UNIVERSITY of York

This is a repository copy of The implementation of binding blocks in the classroom.

White Rose Research Online URL for this paper: <u>https://eprints.whiterose.ac.uk/132634/</u>

Version: Accepted Version

Article:

A J, Wright, Willett, Hannah Victoria, S R, Beanland et al. (3 more authors) (2017) The implementation of binding blocks in the classroom. Physics Education. 054001. ISSN 0031-9120

https://doi.org/10.1088/1361-6552/aa76d5

Reuse

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



eprints@whiterose.ac.uk https://eprints.whiterose.ac.uk/

The implementation of *Binding Blocks* in the classroom

A J Wright¹, H V Willett¹, S R Beanland¹, M Carson², R A Davies¹, G Duffett¹ and A Pastore¹

¹Department of Physics, University of York, Heslington, York Y010 5DD, UK ²Fulford School, Fulfordgate, Heslington Lane, York YO10 4FY, UK

E-mail: Corresponding author: hvw502@york.ac.uk, physics-bindingblocks@york.ac.uk

Abstract. We discuss a series of activities for A level students which can be carried out using the *Binding Blocks* three dimensional chart of nuclides. The planned activities cover four main sections which can be linked to the A level curriculum; nuclear decays (as seen through the different colours on the chart), medical physics (medical isotopes highlighted on the chart), fusion on Earth (binding energy demonstrated through tower heights) and stellar fusion (which has a limit at 56 Fe, illustrated by the decreasing tower heights).

1. Introduction

A level physics curricula typically cover four key areas of nuclear physics: nuclear decay, radioisotopes, nuclear energy and astrophysics [1–4]. In this paper, we will present activities which have been created as interactive lessons covering these four different topics. The activities make use of different aspects of the *Binding Blocks* three-dimensional isotope chart, as presented in [5]. They can be used either individually or together, at the discretion of the user. Typically, there is some overlap between the activities; however they place the physics in different contexts.

The *Binding Blocks* chart uses towers of LEGO[®] bricks[‡] to create the chart of nuclides in three dimensions. The towers are colour-coded according to the mechanism through which each isotope decays, and the height of each tower represents the mass excess per nucleon per kilogram of the material (relative to iron-56, the most stable nucleus). Mass excess is the difference between the actual mass of the nucleus and its mass number in atomic mass units. Each layer of bricks in the tower represents 25 TJ of energy per kilogram of material, so the taller the tower, the more mass a nucleus has per nucleon. See [5] for more details.

 \ddagger LEGO[®] is a trademark of the LEGO Group of companies which does not sponsor, authorise or endorse the present work.

The implementation of Binding Blocks in the classroom

An interactive session using the *Binding Blocks* chart begins with a short introductory talk, which provides an overview of the concepts that will be covered in the chosen activities. Students are then invited to participate in the construction of the chart (the number of plates built depends on the requirements of the activities and the time available). Following this, one or more of the activities can be carried out. We introduce four activities, based on the following topics:

- Section 2: Radioactivity and decay pathways
- Section 3: Medical isotopes
- Section 4: Fusion energy
- Section 5: Nuclear astrophysics.

Each activity is given with a specific set of learning outcomes and practical examples.

2. Radioactivity and decay pathways

This activity is typically for A level students, but can be delivered to higher level GCSE students if certain components are removed. The learning outcomes are as follows:

- Describe the two primary modes of radioactive decay (α, β) that are presented on the chart and are part of the standard A level syllabus, along with their properties;
- Create and explain a variety of possible decay paths with multiple steps for an unstable nucleus;
- Apply knowledge of different decay modes to naturally occurring background radiation and technological applications of radioactive decay;
- Solve nuclear decay problems by applying the rate equation (2);
- Explain the concept of half-life.

The overall focus of the activity is to convey the idea that unstable nuclei decay into more stable ones because it is energetically favourable to do so. The heights of the towers of the chart help the visualisation of a tightly bound nucleus.

Pre-activity

For this activity, building the chart up to the iron region is recommended, but it can be carried out in a limited capacity with only the A1 plate [5]. The introductory talk should provide an overview or recap of the two primary decay modes (α and β). There are five key properties to discuss: degree of ionisation; degree of penetration; the charge of the decay products; the energy of the decay products; and the expected level of activity of a source. Another point to emphasise is that nuclei decay in order to lose energy and become more stable, decreasing their mass excess per nucleon. The decay chain will continue until the product is a completely stable nucleus (represented by black bricks in the centre of the chart).



