This is a repository copy of *Magnetomotive force harmonic reduction techniques for fractional-slot non-overlapping winding configurations in permanent-magnet synchronous machines.*

White Rose Research Online URL for this paper:
http://eprints.whiterose.ac.uk/131402/

Version: Accepted Version

**Article:**

10.23919/CJEE.2017.8048416

© 2017 IEEE. This is an author produced version of a paper subsequently published in Chinese Journal of Electrical Engineering. Uploaded in accordance with the publisher's self-archiving policy.

**Reuse**
Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**
If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.
Magnetomotive Force Harmonic Reduction Techniques for Fractional-Slot Non-Overlapping Winding Configurations in Permanent-Magnet Synchronous Machines

Xiao Chen*, and Jiabin Wang

(Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield, S3 7HQ, UK)

Abstract: Compared to conventional distributed winding configurations, the fractional-slot non-overlapping (concentrated) windings exhibit advantages such as short end-winding length, high copper packing factor (particularly with segmented stator structure), low cogging torque, good field weakening capability, owing to relatively large d-axis inductance, and better fault tolerant capability due to low mutual inductance. However, one of the key problems of employing concentrated windings in Permanent-magnet Synchronous Machines (PMSMs) is the high eddy-current losses in rotor magnets and/or rotor iron due to the presence of a large number of lower and higher order space harmonics in the stator Magnetoe-Motive Force (MMF). These MMF harmonics also result in undesirable effects such as acoustic noise and vibrations, and localized core saturation which tend to reduce reluctance torque. This paper reviews the current state-of-the-art of the MMF harmonic reduction techniques for concentrated winding configurations in PMSMs, including winding split and shift, delta-star connected windings, multiple 3-phase windings, multilayer windings, uneven turn numbers, and stator flux barriers. Their concepts, advantages and disadvantages are presented and assessed.

Keywords: Magnetomotive force (MMF), fractional-slot windings, concentrated windings, permanent-magnet machines.

1 Introduction

Permanent-magnet Synchronous Machines (PMSMs) exhibit high torque density and high energy efficiency over a wide operation range, due to the presence of the permanent magnets. Therefore, they have increasingly been employed in a variety of applications, such as industrial drives, hybrid and electric vehicles, wind turbines, aerospace, marine, and domestic appliances.

Subject to the locations of the permanent-magnets, PMSMs can be classified into Surface-mounted Permanent-magnet Machines (SPMs) and Interior Permanent-magnet Machines (IPMs). With magnets mounted on the rotor surface, SPMs produce torque by the interaction of the magnetic field contributed by permanent-magnets with the armature reaction field produced by the stator magnetic-motive force (MMF). This torque component only depends on the permanent-magnet flux-linkage and q-axis current. Therefore, SPMs exhibit a relatively linear torque-current characteristic, good controllability, and low torque ripple. Nevertheless, the surface-mounted magnets result in a large equivalent air-gap because the relative permeability of magnets is close to 1. Hence, the synchronous inductance of the machine is relatively low, leading to a high characteristic current, defined as the ratio of the PM flux to the synchronous inductance, and thereby low field weakening capability.

However, the fractional-slot non-overlapping (concentrated) winding configurations can be utilized to increase the d-axis inductance and thus improve the field weakening capability of SPMs. A. M. EL-Refaie et al. employed concentrated windings on SPMs and hence developed a 6kW SPM which can deliver constant-power over a 10:1 speed range by increasing the d-axis inductance and meeting the optimum field weakening conditions. The authors also investigated the scalability of the concentrated winding configuration for achieving optimal field weakening in SPMs in [34], and showed that the optimal field weakening can be reached over a wide range of machine power ratings from 1kW to 60kW. J. Cros et al. performed a synthesis of SPMs with concentrated windings and subsequently compared their performances with those of SPMs using conventional distributed windings. It is shown that by using the concentrated winding configuration, the copper loss can be reduced dramatically owing to both the short end-winding length and the high copper packing factor (if the segmented stator structure is employed). Moreover, the cogging torque can also be significantly reduced when the least common multiple of the slot number and the pole number increases. Therefore, SPMs with concentrated windings exhibit advantages such as high torque density, low copper loss, good field weakening capability, and low cogging torque.

However, by employing concentrated winding configurations in SPMs, the rotor magnets are subjected to a large amount of stator MMF harmonics which are asynchronous with the rotor speed, and...
consequently incur high eddy current loss\[^{36}\]. This can further lead to a high rotor temperature particularly at high speeds, and hence the rotor magnets will suffer from a high risk of irreversible demagnetization.

