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Wireless-Powered Device-to-Device-Assisted Offloading in Cellular Networks

Bodong Shang, Student Member, IEEE, Liqiang Zhao, Member, IEEE, Kwang-Cheng Chen, Fellow, IEEE, and Xiaoli Chu, Senior Member, IEEE

Abstract—Offloading cellular traffic to device-to-device (D2D) communications has been proposed to improve the network capacity and to alleviate the traffic burden on base stations (BSs). However, as mobile devices are powered by limited battery energy, there is no obligation for D2D transmitters (D2D-Txs) to offload cellular traffic through D2D content sharing. In this paper, we model and analyze the wireless-powered D2D-assisted offloading (WPDO) in cellular networks, where the D2D-Txs can harvest radio frequency (RF) energy from nearby BSs and utilize the harvested energy to share popular contents with nearby user equipments (UEs). Stochastic geometry is used to characterize the intrinsic relationship between the wireless power transfer (WPT) and the information transmission. Based on the proposed model, we derive the average transmit power at D2D-Tx, the expected minimum transmit power at BS, the D2D outage probability, and the cellular downlink outage probability. We also investigate the energy efficiency of the WPDO network from a system-level perspective. Input and numerical results show that the energy efficiency of the WPDO network can be maximized by optimizing the fraction of time allocated for WPT and it can be further improved by using massive antenna arrays at each BS and by sharing more popular contents between devices.

Index Terms—D2D communications, energy efficiency, traffic offloading, cellular networks, wireless power transfer.

I. INTRODUCTION

WITh the upsurge of mobile data traffic and the explosively increase of mobile devices, cellular networks are facing technical challenges in supporting enormous data flows, high data rate, and large system capacity. In high user density areas, the base stations (BSs) are suffering heavy load burdens. To address the above issues, device-to-device (D2D) communications have been proposed to improve the network capacity and to alleviate the traffic burden on cellular networks by exploiting the physical proximity of mobile devices [2]. In D2D communications, nearby devices can communicate with each other directly without using conventional cellular links, enjoying an improved received signal strength due to the short link distance.

However, as mobile devices are powered by limited battery energy, in general there is no obligation for mobile devices to participate in cellular traffic offloading or D2D content sharing [3]. Dedicated wireless power transfer (WPT) through electromagnetic radiation has emerged as a cost-effective technique to enable on-demand energy supplies and uninterrupted operations [4], [5]. The radio frequency (RF) signal emitted by dedicated energy sources, such as the hybrid access point (HAP) [6], which can provide both the energy and information transmission to/from user equipment (UE), can be used to supply energy over a long distance to UE.

In this paper, we incorporate WPT into D2D communications to facilitate D2D-assisted cellular traffic offloading. Considering the increasing power consumption of wireless networks [7], we propose an energy efficient wireless-powered D2D-assisted offloading (WPDO) network, where the D2D transmitters (D2D-Txs) scavenge RF energy from the nearest BS by pointing beams towards them as well as the ambient RF energy emitted by other BSs, and utilize the harvested energy to share popular contents with content requesters located in the D2D-Txs’ offloading regions. In the offloading regions, the quality-of-service (QoS) requirements of the offloaded UEs (i.e., the D2D receivers) can be guaranteed. By leveraging tools from stochastic geometry, we evaluate the energy efficiency of the WPDO network and provide insights into the network design from a system-level perspective.

A. Related Works

The existing work on cellular traffic offloading can be categorized as follows: traffic offloading through small cells [8], traffic offloading through WiFi networks [9], traffic offloading through D2D communications [10]–[12]. Although offloading traffic from macro cells to small cells provides a convenient way to mitigate cellular network congestions, the decreasing coverage probability due to inter-cell interference [13] and the expensive operating cost for backhaul links hinder the dense deployment of small cells [14]. Different from cellular technologies, WiFi networks provide higher data rates by exploiting wider unlicensed frequency bandwidths and higher-order modulations [15]. However, in WiFi-assisted offloading

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networks, the UE mobility management and the network coverage are limited [9]. In D2D-assisted offloading networks, popular contents can be shared directly among mobile devices as an economical way to alleviate the traffic burden on cellular BSs [10], [11], however, at the cost of increased power/energy consumption at D2D-Txs. In [3], [16], [17], incentive schemes were investigated to stimulate UEs to participate in D2D communications. In [12], [18], the social interactions among UEs, either in real life or in social networks, were exploited in the design of D2D communications.

With the development of the wireless charging techniques [19], it has been proposed to harvest RF energy for powering information transmission in cellular networks [20], [21], as well as in D2D communications [22]–[24]. In [24], energy harvesting D2D communications were designed to maximize the sum-rate for D2D links. However, it has been shown that, the energy harvested from ambient RF signals can only power small sensors with sporadic activities [25], while supplying stable and fully controllable power for D2D communications would require dedicated WPT [5]. In [26], unmanned aerial vehicles were used as dedicated energy sources to provide WPT to UEs, where the resource allocation was optimized to maximize the average throughput. In [27], the sum rate of wireless-powered D2D links was maximized by jointly optimizing beamforming and resource allocation. Note that most of the above works focused on a single cell and ignored the interference between cellular and D2D links, which may significantly affect the performance of both the cellular network and the D2D links. In addition, none of the existing works has studied the energy efficiency of wireless-powered D2D assisted offloading networks while considering the UEs’ QoS requirements.

B. Paper Contributions

In our proposed WPT enabled D2D-assisted offloading networks, a communication time slot is divided into two sub-slots. In the first sub-slot, each BS with a large antenna array wirelessly charges the D2D-Txs located in its coverage area by direct beamforming. In the second sub-slot, D2D-Txs utilize the harvested energy to broadcast popular contents to nearby mobile content requesters, and BSs perform downlink transmissions to their scheduled cellular UEs. We consider the underlay mode of D2D communications and thus the mutual interference between cellular and D2D links.

The main contributions of the paper are summarized as follows:

- We develop a tractable analytical model for the WPDO network. Using stochastic geometry, we derive the expressions for the average transmit power of a typical D2D-Tx and the expected minimum transmit power at a typical BS while meeting cellular UEs data rate requirements, and investigate the relationship between the D2D-Tx average transmit power and the density of D2D-Txs as well as the size of antenna arrays at BSs.
- Based on the D2D-Tx average transmit power, we derive the outage probabilities of D2D and cellular UEs, respectively. The D2D outage probability is analyzed as a function of the time allocation factor (i.e., the fraction of time allocated for WPT) and the D2D UE data rate requirement.
- We define and maximize the network energy efficiency for WPDO by optimizing the time allocation factor, and provide insights into the design of an energy-efficient WPDO network, with respect to the time allocation factor, the popularity of contents shared via D2D, and the size of BS antenna arrays.

C. Paper Organization

The remainder of this paper is organized as follows. In Section II, the system model is presented. Section III derives the average transmit power at a typical D2D-Tx and the BS expected minimum transmit power. Section IV gives the analytical expressions of the outage probabilities for D2D UE and cellular UE, respectively. Section V defines and optimizes the WPDO network energy efficiency. Simulation and numerical results are presented in Section VI. Finally, conclusions are drawn in Section VII.

Notation: $E\{x\}$ denotes the expectation of variable $x$. $\mathbb{P}\{A\}$ denotes the probability that event $A$ happens. Finally, $y^*$ denotes the optimal value of $y$.
plane $\mathbb{R}^2$ with the density $\lambda_B$ and are denoted by the set $\Phi_B = \{b_j, j = 0, 1, 2, \ldots\}$. Each BS has a maximum allowable transmit power $P_m$, and is equipped with $N_t$ antennas. The cell area of the $j$th BS $b_j$ is given by $V_j = \{x \in \mathbb{R}^2 | ||x - b_j|| \leq ||x - b_j||, b_j \in \Phi_B\},$ where $||a - b||$ represents the Euclidean distance between $a$ and $b$ in the plane $\mathbb{R}^2$. Since there is no interference concern in the downlink wireless power transfer phase, each BS can adopt the simple maximal ratio transmission (MRT) beamforming to maximize the wirelessly transferred power to the D2D-Txs in its cell area. Most of the other beamforming methods, such as zero-forcing (ZF) beamforming, are designed to mitigate interference at the cost of reduced radiated power gain [28]. D2D-Txs are distributed following an independent homogeneous PPP denoted by $\Phi_D$ with the density $\lambda_D$. UEs$^1$ are spatially scattered in $\mathbb{R}^2$ following another independent PPP denoted by the set $\Phi_U$ with the density $\lambda_U$, which can be classified into cellular UEs (served by BSs) and D2D UEs (served by D2D links). Each UE is assumed to be equipped with a single antenna.

