Proceedings Paper:

Abdullah, M. and Rossiter, J.A. (2018) A formal sensitivity analysis for Laguerre based predictive functional control. In: Proceedings of 2018 UKACC 12th International Conference on Control (CONTROL). Control 2018: The 12th International UKACC Conference on Control, 05-07 Sep 2018, Sheffield, UK. IEEE , pp. 20-25. ISBN 978-1-5386-2864-5

https://doi.org/10.1109/CONTROL.2018.8516849

A Formal Sensitivity Analysis for Laguerre Based Predictive Functional Control

Muhammad Abdullah* and John Anthony Rossiter†
*†Department of Automatic Control and System Engineering,
University of Sheffield, Mappin Street, S1 3JD, UK.
Email: MAbdullah2@sheffield.ac.uk* and j.a.rossiter@sheffield.ac.uk†
*Department of Mechanical Engineering,
International Islamic University Malaysia, Jalan Gombak, 53100, Kuala Lumpur Malaysia.
Email: mohd_abdl@iium.edu.my*

***Abstract*—A Laguerre Predictive Functional Control (LPFC) is a simple input shaping method, which can improve the prediction consistency and closed-loop performance of the conventional approach (PFC). However, it is well-known that an input shaping method, in general, will affect the loop sensitivity of a system. Hence, this paper presents a formal sensitivity analysis of LPFC by considering the effect of noise, unmeasured disturbance and parameter uncertainty. Sensitivity plots from bode diagrams and closed-loop simulation are used to illustrate the controller robustness and indicate that although LPFC often provides a better closed-loop tracking response and disturbance rejection, this may involve some trade-off with the sensitivity to noise and parameter uncertainty. Finally, to validate the practicality of the results, the sensitivity of the LPFC control law is illustrated on real-time laboratory hardware.**

***Index Terms*—Predictive Control, PFC, Sensitivity Analysis, Laguerre function, Parameter Uncertainty, Noise, Disturbance**

I. Introduction

Model Predictive Control (MPC) is an optimal controller that employs a control action based on a future output prediction. Typically, MPC utilises a finite horizon prediction in the optimisation process and can explicitly take into account different types of constraints in a system [1]. Nevertheless, the implementation of this controller is often more expensive and requires higher computational effort and time compared to its competitors [2]. Hence for low-end applications, it is wiser to consider a simpler controller such as Proportional Integral Derivative (PID) or Predictive Functional Control (PFC).

Developed in 1973, PFC is known as a simplified version of MPC that minimises the output error at a single point instead of over a whole trajectory [3], [4]. With this simplification, PFC only needs simple coding and minimal computation. Although in general, the computed input is not optimal, it still retains some of the core benefits of an MPC approach such as systematic handling of constraints and/or systems with delays [4]. Besides, the use of a target first-order Closed-loop Time Response (CLTR) as one of its tuning parameters, makes the design process more transparent. Currently, this controller is widely used in many industrial applications and has become a prime competitor with PID regulators [4]–[6].

This work is funded by International Islamic University Malaysia and Ministry of Higher Education Malaysia.

Despite its attractive attributes, the simple PFC concept is often unable to provide a consistent prediction [7], accurate constrained solutions [8] and effective handling of systems with challenging dynamics [9], [10]. Several works have modified the traditional PFC framework to tackle these weaknesses either via cascade structures [4], [11], pole-placement [9], [12] or input shaping [8], [10]. However, the derivation of these methods often excludes explicit consideration of uncertainty, and only a few works have systematically discussed or analysed the robustness of PFC [13], [14]. Hence, the main objective of this work is to tackle this issue on one of its alternative structures know as Laguerre PFC (LPFC).

LPFC is defined by shaping the future predicted input trajectory with a first-order Laguerre polynomial [15], [16]. Instead of the constant input assumption of PFC, the future dynamics are now forced to converge gradually to the steady-state value. This modification can improve the prediction consistency and the significance of CLTR as a tuning parameter [16]. Furthermore, due to the well-posed decision making, satisfying constraints within a larger validation horizon becomes more accurate and less conservative [8]. However, this algorithm, as in common in MPC, is utilising the model parameters to estimate the steady state input while improving the loop performance and hence, it is worth investigating its sensitivity concerning noise, disturbances and parameter uncertainty.