Compared with SPMs, IPMs have magnets buried in the rotor iron. The IPM rotor topology gives rise to a saliency in reluctances\[^{37}\]. This difference in the \(d\)- and \(q\)-axis reluctances contributes to reluctance torque which is conducive to reduction of PM volume in the IPM\[^{38,39}\]. To maximize the reluctance torque, the distributed winding configurations are usually employed in IPMs\[^{39}\]. However, compared to the concentrated windings, the conventional distributed windings have lower copper packing factor (slot fill factor), longer end-winding length, higher cogging torque, and less fault tolerant owing to higher mutual inductance and winding overlapping\[^{40}\]. Thus, the concentrated winding configurations are of potential to be employed in IPMs, given that the lower and higher order space harmonics in the stator MMF can be suppressed to a desirable level. This can increase IPMs’ reluctance torque production, reduce the eddy-current losses in both rotor magnets and rotor iron, and suppress acoustic noise and vibrations.

Therefore, both SPMs and IPMs can greatly benefit from the reduction of the stator MMF harmonics in the concentrated windings. This paper reviews the current state-of-the-art of MMF harmonic reduction techniques. The method for analysis of the MMF harmonics in 3-phase machines will be introduced before various MMF harmonic reduction techniques are introduced and discussed.

2 MMF harmonics

This section describes the method for analyzing the MMF harmonics in any 3-phase concentrated winding with any feasible slot-pole combination.

Assuming \(ABC\) windings are symmetric and have the same number of turns, the MMF distribution of each phase due to unit current, known as the winding function, can be expressed as Fourier series given in (1).

\[
\begin{align*}
    n_A(\theta_m) &= \sum_h N_h \cos(h\theta_m + \beta_h) \\
    n_B(\theta_m) &= \sum_h N_h \cos(h\theta_m + \beta_h - h\theta_{phm}) \\
    n_C(\theta_m) &= \sum_h N_h \cos(h\theta_m + \beta_h + h\theta_{phm})
\end{align*}
\]

where \(h\) denotes the harmonic order, \(N_h\) is the magnitude of the \(h\)th order harmonic, \(\beta_h\) is the phase angle of the \(h\)th order harmonic of phase \(A\), \(\theta_{phm}\) is the mechanical angular displacement between two adjacent phases of a 3-phase winding, and \(\theta_m\) denotes the space angle at a point of interest in the air-gap with respect to phase \(A\) axis as shown in Fig.1.

Due to the symmetrical distribution of a 3-phase winding, \(\theta_{phm}=120^\circ\) or \(-120^\circ\), depending on the direction of rotation of the MMF working harmonic.

Assuming the current amplitudes in all the phases are identical, the phase currents are expressed by (2).

\[
\begin{align*}
    i_A &= I \cos(p_0 t + \gamma_d) \\
    i_B &= I \cos(p_0 t + \gamma_d - 120^\circ) \\
    i_C &= I \cos(p_0 t + \gamma_d + 120^\circ)
\end{align*}
\]

where \(I\) is the current amplitude, \(p\) is the pole pair number, \(\omega_r\) is the rotor angular speed, \(t\) is the time, and \(\gamma_d\) is the phase angle between the current vector and the rotor \(d\)-axis as shown in Fig.1.

The combined MMF of all the 3 phase windings is given in (3).

\[
F_s = n_A i_A + n_B i_B + n_C i_C
\]

Substituting (1) and (2) into (3), the forward and backward rotating MMF harmonics can be derived and given in (4) and (5) respectively\[^{41}\].

\[
\begin{align*}
    F_f &= \frac{1}{2} \sum_h [N_h I(1 + 2 \cos(h\theta_{phm} - 120^\circ))] \cdot \\
    & \cdot \cos(h\alpha_m + (h - p)\omega_r t - \gamma_d + \beta_h)] \\
    F_b &= \frac{1}{2} \sum_h [N_h I(1 + 2 \cos(h\theta_{phm} + 120^\circ))] \cdot \\
    & \cdot \cos(h\alpha_m + (h + p)\omega_r t + \gamma_d + \beta_h)]
\end{align*}
\]

where \(\alpha_m=\theta_m-\omega_r t\).