B. UE Association

In our system model, each cellular UE connects to the closest BS. Each D2D-Tx centers at its offloading region with the radius $R_D$, which is set to guarantee the D2D data rate requirement in the offloading region (see Section V). The data rate requirements of cellular UEs and D2D UEs are denoted by $R_c$ and $R_d$ (Mbps), respectively. We define the content popularity of a content available at a D2D-Tx as the probability $P_{\text{pop}}$, that the content is requested by at least one UE. The content popularity can be obtained by the keywords feature extraction method [29] or the machine learning method [30] according to the UEs’ download history. If a content requesting UE is located in the offloading region of a D2D-Tx containing the requested contents, then the UE will be informed by its serving BS to connect with the D2D-Tx.

C. Channel Model

The downlink bandwidth is $B$ MHz, which is shared between cellular downlink and D2D communications. Each BS performs adaptive power control according to the channel state information (CSI) obtained from UE feedback [31] and adopts MRT beamforming to transmit the information to cellular UEs. According to Shannon’s theorem, the transmit power $P_{i,j}^B$ of BS $b_j$ for cellular UE $u_{i,j}$ (i.e., the $i$th cellular UE in the $j$th cell) to achieve the required data rate $R_c$ can be obtained by solving the following equation:

$$R_c = \frac{B}{N_j} \log_2 \left( 1 + \text{SINR} \left( u_{i,j} \right) \right),$$

where

$$\text{SINR} \left( u_{i,j} \right) = \frac{P_{i,j}^B \left| \mathbf{h}_{b_j u_{i,j}} \right|^2 H_\alpha \left| b_j - u_{i,j} \right|^{-\alpha}}{I_{u_{i,j}}^C + \frac{1}{I_{u_{i,j}}^D} + \sigma^2}.$$

$\Phi_B, \Phi_D, \Phi_U$ Sets of cellular BSs, D2D-Txs and UEs

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where $N_c^T$ denotes the total number of cellular UEs served by BS $b_j$. $\mathbf{h}_{b_j u_{i,j}} \in C^{1 \times N_t}$ is the small-scale fading channel vector$^2$, $H_\alpha$ is a frequency dependent constant value [13], which is commonly set as $(\frac{c}{4\pi f_\alpha})^2$ with $c = 3 \times 10^8$ m/s and the carrier frequency $f_\alpha$, $\alpha$ is the path loss exponent. $I_{u_{i,j}}^C$ and $I_{u_{i,j}}^D$ denote the interference power from interfering BSs and from D2D-Txs to $u_{i,j}$, respectively, and $\sigma^2$ is the additive noise. Specifically, we have

$$I_{u_{i,j}}^C = \sum_{b_k \in \Phi_B \setminus b_j} P_{i,j}^B \left| \mathbf{h}_{b_k u_{i,j}} \right|^2 \left| \mathbf{g}_{b_k u_{i,j}} \mathbf{g}_{b_k u_{i,j}}^H \right|^{-\alpha} H_\alpha \left| b_k - u_{i,j} \right|^{-\alpha},$$

and

$$I_{u_{i,j}}^D = \sum_{d_k \in \Phi_D} P_{d_k} d_k \left| \mathbf{h}_{d_k u_{i,j}} \right|^2 \left| \mathbf{g}_{d_k u_{i,j}} \mathbf{g}_{d_k u_{i,j}}^H \right|^{-\alpha} H_\alpha d_k - u_{i,j}^{-\alpha}$$

where $\mathbf{h}_{b_k u_{i,j}} \in C^{1 \times N_t}$ is the interfering small-scale fading channel vector, and $\mathbf{g}_{d_k u_{i,j}} \mathbf{g}_{d_k u_{i,j}}^H$ is the MRT beamforming vector of BS $b_k$, where $\mathbf{g}_{b_k u_{i,j}} \in C^{1 \times N_t}$ is the small-scale fading channel vector from BS $b_k$ to its associated UE $u_{i,j}$. According to eq.(3) of [32] and Proposition 1 of [33], $\mathbf{h}_{b_k u_{i,j}} \mathbf{g}_{b_k u_{i,j}}^H$ is a zero-mean complex Gaussian variable.

D. Wireless Power Transfer

Since we assume that each D2D-Tx is equipped with one antenna either for energy harvesting or for information transmission and to ease the energy consumption burden on D2D-Txs in D2D-assisted offloading, we employ the harvest-then-transmit protocol [21], where the D2D-Tx first harvests

$^1$In this paper, UEs refer to the information receivers which includes the cellular UEs and the D2D UEs (i.e., D2D receivers).

$^2$With a slight abuse of notation we will use $\mathbf{h}_{b_k}$ to denote the small-scale fading channel vector from $x$ to $y$, where the channels are assumed to experience Rayleigh fading such that $\left| \mathbf{h}_{b_k} \right|^2 \sim \text{Gamma}(N_t, 1)$. 

the wireless energy from both the directed power transferred by its nearest BS and the ambient power radiated by other BSs, and then utilizes the harvested energy to send data to D2D UEs. Note that simultaneous energy harvesting and information transmission at the D2D-Tx would require multiple antennas and an integrated circuit architecture, resulting in additional costs and increased complexities in circuit design and antenna array configuration at the UEs [34]. Let $T$ denote the duration of a communication time slot, which is divided into two sub-slots of duration $\theta T$ and $(1-\theta)T$, respectively, where $\theta (0 \leq \theta \leq 1)$ is the time allocation factor. The $\theta T$ sub-slot is allocated for WPT and the $(1-\theta)T$ sub-slot is for information transmission. In WPT, the D2D-Tx in a cell take turns to harvest RF energy from the nearest BS by direct beamforming as well as the ambient RF energy from other BSs for a time duration of $\frac{\theta T}{n_d}$, where $n_d$ denotes the number of D2D-Txs in a cell and $\mathbb{E}(n_d) = \frac{\lambda_s}{\lambda_B}$. Therefore, the instantaneous received power $P_{d,k,j}^{\theta T}$ at the $k^{th}$ D2D-Tx $d_{k,j}$ in the $j^{th}$ cell during the allocated time $\frac{\theta T}{n_d}$ is expressed as

$$P_{d,k,j}^{\theta T} = P_m \left| \mathbf{h}_{b_d,k,j} \right|^2 \beta T \left| \mathbf{h}_{b_d,k,j} \right|^2 \beta T + \sum_{b_n \in \Phi_B \backslash \{b_d\}} \left| \mathbf{h}_{b_n,k,j} \right|^2 \left| \mathbf{h}_{b_n,k,j} \right|^2 \beta T$$

where $\beta$ is the path loss exponent for a WPT link, and $v_1 (v_1 \geq 1)$ is used to avoid singularity at zero distance and to ensure the finite moments of the direct and the ambient RF signals. It is worth noting that the carrier frequencies of WPT and information transmission are different. $H_\beta$ is a frequency dependent constant value of a WPT link.

In addition, during the remaining time of $\frac{(n_d-1)\theta T}{n_d}$, the typical D2D-Tx harvests the ambient RF energy emitted by all BSs in the network. Thus, the instantaneous received power $P_{d,k,j}^{\frac{n_d-1}{n_d} \theta T}$ at D2D-Tx $d_{k,j}$ during $\frac{(n_d-1)\theta T}{n_d}$ is given by

$$P_{d,k,j}^{\frac{n_d-1}{n_d} \theta T} = P_m \left| \mathbf{h}_{b_d,k,j} \right|^2 \beta T \left| \mathbf{h}_{b_d,k,j} \right|^2 \beta T + \sum_{b_n \in \Phi_B \backslash \{b_d\}} \left| \mathbf{h}_{b_n,k,j} \right|^2 \left| \mathbf{h}_{b_n,k,j} \right|^2 \beta T$$

We assume that each D2D-Tx has a rechargeable battery with a sufficiently large storage such that enough harvested energy can be stored at D2D-Txs for supporting stable transmit power. The randomness of the instantaneous received power at a D2D-Tx can be averaged out, and a fixed transmit power up to $P_d$ can be provided [20], [35]. Note that if a D2D-Tx has a small battery storage, the battery may be saturated and the additionally arriving energy will be discarded without being utilized for data transmission [36].