Figure 1: (Colour online). Possible decay paths of ¹³Be to a stable ground state. The colours indicate the decay mode of the isotope according to the *Binding Blocks* chart: neutron emission (dark blue); β^- (light blue); α (yellow); stable (black/grey).

Practical activity

After this background knowledge has been established, the students can move back to the chart and begin consideration of the types of nuclei that undergo each decay and why. Here the colours of the chart will be helpful in distinguishing between different modes of decay (see [5] for details). Questions to pose to the group can include 'How could you identify nuclei which could decay through multiple modes?' and 'What patterns do you see among the colours on the chart?'

The next stage of the activity is more interactive. Firstly, some examples of decay paths are given to the group, such as:

$${}^{14}\text{C} \to {}^{14}\text{N} + e^- + \bar{\nu}_e \quad (T_{1/2} = 126.5 \text{ milliseconds})$$
(1*a*)

$${}^{18}\text{F} \rightarrow {}^{18}\text{O} + e^+ + \nu_e \quad (T_{1/2} = 109.771 \text{ minutes}).$$
 (1b)

In these two equations, we illustrate the two types of β -decay: in (1*a*) one neutron is converted into a proton with emission of an electron e⁻ and an antineutrino $\bar{\nu}_{\rm e}$; while in (1*b*) we have the opposite case, with emission of a positron e⁺ and a neutrino $\nu_{\rm e}$. The students are then split into smaller groups and are given (different) worksheets with incomplete paths (an example of a complete decay chain is illustrated in Figure 1). They follow the decay pathway along the chart by identifying the mode of decay: β, α or neutron emission.

As part of the activity they can add their paths onto the chart, using blue tack and string to link the parent and daughter nuclei. Following this, student groups can be given a list of various scenarios where a given radioactive source is needed, and asked to select a decay mode that fits the requirements.

Mathematical activity

This activity would be for A level groups only and uses the rate equation (2) and decay constant (3) to calculate the activity, A, of various nuclei, given their half-lives $(T_{1/2})$.

$$A = A_0 \exp\left(-\lambda t\right) \tag{2}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} \tag{3}$$

If the students are unfamiliar with the concept of the half-life this activity can be omitted, or a short introduction can be given to explain the principles.

Discussion

At the end of the activity the discussion can be widened to topics related to nuclear radiation in everyday life. The students can be divided into groups and asked to think about where they can find sources of nuclear radiation around them. A few examples include:

- Natural background radiation and radiation coming from space;
- ²³⁸Pu (α emitter) as a source of heat in satellites;
- ²⁴¹Am (α emitter) in smoke alarms;
- 99m Tc (γ emitter) for medical imaging.

Finally, if time permits, it is then possible to introduce the concepts of nuclear safety and the limit to the dose that the human body can absorb without harm.

3. Medical isotopes

The medical isotope activity is aimed at A level students or higher set physics GCSE students. It expands the discussion of radioactive decay to real-world applications by introducing some uses of radioactive isotopes in medicine. The learning outcomes are:

- Explain the difference between the interactions of charged particles and photons with matter;
- Evaluate the constraints on the half-life and decay mode for different applications of nuclear radiation in medicine.

Pre-activity

Building the *Binding Blocks* chart will encourage students to think about different types of radiation and decay modes. Some medical isotopes are marked using pink bricks, and to include them all, the chart should be built up to the seventh plate column. The



Figure 2: (Colour online) Left: the instruction file used to build the nuclide chart. Right: the detail of the plate containing 131 I [5].

isotopes highlighted on the chart are ¹⁸F, ⁶⁷Ga,⁹⁹Tc, ¹²³I and ¹³¹I, commonly used for medical imaging [6] and, in the case of ¹³¹I, treating overactive thyroid glands. In Figure 2 we show the instruction diagram required to build the chart, as well as the E7 plate containing ¹³¹I [5].

Students begin this activity by discussing the features of their ideal medical isotope as a class. In particular they should consider:

- the chemical properties of the element (interaction with human cells);
- the half-life of the element;
- the impact of radiation on the human body;
- transport and production.