Since the general unconstrained PFC framework provides a fixed control law, loop sensitivity can be computed and analysed to assess the controller robustness [3]. The performance of LPFC will be benchmarked against a nominal PFC structure to get some insight into the sort of sensitivity trade-off that ones should expect. The reader is reminded again that the scope of this work is only focused on simple and stable dynamic system; further development of LPFC to deal with challenging or unstable systems constitutes future work and in general is non-simple with a PFC approach.

This paper consists of five main sections. Section II discusses the basic formulation and derivations of sensitivity functions for PFC and LPFC. Section III presents some numerical examples. Section IV illustrates the findings are consistent with those on real-time laboratory hardware and section V gives the conclusions.

II. PFC Structures and Sensitivity Functions

This section presents a brief formulation for both PFC and LPFC together with the derivation of their sensitivity functions. More detailed derivations, theory and concepts are available in these references [3], [4], [6], [7]. Without loss of generality, this work utilises an autoregressive with exogenous terms (ARX) model with an independent model (IM) structure.

A. Conventional PFC

1) Target trajectory: PFC is designed to follow a closed-loop behaviour of the first order system with a delay τ (or h samples) and a time constant T_r [7]. The z-transform of the target trajectory, $r(z)$ with steady-state R is:

$$r(z) = \frac{z^{-h}(1-\lambda)}{1-\lambda z^{-1}} R \tag{1}$$

The representation of target pole, λ in (1) is equivalent to the desired closed-loop time response (CLTR) which is normally used by industrial practitioners [4]. The conversion can be presented by $T_r = CLTR/3$, where $\lambda = e^{\frac{-T}{Tr}}$ with T the sampling period.

2) Coincidence point and degree of freedom: The control objective of PFC is to force the system open-loop prediction, y_p to exactly match the predicted target trajectory of (1) at a selected coincidence point n samples into the future [4]. Consequently, the control law is formulated to enforce the equality:

$$y_{p,k+n|k} = (1-\lambda^n)R + \lambda^n y_{p,k} \tag{2}$$

where $y_{p,k+n|k}$ is the n-step ahead system prediction at sample time k and $y_{p,k}$ is the current process output measurement.

3) Independent model: The independent model (IM) structure is often used in conventional PFC [4], [5] as this is known to provide good sensitivity properties in general, yet it is only applicable to open-loop stable systems. The implementation is equivalent to using a step response model (ignoring truncation errors [1]). Both the model G_m and process G_p run in parallel using the same input u_k (see Fig.1). The error ($d_k = y_{p,k} - y_{m,k}$) between process output y_p and model output y_m is utilised to handle noise, disturbance and parameter uncertainty. Using the unbiased model prediction, the equality (2) is altered to:

$$\begin{aligned} (1-\lambda^n)R + \lambda^n y_{p,k} &= y_{m,k+n|k} + d_k \\ (R - y_{p,k})(1-\lambda^n) &= y_{m,k+n|k} - y_{m,k} \end{aligned} \tag{3}$$

4) Control law: The n-step ahead prediction algebra for an ARX model is well known in the literature, which can be represented using Toeplitz/Hankel form (e.g. [1]), hence only the final form is given here. For input u_k and model outputs $y_{m,k}$, the n-step ahead linear prediction model is:

$$y_{m,k+n|k} = H\underset{\rightarrow}{u_k} + P\underset{\leftarrow}{u_k} + Q\underset{\leftarrow}{y_{m,k}} \tag{4}$$

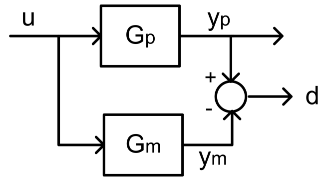

Fig. 1: The independent model structure.

where parameters H, P, Q depend on the model parameters and for a model of order m:

$$\underset{\rightarrow}{u_k} = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+n-1} \end{bmatrix}; \ \underset{\leftarrow}{u_k} = \begin{bmatrix} u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-m} \end{bmatrix}; \ \underset{\leftarrow}{y_{m,k}} = \begin{bmatrix} y_{m,k} \\ y_{m,k-1} \\ \vdots \\ y_{m,k-m} \end{bmatrix} \tag{5}$$

Substituting prediction (4) into equality (3) gives:

$$H\underset{\rightarrow}{u_k} + P\underset{\leftarrow}{u_k} + Q\underset{\leftarrow}{y_{m,k}} - y_{m,k} = (R - y_{p,k})(1-\lambda^n) \tag{6}$$