In addition, $P_d$ is expressed as follows

$$P_d = \eta \frac{1}{(1-\theta)T} \mathbb{E} \left\{ E_{d,k,j}^{\theta T} \right\}$$

where $\eta (0 < \eta < 1)$ is the RF-to-DC conversion efficiency [37] and $\mathbb{E} \left\{ E_{d,k,j}^{\theta T} \right\}$ is the expectation of the total received RF energy at a typical D2D-Tx $d_{k,j}$ during $\theta T$, and we have

$$E_{d,k,j}^{\theta T} = \theta T P_m \frac{1}{n_d} \left( P_{d,k,j}^{\theta T} + (n_d-1) \theta T P_{d,k,j}^{\frac{n_d-1}{n_d} \theta T} \right).$$

To achieve a reliable transmit power and to avoid the interruptions caused by energy shortage at a D2D-Tx, we assume that the energy consumed for information transmission of a D2D-Tx should not exceed the harvested energy [35].

### E. D2D Information Transmission

In D2D information transmission, the signal-to-interference-plus-noise (SINR) ratio at the $i^{th}$ D2D UE $u_{i,k,j}^d$ connecting with $d_{k,j}$ is given by

$$SINR(u_{i,k,j}^d) = \frac{P_{C_i}^d h_{d_{k,j}}^d u_{i,k,j}^d H_\alpha \| d_{k,j} - u_{i,k,j}^d \|^{-\alpha}}{I_{C_i}^d + I_{D_i}^d + \sigma^2},$$

where $P_{C_i}^d h_{d_{k,j}}^d u_{i,k,j}^d$ denotes the interference power from D2D communications, $I_{D_i}^d$ denotes the interference power from cellular transmissions, and $\sigma^2$ is the channel power gain.

A D2D outage occurs when the data rate of a D2D link with a distance $R_D$ falls below the D2D data rate requirement $R_d$ during a communication time slot $T$. The outage probability of D2D communications is given by

$$P_d^\text{out} = 1 - \mathbb{P} \left\{ \frac{1}{T} \mathbb{E} (\lambda_B) T B \left( 1 + \frac{1}{T} \mathbb{E} (\lambda_B) T B \right)^{-1} \right\}.$$

The D2D outage probability needs to be kept below a certain threshold $\varepsilon$ (i.e., $P_d^\text{out} \leq \varepsilon$).

### III. System-level Performance Evaluation

In this section, to evaluate the WPT efficiency and the performance of subsequent D2D information transmission, we characterize the average transmit power of a typical D2D-Tx and the BS expected minimum transmit power based on a system-level analysis.

#### A. Average Transmit Power Of D2D-Tx

**Proposition 1.** In the WPDO network, given the BS density $\lambda_B$, the BS antenna array size $N_t$, the transmit power $P_m$ in
where

\[ \frac{(\pi \lambda_B)^{\frac{\theta}{2}}}{e^{\pi \lambda_B \psi}} + \left( \frac{\pi \lambda_B \psi^{\frac{\theta}{2}}}{\psi - 2} \right) \frac{1}{1 - N_t} \left( 1 - e^{-\pi \lambda_B \psi^{\frac{\theta}{2}}} \right) \]

and

\[ \Delta = \theta \Gamma \left( \frac{2 - \theta}{2}, \pi \lambda_B \psi^{\frac{\theta}{2}} \right) + 2 \Gamma \left( \frac{4 - \theta}{2} \right) \]

Lemma 1. The average transmit power of a typical BS in the WPDO network, given the BS density \( \lambda_B \), the BS antenna array size \( N_b \), and the content popularity \( \pi \), is given by [16]

\[ \lambda_U = e^{-\pi \lambda_B \psi^{\frac{\theta}{2}}}. \]

Proof: Please refer to Appendix A in [16].

Proposition 2. In the WPDO network, given the BS density \( \lambda_B \), the BS antenna array size \( N_b \), and the cellular UE data rate requirement \( R_c \), the expected minimum transmit power of BSs is given by

\[ \exp \left\{ \frac{1}{N_t} \left[ \frac{H_{D_1}}{\pi \lambda_B^{\frac{\theta}{2}}} + \frac{2 P_m H_{\alpha}}{\pi \lambda_B^{\frac{\theta}{2}}} \right] \right\} \]

where \( \lambda_U \) is given in Lemma 1.

Proof: Please refer to Appendix B in [16].

Proposition 3. In the WPDO network, given the BS density \( \lambda_B \), the BS antenna array size \( N_b \), and the cellular UE data rate requirement \( R_c \), the expected minimum transmit power of BSs is given by

\[ \exp \left[ \frac{1}{N_t} \left( \frac{H_{D_1}}{\pi \lambda_B^{\frac{\theta}{2}}} + \frac{2 P_m H_{\alpha}}{\pi \lambda_B^{\frac{\theta}{2}}} \right) \right] \left\{ \frac{\pi \lambda_B \psi^{\frac{\theta}{2}}}{\psi - 2} \frac{1}{\lambda_B} \right\} \]

where \( \lambda_U \) is given in Lemma 1.

Proof: Based on (15) and (17), we have

\[ \exp \left[ \frac{1}{N_t} \left( \frac{H_{D_1}}{\pi \lambda_B^{\frac{\theta}{2}}} + \frac{2 P_m H_{\alpha}}{\pi \lambda_B^{\frac{\theta}{2}}} \right) \right] \left\{ \frac{\pi \lambda_B \psi^{\frac{\theta}{2}}}{\psi - 2} \frac{1}{\lambda_B} \right\} \]

where \( \lambda_U \) is given in Lemma 1.

Proof: Please refer to Appendix B for the proof.

Recall that a cellular UE can be offloaded to D2D link if the following two conditions are both satisfied. First, the cellular UE locates within the offloading region of a D2D-Tx. Second, the UE’s requested contents are available at that D2D-Tx. Accordingly, we give the following Lemma to calculate the density of residual cellular UEs that are unable to be offloaded to D2D links in the WPDO network.

Lemma 1. In the WPDO network, given the BS density \( \lambda_B \), the offloading radius \( R_D \) of D2D-Txs, and the content popularity \( \pi \), the density of residual cellular UEs \( \lambda_U \) is given by [16]

\[ \lambda_U = e^{-\pi \lambda_B \psi^{\frac{\theta}{2}}}. \]

Proof: Please refer to Appendix B in [16].

IV. OUTAGE PROBABILITY

In this section, we derive the outage probability of information transmission for D2D and cellular UEs in the WPDO network, which will be used for the evaluation of network energy efficiency in Section V.
A. Outage Probability Of A D2D Link

Following (12), the outage probability of a D2D link can be rewritten as

$$ P_{d}^{\text{out}} = 1 - P \left\{ \text{SINR}(w_{i,k,j}) \geq \gamma_{th}(\theta) | R_{D} \right\} \quad (21) $$

where the SINR threshold $\gamma_{th}(\theta)$ is given by

$$ \gamma_{th}(\theta) = 2 \pi \theta n_{m} - 1. \quad (22) $$

In the following Proposition, we obtain the outage probability $P_{d}^{\text{out}}$ of a typical D2D UE.

**Proposition 4.** In the WPDO network, given the time allocation factor $\theta$ and the D2D UE data rate requirement $R_{d}$, the outage probability of a typical D2D UE is given by

$$ P_{d}^{\text{out}} = 1 - \exp \left\{ - \frac{2 \lambda \pi^{2}}{\alpha \sin \left( \frac{2 \pi}{\alpha} \right)} \sigma_{\text{out}}^{\psi}(\theta, R_{d}) - \frac{\gamma_{th}(\theta)}{\alpha \sin \left( \frac{2 \pi}{\alpha} \right)} \right\} \quad (23) $$

and $\psi(\theta, R_{d}) = \frac{\gamma_{th}(\theta)}{\alpha \sin \left( \frac{2 \pi}{\alpha} \right)}$ where $\theta_{th} = 2 \pi \theta n_{m} - 1$.

