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A Modification to the WPC Model

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Abstract:
We modify the Barclay and Warner (1993) Weighted Price Contribution model - which measures market participants' contribution to price discovery process - to incorporate price movements that extend beyond the final price. We validate our model with an empirical illustration.

Keywords: Weighted Price Contribution, Price Discovery, High Frequency Trading.
JEL classification: G1

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1. Introduction

One of the core functions of financial markets is the facilitation of price discovery – a process through which market players determine asset prices. A key issue concerning this process is how to empirically measure the impact that market players have on price discovery. One of the most commonly used methods is the application of the Weighted Price Contribution model (WPC), originally designed by Barclay and Warner (1993) to examine which trade moves prices first[1] and adapted by Huang (2002) to measure individual market players' contribution to price discovery. The advantage of the WPC over other commonly used price discovery models is that it provides "a single measure that can be used in cross-sectional analyses" (Huang, 2002). However, one drawback of the WPC - especially given the increasing importance of high frequency trading – is that it may over- and/or underestimate the contribution of certain trades to price discovery. Specifically, overestimation occurs when prices are overshot in the direction, yet, outside the range of the total price movement; whereas the contributing factor of the trades that correct this overshoot is underestimated.

In this paper we present a Modified Weighted Price Contribution model (MWPC), which corrects for the above misestimation by incorporating the location of price movement relative to the final price. We further offer some empirical evidence on the validity of the MWPC.

2. Original WPC, its Limitation and Empirical Validation

2.1. Original WPC

Huang (2002) defines the WPC for dealer *j* in stock *i* as follows:

$$WPC_{i,j} = \sum_{t=1}^{T} \left(\frac{|\Delta p_t^i|}{\sum_{t=1}^{T} |\Delta p_t^i|} \right) \left(\frac{\Delta p_t^{i,j}}{\Delta p_t^i} \right), \tag{1}$$

where T is total number of periods in a day, Δp_t^i is the price change of an asset *i* in period *t* and $\Delta p_t^{i,j}$ is the sum of all price changes that dealer *j* is responsible for, which can be expressed as follows:

$$\Delta p_t^{i,j} = \sum_{n=1}^{N_t} (p_n^j - p_{n-1}), \tag{2}$$

where N_t is the number of trades or quotes in period *t*, p_n^j is the current price disseminated by dealer *j* and p_{n-1} is the previous price (by any dealer).

Consider a scenario of price formation, where, in a period *t*, the final price ($p_f^{j_4}$) is higher than the start price (p_s), as shown in Figure 1. Based on the WPC definition, any dealer that has disseminated a price

[1] For examples of the application of WPC model, see, Barclay and Hendershott, 2003; Huang, 2002; Madureira and Underwood, 2008; O'Hara et al., 2014.

- relatively to the previous price - in the same (opposite) direction of the price movement from p_s to $p_f^{j_4}$, has positively (negatively) contributed to the price discovery process. In this scenario the positive contributors are agents that move the prices from p_s to p^{j_1}, p^{j_1} to p^{j_2} and p^{j_3} to $p_f^{j_4}$, whereas the negative contribution is the price movement from p^{j_2} to p^{j_3}.

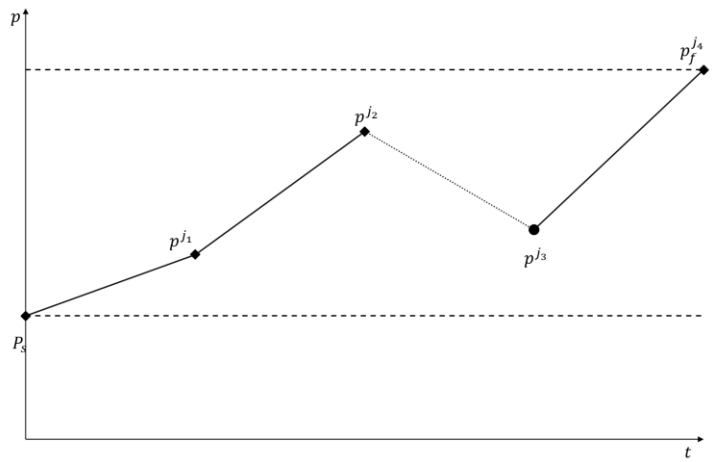

Figure 1. An illustration of a price formation process where the start price (p_s) is lower than the final price ($p_f^{j_4}$). The horizontal dash lines present the range of the start and final prices.