The constant future input assumption of PFC [3], [4] means that $u_{k+i|n} = u_k$ for $i > 0$, hence defining $h = \sum(H)$, the control law reduces to:

$$u_k = \frac{1}{h}\left[(1-\lambda^n)R - (1-\lambda^n)y_{p,k} - Q\underset{\leftarrow}{y_{m,k}} + y_{m,k} - P\underset{\leftarrow}{u_k}\right] \tag{7}$$

The control law can be represented in a vector form by rearranging (7) in terms of parameters F_p, N_p, M_p and $\hat{D}_p$ with obvious definitions:

$$u_k = F_p R - N_p \underset{\leftarrow}{y_{m,k}} - M_p y_{p,k} - \hat{D}_p \Delta\underset{\leftarrow}{u_k} \tag{8}$$

Remark 1: Conventional PFC can work well with low order and simple dynamical systems, especially when the coincidence point is selected properly [7]. However, with the restricted degree of freedom (d.o.f) in its future input dynamics, an inconsistency between open-loop and closed-loop predictions will occur [7], [16]. Since the current decision making could then be ill-posed, the accuracy of a constrained solution might also be affected, especially when the validation horizon is selected far beyond the coincidence point [8].

B. Laguerre based PFC (LPFC)

1) Future input dynamics: The main difference between LPFC and PFC is that the future predicted input dynamics are shaped via a first-order Laguerre polynomial (in effect, a simple exponential decay function with pole a) so that it will converge to the expected steady state input u_{ss} [15], [16]. Thus, instead of the constant dynamics assumption of PFC, the future input is modified to

$$\underset{\rightarrow}{u_k} = u_{ss} + L\eta \tag{9}$$

where L is the vector ($L = [1, a, a^2, ...a^{n-1}]^T$) and η is a degree of freedom. For a general transfer function $G_m(z) = B(z)A(z)^{-1}$, the value u_{ss} is estimated as:

$$u_{ss} = G_m(z)^{-1}(R - d_k) \tag{10}$$

The inclusion of error term d_k in (10) is to ensure an unbiased estimation.

Remark 2: For a first-order system, a should be equal to λ to ensure consistent dynamics with the target trajectory [16]. Although for higher-order systems, the value of a can be tuned for faster convergence [15], this work will only use $a = \lambda$ to keep the sensitivity analysis transparent.

2) LPFC control law: The output prediction of (4) is modified with the new input dynamics of (9) to give:

$$y_{m,k+n|k} = H(u_{ss} + L\eta) + P\underset{\leftarrow}{u_k} + Q\underset{\leftarrow}{y_{m,k}} \tag{11}$$

The equality of (6) now becomes:

$$HL\eta + hu_{ss} + P\underset{\leftarrow}{u_k} + Q\underset{\leftarrow}{y_{m,k}} - y_{m,k} = (r - y_{p,k})(1-\lambda^n) \tag{12}$$

and the control law is computed by solving for η as:

$$\eta = \frac{1}{HL}\left[(1-\lambda^n)r - (1-\lambda^n)y_{p,k} - hu_{ss} - Q\underset{\leftarrow}{y_{m,k}} + y_{m,k} - P\underset{\leftarrow}{u_k}\right] \tag{13}$$

Due to the receding horizon principle [3] and the definition of $L(z)$, the current input is defined as:

$$u_k = u_{ss} + \eta \tag{14}$$

Noting the structure of u_{ss} in (10) and η in (13), the manipulated input u_k in (14) can be altered into vector form simply by rearranging the algebra and grouping the common terms into parameters F_l, N_l, M_l and $\hat{D}_l$ so that:

$$u_k = F_l r - N_l \underset{\leftarrow}{y_{m,k}} - M_l y_{p,k} - \hat{D}_l \Delta \underset{\leftarrow}{u_k} \tag{15}$$

Remark 3: It has been shown in [16] that LPFC law of (15) manages to improve the prediction consistency and the efficacy of λ as tuning parameter compared to the conventional PFC law of (8). In addition, the constrained solution becomes more accurate and less conservative [8].

C. General Sensitivity function for IM structure

From the previous subsections, it is clear that both PFC and LPFC can be represented by a fixed control law as in (8) and (15). These are used in the derivation of sensitivity functions presented next to analyse their respective robustness [1].