These ideas can then be brought forward into later stages of the activity.

Practical activity

In groups of between three and five, students design their ideal medical isotope. They make a list of the ideal features a medical isotope would have, following on from the previous discussion. They are encouraged to consider which of the features they have listed contradict each other (such as activity vs. half life). These data can be extracted using the interactive nuclear chart in [7]. An example is given in Figure 3 for the region of the chart around iodine.

The students are then given the opportunity to compare the features of their designed isotope to one that is used in medicine, such as ¹³¹I (researching a commonly-used medical isotope could be set as a home learning activity prior to the lesson). They are encouraged to compare and contrast the compromises they have made with their designed isotope to the real-world example. To close the activity, the students present their designed isotopes to the class, comparing them to the real-world medical isotope that most closely matches their ideal requirements.

The implementation of Binding Blocks in the classroom

¹³⁴ Cs	¹³⁵ Cs	¹³⁶ Cs	¹³⁷ Cs	¹³⁸ Cs	¹³⁹ Cs	¹⁴⁰ Cs	¹⁴¹ Cs	¹⁴² Cs	¹⁴³ Cs	¹⁴⁴ Cs	¹⁴⁵ Cs	
¹³³ Xe		¹³⁵ Xe		¹³⁷ Xe	¹³⁸ Xe	¹³⁹ Xe	¹⁴⁰ Xe	¹⁴¹ Xe	¹⁴² Xe	¹⁴³ Xe	¹⁴⁴ Xe	
¹³²	¹³³	¹³⁴	¹³⁵	¹³⁶	¹³⁷	¹³⁸	¹³⁹	¹⁴⁰	¹⁴¹	¹⁴²	¹⁴³	
¹³¹ Te	¹³² Te	¹³³ Te	¹³⁴ Te	¹³⁵ Te	¹³⁶ Te	¹³⁷ Te	¹³⁸ Te	Telluri Z=52	um			
¹³⁰ Sb	¹³¹ Sb	¹³² Sb	¹³³ Sb	¹³⁴ Sb	¹³⁵ Sb	¹³⁶ Sb	¹³⁷ Sb	¹³⁸ Sb	¹³⁹ Sb Antimony Z=51			
¹²⁹ Sn	¹³⁰ Sn	¹³¹ Sn	¹³² Sn	¹³³ Sn	¹³⁴ Sn	¹³⁵ Sn	¹³⁶ Sn	¹³⁷ Sn	Tin Z=50			
¹²⁸ In	¹²⁹ In	¹³⁰ ln	¹³¹ ln	¹³² ln	¹³³ In	¹³⁴ ln	¹³⁵ ln	Indium Z=49	ı			
¹²⁷ Cd	¹²⁸ Cd	¹²⁹ Cd	¹³⁰ Cd	¹³¹ Cd	¹³² Cd	¹³³ Cd	Cadm Z=48	ium				
¹²⁶ Ag	¹²⁷ Ag	¹²⁸ Ag	¹²⁹ Ag	¹³⁰ Ag	Silver Z=47		Half-life [s]					
Pallad Z=46	ium				10 ⁻¹⁰ 10 ^o 10 ^o 10 ^o							1

Figure 3: (Colour online). Illustration of the iodine section of the online interactive nuclear chart, coloured according to the half lives of the isotopes [7].

Mathematical activity

To better understand the requirements for designing an optimal isotope for medical purposes, the students can estimate the absorbed dose of radiation for β particles and photons (γ radiation). Using simple estimates from [8, 9], the absorbed dose for a β emitter is:

$$D_{\beta} = 7380 \, C E_{\beta} \, T_{1/2} \tag{4}$$

where D_{β} is the absorbed dose of radiation per kilogram of material, measured in J kg⁻¹. E_{β} is the mean energy emitted per decay of the nuclide in J, and $T_{1/2}$ is the half-life of the nuclide in the tissue in seconds. C is the concentration of the nuclides expressed in microcuries per gram (where a curie is equivalent to 37 billion decays per second).