For a machine with \(Q\) slots and \(p\) pole pairs, its periodicity \(r\) is subject to the greatest common divider between \(Q\) and \(p\), as described in (6).

\[
r = \text{GCD}\{Q, p\}
\]

The orders of harmonics with non-zero magnitudes for the single phase winding function in (1) are summarized in Table 1, where \(Z\) denotes the positive integer set. By way of example, in a double layer winding with even \(Q/r\), the orders of harmonics with non-zero magnitudes for the single phase winding function are odd numbers \((2h-1)r\) except for \(jQ, j \in \mathbb{Z}\). This is because the even \(Q/r\) can form a symmetric pattern with opposite phasors that are \(\pi\) radians out of phase. The harmonics associated with the integer multiples of the slot number, \(jQ\), are cancelled out by the mechanical periodicity. Since 3-phase windings are uniformly displaced in space with respect to each other by \(120^\circ\) electrical degrees all triplen MMF harmonics are eliminated in \(Q/r\) slots. This can also be observed in
configuration. Given that \( jQ \) is a subset of \( 3j/2 \), the orders of MMF harmonics with non-zero magnitudes for a double layer 3-phase winding with an even \( Q/r \) are odd numbers \((2h-1)r\) except for \( 3j/2 \). The harmonic orders in the other scenarios in Table 1 can also be derived in the same way.

The forward and backward MMF harmonic expressions shown in (4) and (5) can be used to predict any stator MMF harmonic of a conventional 3-phase winding configuration. The magnitude of the \( h \)th order winding function harmonic \( N_h \) can be obtained by Fast Fourier Transform (FFT) of the winding function waveform of a given winding configuration.

By way of example, for a conventional 3-phase 12-slot 10-pole winding configuration, phase \( B \) lags phase \( A \) by 8 slots with respect to the rotating direction. Thus, \( \theta_{\text{phm}}=120^\circ \). Table 2 lists the values for \( N_h \), \( 1+2\cos(\theta_{\text{phm}})=120^\circ \), and \( 1+2\cos(\theta_{\text{phm}}+120^\circ) \) shown in (4) and (5) to determine the forward and backward MMF harmonics of the 12-slot 10-pole winding configuration up to the 19th order. \( N_h \) is normalized against its 5th order working harmonic. The resultant MMF harmonics are calculated using (4) and (5), and plotted in Fig.2 in which ‘F’ and ‘B’ denote forward and backward harmonics respectively. Their magnitudes are normalized against that of the 5th order working harmonic. It can be observed that the most detrimental harmonic is the 7th order harmonic, because it not only has a high amplitude but also exhibit a very high frequency seen from the rotor (i.e. 12 times the mechanical frequency).

### Table 1 The orders of MMF harmonics with non-zero magnitudes

<table>
<thead>
<tr>
<th>Winding type</th>
<th>Condition</th>
<th>Double layer winding</th>
<th>Single layer winding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Q/r ) is even</td>
<td>( Q/r ) is odd</td>
<td></td>
</tr>
<tr>
<td>Non-zero harmonic orders for winding function</td>
<td>((2h-1)r ) except ( jQ, j \in \mathbb{Z} )</td>
<td>( hr ) except ( jQ, j \in \mathbb{Z} )</td>
<td></td>
</tr>
<tr>
<td>Non-zero harmonic orders for 3-phase MMF</td>
<td>((2h-1)r ) except ( 3jr, j \in \mathbb{Z} )</td>
<td>( hr ) except ( 3jr, j \in \mathbb{Z} )</td>
<td></td>
</tr>
<tr>
<td>Winding type</td>
<td>( Q(2r) ) is even</td>
<td>( Q(2r) ) is odd</td>
<td>( Q/r ) is odd &amp; ( r ) is even</td>
</tr>
<tr>
<td></td>
<td>( (2h-1)r ) except ( jQ, j \in \mathbb{Z} )</td>
<td>( hr ) except ( jQ, j \in \mathbb{Z} )</td>
<td>( hr/2 ) except ( jQ, j \in \mathbb{Z} )</td>
</tr>
<tr>
<td>Non-zero harmonic orders for winding function</td>
<td>( (2h-1)r ) except ( jQ, j \in \mathbb{Z} )</td>
<td>( nr ) except ( 3jr, j \in \mathbb{Z} )</td>
<td>( 3nr/2, j \in \mathbb{Z} )</td>
</tr>
<tr>
<td>Non-zero harmonic orders for 3-phase MMF</td>
<td>( (2h-1)r ) except ( 3jr, j \in \mathbb{Z} )</td>
<td>( hr ) except ( 3jr, j \in \mathbb{Z} )</td>
<td>( hr/2 ) except ( 3jr, j \in \mathbb{Z} )</td>
</tr>
</tbody>
</table>

### Table 2 MMF harmonic expression breakdown of a conventional 3-phase 12-slot 10-pole winding configuration

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>( N_h )</th>
<th>( 1+2\cos(\theta_{\text{phm}}-120^\circ) )</th>
<th>( 1+2\cos(\theta_{\text{phm}}+120^\circ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.36</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0.89</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.72</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.30</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>0.03</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>0.18</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>0.29</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>0.26</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

The foregoing technique can be used to analyze MMF harmonic distributions of all 3-phase and multiple 3-phase windings, including most of the winding configurations designed to reduce MMF harmonics that will be described and discussed in the subsequent sections.