2.2. WPC Limitation

The scenario described above is ideal, as all the prices are within the range of the start and final prices. In financial markets, however, agents are heterogeneous, and an overshoot (undershoot) in prices may occur. In essence, the over and/or underestimation of the contribution to price discovery occurs if at least one of the prices - p_{n-1} and/or p_n^j- are higher (lower) than the final price, where $p_f > p_s$ ($p_f < p_s$). We formalise the latter expression as follows:

$$\begin{aligned} &\text{Max}\{p_{n-1}, p_n^j\} > p_f, \text{ where } p_f > p_s \\ &\text{Min}\{p_{n-1}, p_n^j\} < p_f, \text{ where } p_f < p_s \end{aligned} \tag{3}$$

To illustrate the limitation of the WPC, we present a scenario of a single period price discovery process in Figure 2. As before, the start price (p_s) is lower than the final price ($p_f^{j_7}$) and the prices p^{j_1} to $p_f^{j_7}$ are disseminated by dealers j_1 to j_7, respectively.

While the price changes disseminated by dealers j_1, j_2 and j_7 fall outside the ranges expressed in Eq. (3) the WPC misestimates the contribution to price discovery for dealers j_3 to j_6. Dealer j_3 (j_6) has traded in the same (opposite) direction of the price movement (p_s to $p_f^{j_7}$), however, at a price higher (lower) than the final price $p_f^{j_7}$. According to the WPC, a positive (negative) contribution to price discovery of $\frac{(p^{j_3} - p^{j_2})}{(p_f^{j_7} - p_s)}$

$\left(\frac{(p^{j_6}-p^{j_5})}{(p_f^{j_7}-p_s)}\right)$ is produced by j_3 (j_6). Consequently, the contribution of dealer j_3 (j_6) is overestimated (underestimated) due to the overshoot (partial price correction) of $|p^{j_3}-p_f^{j_7}|$ ($|p^{j_5}-p_f^{j_7}|$).

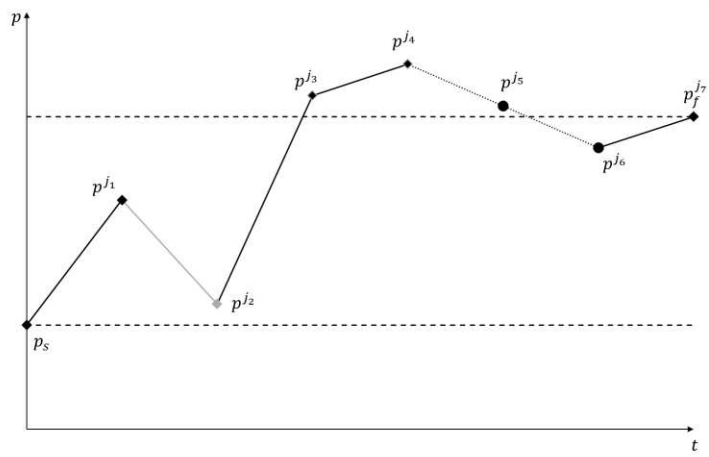

Figure 2. An illustration of a price formation process where the start price (p_s) is lower than the final price ($p_f^{j_7}$). The horizontal dash lines present the range of the start and final prices.

Even more striking are the premises about dealers j_4 and j_5. Based on the WPC, a positive contributor is dealer j_4, who raised the price from p^{j_3} to p^{j_4}, furthering the price from the final price by $|p^{j_4}-p^{j_3}|$, while dealer j_5, who has partially corrected the overreaction of dealer j_4 and has brought the price closer to its final price by $|p^{j_5}-p^{j_4}|$, obtains a negative WPC.

2.3. WPC Limitation Empirical Validation

Our premise for the requirement to modify the WPC is the notion that a considerable amount of price movements falls under the 'troublesome' price movements, as expressed in Eq. (3). We, therefore, present in Table 1 the summary statistics of these 'troublesome' price movements, based on high frequency one day[2] trade data for four stocks (Apple, Bank of America, J.P. Morgan, Microsoft) and the EUR:USD[3] exchange rate. The summary statistics include the number of total ('troublesome') 1-minute intervals for the given asset on the day, shown in Column (I) ((II)). The results show that from the 390 (480) minute intervals observed for the stock (currency) data, a greater proportion incurred at least one 'troublesome' price movement.

We further report the mean, median and the maximum number of the 'troublesome' price movements within the samples of 'troublesome' minutes in Columns (III) to (V), respectively. The results show the mean (median) number of 'troublesome' price movements ranges from 23.09% to 31.50%

[2] We obtain the trade data on equity and quote data on the currency pair for March 12th, 2018 from Bloomberg and Thomson Reuters Eikon, respectively.

[3] Following Chaboud et al., (2014) we focus on the most active time of the market that is 8 to 16 GMT.

(17.67% to 25.76%) across different assets. Finally, in at least one of the 'troublesome' minutes, the ratio of 'troublesome' price movements exceeds 90% (80%) for stock (currency) data.

