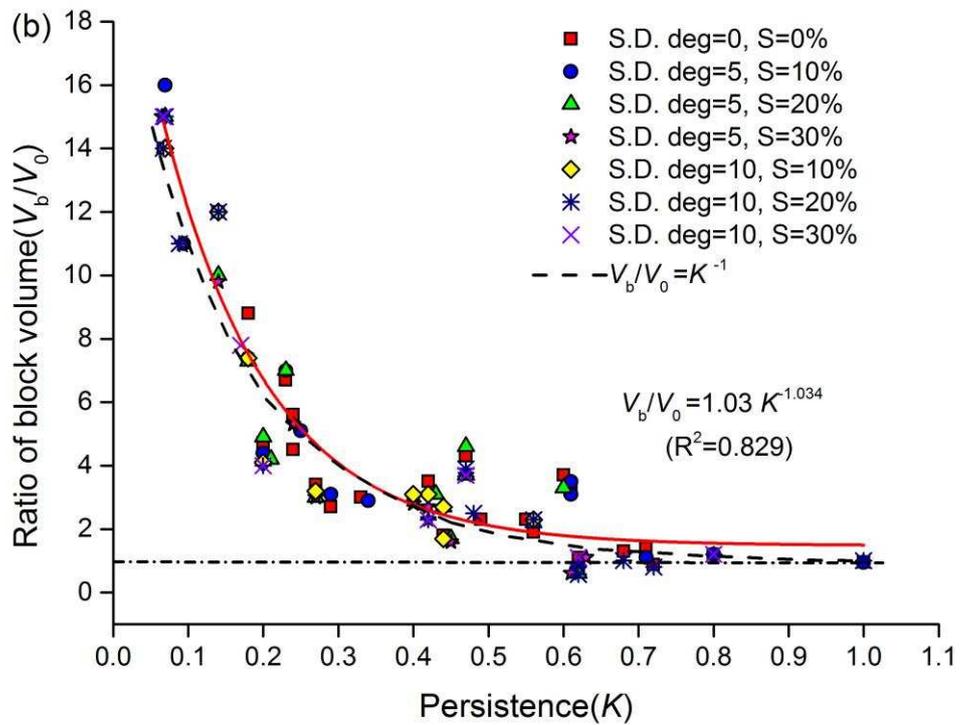
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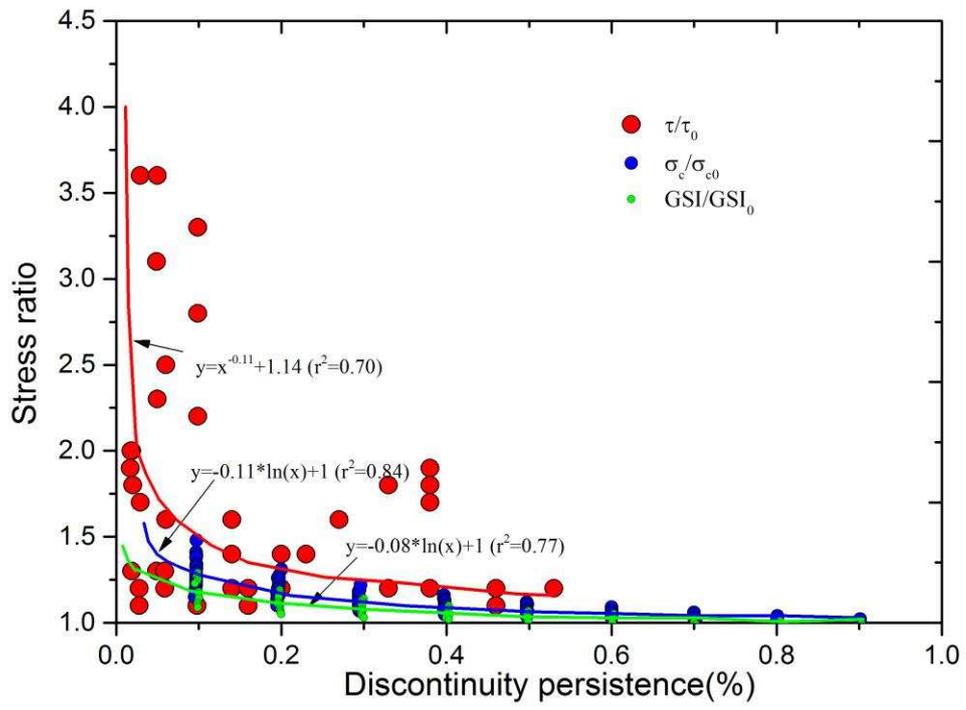


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1043 Fig 9



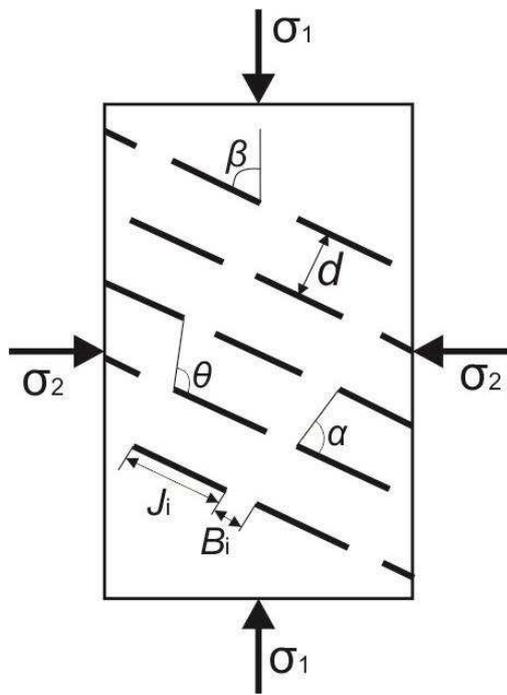
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1045 Fig 10

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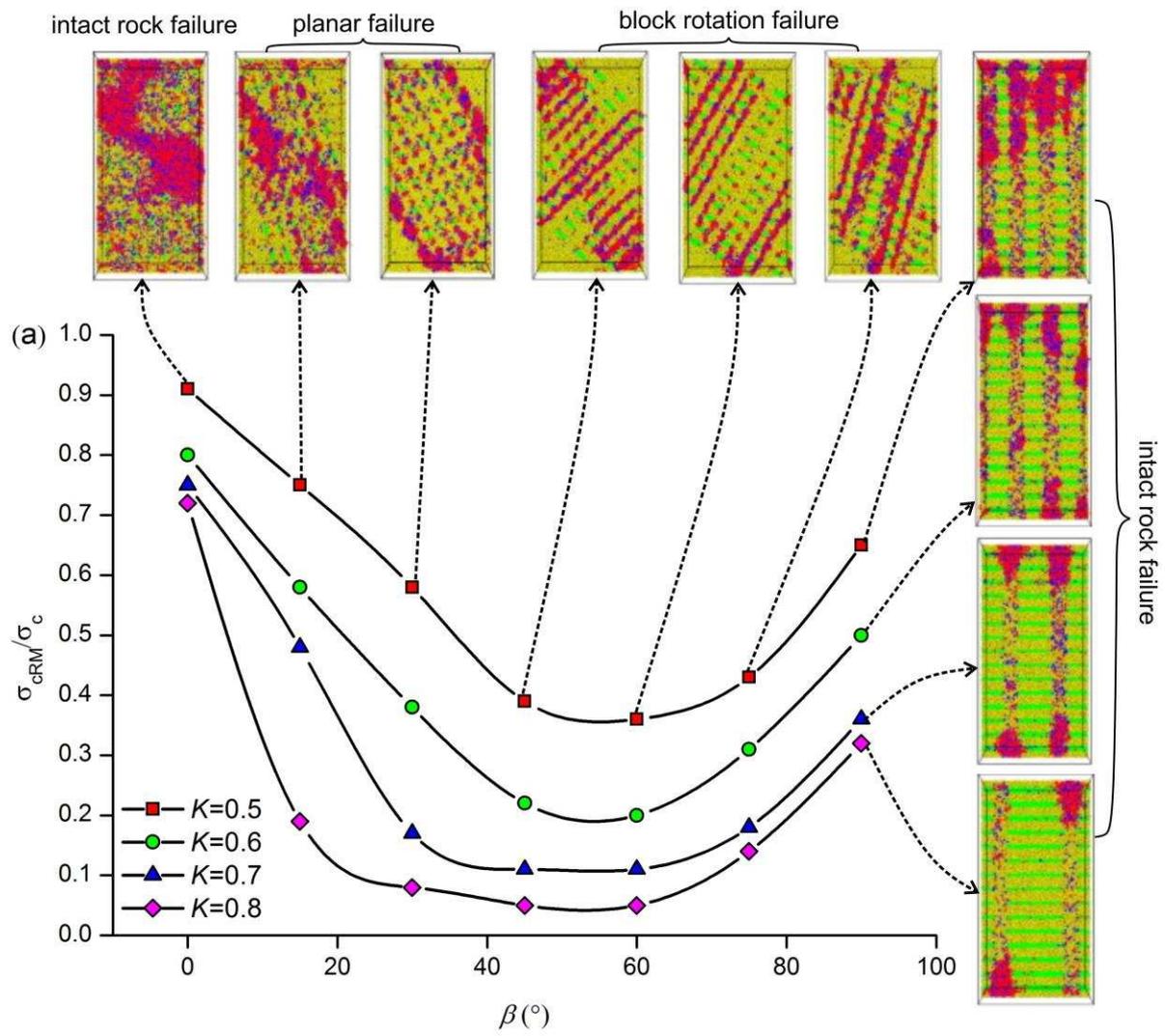
Parameter keys

- β Joint orientation relative to loading axis
- θ Joint step angle
- α Joint tip to tip angle
- d spacing