**Proof:** Please refer to Appendix C for the proof. ■

In (23), we can observe that $P_{d}^{\text{out}}$ approaches to 1 when $\theta \to 0$ or $\theta \to 1$, which is in line with the intuition. More specifically, $\theta \to 0$ indicates that there is no time allocated for WPT, and thus $P_{d} \approx 0$ and $P_{d}^{\text{out}} \to 1$. Besides, $\theta \to 1$ indicates that no time is allocated for information transmission, which pushes the SINR threshold $\gamma_{th}(\theta)$ to infinity, and thus $\psi(\theta, R_{d}) \to \infty$ and $P_{d}^{\text{out}} \to 1$.

B. Outage Probability Of A Cellular Downlink

We define the outage probability of a cellular UE as follows

$$ P_{c}^{\text{out}} = P \left\{ P_{j}^{B} > P_{m} \right\} \quad (24) $$

which is the probability that the BS’s expected maximum transmit power exceeds the maximum allowable transmit power $P_{m}$ for guaranteeing the cellular UE’s required data rate $R_{c}$.

The outage probability of a typical cellular UE in the WPDO network is presented in the following proposition.

**Proposition 5.** In the WPDO network, given the time allocation factor $\theta$, the cellular UE data rate requirement $R_{c}$, and the D2D offloading radius $R_{D}$, the outage probability of a typical cellular UE is given by

$$ P_{c}^{\text{out}} = \frac{\alpha \lambda_{B} \Gamma \left( \frac{N_{t}}{2} \right)}{\Gamma \left( N_{t} \right) \exp \left( \frac{\lambda_{B}}{\lambda_{m}} \right)} \sum_{n_{1}=1}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \sin \left( t \sin \left( \frac{2 \pi}{\alpha} \right) \right) y^{\frac{N_{t}}{2} - 1} e^{-\frac{\lambda_{B} y}{\lambda_{m}} - 1} \pi \lambda_{B} \xi \left( -\left( \frac{2}{\alpha} \right)^{2} T(n, y)^{2} + 1 \right) e^{-\pi \lambda_{B} y^{2}} dT dy $$

$$ + \frac{2 \lambda_{D} \xi}{\alpha \sin \left( \frac{2 \pi}{\alpha} \right)} \pi (n, T(n)) T(n) = 2 \frac{n_{m}}{\pi \theta n_{m} - 1} \quad (25) $$

where $\xi = \frac{2 \lambda_{D} \xi}{\alpha \sin \left( \frac{2 \pi}{\alpha} \right)}$ and $\lambda_{B}$ is given in (16).

**Proof:** Please refer to Appendix D for the proof. ■

The result in (25) involves two integrations and a summation of series which can be obtained by numerical calculations. We can observe from the expression in (25) that the outage probability of a typical cellular UE decreases with the increasing number of antennas used at each BS.

V. NETWORK ENERGY EFFICIENCY

The WPDO network energy efficiency ($\eta_{EE}$) is defined as the ratio of area spectral efficiency (ASE) to area power consumption (APC) as follows

$$ \eta_{EE} = \frac{\text{ASE}}{\text{APC}} \quad (26) $$

where the ASE refers to the sum rate of both cellular and D2D UEs per unit area per Hz, while the APC is the total power consumption per unit area.

More specifically, we have

$$ \text{ASE} = \lambda_{B} \left( 1 - P_{d}^{\text{out}} \right) R_{c} + \left( \lambda_{U} - \lambda_{U}^{\text{th}} \right) \left( 1 - P_{d}^{\text{out}} \right) R_{D} $$

$$ \text{APC} = (1 - \theta) \lambda_{B} \text{E} \left\{ P_{j}^{B} \right\} + \theta \lambda_{B} P_{m} $$

where the APC includes the BS power consumptions for downlink information transmission and for WPT towards D2D-Txs, and $\text{E} \left\{ P_{j}^{B} \right\}$ is given in (18).

Note that the time allocation factor $\theta$ should be carefully selected for achieving a high WPDO network energy efficiency. In the following, based on our analytical results from previous sections, we propose an algorithm to maximize the WPDO network energy efficiency while guaranteeing that the D2D outage probability is below a certain value $\varepsilon$, i.e., the maximum acceptable outage probability. Based on (23), letting $P_{d}^{\text{out}} = \varepsilon$ (as shown in (29) at the bottom of next page) and solving it for $R_{D}$ by numerical methods, we can obtain the D2D-Tx’s offloading radius $R_{D}$ for a given $\theta$. Based on $R_{D}$, (13), (16), (18), (23) and (25-28), we can obtain the maximum WPDO network energy efficiency $\eta_{EE}^{\text{opt}}$ and can acquire the near-optimal time allocation factor $\theta^{*}$ by an exhaustive search. In Algorithm 1, we summarize the main steps of obtaining $\theta^{*}$ and $\eta_{EE}^{\text{opt}}$, where the searching space is $(0, 1)$ and the searching step size is $\pi$.

Please note that in this paper, we focus on the modeling and analysis of an energy efficient WPDO network from a system-level perspective. The optimization algorithm design for obtaining the optimal value of $\theta$ is beyond the scope of this paper.

Next, we present the closed-form expression of the D2D-Tx’s offloading radius $R_{D}$ for two special cases and provide insights into the design of energy-efficient WPDO networks.
Special case 1: For the path loss exponent $\alpha = 4$, $R_D$ in (29) becomes
\[
R_D = \left\lceil \frac{\sqrt{\frac{\Lambda^2\pi^2}{4} - 4\sigma^2 \ln(1 - \varepsilon) - \frac{\Lambda^2}{2}}}{2\sigma^2 \left\langle \frac{\gamma_{\text{th}}(\theta)}{P_d H_\alpha} \right\rangle^\frac{1}{2}} \right\rceil \tag{30}
\]
where $\Lambda$ and $\gamma_{\text{th}}(\theta)$ are given in (23).

Special case 2: For an interference limited network, i.e., $\sigma^2 = 0$, $R_D$ in (29) reduces to
\[
R_D = \frac{\sqrt{-\ln(1 - \varepsilon)}}{\pi \alpha \sin\left(\frac{2\pi}{\alpha}\right) \left\langle \frac{\gamma_{\text{th}}(\theta)}{P_d H_\alpha} \right\rangle^\frac{1}{2}} \tag{31}
\]
where $\Lambda$ and $\gamma_{\text{th}}(\theta)$ are given in (23).

Remark: In (30) and (31), since $-\ln(1 - \varepsilon)$ is a monotonically increasing function of $\varepsilon$ ($0 < \varepsilon < 1$), the D2D-Tx offloading radius increases with the value of the maximum acceptable outage probability $\varepsilon$. In addition, we observe that $R_D \approx 0$ when $\theta \to 0$ or $\theta \to 1$, since $R_D \approx 0$ when $\theta \to 0$, and $\gamma_{\text{th}}(\theta) \to \infty$ when $\theta \to 1$. This indicates that there exists a $\theta$ which can maximize the D2D-Tx offloading radius.

VI. SIMULATION AND NUMERICAL RESULTS

In this section, we provide simulation results to evaluate the energy efficiency of the WPDO network. The network operates at $B = 10$MHz, $\lambda_B = 1 \times 10^{-5}$BSs/m$^2$, $\lambda_D = 1.5 \times 10^{-3}$UEs/m$^2$, $\lambda_U = 4 \times 10^{-4}$D2D-Txs/m$^2$, $P_m = 24$dBm, $N_i = 128$, $v_1 = 1$m, $v_2 = 5$m, $\alpha = 3$, $\beta = 2.5$, $\sigma^2 = 1 \times 10^{-11}$W, $\eta = 1$, $\varepsilon = 0.3$, unless otherwise stated.

In Fig.2, the average transmit power at a typical D2D-Tx is shown versus the density of D2D-Txs, where $\theta = 0.5$. We observe that the average transmit power at a typical D2D-Tx reduces with increasing the density of D2D-Txs, since the BS needs to wirelessly power more D2D-Txs in a certain communication time slot $T$ while the total energy is limited. In addition, the average transmit power at a typical D2D-Tx increases with the BS antenna array size $N_i$ as the radiated energy can be concentrated in a narrower beam and directly point the target D2D-Tx for WPT. Besides, the analytical results derived in this paper are validated by the Monte Carlo (MC) simulations.