A dose of $1 \,\mathrm{J\,kg^{-1}}$ will typically produce no immediate effect. A dose of $1-2 \,\mathrm{J\,kg^{-1}}$ in less than a day will start inducing some serious symptoms in the human body, and above $10 \,\mathrm{J\,kg^{-1}}$ the dose become fatal. The students can thus calculate the absorbed dose for the isotopes they have identified and estimate the effects on the human body. The discussion can also be extended to the varying penetration of radiative particles into human tissue: from alpha particles, which are stopped almost immediately, to gamma rays, which can travel through the entire body.

Discussion

After laying the foundations of the medical applicability of different decay modes, techniques such as positron emission tomography (PET) scans can be outlined [10]. The discussion should highlight that nuclear radiation can be used for civil applications apart from energy production and military applications.

4. Fusion energy

This activity places the process of nuclear fusion into the context of the wider search for a clean energy source for the future. It is hoped that fusion energy will be a realistic source of commercial energy by approximately 2050 [11]. The learning outcomes for this section are:

- Describe how energy is gained by fusing nuclei;
- Calculate the energy released in different fusion reactions;
- Demonstrate an understanding of the sustainability of fusion as an energy source for the future.

Pre-activity

The activity requires the first (A1) plate of the *Binding Blocks* chart, but can also involve the plates up to the iron group elements (C plates). The longer chart can be used to demonstrate the end point of fusion in stars using a tennis ball (see Section 5). However, for the sake of the fusion energy activity alone, this is not necessary and the A1 plate is sufficient.

The introductory talk should provide an overview of the concepts involved in fusion energy; primarily focusing on magnetic confinement fusion and the tokamak reactor design (e.g. ITER [12]). The fuel in a tokamak has to be at extremely high temperatures and pressures, so is confined in a vacuum chamber by strong magnetic fields. At this point the students could be introduced to the plasma state (the fourth state of matter). The preliminary discussion should emphasise the need for alternative energy sources to replace fossil fuels.

Practical activity

The activity will be presented as an opportunity to 'design a tokamak'. The students will consider a variety of fusion reactions involving a combination of the isotopes present on the A1 plate. Using their physics knowledge, they need to determine appropriate fuels for a tokamak, taking into consideration both the energy yield of the reactions and how the isotopes may behave in an reactor environment.

The students are encouraged to come and interact with the constructed chart so they can visually compare the tower heights. However, as there is one chart between the whole group, a worksheet of the A1 plate with all the tower heights included is provided. The worksheet will list a group of combinations of isotopes and one of the products, e.g. a neutron. From this, they will have to work out the other product and the energy gained from the fusion reaction. The available 'fuels' on the worksheet will be mainly stable isotopes from the A1 plate, with the exception of neutrons and tritium. Examples of reactions include:

$${}^{2}_{1}D + {}^{2}_{1}D \rightarrow {}^{3}_{2}He + {}^{1}_{0}n + 3.2 \text{ MeV}$$
$${}^{1}_{1}H + {}^{11}_{5}B \rightarrow {}^{4}_{2}He + {}^{4}_{2}He + {}^{4}_{2}He + 8.7 \text{ MeV}.$$

Discussion

The students can discuss as a class the reactions that they decide are most appropriate. They can then be introduced to the reaction that has been chosen for fusion energy research (between deuterium and tritium):

$$^{2}_{1}D + ^{3}_{1}T \rightarrow ^{4}_{2}He + ^{1}_{0}n + 17.6 \text{ MeV}.$$

This interaction has the advantages of both a high energy yield, and a relatively high reaction rate at temperatures that are achievable in a laboratory.

To demonstrate the energy released in this reaction, four LEGO[®] brick towers are required, which represent 1 kg of deuterium (30 bricks high), 1.5 kg of tritium (36 high), 2 kg of helium (14 high) and 0.5 kg of neutrons (18 high). By 'fusing' the deuterium and tritium towers and comparing this to the total height of the helium and neutron towers, the energy released in the deuterium-tritium reaction can be visualised. The tower height difference is 34 bricks, and each brick represents roughly 25 TJ kg⁻¹, meaning that approximately 850 TJ is produced for every kilogram of deuterium burned.