First consider a generic formulation of the control law within an IM structure:

$$u_k = Fr - N\underset{\leftarrow}{y_{m,k}} - My_{p,k} - \hat{D}\Delta\underset{\leftarrow}{u_k} \tag{16}$$

This can be represented in a transfer function form, where the vectors of

$$\begin{aligned} N &= [N_0, N_1, N_2, ..., N_n] \\ \hat{D} &= [\hat{D}_0, \hat{D}_1, \hat{D}_2, ..., \hat{D}_n] \end{aligned} \tag{17}$$

are defined in the z domain as:

$$\begin{aligned} N(z) &= N_0 + N_1 z^{-1}, N_2 z^{-2} + ... + N_n z^{-n} \\ \hat{D}(z) &= \hat{D}_0 + \hat{D}_1 z^{-1}, \hat{D}_2 z^{-2} + ... + \hat{D}_n z^{-n} \\ D(z) &= 1 + z^{-1}\hat{D}(z) \end{aligned} \tag{18}$$

Noting the definitions of $\underset{\leftarrow}{u_k}$ and $\underset{\leftarrow}{y_{m,k}}$ in (5), the sensitivity functions are derived based on a closed-loop form of:

$$D(z)u_k = F(z)r - N(z)y_{m,k} - M(z)y_{p,k} \tag{19}$$

alongside the model/plant equations (e.g. $y_{m,k} = B(z)A(z)^{-1}u_k$) and hence equation (19) can be replaced by:

$$\underbrace{\left[D(z) + N(z)B(z)A(z)^{-1}\right]}_{D_i(z)} u_k = F(z)r - M(z)y_{p,k} \tag{20}$$

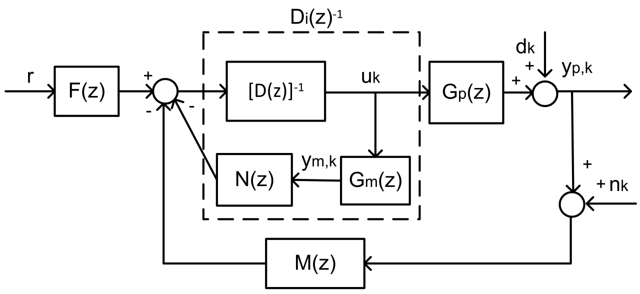

Fig. 2: PFCI control loop.

Fig. 2 indicates the equivalent block diagram with the addition of measurement noise n_k and output disturbance d_k. From the structure, the effective control law can be simplified to $K(z) = M(z)[D_i(z)\Delta]^{-1}$. Assuming system $G(z) = B(z)A(z)^{-1}$, the closed-loop pole polynomial $P_i(z) = 1 + K(z)G(z)$ is represented as:

$$P_i(z) = D_i(z)A(z) + M(z)B(z) \tag{21}$$

The sensitivity of the input to noise is derived by finding the transference from $n(z)$ to $u(z)$ (refer to Fig. 2):

$$S_{un} = K(z)[1 + K(z)G(z)]^{-1} = M(z)P_i(z)^{-1}A(z) \tag{22}$$

Similarly, the sensitivity of output to disturbance is obtained by solving the transference from $d(z)$ to $y(z)$:

$$S_{yd} = [1 + K(z)G(z)]^{-1} = A(z)P_i(z)^{-1}D_i(z) \tag{23}$$

Finally, the multiplicative uncertainty is modelled as $G(z) \rightarrow (1+\delta)G(z)$, for δ a scalar (possibly frequency dependent). Thus the closed-loop pole sensitivity to multiplicative uncertainty becomes:

$$\begin{aligned} P_c &= [1 + G(1+\delta)K] = 0 \\ S_g &= GK[1 + K(z)G(z)]^{-1} = M(z)P_i(z)^{-1}B(z) \end{aligned} \tag{24}$$

D. Summary of Control Laws

Table I summarises some of the sensitivity functions for PFC and LPFC. It is noted that the structures of all the sensitivity functions are same, but obviously with different parameters and hence, different sensitivity responses should be expected.

TABLE I: Sensitivity functions for PFC and LPFC.