In Fig.3, the expected minimum transmit power at a typical BS given in (18) is expressed regarding to the cellular UE data rate requirement $R_c$. Note that the BS expected minimum transmit power increases with the density of cellular UEs and the data rate requirement. When the density of cellular UEs increases, the allocated communication resources for each cellular UE will be reduced. Therefore, to guarantee the cellular UE’s required data rate, BS needs to provide more power to compensate for the lessened allocated bandwidth.

In Fig.4, the outage probability of a typical D2D UE given in (23) is presented against the time allocation factor $\theta$. We observe that the outage probability of a typical D2D UE can be minimized by adjusting the parameter $\theta$. This is because of the fact that, for a small value of $\theta$, increasing $\theta$ improves the average transmit power at D2D-Tx and thus enhances the D2D information transmission. However, after the optimal $\theta$ for D2D outage probability, increasing $\theta$ decreases the time allocated for the D2D information transmission, and the

\[
\ln(1 - \varepsilon) + \frac{2\lambda_D\pi^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right) H_\alpha} \left(2\pi^2 \gamma_{\text{th}} - 1\right)^\frac{1}{2} \frac{R_D^2}{P_d H_\alpha} + \frac{2\gamma_{\text{th}}}{P_d H_\alpha} \sigma^2 R_D^\alpha = -\frac{2\lambda_B}{\alpha \sin\left(\frac{2\pi}{\alpha}\right) N_i^\frac{1}{2} \alpha \left(2\pi^2 \gamma_{\text{th}} - 1\right)^\frac{1}{2} \frac{R_D^2}{P_d H_\alpha} \left\{\pi\lambda_D P_d H_\alpha \alpha + \sigma^2 \left(\frac{\gamma_{\text{th}}}{P_d H_\alpha} \right)^\frac{1}{2} \Gamma\left(\frac{\alpha}{2} + 1\right) \frac{P_m H_\alpha}{\alpha - 2}\right\} \tag{29}
\]

**Algorithm 1** Energy efficiency of WPDO network.

**Input:**
- The network parameters, $\lambda_B$, $\lambda_D$, $\lambda_U$, $\alpha$, $\beta$, $B$, $\mathbb{P}_{\text{con}}$, $\eta$, $P_m$, $N_i$, $H_\alpha$, $H_\beta$.
- The required service data rates of cellular UEs and D2D UEs, $R_c$, $R_d$.
- The maximum acceptable outage probability of D2D UE, $\varepsilon$.
- The step length, $\omega$.

**Output:**
- The near-optimal time allocation factor for WPT, $\theta^*$.
- The maximum WPDO network energy efficiency, $\eta_{EE}^\star$.

1: Initializing $\theta$ with $\omega$;
2: Calculating the average transmit power at D2D-Tx $P_d$ based on (13) for a given value of $\theta$;
3: Based on $\theta$ and $P_d$, quantifying $R_D$ according to the equation of (29);
4: Obtaining the WPDO network energy efficiency $\eta_{EE}$ by combining equation (16), (18), (23) and (25-28);
5: for $0 < \theta < 1$ do
6: $\theta_n = \theta_{n-1} + \omega$;
7: $\theta = \theta_n$, repeating steps 2, 3, 4 and obtaining $\eta_{EE}^\prime$;
8: if $\eta_{EE}^\prime > \eta_{EE}$ then
9: Substituting the value of $\eta_{EE}^\prime$ into $\eta_{EE}$;
10: else
11: The near-optimal value of $\theta$ is $\theta_{n-1}$, $\theta^* = \theta_{n-1}$;
12: The maximum WPDO network energy efficiency is $\eta_{EE}^\star = \eta_{EE}$;
13: Break;
14: end if
15: end for
16: return The near-optimal time allocation factor, $\theta^*$;
- The maximum WPDO network energy efficiency, $\eta_{EE}^\star$.
aggregated interference power goes up due to the higher average transmit power at D2D-Txs, which dramatically degrades the communication performance. Simulations are conducted to verify our theoretical results. The minor mismatches are resulted from that, in the theoretical results, the aggregated interference power is calculated in an infinite region, while in the simulations it is evaluated in a finite region, which results in the minor difference of the outage probability. Furthermore, another interesting observation can be found that, when D2D UE data rate requirement $R_d$ gets large, it is desirable to divert larger fraction of time in a communication time slot to the information transmission at D2D-Tx in order to lower the outage probability of D2D UE, while a larger fraction of time needs to be portioned for the WPT when $R_d$ is small.

In Fig.5, we compare the offloading radius of a typical D2D-Tx against the time allocation factor $\theta$, where the maximum acceptable outage probability $\varepsilon$ is 0.3. The theoretical results are obtained according to the equation (29). We see that there exists a maximum offloading radius by selecting an appropriate $\theta$, where the offloaded traffic in the WPDO network is maximized at this point. This is because of the fact that, when
θ is small, increasing θ results in a higher average transmit power at D2D-Tx and thus increases the offloading radius \( R_d \). However, when θ becomes large, the time allocated for D2D information transmission is reduced. Therefore, in this case, the link distance of D2D communications should be shrunk to guarantee the D2D UE data rate requirement \( R_c \). In Fig.5, we also note that the offloading radius \( R_d \) decreases with the data rate requirement \( R_c \), which is in line with the intuition. In addition, the offloading radius \( R_d \) decreases with the density of D2D-Txs, since the average transmit power is reduced at a D2D-Tx in accordance with Fig.2.

In Fig.6, the outage probability of a typical cellular UE given in (25) is shown against the time allocation factor θ. We observe that the outage probability of a typical cellular UE increases with the time allocation factor θ. This is because of the fact that increasing θ increases the aggregated interference power from D2D-Txs and reduces the time of BS information transmission, which degrades the performance of cellular link and thus improves the outage probability of a cellular UE. Further, we also see that the outage probability of a typical cellular UE increases with the cellular UE data rate requirements, and it approaches to 1 when θ becomes large.

Fig.7 depicts the WPDO network energy efficiency \( \eta_{\text{EE}} \) versus θ for different values of content popularity \( P_{\text{con}} \). As can be seen from Fig.7, the maximum WPDO network energy efficiency \( \eta^*_{\text{EE}} \) is obtained by optimizing the parameter θ. Specifically, when θ is small, the offloaded traffic is substantially rare which leads to the low energy efficiency. However, when θ gets large, the total energy consumption rises up due to the increased energy for WPT and the increased transmit power for BS information transmission, which results in a low network energy efficiency. Furthermore, it is interesting to see that the network energy efficiency is improved when the shared contents become more popular. In addition, the optimal time allocation factor θ* increases with the content popularity \( P_{\text{con}} \) of the shared contents. This indicates that the BSs should allocate more time fraction for WPT during a communication time slot to acquire a higher average transmit power at D2D-Txs as well as a larger offloading radius when the shared contents become popular.

In Fig.8, the maximum WPDO network energy efficiency \( \eta^*_{\text{EE}} \) is compared against the BS antenna array size \( N_t \). We observe that \( \eta^*_{\text{EE}} \) increases with the number of elements in the BS antenna array. This suggests that the performance of the WPDO network is greatly improved by using the
where content popularity $P$ performance in terms of the network energy efficiency.

The results in Fig. 8 also illustrate that $\eta_{EE}$ increases with the content popularity $P_{con}$; since more traffic can be offloaded to D2D links, which is a cost-effective way and improves the network capacity. In addition, in Fig. 8, we also compare the proposed scheme, i.e., energy efficient WPDO network, with the case that the time allocation factor is fixed at 0.5. The results demonstrate that our proposed scheme achieves better performance in terms of the network energy efficiency.

**VII. CONCLUSIONS**

In this paper, we have modeled and analyzed wireless-powered D2D-assisted offloading (WPDO) in cellular networks. Considering the interference between cellular downlinks and underlaid D2D links and using stochastic geometry, we have derived the closed-form expressions of the average transmit power at D2D-Tx, the BS expected minimum transmit power, the D2D outage probability, and the cellular downlink outage probability. Based on the above analytical results, we have proposed an algorithm to maximize the WPDO network energy efficiency by optimizing the time allocation factor for WPT. The analytical and simulation results demonstrate that the WPT time allocation factor can be optimized to minimize the D2D outage probability and to maximize the WPDO network energy efficiency as well as the D2D offloading region. In addition, the WPDO network energy efficiency can be dramatically improved by using massive antenna arrays at BSs and by caching highly popular contents at D2D-Txs for content sharing. We have also provided useful insights into the design of an energy efficient WPDO network from a system-level perspective.