### 3 MMF harmonic reduction techniques

In an ideal 3-phase machine, only MMF working harmonic should be present and all other harmonics eliminated. In this section, the techniques to reduce undesirable MMF harmonics reported in literature are outlined, including winding split and shift, delta-star connected windings, multiple 3-phase windings, multilayer windings, uneven turn numbers, and stator flux barriers. The other techniques which are of obvious disadvantage have not been discussed in this paper, such as uneven tooth width and uneven back-iron.

#### 3.1 Winding split and shift

G. Dajaku and D. Gerling\cite{42-43} proposed a 24-slot 10-pole winding configuration, as shown in Fig.3, to minimize the 1st and 7th order MMF harmonics of the conventional 12-slot 10-pole winding configuration. The MMF spectrum of the conventional 12-slot 10-pole winding configuration is shown in Fig.2. The concept is to divide the original 12-slot 10-pole windings into 2 sets, arrange them with a specific mechanical phase shift, and connect them in series, as illustrated in Fig.3. The mechanical phase shift, \( \alpha_m \), between these 2 sets of windings is determined by the order of MMF harmonic to be eliminated. Mathematically, this concept is to reshape the single phase winding function to make the magnitude of the \( h \)th order harmonic of the winding function, \( N_h \) in (1), to be zero.
To eliminate the 7th order MMF harmonic shown in Fig.2, the optimal mechanical phase shift $\alpha_w$ is selected to be slightly higher than 2.5 times the tooth pitch angle. Thus, uneven tooth width technique has to be utilized. Although with this technique the 7th order MMF harmonic has been significantly reduced, the reduction in the 1st order MMF harmonic is only 20% approximately. Therefore, to attenuate the 1st order MMF harmonic, uneven turn numbers for the neighboring phase coils are also employed. Fig.4 illustrates the MMF spectrum of the proposed 24-slot 10-pole winding configuration with winding split and shift, uneven tooth widths, and also uneven turn numbers techniques. A significant reduction in the 1st and 7th order MMF harmonics can be observed. In doing so, however, the winding factor of the fundamental is slightly reduced compared with the conventional 12-slot, 10-pole winding configuration.

V. I. Patel and J. Wang et al [44-45] proposed a 6-phase 18-slot 10-pole winding configuration whose schematic can be found in Fig.5. The concept is to split the 3-phase 9-slot 8-pole windings into two sets and apply appropriate phase shift for the specific harmonic cancellation. To enhance the machine fault-tolerant capability, 6-phase (two 3-phase) technique is employed and these two 3-phase windings are fed by two separated inverters. The mechanical phase shift between these two sets of 3-phase windings is selected to eliminate the most MMF harmonics. The currents fed into these two sets of 3-phase windings have no phase shift in time domain.

Fig.6 compares the MMF spectra of the 6-phase 18-slot 8-pole winding configuration with those of the conventional 9-slot 8-pole windings whose MMF spectrum can be calculated using (4) and (5). It can be seen that 1st, 5th, 7th, 11th, 13th, 17th, 19th, 23rd … orders of MMF harmonics are completely eliminated.

J. Wang et al [46] also devised all the feasible slot-pole combinations with the foregoing technique. The optimal phase shift between these two 3-phase windings for various slot-pole combinations are given.

P. B. Reddy et al [47] derived a generic approach to the selection of the optimal mechanical phase shift of the two sets of windings after splitting the phase windings. Uneven tooth width design is considered to reach the best harmonic cancellation effects.

A. S. Abdel-Khalik et al [48] developed a 6-phase 24-slot 10-pole winding configuration whose schematic is shown in Fig.7. The concept is also to split phase windings into two 3-phase sets. The phase shifts in both space and time between the two 3-phase phase sets are selected to eliminate the most undesirable MMF harmonics. Fig.8 compares the MMF spectra of the 6-phase 24-slot 10-pole winding configuration with those of the 3-phase 24-slot 10-pole windings. It can be seen that with the introduced phase shift between the two 3-phase sets in time domain, the 1st order MMF harmonic which is present in the 3-phase machines has been cancelled out.