- J_i Persistent joint segment
- B_i Rock bridge segment
- σ_1 Principle stress
- σ_2 Intermediate stress

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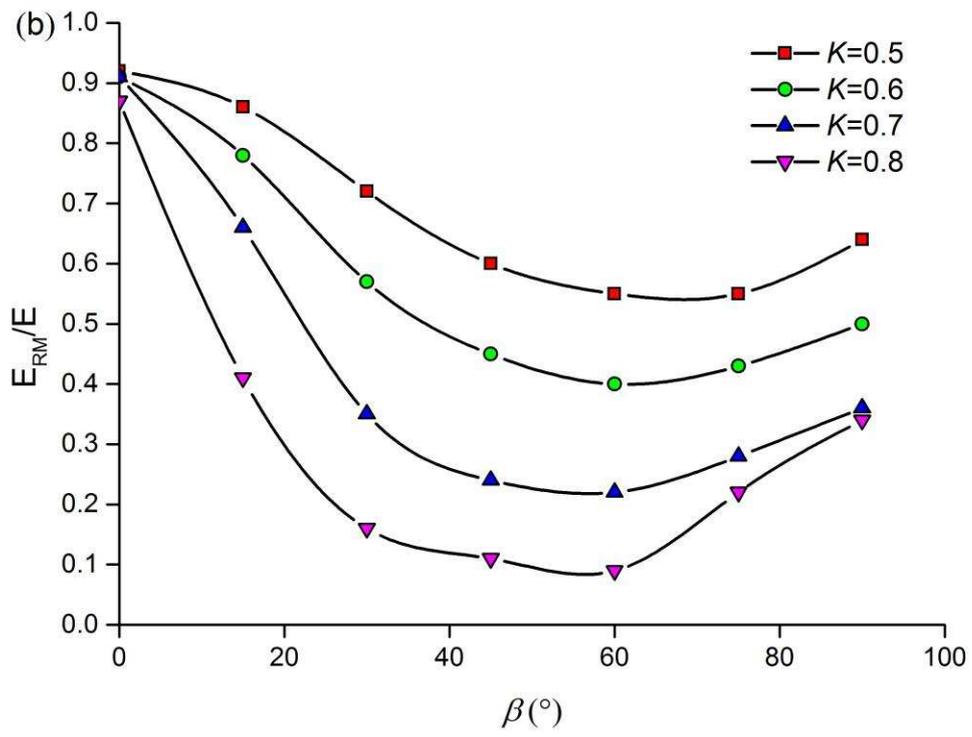
1050 Fig 11

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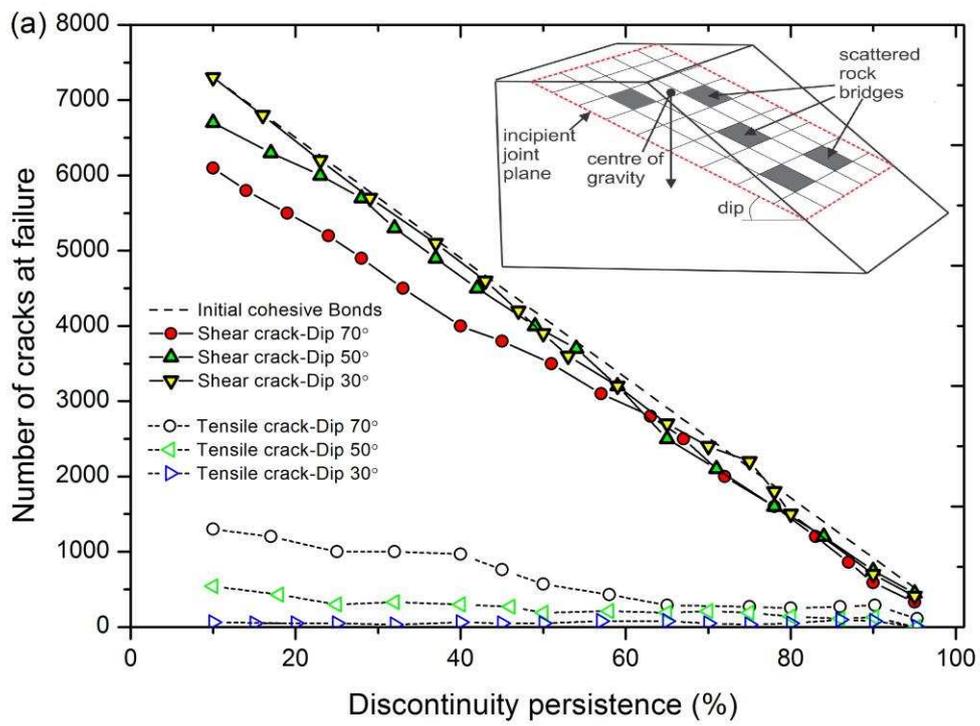
1053 Fig 12a

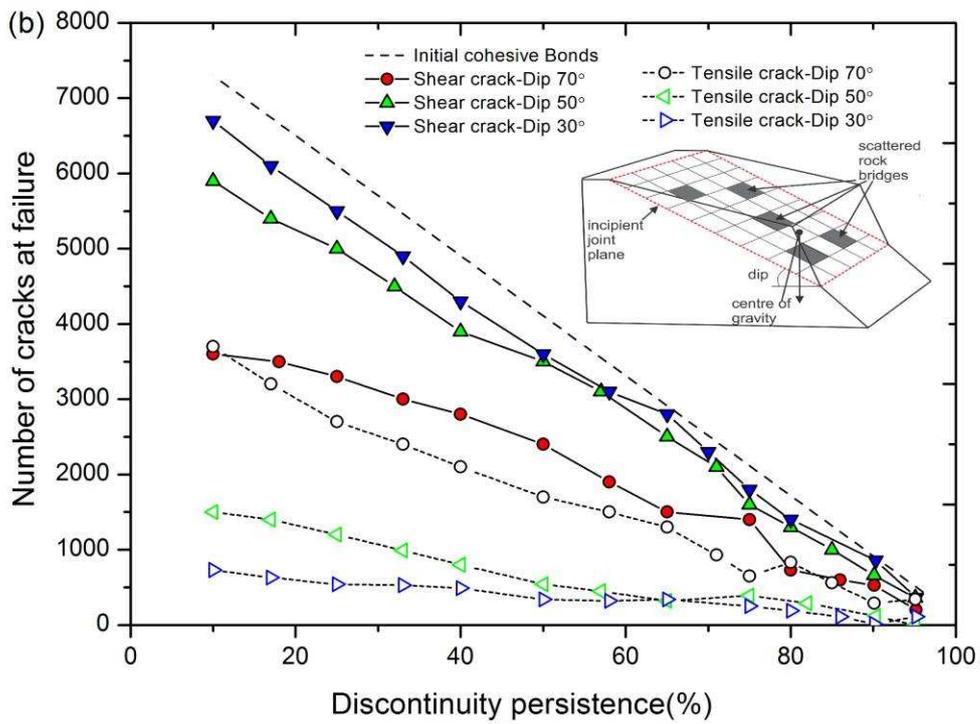


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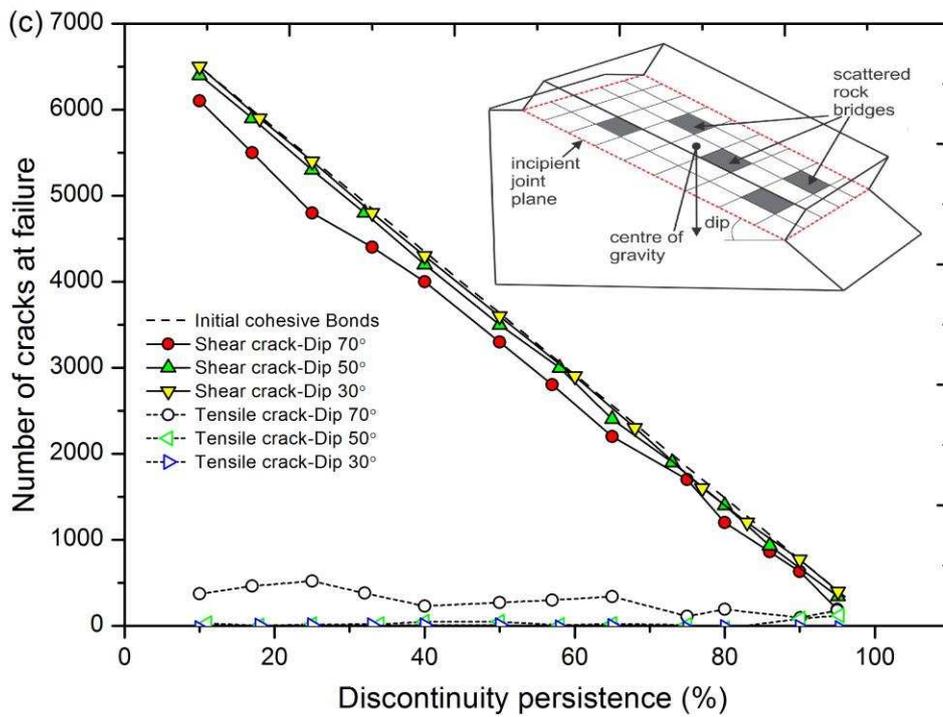
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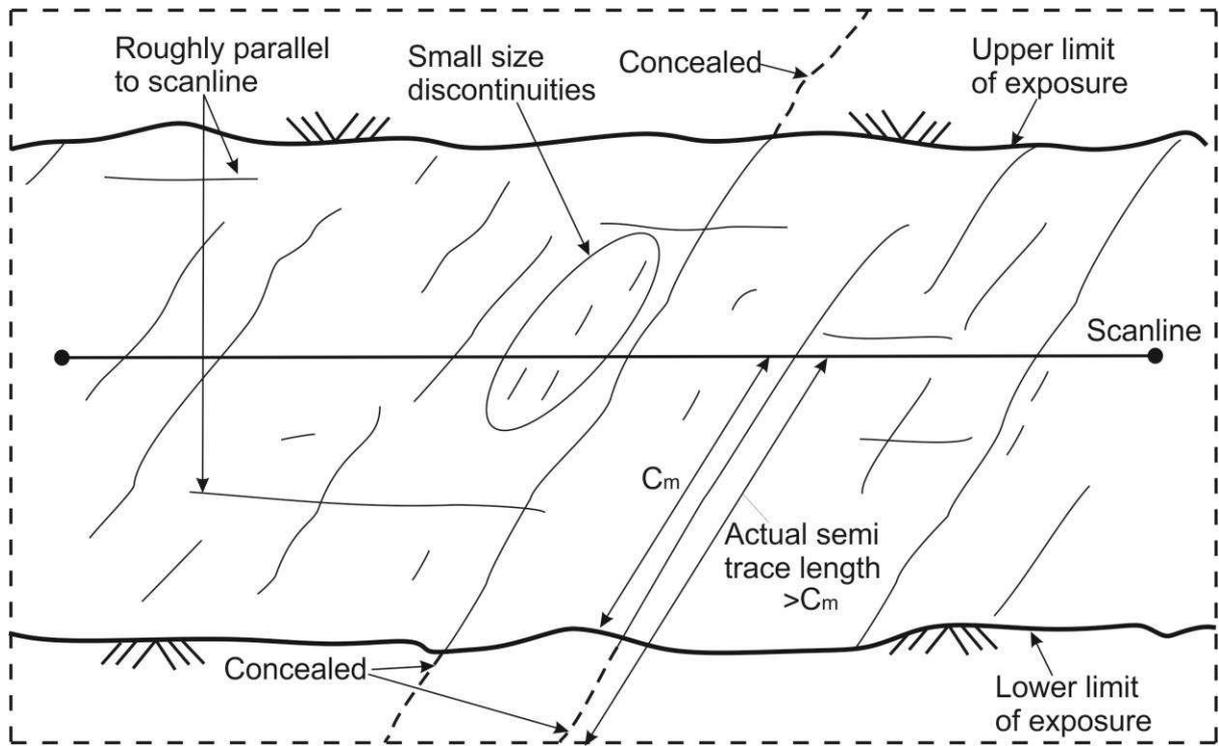