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**APPENDIX A**

**PROOF OF PROPOSITION 1**

According to (5), (6), (7) and (8), we have

$$P_d = \frac{1}{1 - \theta} \eta_{EE} \eta \Theta \left( P^{t, \frac{1}{\pi} \theta T} \right) = \eta_\theta \eta \Theta \left( P^{t, \frac{1}{\pi} \theta T} \right) + \left( n_d - 1 \right) \left( P^{t, \frac{1}{\pi} \theta T} \right), \tag{32}$$

where

$$\Theta \left( P^{t, \frac{1}{\pi} \theta T} \right) = \mathbb{E} \left\{ P^{t, \frac{1}{\pi} \theta T} \right\} = \mathbb{E} \left\{ P^{t, \frac{1}{\pi} \theta T} \right\} + \mathbb{E} \left\{ P^{t, \frac{1}{\pi} \theta T} \right\}. \tag{33}$$

Based on (5), the average received power $P^{t, \frac{1}{\pi} \theta T}$ from the nearest BS $b_j$ by pointing beam is given by

$$\mathbb{E} \left\{ P^{t, \frac{1}{\pi} \theta T} \right\} = \mathbb{E} \left\{ P_{m} \left\| h_{b_d k,j} \right\|^{2} H_\beta \left( \max \left\{ \left\| b_d - k,j \right\|, v_1 \right\} \right)^{-\beta} \right\} = P_{m} N_{t} H_\beta \left[ \int_{0}^{v_1} v_1^{-\beta} f_{\left\| b_{j} - d_{k,j} \right\|} (x) dx \right] + \int_{v_1}^{\infty} x^{-\beta} f_{\left\| b_{j} - d_{k,j} \right\|} (x) dx \tag{34}$$

and

$$f_{\left\| b_{j} - d_{k,j} \right\|} (x) = 2\pi \lambda \pi x e^{-\pi \lambda x^2} (x > 0) \tag{35}$$

where $f_{\left\| b_{j} - d_{k,j} \right\|} (x)$ is the probability density function (PDF) of the distance $\left\| b_{j} - d_{k,j} \right\| [39]$, and (a) in (34) is obtained by using $\mathbb{E} \left\{ \left\| h_{b_d k,j} \right\| \right\} = N_{t} [32]$. In step (b), $\Theta \left( \cdot, \cdot \right)$ is the incomplete Gamma function.

In addition, the average received power $P^{t, \frac{1}{\pi} \theta T}$ from other BSs during $\frac{\theta T}{\pi}$ is calculated by, which is the second term of the right hand of the equation (33),

$$\mathbb{E} \left\{ P^{t, \frac{1}{\pi} \theta T} \right\} = \sum_{b_n \in \Phi_b \setminus b_j} \mathbb{E} \left\{ P_{m} \left\| h_{b_a k,j} \right\| \left\| g_{b_a d_{l,n}} \right\|^{2} H_\beta \right\} \cdot \mathbb{E} \left\{ \left( \max \left\{ \left\| b_n - d_{l,n} \right\|, v_1 \right\} \right)^{-\beta} \right\} \tag{36}$$

where (a) follows from $\left\| h_{b_a k,j} \right\| \left\| g_{b_a d_{l,n}} \right\|^{2} \sim \exp \left\{ 1 \right\}$, $\Gamma \left( \cdot, \cdot \right)$ is the standard Gamma function, and more specifically we have

$$\mathbb{P} \left\{ X > v_1 \right\} = \int_{v_1}^{\infty} f_{\left\| b_{j} - d_{k,j} \right\|} (x) dx = e^{-\pi \lambda x^2}, \tag{37}$$
\[ P \{ x < v_1 \} = 1 - P \{ x > v_1 \} \]

where \( f_{\|b_j - d_{k,j}\|}(x) \) is given in (35).

Based on (6), the average received power \( P_{d_{k,j}}^{r,n_{d,j}-1} \) at \( d_{k,j} \) during \( \frac{(n_{d,j}-1)\sigma_T}{n_{d,j}} \) is given by

\[
E \left\{ P_{d_{k,j}}^{r,n_{d,j}-1} \right\} = \sum_{b_n \in \Phi_B, b_n \neq d_{k,j}} E \left\{ \left\| h_{b_{n},d_{k,j}} \right\|^2 \left\| g_{b_{n},d_{l,n}} \right\|^2 \right\}
\]

\[
= \sum_{b_n \in \Phi_B} E \left\{ \left( \max \{ \left\| b_n - d_{l,n} \right\|, v_1 \} \right)^{-\beta} \right\}
\]

\[
= P_m H_{\beta} E_{\Psi} \left\{ \left( \max \{ \left\| b_n - d_{l,n} \right\|, v_1 \} \right)^{-\beta} \right\}
\]

\[
= P_m H_{\beta} \pi \lambda_B \int_0^{\infty} \left( \max \{ r, v_1 \} \right)^{-\beta} dr
\]

\[
= P_m H_{\beta} \pi \lambda_B v_1^{-2\beta} - \frac{1}{2} v_1^{2-\beta} + P_m H_{\beta} 2 \pi \lambda_B v_1^{2-\beta} - \frac{2}{2} v_1^{2-\beta}
\]

\[
= P_m H_{\beta} \pi \lambda_B v_1^{2-\beta} \left( 1 + \frac{2}{\beta - 2} \right).
\]

Combining (33), (34), (36) and (39) into (32) and with some mathematical manipulation, we have the desired result in (13).

**APPENDIX B**

**PROOF OF PROPOSITION 2**

Given a typical cellular UE \( u_{i,j} \) which requests data rate \( R_c \) during the communication time slot \( T \), the expected minimum transmit power at its serving BS \( b_j \) during the sub-slot (1-\( \theta \)) is given by a transformation of (1) as follows

\[
E_I \left\{ P_{i,j}^{B} \mid N_j^c, \| b_j - u_{i,j} \| \right\}
= \frac{2 \pi \lambda_B N_j^c \beta}{\| b_j - u_{i,j} \|} \frac{1}{N_t} E \left\{ I_{u_{i,j}}^I + I_{u_{i,j}}^D + \sigma^2 \right\}
\]

where \( E_I [x] \) denotes taking expectation of variable \( x \) on the interference power \( I \), and we have utilized \( E \left\{ \left\| h_{b_{u_i},u_{i,j}} \right\|^2 \right\} = N_t \), which characterizes the average performance in channel.

We consider the worst-case scenario, where the interfering BSs transmit at the maximum allowable transmit power, and thus we have

\[
E \left\{ I_{u_{i,j}}^I \right\} = E_{h_{u_i},u_{i,j}} \left\{ \| b_{u_i} - u_{i,j} \| \right\}
\]

\[
\leq \sum_{b_n \in \Phi_B \setminus b_j} \left\| h_{b_{u_i},u_{i,j}} \right\|^2 \left\| g_{b_{u_i},u_{i,j}} \right\|^2 \left\| b_{u_i} - u_{i,j} \right\|^{\alpha_0} \left\| b_{u_i} - u_{i,j} \right\|^{\alpha_0}
\]

\[
= \frac{P_m H_{\alpha}}{2 \pi \lambda_B} \int_{y_{i,j}}^{\infty} x^{1-\alpha} dx
\]

\[
= \frac{2 \pi \lambda_B P_m H_{\alpha}}{(\alpha - 2)} \left( y_{i,j}^{\alpha - 2} \right)
\]

where (a) is obtained by using Campbell’s Theorem, and we have utilized \( E \left\{ h_{b_{u_i},u_{i,j}} \right\} = 1 \).