Fig.4 MMF spectrum of the proposed 24-slot 10-pole winding configuration (with uneven turn numbers) in [42-43]

Fig.5 Schematic of the 6-phase 18-slot 8-pole winding configuration proposed in [44-45]

Fig.6 MMF spectra comparison between the conventional 9-slot 8-pole and the 6-phase 18-slot 8-pole winding configuration proposed in [44-45]

Fig.7 Schematic of the 6-phase 24-slot 10-pole winding configuration (12 slots displayed) proposed in [48]

Fig.8 MMF spectra comparison of the 3-phase and 6-phase 24-slot 10-pole winding configurations proposed in [48]
All of the foregoing techniques can reduce or eliminate certain MMF harmonics, particularly the most detrimental harmonic whose order is close to the working harmonic. However, by splitting the phase windings into two 3-phase sets, slot and coil numbers are doubled. In order to maintain a decent fundamental winding factor, a coil span of 2 slots is employed. Thus, these proposed windings are no longer concentrated or tooth-wound windings in conventional sense, thereby compromising the advantages of the concentrated windings.

### 3.2 Delta-star windings

G. Dajaku and D. Gerling et al\textsuperscript{[49-50]} proposed an 18-slot 10-pole concentrated winding configuration with a delta-star connection. The idea is to create two sets of windings, one in star connection and other in delta connection. By exploiting the phase shift between the MMFs produced by the two sets of windings some undesirable harmonics can be eliminated with appropriate selection of the number of coils in each winding set. Meanwhile, the number of slots is increased by $k_n m$ where $k_n$ is the number of winding sets and $m$ is the number of phases. By way of example, Fig. 9 (a) shows the coil distribution and connection of phase $A$ winding in the conventional 12-slot 10-pole machine. By expanding 12 slots to 18 slots, phase $A$ now has 6 coils which are grouped into $A1$ and $A2$ as shown in Fig. 9(b). Thus, this winding arrangement maintains the concentrated winding configuration in which the coil span equals 1 slot pitch. However, for the same pole pairs, the coil span in electrical degree is much reduced. This greatly penalizes the fundamental winding factor and thus the torque production capability is compromised.

These two sets of windings shown in Fig. 9(b) are connected using delta-star combinations to produce the phase shift in the currents flowing in them. Fig. 10 illustrates the schematics of the developed 18-slot 10-pole winding and its delta-star connection diagram. The delta and star windings in each phase have 4 and 2 coils respectively. The electrical phase shift between the delta and star connected windings is $30^\circ$, while the current magnitude in the star connected windings are $\sqrt{3}$ times those in the delta connected windings. Thus, the number of turns per phase of the delta windings needs $\sqrt{3}$ times greater than those in the star windings to balance the MMF magnitudes of the two sets of windings and cancel out the unwanted MMF harmonics.

Fig. 11 compares the MMF spectra of the 18-slot 10-pole winding configuration with those of the conventional 12-slot 10-pole windings. It can be seen that the 1st, 7th, 11th, 13th, 17th, 19th … orders of MMF harmonics have been eliminated whereas the 13th order harmonic increases.
A. S. Abdel-Khalik et al.\cite{51} proposed a generic dual \( n \)-phase winding configuration with delta-star connections for \( n \)-phase PM machine occupying \( 4n \) slots, to cancel out all the sub-harmonics of the winding MMF. Compared to the techniques proposed in \cite{49-50}, the slot and coil numbers are not increased. Instead, the original coils are split into 2 sets of windings which are arranged with a specific phase shift in space. Thus, the fundamental winding factor is not penalized. The delta-star connection is employed to generate a phase shift in time domain for the cancellation of some undesirable MMF harmonics.

The winding configuration are applied in \cite{51} to a 3-phase 12-slot 10-pole winding configuration with delta-star connection and to a 5-phase 20-slot 18-pole winding configuration with pentagon-star connection, as shown in Fig.12 and Fig.13, respectively.

Fig.14 shows the MMF spectra of the 3-phase 12-slot 10-pole and 5-phase 20-slot 18-pole windings proposed in \cite{51}. It can be observed that the sub-harmonics and some of the higher order MMF harmonics have been eliminated while the working harmonic and some higher order harmonics are increased due to slightly higher winding distribution factors.