Table 1.
Summary statistics of the 'troublesome' minutes and 'troublesome' price movements for a sample of high-frequency data on March 12th, 2018 at 1-minute intervals.

	(I)	(II)	(III)	(IV)	(V)
	Total	Troublesome	Mean	Median	Max
	Minutes		Troublesome Price Movements		
Apple	390	330	26.94%	19.03%	96.02%
Bank of America		258	25.42%	17.67%	93.98%
J.P. Morgan		271	31.50%	25.71%	95.00%
Microsoft		301	22.88%	13.75%	95.12%
EUR:USD (Ask)	480	262	24.43%	18.83%	83.00%
EUR:USD (Bid)		255	23.09%	18.24%	83.06%

Overall, we show that the problem of over- and/or underestimation by WPC can materialize at different studied frequencies[4], specifically when high volatility or price reversal exist.

3. The Modified WPC

3.1. The Model

In our Modified WPC (MWPC), rather than comparing the current price to the previous price, we measure the change in the distance to the final price. To do so, we first adjust the numerator of the right hand side of Eq. (1). More formally, we express the modified calculation of $\Delta p_t^{i,j}$ as follows:

$$\nabla p_t^{i,j} = \sum_{n=1}^{N_t} (\underbrace{|p_{n-1}^i - p_f^i|}_{distance\ I} - |\underbrace{p_n^{i,j} - p_f^i}_{distance\ II}|) , \tag{4}$$

where ∇ is used instead of Δ to highlight that convergence to the final price rather than the total price movement should be attributed to agent *j*, and Distance *I* (*II*) shows the distance of p_{n-1} to p_f (p_n^j to p_f). Consequently, the MWPC yields a negative value (contribution) to agent *j* for price movements where p_n^j is further from p_f than the distance of p_f to p_{n-1}. Whereas, whenever agent *j* reduces the distance to p_f a positive contribution to price discovery will be obtained.

Note here, that with our modification, we can now ignore the direction of the start to final price movement. This is because we propose to focus on price convergence to the final price, rather than the actual price change. In addition MWPC does not violate the prerequisite of the WPC that all agents' contribution to price discovery sums up to 100%. In essence, we change the final and start price difference (Δp_t^i) to its absolute value ($|\Delta p_t^i|$). The formal expression of the MWPC is as follows:

[4] We extend our analysis using alternative to 1-minute time intervals and find similar results, however, due to brevity, do not report these results.

$$MWPC_{i,j} = \sum_{t=1}^{T} \left(\frac{|\Delta p_t^i|}{\sum_{t=1}^{T} |\Delta p_t^i|} \right) \left(\frac{\nabla p_t^{i,j}}{|\Delta p_t^i|} \right) = \sum_{t=1}^{T} \left(\frac{\nabla p_t^{i,j}}{\sum_{t=1}^{T} |\Delta p_t^i|} \right) \quad (4)$$

3.2. Empirical MWPC Illustration

To illustrate the economic significance of the modification to the WPC, we compare the price discovery estimates derived by the two models for ten most active market dealers in the EUR:USD currency pair[5]. We report in Table 2 the mean (median) absolute differences between the estimates for ask and bid price discovery separately, with t-values (z-values) of the mean (median) equality tests provided in the parentheses below.

Table 2.
Summary of the absolute differences between WPC and MWPC estimates for the top 10 most active dealers in EUR:USD, on March 12th, 2018 at 1-minute frequency.

		Mean(%)	Median(%)
EUR:USD	Ask	6.53***	5.61***
		(4.22)	(2.81)
	Bid	5.24**	2.44***
		(2.73)	(2.80)

The results provide evidence that the two models yield significantly different results, as indicated by statistically significant mean and median differences for both, ask and bid prices. In essence, we show that the MWPC corrects the over-/under-estimation of WPC. Unreported tests further show that the results are robust to the use of various settings (i.e. using different intervals, including all dealers in the sample, etc.).

4. Conclusions

The emergence of high frequency trading has changed the nature of the price discovery process. How this phenomena affected the application of the Weighted Price Contribution model, however, has been unexplored. In this paper we re-examine the WPC and theoretically and empirically show that when a dealer submits a price beyond the final price the WPC over- and/or underestimates a market participant's contribution to price discovery. This is especially more likely to occur when the number of trades, coupled with higher price volatility and growth in price reversal trends, increases. We address this issue with our Modified WPC model by estimating the contribution to price discovery based on convergence of prices towards the final price. Using high frequency data, we further our argument with some empirical evidence on the different estimates the two models yield for the top 10 most active dealers in the EUR:USD currency pair.

[5] We chose the currency pair data for the illustration because it contains dealers' identity. This allows us to compare WPC and MWPC for individual dealers.

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