The distance between \( u_{i,j} \) and its associated BS. In addition, we have

\[
E \left\{ I_{u_{i,j}}^D \right\}
= \frac{\pi \lambda_D}{2 \pi \lambda_B} \int_{y_{i,j}}^{\infty} x^{1-\alpha} dx
\]

\[
= \frac{2 \pi \lambda_D P_m H_{\alpha}}{(\alpha - 2)} \left( y_{i,j}^{\alpha - 2} \right)
\]

where (a) follows from \( h_{d_{k,j}} \sim \exp(1) \).

Combining (41) and (42) into (40), we obtain the upper bound expression of \( E \left\{ P_{i,j}^{B} \mid N_j^c, y_{i,j}^c \right\} \) as follows

\[
E \left\{ P_{i,j}^{B} \mid N_j^c, y_{i,j}^c \right\}
\leq \frac{2 \pi \lambda_B R_{c} N_j^c}{(\alpha - 2) N_t} \left[ \frac{2 \pi \lambda_B P_m H_{\alpha}}{(\alpha - 2)} + \frac{\pi \lambda_D P_m H_{\alpha}}{(\alpha - 2) \sigma^2} \right]
\]

\[
\int_{y_{i,j}}^{\infty} y^{\alpha - 1} e^{-\pi \lambda_B y^2} dy + 2 \pi \lambda_B P_m H_{\alpha} \int_{y_{i,j}}^{\infty} y^{\alpha - 1} e^{-\pi \lambda_B y^2} dy
\]

where (a) is obtained by using Campbell’s Theorem, and we have utilized \( E \left\{ h_{b_{u_i},u_{i,j}} \right\} = 1 \).

The distance between \( u_{i,j} \) and its associated BS. In addition, we have

\[
E \left\{ I_{u_{i,j}}^D \right\}
= \frac{\pi \lambda_D}{2 \pi \lambda_B} \int_{y_{i,j}}^{\infty} x^{1-\alpha} dx
\]

\[
= \frac{2 \pi \lambda_D P_m H_{\alpha}}{(\alpha - 2)} \left( y_{i,j}^{\alpha - 2} \right)
\]

where (a) follows from \( h_{d_{k,j}} \sim \exp(1) \).

Combining (41) and (42) into (40), we obtain the upper bound expression of \( E \left\{ P_{i,j}^{B} \mid N_j^c, y_{i,j}^c \right\} \) as follows

\[
E \left\{ P_{i,j}^{B} \mid N_j^c, y_{i,j}^c \right\}
\leq \frac{2 \pi \lambda_B R_{c} N_j^c}{(\alpha - 2) N_t} \left[ \frac{2 \pi \lambda_B P_m H_{\alpha}}{(\alpha - 2)} + \frac{\pi \lambda_D P_m H_{\alpha}}{(\alpha - 2) \sigma^2} \right]
\]

\[
\int_{y_{i,j}}^{\infty} y^{\alpha - 1} e^{-\pi \lambda_B y^2} dy + 2 \pi \lambda_B P_m H_{\alpha} \int_{y_{i,j}}^{\infty} y^{\alpha - 1} e^{-\pi \lambda_B y^2} dy
\]

By calculating (44), we have the result in (15), which completes the proof.

**APPENDIX C**

**PROOF OF PROPOSITION 4**

According to (21) and (9), we have

\[
E \left\{ P_{d,q}^{out} \mid R_{D}, I_{u_{i,k,j}}^C, I_{u_{i,k,j}}^D \right\}
= 1 - \Pr \left\{ \frac{\pi \lambda_D P_{d_{k,j}} H_{\alpha} R_{D}^{-\alpha}}{I_{u_{i,k,j}}^C + I_{u_{i,k,j}}^D + \sigma^2} \geq \psi(\theta, \theta) \right\}
\]

\[
= 1 - \Pr \left\{ \frac{h_{d_{k,j},u_{i,k,j}} \psi(\theta, \theta)}{\psi(\theta, \theta)} \right\}
\]

\[
= 1 - e^{-\psi(\theta, \theta) \frac{\pi \lambda_D P_{d_{k,j}} H_{\alpha} R_{D}^{-\alpha}}{\psi(\theta, \theta)}}
\]

\[
= 1 - e^{-\psi(\theta, \theta) \frac{\pi \lambda_D P_{d_{k,j}} H_{\alpha} R_{D}^{-\alpha}}{\psi(\theta, \theta)}}
\]

(45)
where $\psi(\theta, R_d) = \frac{\gamma_n}{P_d H_\alpha R_d - \alpha}$ is given in (23), (a) follows from $h_{d_{i,k,j}} u_{i,k,j}^d \sim \exp(1)$, $L_{I_C}^{\{u_{i,k,j}^d\}}$ and $L_{I_D}^{\{u_{i,k,j}^d\}}$ denote the Laplace transform of $I_{u_{i,k,j}^d}^C$ and $I_{u_{i,k,j}^d}^D$, respectively.

In addition, we have

$$
L_{I_C}^{\{u_{i,k,j}^d\}} \{s\} = \mathbb{E} \left\{ \exp \left( -s \sum_{b_n \in \Phi_B} P_{i,n} \right) \cdot h_{b_n u_{i,k,j}^d} \left\| \frac{E}{H_\alpha \| b_n - u_{i,k,j}^d \|^{-\alpha}} \right\|^2 \right\}
$$

$$
= \mathbb{E}_{\Phi_B} \left\{ \prod_{b_n \in \Phi_B} \left( 1 + s P_{i,n} H_\alpha \| b_n - u_{i,k,j}^d \|^{-\alpha} \right) \right\}
$$

$$
\approx \exp \left\{ -2\pi \lambda_B \int_0^\infty \left( 1 - \frac{1}{1 + s \mathbb{E} \{ P_{i,j}^B \} H_\alpha x^{-\alpha}} \right) x dx \right\}
$$

$$
= \exp \left\{ -2\pi \lambda_B \left( \mathbb{E} \{ P_{i,j}^B \} \right) \frac{\pi^2}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \right\}
$$

(46)

where in (a) we have utilized $\mathbb{E} \{ P_{i,j}^B \}$ given in (18) to denote the interfering power from cellular BSs.

Furthermore, $L_{I_D}^{\{u_{i,k,j}^d\}} \{s\}$ is given by

$$
L_{I_D}^{\{u_{i,k,j}^d\}} \{s\} = \exp \left\{ -s \sum_{d_n \in \Phi_D} P_{d,n} H_{\alpha} \| d_n - u_{i,k,j}^d \|^\alpha \right\}
$$

$$
= \exp \left\{ -2\pi \lambda_D \int_0^\infty \left( 1 - \frac{1}{1 + s P_{d,n} H_\alpha x^{-\alpha}} \right) x dx \right\}
$$

$$
= \exp \left\{ -2\pi \lambda_D \left( P_{d,n} \right) \frac{\pi^2}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \frac{1}{s} \right\}
$$

(47)

where $\Phi_D^{i\{u_{i,k,j}^d\}}$ is the set of D2D-Txs that are located in $B \left( o, l_{i,k,j}^d \right)$ which is a circular region centered at the origin with radius $l_{i,k,j}^d$, and $l_{i,k,j}^d$ is the distance between the D2D UE $u_{i,k,j}^d$ and its connecting D2D-Tx $d_{i,k,j}$ in $j^{th}$ cell. Besides, $\Phi_D^{i\{u_{i,k,j}^d\}}$ is the set of D2D-Txs that are located outside the region of $B \left( o, l_{i,k,j}^d \right)$. In (47), we approximate that the set of interfering D2D-Txs follows a PPP distribution with density $\lambda_D$ and then utilize the probability generating function to calculate the Laplace transform of $I_{u_{i,k,j}^d}^D$.

Combining (46) and (47) into (45), we have

$$
P_{out} = 1 - \exp \left\{ -2 \left[ \lambda_B \left( \mathbb{E} \{ P_{i,j}^B \} \right) \frac{\pi^2}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \right] \frac{1}{P_d H_\alpha R_D - \alpha} \right\}
$$

$$
\cdot \left( \frac{\gamma_n}{P_d H_\alpha R_D - \alpha} \right)^{\frac{\pi}{\alpha}} - \gamma_n \left( \frac{\pi}{\alpha} \right) \frac{2\pi^2}{\alpha \sin \left( \frac{\pi}{\alpha} \right)}
$$

(48)

where $P_d$ and $\mathbb{E} \{ P_{i,j}^B \}$ are given in (13) and (18), respectively, which completes the proof.