However, the MMF harmonic reduction by this winding technique is limited because the electrical phase shift between the delta and star windings is restricted to 30° and that between the pentagon and star windings to 18°. Consequently the most detrimental harmonic whose order is close to the working harmonic is not affected.

X. Chen and J. Wang et al.\cite{41, 52} proposed a generic approach to reduction of MMF harmonics using multiple 3-phase winding configurations, and subsequently devised a 9-phase 18-slot 14-pole machine. Fig.15 shows the MMF harmonic cancellation principle using multiple 3-phase winding configurations. The three 3-phase windings are uniformly arranged in space but with a specific spatial phase angle with respect to each other, as shown in Fig.15(a). The phase shift in time for
the currents in each 3-phase set can also be selected in order to eliminate unwanted harmonics. The condition to cancel out a given order MMF harmonic is to select appropriate space and time phase shifts so that the MMF vectors produced by the three 3-phase windings form a balanced 3-phase system, as illustrated in Fig.15(c), while their working harmonic vectors overlap with each other, as shown in Fig.15(b). Indeed, this multiple 3-phase winding configuration introduces an additional ‘degree of freedom’ in the MMF harmonic cancellation mechanism.

The generic mathematic expressions for the forward and backward MMF harmonics in a multiple 3-phase windings have been derived based on (4) and (5), and they are given in (7) and (8), respectively.

\[ F_f = \frac{1}{2} \sum_{k} \{ N_k I [1 + 2 \cos(h \theta_{phm} - 120^\circ)] \sum_k \cos[h \alpha_m + (h - p) \omega t - \gamma_d + \beta_{kh} + (k-1)\theta_{\Delta}] \} \]  

\[ F_b = \frac{1}{2} \sum_{k} \{ N_k I [1 + 2 \cos(h \theta_{phm} + 120^\circ)] \sum_k \cos[h \alpha_m + (h + p) \omega t + \gamma_d + \beta_{kh} - (k-1)\theta_{\Delta}] \} \]

where \( k \) denotes the \( k \)th 3-phase set, \( \beta_{kh} \) is the phase angle of the \( k \)th order spatial harmonic of the \( k \)th set 3-phase winding, and \( \theta_{\Delta} \) is the phase shift angle of the currents between the \((k+1)\)th and \( k \)th 3-phase sets.

The condition to eliminate the \( k \)th order MMF harmonic is that the combined phase shift angles between the \( k \)th and \((k+1)\)th 3-phase sets denoted as \( \theta_{fw} \) and \( \theta_{bw} \) for the forward and backward MMF harmonics satisfy (9) and (10), respectively.

\[ \theta_{fw} = \beta_{(k+1)h} - \beta_{kh} + \theta_{\Delta} = \pm 360^\circ / K + q_f \cdot 360^\circ \quad q_f \in \mathbb{Z} \]  

\[ \theta_{bw} = \beta_{(k+1)h} - \beta_{kh} - \theta_{\Delta} = \pm 360^\circ / K + q_b \cdot 360^\circ \quad q_b \in \mathbb{Z} \]

where \( K \) is the number of 3-phase sets. The above conditions lead to the second summation in (7) and (8) becoming zero and hence the \( k \)th order harmonic is eliminated.

Based on the foregoing approach, a 9-phase 18-slot 14-pole winding configuration for IPM machines is developed and its schematic is illustrated in Fig.16. The MMF spectra comparison of the conventional 3-phase and 9-phase 18-slot 14-pole winding configurations is shown in Fig.17. The MMF spectrum of the conventional 3-phase winding is calculated using (4) and (5). It can be seen that by employing the multiple 3-phase winding configuration, the 1st, 5th, 13th, 17th, 19th, 23rd \(
\ldots\) order harmonics are cancelled out while the 7th (working harmonic for 14-pole), 11th, 25th, 29th \(
\ldots\) order harmonics slightly increase due to higher winding distribution factors.

This technique also contributes to a degree of fault tolerance in that the machine can continue to operate even if one 3-phase windings/drive fails.

The use of multiple 3-phase winding configurations and inverters may not be a disadvantage as the reduction of current rating in each 3-phase system would facilitate inverter heat dissipation and integration into machine housing, particularly for high power drives.