Algorithm	PFC	LPFC
S_{un}	$M_p(z)P_{i,p}(z)^{-1}A(z)$	$M_l(z)P_{i,l}(z)^{-1}A(z)$
S_{yd}	$A(z)P_{i,p}(z)^{-1}D_{i,p}(z)$	$A(z)P_{i,l}(z)^{-1}D_{i,l}(z)$
S_g	$M_p(z)P_{i,p}^{-1}B(z)$	$M_l(z)P_{i,l}^{-1}B(z)$

The polynomials $M(z), D(z), P_i(z)$ used a subscript p for PFC, while for LPFC the subscript is l.

III. Numerical examples

This section presents the sensitivity analysis of unconstrained second order over-damped process (25) as constraint handling would imply non-linear control. In fact, if the loop structure has low sensitivity in the nominal case, it is likely to carry over for the constrained case. For the first example, both PFC and LPFC are tuned using a faster λ compared to the slowest open-loop pole. The second example demonstrates the effect of loop sensitivity when the controllers are tuned to have almost similar closed-loop poles. The outcome of this analysis is then validated with the closed-loop simulation using Matlab.

$$G_1 = \frac{0.1z^{-1} + 0.4z^{-2}}{(1-0.5z^{-1})(1-0.9z^{-1})} \tag{25}$$

A. First example

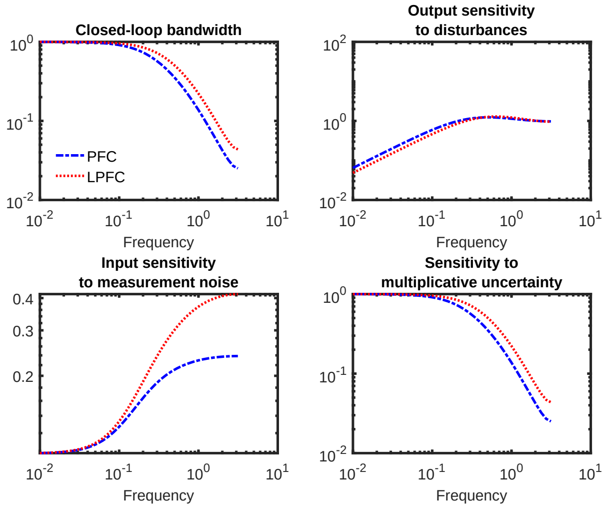

Fig. 3: Sensitivity plot for process G_1 with $\lambda = 0.7$ and $n = 7$.

In this example, the system (25) is considered to track a unit set point. The desired pole is set to $\lambda = 0.7$, while the coincidence point is tuned at $n = 7$ using conjecture presented in [7], that is corresponding to 40% to 80% rise of the step response to the steady-state value.

To analyse the trade-off between performance and robustness of PFC and LPFC, the Bode plots of each sensitivity function are plotted together with their closed-loop bandwidth (see Fig. 3). It can be observed that:

- for this particular selection of tuning parameters, LPFC (red dotted line) has a higher bandwidth compared to PFC (blue dashed line). Since LPFC has a faster dynamics, it becomes less sensitive in rejecting low-frequency disturbance..
- However, higher bandwidth requires more aggressive input activity, and thus LPFC becomes more sensitive to measurement noise and modelling uncertainty compared to conventional PFC.

One could argue that PFC has failed to deliver the desired bandwidth and if LPFC were to be tuned to give an equivalent lower bandwidth, in all likelihood, the sensitivities would be similar.

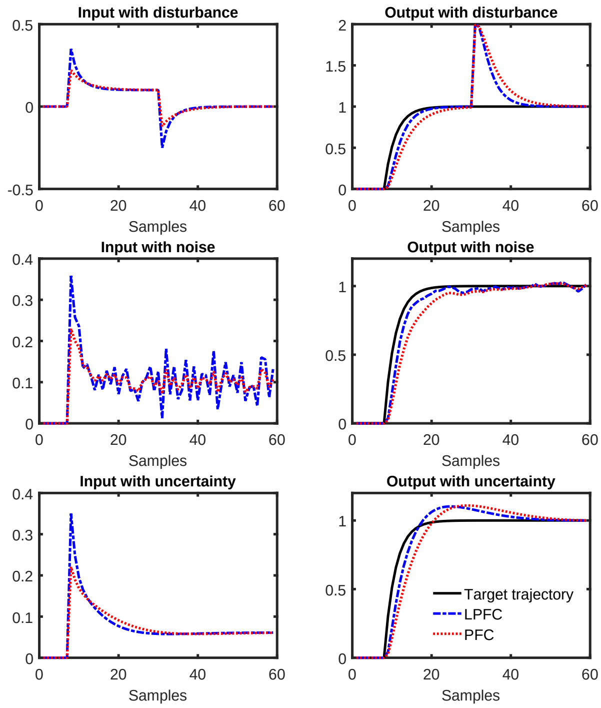

Fig. 4: Closed-loop response of process G_1 with $\lambda = 0.7$ and $n = 7$ in the presence of disturbance, noise, and uncertainty.