APPENDIX D

PROOF OF PROPOSITION 5

According to (24), the outage probability of a typical cellular UE conditioned on the link distance $y_{c_{i,j}}$, channel power gain $\| h_{b_j u_{i,j}^c} \|^2$ and the number of cellular UEs $N_{c_i}^c$ in its cell is obtained as follows

$$
\mathbb{E} \left\{ P_{out_{c_{i,j}}} \left( \| h_{b_j u_{i,j}^c} \|^2 , N_{c_i}^c \right) \right\}
$$

$$
= 1 - \mathbb{P} \left\{ P_{i,j}^B \left( R_c \right) \leq P_m \| h_{b_j u_{i,j}^c} \|^2 \right\}
$$

$$
= 1 - \mathbb{P} \left\{ \frac{I_{agg_{u_{i,j}^c}}}{\| h_{b_j u_{i,j}^c} \|^2} \leq \frac{P_m}{\left( \frac{\nu_i N_{c_i}^c}{2 \left( \frac{\pi^2}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \right) - 1 \right)} \left( y_{c_{i,j}} \right)^{-\alpha} \right\}
$$

(49)

where $F_{I_{agg_{u_{i,j}^c}}}$ is the Cumulative Distribution Function (CDF) of $I_{agg_{u_{i,j}^c}}$ and $I_{agg_{u_{i,j}^c}} = I_{agg_{u_{i,j}^d}} + I_{agg_{u_{i,j}^d}}$ denotes the aggregated interference power from cellular and D2D links at $u_{i,j}^c$. In addition, we have

$$
L_{I_{agg_{u_{i,j}^c}}} \{s\} = \mathbb{E} \left\{ \exp \left\{ -s I_{agg_{u_{i,j}^c}} \right\} \right\} = \int_0^\infty e^{-st} f_{I_{agg_{u_{i,j}^c}}} (t) dt,
$$

and

$$
f_{I_{agg_{u_{i,j}^c}}} (t) = \mathcal{L}^{-1} \left\{ \mathcal{L}_{I_{agg_{u_{i,j}^c}}} \{s\} \right\}
$$

(51)

where $f_{I_{agg_{u_{i,j}^c}}} (t)$ denotes the PDF of $I_{agg_{u_{i,j}^c}}$ and $\mathcal{L}^{-1} \{ \cdot \}$ represents the inverse Laplace transform.

Based on the properties of Laplace transform, $L_{I_{agg_{u_{i,j}^c}}} \{s\}$ can be expressed as

$$
L_{I_{agg_{u_{i,j}^c}}} \{s\} = L_{I_{agg_{u_{i,j}^d}}} \{s\} L_{I_{agg_{u_{i,j}^d}}} \{s\}
$$

(52)

where $L_{I_{agg_{u_{i,j}^d}}} \{s\}$ denotes the Laplace transform of the aggregated interference power from cellular BSs, while $L_{I_{agg_{u_{i,j}^d}}} \{s\}$
is the Laplace transform of the aggregated interference power from D2D-Txs. More specifically, we can obtain

\[
\mathcal{L}_{I_{\text{agg}}^{D}}\{s\} = \mathbb{E}\left\{ \exp\left(-s \sum_{b_n \in \Phi_B \setminus b_j} H_n F_B^{D} \right) \cdot \left| \frac{h_{b_n u_{ij}}}{\|G_{b_n u_{ij}}\|^2} \right|^{2} \left| b_n - u_{ij}^{e} \right|^{-\alpha}\right\}
\]

\[
\approx \exp\left\{ -\pi \lambda_B \zeta \{ s, y_{ij}^{e}, P_m \} \right\}
\]

where \( \zeta \{ s, y_{ij}^{e}, P_m \} = \frac{2sP_m(y_{ij}^{e})^{2-\alpha}}{\alpha - 2} \cdot \frac{\alpha}{\alpha - 2} \cdot \frac{2 \pi}{\alpha} \cdot \frac{sP_m}{\left( y_{ij}^{e} \right)^{\alpha}} \)

(54)

\[
\mathcal{L}_{I_{\text{agg}}^{D}}\{s\} = \mathbb{E}\left\{ \exp\left(-s \sum_{d_n \in \Phi_D} \overline{P_d} h_{d_n u_{ij}} H_n \right) \right\}
\]

\[
= \exp\left\{ -\frac{2 \pi \lambda D (\overline{P_d})^{2} \pi^{2}}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \cdot \frac{s^{2}}{2} \right\}
\]

where \( \overline{P_d} \) is given in (13).

Therefore, the Laplace transform of \( I_{agg}^{D} \) is given by

\[
\mathcal{L}_{I_{\text{agg}}^{D}}\{s\} = \mathbb{E}\left\{ \exp\left(-s \sum_{d_n \in \Phi_D} \overline{P_d} h_{d_n u_{ij}} H_n \right) \right\}
\]

\[
= \exp\left\{ -\frac{2 \pi \lambda D (\overline{P_d})^{2} \pi^{2}}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \cdot \frac{s^{2}}{2} \right\}
\]

(55)

The Laplace transform of \( I_{agg}^{D} \) is obtained as follows

\[
\mathcal{L}_{I_{\text{agg}}^{D}}\{s\} = \mathbb{E}\left\{ -\left[ \pi \lambda_B \zeta \{ s, y_{ij}^{e}, P_m \} + \xi s^{2} \right] \right\}
\]

(56)

where \( \xi = \frac{2 \pi \lambda D (\overline{P_d})^{2} \pi^{2}}{\alpha \sin \left( \frac{\pi}{\alpha} \right)} \) being same with the expression in (25).

According to (51), we obtain the CDF of \( I_{agg}^{D} \) as

\[
F_{I_{\text{agg}}^{D}}(x) = \mathbb{P}\{I_{agg}^{D} \leq x\}
\]

\[
= \int_{0}^{x} \frac{1}{2 \pi t} \int_{-\infty}^{+\infty} e^{-\frac{t}{2}} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds dt
\]

\[
= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

\[
= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

\[
= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

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= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

\[
= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

\[
= \frac{1}{2 \pi s} \int_{-\infty}^{+\infty} s^{2} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

(57)

Considering equation (57) has a branch point at the origin, we use the Bromwich inversion method with a specified contour to calculate the integral [40] as follows

\[
F_{I_{\text{agg}}^{D}}(x) = \lim_{R \to \infty} \int_{-\infty}^{+\infty} e^{-\frac{t}{2}} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds
\]

\[
= \frac{1}{2 \pi i} \left\{ \int_{-\infty}^{+\infty} e^{-\frac{t}{2}} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds \right\}
\]

\[
= \frac{1}{2 \pi i} \left\{ \int_{-\infty}^{+\infty} e^{-\frac{t}{2}} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds \right\}
\]

\[
= \frac{1}{2 \pi i} \left\{ \int_{-\infty}^{+\infty} e^{-\frac{t}{2}} \mathcal{L}_{I_{\text{agg}}^{D}}\{s\} ds \right\}
\]

(58)

where in the last step we have utilized \( t = \xi u^{2} \).

Now we are in the position of computing the outage probability of a typical cellular UE as follows

\[
E\left\{ P_{out}^{D} \right\}
\]

\[
= \int_{0}^{\infty} \int_{0}^{\infty} \mathbb{E}\left\{ P_{out}^{D} \right\} \cdot \|h_{b_{ij}} u_{ij}^{e} \|^2 \cdot \mathbb{E}\left\{ P_{out}^{D} \right\} \cdot \|h_{b_{ij}} u_{ij}^{e} \|^2
\]

(59)

where \( f_{\|h_{b_{ij}} u_{ij}^{e} \|^2} \) is the PDF of \( \|h_{b_{ij}} u_{ij}^{e} \|^2 \) which follows from the Gamma distribution as

\[
f_{\|h_{b_{ij}} u_{ij}^{e} \|^2} = \frac{1}{\Gamma(N_{t})} N_{t}^{N_{t}-1} e^{-h}.
\]

(60)
Finally, we have the desired result by calculating the following summation of series

$$P_{out} = \sum_{n=1}^{\infty} \mathbb{E} \left\{ P_{out} \mid N_j \right\} g_N(n)$$

(61)

which completes the proof.

REFERENCES