### 3.4 Multilayer windings

M. V. Cistelecan et al\cite{53-54} proposed concentrated multilayer winding configurations whose schematics are illustrated in Fig.18. Two conventional windings, 12-slot 10-pole and 9-slot 8-pole, are chosen to demonstrate the MMF harmonic cancellation effects with the multilayer winding techniques. The concept is to split the coils into two sets and shift one set of coils

![Fig.16 Schematic of the 9-phase 18-slot 14-pole winding configuration proposed in [41, 52]](image)

![Fig.17 MMF spectra comparison of the conventional 3-phase and 9-phase 18-slot 14-pole winding configurations proposed in [41, 52]](image)
by one or more slots, while maintaining the same number of slot for a given number of pole pairs. Initially four layers of windings are devised. By merging the coils which are in the same slot and also in the same phase, a triple layer winding configuration, as shown in Fig.18(a) can be developed. If there are no coils to satisfy both conditions, the quadruple layer winding configuration, as shown in Fig.18(b), will be employed. The number of turns per coil can be adjusted to reduce the most MMF harmonics. By way of example, the number of turns per coil of the top layer (adjacent to the back-iron) windings in Fig.18(a) is designed to be $\sqrt{3}$ times of those of the other layers.

Fig.19(a) and (b) compare the MMF spectra of the developed triple layer 12-slot 10-pole and quadruple 9-slot 8-pole windings with those of their double layer counterparts whose MMF spectra can be calculated using (4) and (5). It shows that the sub-harmonics and some higher order harmonics can be eliminated or greatly diminished.

A. S. Abdel-Khalik et al\cite{55} proposed a triple layer 12-slot 10-pole winding configuration with the delta-star connection as shown in Fig.20(d), and thereafter performs a comparative study with the other three 12-slot 10-pole winding types including the conventional double layer winding denoted as ‘winding 1’ in Fig.21(a), double layer winding with delta-star connection denoted as ‘winding 2’ in Fig.20(b), and triple layer winding denoted as ‘winding 3’ in Fig.20(c).

Fig.21 compares the MMF spectra of these four types of 12-slot 10-pole winding configurations. The MMF spectrum of ‘winding 1’ can be calculated using (4) and (5). It can be observed that both the double layer winding with delta-star connection and the proposed triple layer winding with delta-star connection have the best MMF harmonic cancellation effects, eliminating the sub-harmonics and some of higher order harmonics. However, the proposed triple layer winding with delta-star connection compromises the fundamental winding factor and thus slightly penalizes the torque production, although it may maintain the benefits of multilayer designs, such as higher saliency ratio and reluctance torque component.

The foregoing multilayer winding configurations increases the complexity of connection and results in lower fundamental winding factor and hence low torque capability. In addition, the most detrimental harmonic whose order is close to that of the working harmonic is not significantly reduced.
3.5 Uneven turn numbers

G. Dajaku and D. Gerling [56-57] developed a 12-slot 10-pole winding configuration with uneven number of turns in the neighboring slots, as illustrated in Fig. 22. The mathematic relation between \( n_1 \) and \( n_2 \) shown in Fig. 22 is determined by the order of MMF harmonic to be suppressed. To easily realize uneven turn numbers, \( n_1 \) and \( n_2 \) differ by 1. Thus, the slot with less turn number should not complete the last turn, as illustrated in Fig. 22. The ratio of \( n_1 \) to \( n_2 \) determines the MMF harmonic cancellation effect. By way of example, the 1st order MMF harmonic can be eliminated when the ratio of \( n_1 \) to \( n_2 \) equals 0.87. Therefore, in this application, \( n_1 \) and \( n_2 \) are designed to be 7 and 8 respectively. Thus, the slot with less turn number should not complete the last turn, as illustrated in Fig. 22. The ratio of \( n_1 \) to \( n_2 \) determines the MMF harmonic cancellation effect. By way of example, the 1st order MMF harmonic can be eliminated when the ratio of \( n_1 \) to \( n_2 \) equals 0.87. It can also be inferred that the MMF harmonic cancellation effects are negligible when the number of turns per coil relatively high because the ratio of \( n_1 \) to \( n_2 \) will be close to be 1 if \( n_1 \) and \( n_2 \) only differ by 1.

Fig. 23 compares the MMF spectra of the conventional 12-slot 10-pole winding configuration and that with uneven number of turns proposed in [56-57]. It can be seen that the 1st, 11th, 13th order MMF harmonics has been greatly diminished. However, the most detrimental 7th MMF harmonic is not reduced.

The foregoing technique restricts the difference in the uneven turn numbers to be only 1 turn. Therefore, the ability to reduce MMF harmonics by this technique is quite limited when the turn number has to be designed high. On the other hand, it is not practical to have the difference in the number of turns on two sides of a coil being greater than 1. Further, uneven Ampere turn distribution in slots lead to less effective utilization of the slot areas and localized saturation in the stator teeth, compromising the electromagnetic performances.