Two facts can be used to put this number into context. Firstly, burning coal releases around 25 MJ for every kilogram of fuel (roughly thirty million times smaller). Secondly, the energy that would be obtained from the deuterium from half a bathtub of seawater and the lithium in a laptop battery would supply one person with enough electricity for 30 years of their life. A further discussion point is provided by the inclusion of lithium rather than tritium here: as can be seen from the *Binding Blocks* chart, tritium is itself radioactive (with a half-life of 12.3 years), and so a fusion reactor must create its own tritium in order to be sustainable. Neutrons from the fusion reactions will interact with lithium in 'breeding blanket' modules to produce tritium:

$${}_{3}^{6}\text{Li} + {}_{0}^{1}\text{n} \rightarrow {}_{2}^{4}\text{He} + {}_{1}^{3}\text{T}.$$

5. Nuclear astrophysics

The stellar physics activity places aspects of nuclear physics into the context of the life cycle of a star, and relates this to the nuclear chart. There are four learning outcomes for this activity, linking in with A level Physics curricula:

• Demonstrate an understanding of how stars generate energy;

- Qualitatively describe the formation of different elements;
- Use the $E = mc^2$ equation to calculate the energy released in fusion;
- Investigate the stability of different nuclei, and how this affects the stellar life cycle.

The activity is organised into sections, each covering a different type of reaction that takes place in a star such as the Sun, leading to the release of energy. This activity is related to the fusion activity illustrated in Section 4, but in this case the concept of fusion should be used only as a tool to explain the basic astrophysical processes occuring into stars and thus the production of heavy elements.

Ideally, this activity will make use of the entire chart (particularly for the creation of heavy elements). Printed copies of the chart, with tower heights indicated, can be used in lieu of the full chart, especially for large groups of students. In the following we identify three main topics to be discussed in the activity. For the proton-proton chain and helium burning, only the A1 plate is necessary.

Proton-proton chain

The proton-proton chain is the basic mechanism which produces helium in stars by fusing light nuclei together:

To simplify the notation, we have omitted e^+ , ν_e and γ particles, although the electric charge and lepton number should be conserved at each stage [13]. The students, using the tower heights on the chart, should be able to calculate the energy balance of the reactions above and show that this process is energetically favourable (i.e. energy is released).

Helium burning

After the early stages of the life of the star, thanks to the proton-proton chain process, helium accumulates in the core. We can therefore access new reactions, such as:

$$\begin{array}{rcl} {}^4_2\mathrm{He} + {}^1_1\mathrm{H} & \rightarrow {}^5_3\mathrm{Li} \\ {}^4_2\mathrm{He} + {}^4_2\mathrm{He} \rightarrow {}^8_4\mathrm{Be} \\ {}^4_2\mathrm{He} + {}^4_2\mathrm{He} + {}^4_2\mathrm{He} \rightarrow {}^{12}_6\mathrm{C} \end{array}$$

Students are asked to work out the likely fusion products, discuss the decays of these products, and discuss the difficulties of fusion beyond ${}_{2}^{4}$ He. We refer to the article by Diget et al. in the current volume for further details on this particular aspect of the stellar cycle [14].



Figure 4: (Colour online). Mass excess per nucleon, rescaled to ⁵⁶Fe and given in megaelectronvolts (MeV).

Fusion end point and creation of heavy elements

This activity can be done by using the full chart up to Fe, but ideally will extend beyond this point to focus on the creation of heavy elements in the universe. A demonstration of the fusion endpoint (at 56 Fe) can be given by dropping a tennis ball from the high point of the chart, and watching it roll towards and stop at the valley of the chart. In Figure 4, we show the values mass excess (expressed in MeV) for stable nuclei, scaled to 56 Fe, to illustrate the valley of stability.

This then leads onto the question of how heavier elements are formed, since the mass excess increases again beyond 56 Fe. The main processes are:

- Proton capture (*p*-process)
- Neutron capture (s- or r-process).

By considering the nuclear half-life data given in [7], the students should suggest two possible capture paths along the *Binding Blocks* chart for a typical *s*-process scenario (timescale $\approx 10^{12}$ s) and *r*-process (timescale $\approx 10^{-3}$ s).

Discussion

At the end of the activity, the different astrophysical environments where the above processes take place can be introduced. For example, in the case of the first two activities, the demonstrator can briefly discuss the evolution of a star according to the different stages of burning of nuclear fuel, and thus the current status of our Sun. For the last activity, possible discussion topics include different astrophysical sites, e.g. supernovae explosions [15], or more exotic locations such as neutron star mergers or black-hole mergers [16]. The latter could be linked to the recent discoveries of gravitational waves [17].