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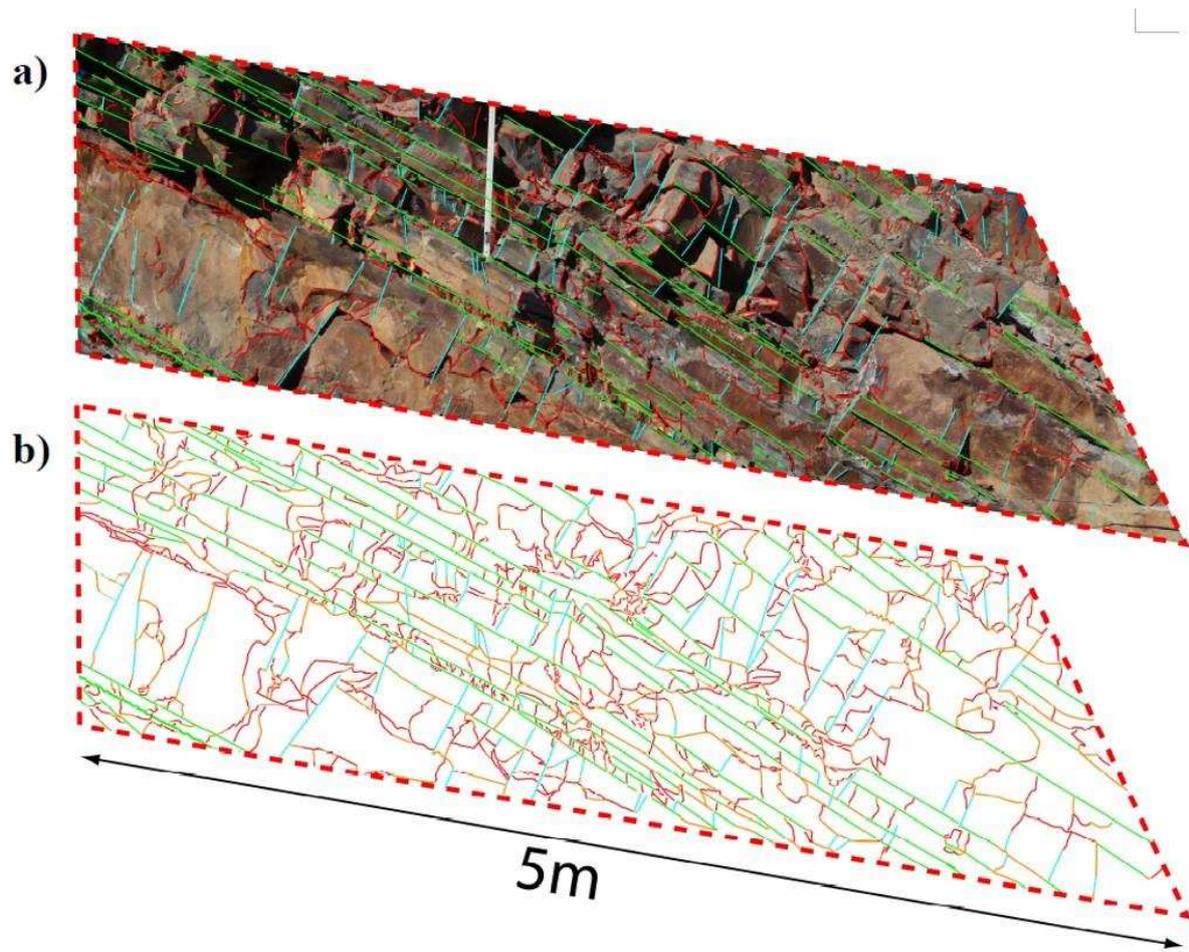
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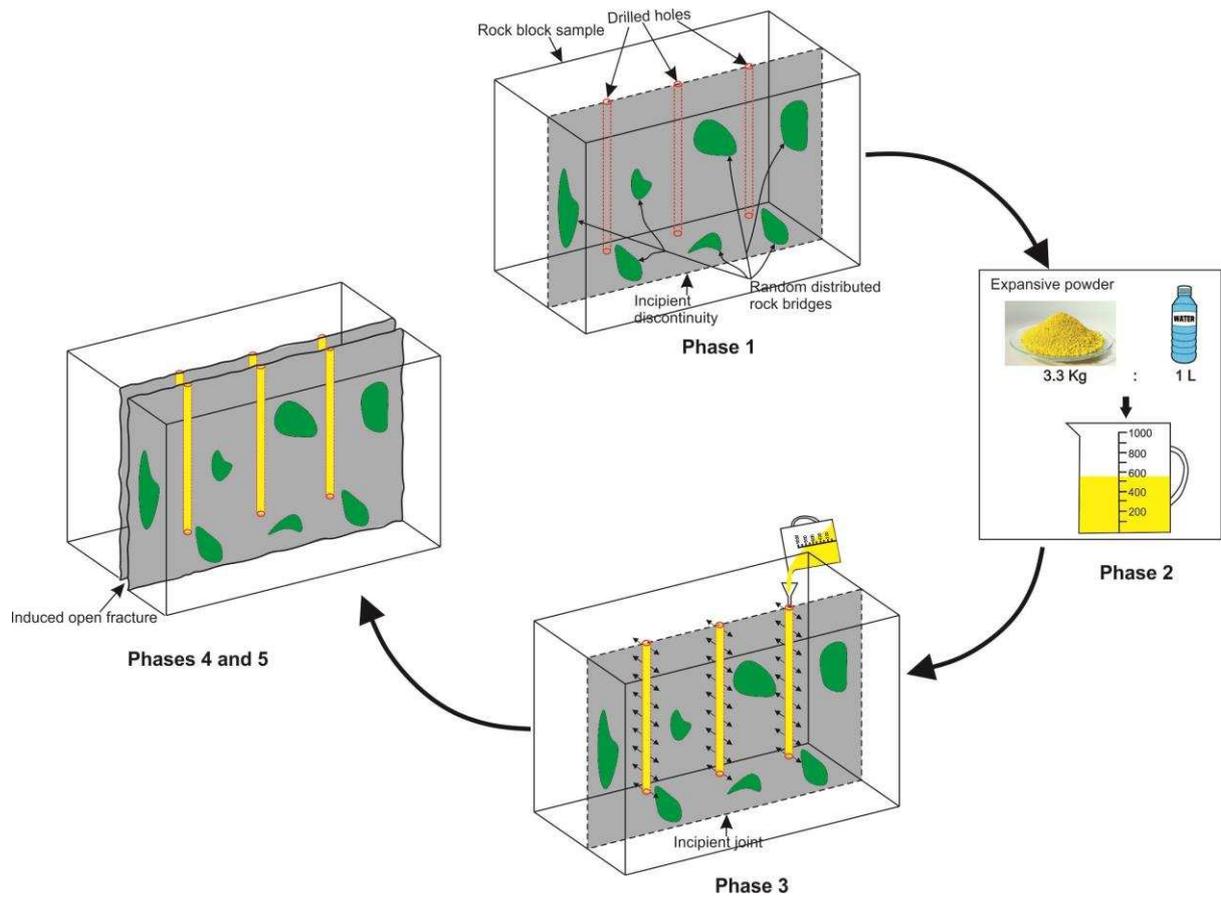
1064 Fig 14



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Table 1 Representative contributions to discontinuity size (trace length) estimation from censored measurements.

Methodologies	Major contributions	Remarks	Sampling methods	References
Censored exponential distribution	Field procedure was devised to provide a method for characterizing and estimating trace length. Data requirements dramatically reduced.	