To validate this analysis, a closed-loop control (see Fig. 4) is simulated with three different conditions:

1) A step output disturbance ($d = 1$) is added to the 30th sample.
2) The output measurement is corrupted by Gaussian random white noise with variance of 0.1.
3) System $G_{1,m}$ (26) is used to predict the future dynamics instead of G_1 to demonstrate the effect of uncertainty.

$$G_{1,m} = \frac{0.12z^{-1} + 0.37z^{-2}}{1 - 1.37z^{-1} + 0.4z^{-2}} \tag{26}$$

The simulation outcomes reflect the previous sensitivity analysis whereby:

- LPFC converges approximately 2 samples faster in tracking the target and rejecting the output disturbance with almost similar overshoot ($y_{max} = 2$) compared to PFC.
- On the other hands, LPFC reacts more to the noise in the input compared to conventional PFC.
- For parameter uncertainty, both controllers manage to converge towards the steady-state value but with apparent differences in their closed-loop response.

In this example, it is clear that LPFC is slightly less robust than PFC in handling noise and uncertainty, yet better in rejecting disturbance and tracking the target, but that observation is most likely linked to the difference in implied closed-loop poles with LPFC delivering the desired pole and PFC not doing so.

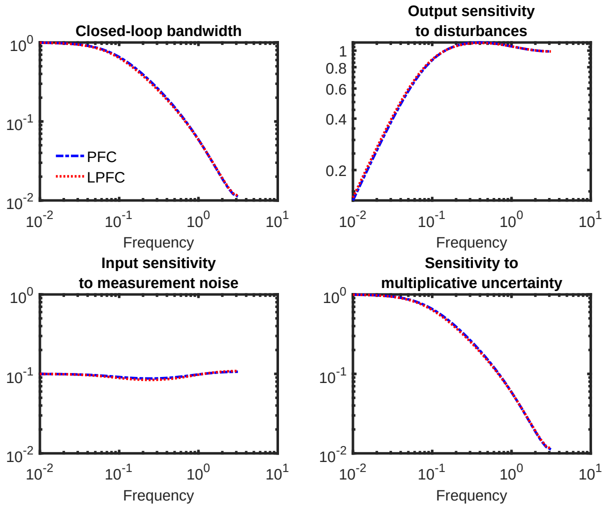

Fig. 5: Sensitivity plot for process G_1 with $\lambda = 0.92$ and $n = 9$.

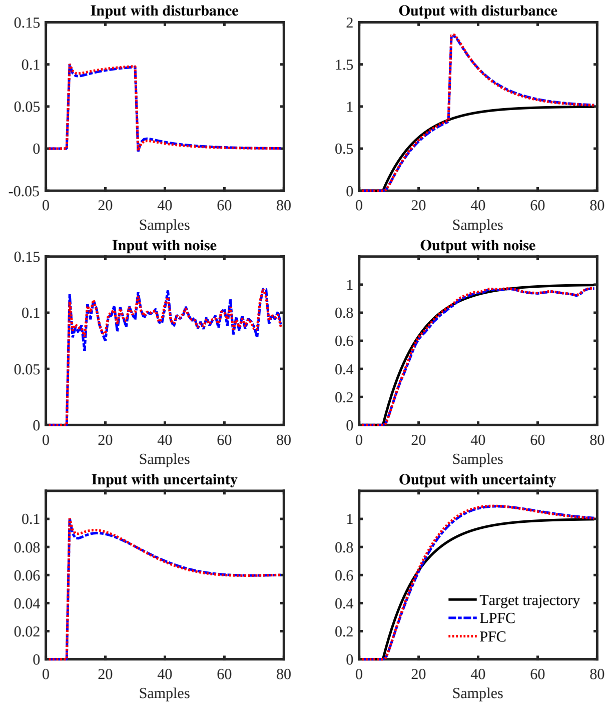

Fig. 6: Closed-loop response of process G_1 with $\lambda = 0.92$ and $n = 9$ in the presence of disturbance, noise and uncertainty.