3.6 Stator flux barriers

G. Dajaku et al [58] proposed a new 12-slot 10-pole machine topology with flux barriers located at the stator back-iron. The introduced flux barriers significantly increase the reluctance to the 1st order MMF harmonic while the reluctance to other higher order harmonics are less affected. Consequently, the air-gap flux density due to the 1st MMF harmonic is reduced.

The proposed single and double layer 12-slot 10-pole machines with stator flux barriers are illustrated in Fig. 24 (a) and (b) respectively. The sizes of these flux barriers can be optimized to obtain the best harmonic cancellation effect.

Fig. 25 compares the air-gap flux density spectra of the conventional 12-slot 10-pole winding configuration and that with stator flux barriers proposed in [58] under excitations of the stator currents only. It shows that the 1st, 11th and 13th harmonics of air-gap flux density of the single layer windings are reduced by approximately 50%. However, these air-gap flux density harmonics of the double layer windings are eliminated, whereas the 5th working harmonic is slightly penalized.

While the concept is simple to be implemented there a number of disadvantages associated with this technique. First the harmonic cancellation effects may be compromised at heavy load condition since the saturation in the stator core affects the reluctance in the magnetic circuit. The fundamental winding factor and
torque production may be compromised without careful design. In addition the flux linkages generated by permanent-magnets are slightly reduced due to higher reluctances caused by the flux barriers in the stator, and thus the magnetic loading is reduced with the same amount of permanent-magnets.

4 Conclusion

This paper reviews the current state-of-the-art of the MMF harmonic reduction techniques for concentrated winding configurations in PMSMs, including winding split and phase shift, delta-star windings, multiple 3-phase windings, multilayer windings, uneven turn numbers, and stator flux barriers. Their concepts, advantages and disadvantages have been assessed and discussed.

The technique of employing winding split and phase shift can eliminate the most detrimental MMF harmonic whose order is close to that of the working harmonic. However, to maintain a decent fundamental winding factor, a coil span of 2 slots has to be employed. Therefore, the neighboring coils overlap with each other, and thus this technique no longer possess some advantages of the conventional concentrated windings.

The other techniques maintain the concentrated winding configurations, yet cannot significantly reduce the most detrimental MMF harmonic whose order is close to that of the working harmonic, without compromising the fundamental winding factor and torque capability. The delta-star winding configuration can only produce a specific phase shift between the delta and star connected winding sets. The multiple 3-phase winding configuration requires the use of the multiple 3-phase inverters. The multilayer winding configuration increases the complexity of connection and also results in lower fundamental winding factor and hence low torque capability. The winding configuration with uneven number of turns has limited MMF harmonic cancellation effects and results in the uneven Ampere turn distribution in slots. This also leads to less effective utilization of the slot areas and cause localized saturation in the stator teeth, compromising the electromagnetic performances. The stator flux barrier technique requires more delicate balance between the electrical and magnetic loadings in the stator design, and may compromise the torque production capability while its harmonic cancellation effects at heavy load condition is limited.

Selection of the most appropriate technique is application dependent. For example, in high power drives, use of multiple 3-phase windings would reduce inverter rating for each 3-phase set and facilitates heat spreading and dissipation. Thus, the technique with multiple 3-phase winding configuration can be readily exploited without increase in cost.

References


Xiao Chen received B.Eng. degree in electrical engineering from Harbin Institute of Technology at Weihai, Weihai, China, in 2009, M.Eng. degree in electrical engineering from Harbin Institute of Technology, Harbin, China, in 2011, and Ph.D degree from University of Sheffield, Sheffield, UK in 2015 respectively. He is currently a research associate in Dept. of Electronic and Electrical Engineering, University of Sheffield, UK. His current research interests include the modeling, design and analysis of permanent-magnet synchronous machines for traction applications.


Currently, he is a Professor in Electrical Engineering at the University of Sheffield, Sheffield, U.K. From 1986 to 1991, he was with the Department of Electrical Engineering at Jiangsu University of Science and Technology, where he was appointed a Lecturer in 1987 and an Associated Professor in 1990. He was a Postdoctoral Research Associate at the University of Sheffield, Sheffield, U.K., from 1996 to 1997, and a Senior Lecturer at the University of East London from 1998 to 2001. His research interests range from motion control and electromechanical energy conversion to electric drives for applications in automotive, renewable energy, household appliances and aerospace sectors.

He is a fellow of the IET and a senior member of IEEE.