The analysis does not consider type of discontinuity termination and tends to overestimate larger trace length.	SS	Cruden 1977
Moment estimate	Moment estimation of unconditional radius distribution of joints was presented.	Reliability of results depends on the probability function assumed.	SS	Baecher and Lanney (1978)
Probability distribution analysis	Four simple probability distributions were used to study bias in scanline sampling. The relations between these distributions provide analytical methods of estimating mean discontinuity trace length.	Reliability of results depends on the probability function assumed.	SS	Priest and Hudson (1981)
Probability distribution function	A technique was proposed for estimating mean trace length on infinite exposures. Does not require lengths and density function of observed traces.	Only applicable to discontinuities whose orientation is described by a probability distribution function.	WS	Kulatilake and Wu (1984)
Distribution-free methods	Simple estimators were developed for the estimation of variably oriented fracture trace length as well as trace density.	Reliability of results depend on the probability function assumed; underlying distribution of trace length is generally unknown.	WS	Mauldon (1998)
Probability analysis, numerical and analytical methods	Joint trace length distribution was estimated for the Poisson disc joint model. Joint diameter distribution was also numerically and analytically investigated	Relies on the assumption that joint lengths are similar in strike and dip directions.	WS	Song and Lee (2001)
Stereological relation analysis	Stereological analysis used to estimate size distributions of elliptical discontinuity from true trace length distribution.	Discontinuity assumed planar and elliptical in shape.	SS and WS	Zhang and Einstein (2002)
Maximum likelihood method	Extends previous methods to include arbitrary joint set and sampling plane orientations.	Derived results only apply for joint traces normal to top and bottom of sampling window.	WS	Lyman (2003)
Statistical graphical approach	A flexible method for inference of trace length using statistical graphical model based on observations at rock outcrops.	--	WS	Jimenez-Rodriguez and Sitar (2006)

Probability weighted moments (PWM) and L-moments	A distribution-free method to estimate fracture trace length distributions in the light of the estimation of PWM and L-moments of true trace length.	--	WS	Li et al. (2014)
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SS: Scanline Sampling; WS: Window Sampling

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