B. Second example

The next example looks at the effect of sensitivity when the process (25) is tuned using slower $\lambda = 0.92$ (almost similar with the slowest open-loop pole). Based on the same procedure [7], the coincidence point $n = 9$ is selected to track a unit set point. It can be observed that (see Fig. 5):

- With the selected tuning parameters, LPFC and PFC have almost a similar bandwidth.
- As a consequence, both controllers are giving a close sensitivity outcome with respect to disturbance, noise and modelling uncertainty.

Again to validate the sensitivity analysis, the closed-loop simulation is run to track a unity set point for three different cases (similar as previous example). The outcomes in Fig. 6 demonstrates that:

- PFC and LPFC converge at the same rate and very close to the target trajectory while rejecting the disturabce with overshoot approximately around $y_{max} = 1.8$.
- Similar observation can be seen with the presence of noise and modelling uncertainty where both controllers performance are almost same.

C. Summary

In summary, for the two cases given, the controller sensitivity is related to the achieved closed-loop bandwidth. LPFC is better at delivering the target λ whereas PFC often gives a slower response than desired when large n is required. In consequence, for the same λ, LPFC is usually more highly tuned and thus more sensitive to noise and modelling uncertainty. However, where the two control laws give similar closed-loop poles (perhaps by deploying different λ), their sensitivities are similar. Therefore, LPFC is a better base on which to explore the trade-offs in the sensitivity, as there is a stronger connection between the tuning parameters and the achieved closed-loop performance [16] in addition to a better constraint handling due to its well-posed decision and prediction consistency as discussed in [8].

Fig. 7: Quanser SRV02 servo based unit.

IV. Real Time System Implementation

This section demonstrates the practicality of LPFC to control a real system, that is a Quanser SRV02 servo based unit [17]. The servo is powered by a Quanser VoltPAQ-X1 amplifier that comes with National Instrument ELVIS II+ multifunctional data acquisition device. The controller

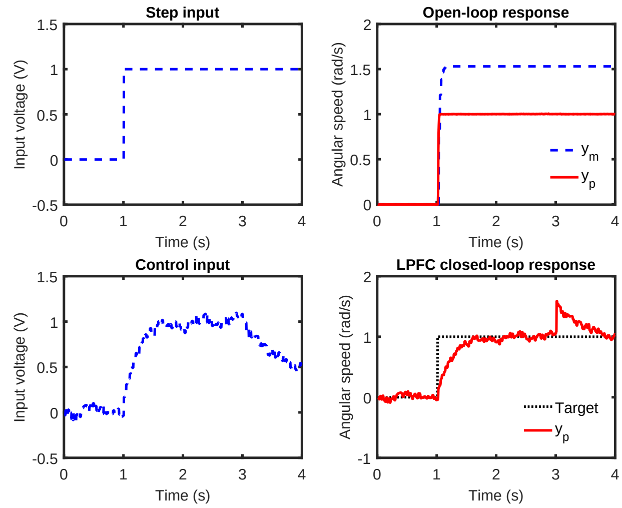

Fig. 8: Step response and LPFC closed-loop behaviour for process G_2.

is run by National Instrument LabVIEW software via USB connection (see Fig. 7). The objective is to track the desired servo angular speed, $\dot{\theta}(t)$ by regulating the supplied voltage, $V(t)$. The mathematical model is given as [17]:

$$0.0254\ddot{\theta}(t) = 1.53V(t) - \dot{\theta}(t) \tag{27}$$

where $\ddot{\theta}(t)$ is the servo angular acceleration. Converting the model (27) to discrete form with sampling time $0.02s$, the transfer function of angular speed to voltage input becomes:

$$G_2 = \frac{0.8338}{1 - 0.455z^{-1}} \tag{28}$$

The upper Fig. 6 shows the modelling uncertainty between the process y_p and model y_m subjected to a step input u. To track the angular speed at 1 rad/s, LPFC is tuned with $n = 1$ (often a sensible choice for a first-order system [7]) with desired CLTR at $0.5s$ (equivalent to $\lambda = 0.89$). It is noted that at $3s$, there is a step output disturbance ($d = 2$) entering the system while the measurement is corrupted by Gaussian white noise with variance of 0.5. The closed-loop response (see lower Fig. 8) shows that:

- LPFC manages to reduce some noise transmission to the input with approximate 0.2 variance from 0.5, while rejecting the output disturbance.
- Although there is modelling uncertainty, the selected CLTR is still achieved at $0.5s$ with minimum offset error.

