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Multi-commodity flow and station logistics resolution for train unit scheduling

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1 Introduction

In UK and many other countries, self-propelled train vehicle units with a fixed number of carriages, e.g. 2-car and 4-car units, are commonly used. This is in contrast to locomotive pulled formations with a variable number of carriages. Scheduling a wide-area rail network with route and time-of-the-day dependent seat demands, optimizing the coupling and decoupling of train units is often a key feature. Train unit movements are quite restrictive because the tracks they run on are shared. Therefore rather than to create a new empty running journey to redistribute a unit to elsewhere, the unit may be scheduled to be attached to some trains already in the timetable. Also, no additional drivers would be needed. However, a side consequence is that some train trips will be overprovided with seats by such train units in transit to serve the real high demand trips. Since seat demand data is often not easy to determine, inference from schedules in the past is heavily relied upon and overprovision in the current round of scheduling therefore may have long lasting effects on future schedules.

To achieve an optimized flow of train unit resources over the rail network during a working day is complex and difficult. The problem is made more complex in ensuring all the train movements are conflict free at individual train stations. Typical station layouts include tracks that are blind-ended or through running. Some platforms may be short limiting how many train units are allowed to be coupled. To achieve an operable blockage-free schedule may involve reassigning some linkages, re-ordering how multiple units are coupled, shunting units between platforms and sidings, etc. The detailed logistics at each train station obviously could have some rippling effects across all other stations in the network.

A multi-stage approach is proposed for the train unit scheduling problem. In the first stage, a multi-commodity flow problem is solved temporarily regarding each station as a single point with no infrastructure details and without fixing how multi-units are ordered when they are coupled to serve a train trip. The second stage resolves potential station conflicts in the first stage solutions. The resolution process is performed on each individual train station. The results of the second stage would include some alternative resolution plans so that the third
stage process can finalize compatible station plans across the whole network. It is anticipated that because of the real world nature of the scheduling problem, a fourth stage would be needed allowing the human planners to assess and fine tune the schedules interactively.

This paper presents an overall framework and the on-going research on its component stages. The research has been carried out in collaboration with some UK train operating companies and tested on ScotRail, Transpennine Express, Great Western Railway and some past rail franchise bid datasets. Research progress status and relevant results will be presented at the conference.

2 Multi-commodity flow

Given a (tentatively) fixed timetable of train trips for one working day, a directed acyclic graph (DAG) is constructed where the train trips are represented by nodes. A source node and a sink node represent the beginning and end of the working day. Arcs represent potential linkages between a pair of nodes. Different train unit types are multi-commodities. The problem is to select paths from source to sink in the graph such that all the train trips are covered meeting the seat demands. Where the paths overlap, the corresponding train units are coupled subject to compatibility constraints and some coupling bounds.

The multi-commodity flow model is formulated as an integer linear program (ILP) [1]. Apart from minimizing the number of train units used, other quality measures such as total mileage are also incorporated in the objective function. The ILP is computationally very challenging to solve. Therefore, a specialized solver has been derived based on branch-and-price [2]. The main features of the solver include local convex hull techniques [3] for more efficient computation regarding constraints on seat demands and unit coupling type compatibility and bounds. Some specialized branching techniques have also been derived.

In practice, train operators often cannot specify seat demands precisely for each train trip. Deviations from the norm may be caused by many different circumstances and factors. Passenger count surveys are only snapshots that may not always yield accurate inferences. Seat capacities provided in historic schedules may also be unreliable because the scheduling process might have deviated from the seat demands originally specified. Hence, bi-level seat demands are accommodated [4]. For each train trip, the lower minimum seat demand is a hard constraint. A higher seat demand may also be specified such that they would be satisfied as much as possible without using additional train units.

The above solver has demonstrated the ability to solve small to medium sized real-life problem instances within practical time. For larger and harder instances, a hybridized algorithm called SLIM [5] has been developed. SLIM is driven by an iterative improvement heuristics, which aims at converging from a low quality initial reduction of the DAG to a minimally sized DAG that is sufficient to yield a (near) optimal solution. In every iteration, the size-reduced DAG is passed to the core ILP solver above to derive a solution. Because of the aggressive reduction of the DAG, the ILP solver needs little computational time in each iteration. SLIM also benefits from being well suited for parallelization.

On-going research on the ILP solver above includes coping with integer fixed charge variables more efficiently and catering for a richer variety of real-life problem variations and constraints. For SLIM, the focus is on maintaining a good balance between DAG size and search intensification/diversification.

3 Station level logistics

The multi-commodity flow schedule yielded in section 2 has left two operational aspects open to be determined before the solution can be fully operable. First is the unit coupling order in a trip served by multi-units. Second is the precise activity plan required to implement a linkage between an arrival and a departure. For example, suppose a unit arrives on route A and is scheduled to departure on route B next. And suppose route B uses a different platform, the
re-platforming implies some movements of the unit within the station that must not be blocked in any way.

The multi-commodity flow network level solution can be easily transformed into a station-by-station view. At each station, the partial solution consists of a list of arrivals, a list of departures, and a set of linkages connecting the two lists. A linkage also includes information about the unit(s) and platforms assigned. The movements of a unit to implement a linkage is called a “linkage shunting plan” and the collection of linkage shunting plans at a station is called a “station shunting plan”. For example, the assignment of a unit on arrival to serve a departure 20 minutes later is a linkage; and if the arrival and departure concerned take place at different platforms, a linkage shunting plan would be needed to re-platform the unit – feasibility of such an activity depends on the time gap available and whether the path of movement is free. Within a station shunting plan, all its linkage shunting plans must be conflict free, i.e. not blocking each other. Since each linkage can have many possible linkage shunting plans, their possible combinations in forming a station shunting plan would be prohibitively numerous to be fully enumerated. Hence, an estimation approach is proposed [6]. The linkages are classified according to characteristics of unit coupling, platform, track type (blind-end or through track) for both the arrival and the departure linked. Each classified linkage type has some associated shunting rules and parameters for determining an estimated minimum shunting time required. Those linkages having time gaps below their corresponding minimum shunting times are deemed infeasible, but they would have been prevented when the DAG was formed. On the other hand, time gaps well above minimum are deemed to pose no problem in deriving a suitable linkage shunting plan. Precise linkage shunting plans are then sought for the remaining linkages, during which relevant unit coupling orders will also be resolved. Finally, any other undetermined unit coupling orders will be determined.

Station logistics requires comprehensive studies of real operations to abstract. Hence, investigations and station site visits with collaborating operators have been carried out and their analyses are on-going.

4 Optimized and operable network-wide train unit schedules

Many UK train operating companies are already using the interactive TRACS-RS [7] system without an optimizer for train unit scheduling. Trials with some collaborating operators have demonstrated that the optimized multi-commodity flow solutions this research produced can be uploaded onto TRACS-RS and the station logistics can be resolved through its interactive facilities. The research described in section 3 will lead to minimal need for interactive station logistic resolution. On-going research is investigating a mathematical approach for finally integrating the prospective individual station logistics. In this approach, the original DAG is transformed into a multi-graph in which some nodes will be extended into multiple nodes representing alternative coupling orders. The objective is to find an optimal selection of alternative coupling order nodes and their associated arcs to be used across the network.

Data statement Part of the data used for this research may be commercially sensitive. Where possible, the data that can be made publicly available is deposited in http://archive.researchdata.leeds.ac.uk/.

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