6. Conclusions

In this article, we have discussed four activities related to important aspects of A level curricula on nuclear physics. These activities have been developed in the form of thematic lectures and they can be combined with the construction of a three-dimensional chart of nuclear isotopes [5]. The objective of this article is to provide ideas and material to teachers to increase the engagement of students toward these specific aspects of the scientific curriculum. We also note that an electronic version of the three-dimensional chart will be made available [18], which can be used with the activities described in the present article without the need to transport the LEGO[®] chart.

Acknowledgments

We kindly acknowledge the contribution of T. Sanders, T. Haylett, M. Langley and J. Parvin for interesting discussions and suggestions that helped us develop the content of this paper. We also thank E. Simpson for giving us access to the interactive nuclear chart. The *Binding Blocks* project has been funded by EPSRC and University of York through an EPSRC Impact Acceleration Award, and by an STFC Public Engagement Small Award ST/N005694/1 and ST/P006213/1.

References

- [1] "AQA A level Physics syllabus." http://www.aqa.org.uk/subjects/science/as-and-a-level/ physics-7407-7408, 2015.
- [2] "OCR A level Physics syllabus." http://www.ocr.org.uk/qualifications/ as-a-level-gce-physics-a-h156-h556-from-2015/, 2015.
- [3] "Pearson Edexcel A level Physics syllabus." https://qualifications.pearson.com/en/ qualifications/edexcel-a-levels/physics-2015.html, 2015.
- [4] "WJEC A level Physics syllabus." http://www.wjec.co.uk/qualifications/science/ as-a-level/physics-as-a-level-2015/, 2015.
- [5] C. Aa Diget, A. Pastore, K. Leech, T. Haylett, S. Lock, T. Sanders, M. Shelley, H. V. Willett, J. Keegans, L. Sinclair, E. C. Simpson, and the Binding Blocks Collaboration, "Binding blocks: building the universe one nucleus at a time," *Physics Education*, vol. 52, no. 2, p. 024001, 2017.
- [6] F. A. Mettler Jr and M. J. Guiberteau, Essentials of nuclear medicine imaging. Elsevier Health Sciences, 2011.
- [7] "Interactive chart by E. Simpson." http://people.physics.anu.edu.au/~ecs103/chart/ index.php.
- [8] L. Marinelli, E. H. Quimby, and G. J. Hine, "Dosage determination with radioactive isotopes; practical considerations in therapy and protection.," *The American Journal of Roentgenology, Radium Therapy and Nuclear Medicine*, vol. 59, no. 2, p. 260, 1948.
- [9] M. Stabin, "Nuclear medicine dosimetry," Physics in Medicine and Biology, vol. 51, no. 13, 2006.

- [10] D. L. Bailey, D. W. Townsend, P. E. Valk, and M. N. Maisey, *Positron emission tomography*. Springer, 2005.
- [11] "European Fusion Development Agreement, Fusion electricity: a roadmap to the realisation of fusion energy." https://www.euro-fusion.org/eurofusion/ the-road-to-fusion-electricity/, 2012.
- [12] "ITER website." http://www.iter.org/.
- [13] E. Segre et al., Nuclei and particles: an introduction to nuclear and subnuclear physics (Dover books on physics). Dover Publ., 2013.
- [14] "CNO cycle educational resources." https://www.stem.org.uk/elibrary/resource/163716/ making-stars-fusion, 2017.
- [15] F. R. Stephenson and D. A. Green, "Historical supernovae and their remnants," Historical supernovae and their remnants, by F. Richard Stephenson and David A. Green. International series in astronomy and astrophysics, vol. 5. Oxford: Clarendon Press, 2002, ISBN 0198507666, vol. 5, 2002.
- [16] S. L. Shapiro and S. A. Teukolsky, "Black holes," Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects, pp. 335–369, 1983.
- [17] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. Adhikari, et al., "Observation of gravitational waves from a binary black hole merger," *Physical Review Letters*, vol. 116, no. 6, p. 061102, 2016.
- [18] M. Shelley and E. Simpson, "Nuclear cartography: patterns in binding energies and subatomic structure," *Physics Education*, 2017.