V. CONCLUSIONS

This work provides a formal sensitivity analysis of LPFC in the presence of noise, disturbance and modelling uncertainty. The performance is then compared with the conventional PFC control law. Indeed it is clear that when using LPFC, a user need to pay a small trade-off by having a more sensitive controller to noise and uncertainty since it is highly tuned with a larger bandwidth than conventional PFC. However, both controllers may typically have similar sensitivities if giving similar closed-loop poles which would indicate a preference for LPFC in general due to easier tuning and other advantages as discussed in *Remark 3*.

Future work will consider the analysis of different PFC structures that deal with more challenging dynamics and unstable systems as PFC is currently has a number of ad-hoc constructive methods to improve its closed-loop behaviour. In addition, a core issue that also needs to be considered is the impact of modelling assumptions on sensitivity. This paper assumes an IM model of Fig. 1, so it would be interesting to consider how sensitivity might change with alternative prediction models such as T-filter [14].

ACKNOWLEDGMENT

The first author would like to acknowledge International Islamic University Malaysia and Ministry of Higher Education Malaysia for funding this work.

REFERENCES

[1] J. A. Rossiter, *Model predictive control: a practical approach*, CRC Press, 2003.

[2] L. G. Bleris and M. V. Kothare, "Real-Time implementation of Model Predictive Control," American Control Conference, Portland, USA, 2015.

[3] J. Richalet, A. Rault, J.L. Testud and J. Papon, Model predictive heuristic control: applications to industrial processes, *Automatica*, 14(5), 413-428, 1978.

[4] J. Richalet, and D. O'Donovan, *Predictive Functional Control: principles and industrial applications*. Springer-Verlag, 2009.

[5] J. Richalet, and D. O'Donovan, "Elementary Predictive Functional Control: a tutorial," Int. Symposium on Advanced Control of Industrial Processes, 2011, pp. 306-313.

[6] R. Haber, J.A. Rossiter and K. Zabet, "An Alternative for PID control: Predictive Functional Control - A Tutorial," American Control Conference (ACC), 2016, pp. 6935-6940

[7] J. A. Rossiter, and R. Haber, "The effect of coincidence horizon on predictive functional control," *Processes*, 3, 1, pp. 25-45, 2015.

[8] M. Abdullah, J. A. Rossiter and R. Haber, "Development of constrained predictive functional control using laguerre function based prediction," IFAC World Congress, 2017.

[9] J. A. Rossiter, R. Haber, and K. Zabet, "Pole-placement predictive functional control for over-damped systems with real poles", *ISA Transactions*, vol. 61, pp. 229-239, 2016.

[10] J. A. Rossiter, "Input shaping for PFC: how and why?," *J. Control and Decision*, pp. 1-14, Sep. 2015.

[11] M. Khadir and J. Ringwood, "Extension of first order predictive functional controllers to handle higher order internal models, *Int. Journal of Applied Mathematics and Comp. Science*, vol. 18, no. 2, pp. 229-239, 2008.

[12] K. Zabet, J. A. Rossiter, R. Haber, and M. Abdullah, "Pole-placement Predictive Functional Control for under-damped systems with real numbers algebra", *ISA Transactions*, In Press., 2017.

[13] K. Zabet and R. Haber, "Robust tuning of PFC (Predictive Functional Control) based on first- and aperiodic second-order plus time delay models", *Journal of Process Control*, vol. 54, pp. 25-37, 2017.

[14] M. Abdullah and J. A. Rossiter, "The effect of model structure on the noise and disturbance sensitivity of Predictive Functional Control", under review for European Control Conference, 2018.

[15] M. Abdullah and J. A. Rossiter, "Alternative Method for Predictive Functional Control to Handle an Integrating Process", under review for Advances in PID, 2018.

[16] M. Abdullah and J. A. Rossiter, "Utilising Laguerre function in predictive functional control to ensure prediction consistency," 11th Int. Conf. on Control, Belfast, UK, 2016.

[17] *Quanser user manual SRV02 rotary servo based unit set up and configuration*. Quanser Inc, 2012.