# What does a universal initial mass function imply about star formation?

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## **ABSTRACT**

We show that the same initial mass function (IMF) can result from very different modes of star formation from very similar underlying core and/or system mass functions. In particular, we show that the canonical IMF can be recovered from very similar system mass functions, but with very different mass ratio distributions within those systems. This is a consequence of the basically lognormal shapes of all of the distributions. We also show that the relationships between the shapes of the core, system and stellar mass functions may not be trivial. Therefore, different star formation in different regions could still result in the same IMF.

**Key words:** binaries: general – stars: formation – ISM: clouds.

## 1 INTRODUCTION

Finding the form of the mass function of stars, in particular the initial mass function (IMF), has been a major goal of stellar and galactic astrophysics since Salpeter's (1955) seminal study (also see Kroupa 2002; Chabrier 2003b; Bonnell, Larson & Zinnecker 2007). The origin of the IMF has recently been a subject of intense interest (see Bonnell et al. 2007 for a review). Observationally, studies of the core mass function (CMF) have shown it to have a similar form to the IMF (Motte, André & Neri 1998; Testi & Sargent 1998; Johnstone et al. 2000, 2001; Motte et al. 2001; Johnstone & Bally 2006; Young et al. 2006; Alves, Lombardi & Lada 2007; Nutter & Ward-Thompson 2007; Enoch et al. 2008; Simpson, Nutter & Ward-Thompson 2008) suggesting a link between the two (see, in particular, Alves et al. 2007; Goodwin et al. 2008).

It is often assumed that if the IMF of two regions is the same, then star formation in those two regions must have been basically the same. However, the IMF is the mass function of *individual* stars. Therefore, the IMF – taken in isolation – ignores the fact that many stars are in multiple systems, and that most/many stars are thought to have formed as multiples (e.g. Goodwin & Kroupa 2005; Kouwenhoven et al. 2005, 2007; Goodwin et al. 2007, 2008 and references therein; see also Lada 2006). The IMF thus ignores a large amount of information related to the star formation process that is stored in the binary population.

During star formation, many cores must collapse and fragment into a multiple system (Goodwin & Kroupa 2005; Goodwin et al. 2007). Therefore, it is the system mass function (SMF) that we should expect to follow the CMF (as emphasized by Goodwin et al. 2008). The IMF is produced by splitting these multiple systems into their component parts. How the masses of the individual stars in the IMF are distributed depends not only upon the

In this Letter, we show that very different models for the conversion of the CMF to the SMF and from the SMF to the IMF can produce very similar IMFs. In Section 2, we outline our method and the results, and discuss the implications in Section 3.

#### 2 FROM THE CMF TO THE IMF

## 2.1 Method

Following Goodwin et al. (2008), we have constructed a simple model of multiple star formation from 'cores'. We assume a universal CMF with a lognormal form. This form is based on the observations of Alves et al. (2007) and Nutter & Ward-Thompson (2007). We take the average mass to be  $\sim\!\!1\,M_\odot$ , and the width to be  $\sigma_{\rm log_{10}M}\sim0.5$ . We then randomly sample cores from this CMF in a Monte Carlo simulation (see also earlier studies by Larson 1973; Elmegreen & Mathieu 1983; Zinnecker 1983).

We note that taking a lognormal form means that we fail to reproduce the power-law tail at high mass that is observed and expected theoretically (e.g. Padoan & Nordlund 2002; Hennebelle & Chabrier 2008), and so we do not expect to fit the IMF properly at high masses.

We create an SMF from a CMF by converting each core into a system with a particular efficiency. Thus, a core of mass  $M_{\rm C}$  will become a system of mass  $M_{\rm S} = \epsilon M_{\rm C}$ , where  $\epsilon$  is the core-to-star efficiency (CSE). The CSE may depend on the mass of the core, and even upon the mass ratio of the stars within the core.

SMF, but also on the binary fraction and mass ratio distribution (MRD) of systems. Moreover, these quantities may well be mass-dependent.

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<sup>&</sup>lt;sup>1</sup> Such cores may be classical isolated cores (e.g. Ward-Thompson et al. 2007) or just dense regions in which stars form (i.e. the star-forming clumps; see in simulations such as those of Bate 2009a,b).

We show below that, depending on the CMF and the CSE, many different SMFs can be formed. Therefore, we use an assumed SMF and then convert the mass of the system  $M_S$  into stars of masses  $M_1$  and  $M_2$ , or into a single star, depending on the binary fraction to create an IMF. The mass ratio  $q = M_2/M_1$  is drawn from an MRD in the range  $0 < q \le 1$ . The corresponding masses of the primary and companion star are then  $M_1 = M_S(q+1)^{-1}$  and  $M_2 = M_S(q^{-1}+1)^{-1}$  (see Kouwenhoven et al. 2009 for a detailed study of pairing functions in binary systems). For simplicity, we will only consider single stars and binary systems.

Finally, the IMF is given by the distribution of masses of all *individual* stars (single stars, primaries, companions) in all of the systems. We then compare this derived IMF to the 'canonical' Chabrier (2003) lognormal IMF.

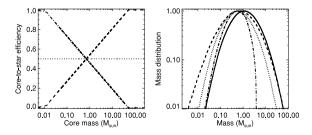
In summary, we have an initial CMF. A combination of the CMF and the CSE gives the SMF. Combining the SMF and the MRD then gives the stellar IMF. Note that the CSE may depend on the MRD as we shall discuss below.

We have assumed at the first step that the CMF is universal (and has a lognormal form). Clearly, this assumption may well be wrong. First, determinations of the CMF are very difficult, and it is not clear if the observed CMF is indeed the CMF that should be used as the underlying distribution from which stars form (Clark, Klessen & Bonnell 2007; Hatchell & Fuller 2008; Smith, Clark & Bonnell 2008; Swift & Williams 2008). However, for the purposes of this Letter we wish to show that *even if* the CMF is universal, we are able to produce the canonical IMF through different modes of star formation. However, as we shall see later, the assumed initial lognormal form of the CMF is one of the main reasons why the following distributions also maintain a lognormal form.

## 2.2 From the CMF to the SMF

In Fig. 1, we show how it is possible to change the width and shape of the SMF formed from the CMF by changing how the CSE varies with core mass. If high-mass cores are more efficient at converting gas to stars than low-mass cores, the SMF is broader than the CMF. However, if low-mass cores are more efficient, then the SMF is narrower than the CMF. But note that the resultant SMF always has a roughly lognormal form, often with just a small degree of skewness added.

Presumably, the CSE is not independent of mass. The higher the level of feedback from stars, the lower we might expect the CSE to be. However, this may not be a trivial relationship. More massive



**Figure 1.** In the right-hand panel, we show the SMFs (dotted, dashed and dash-dotted lines) produced from a single CMF (solid line), produced by different CSEs with mass shown in the left-hand panel (with corresponding dotted, dashed and dash-dotted lines).

stars presumably produce more feedback,<sup>2</sup> and so be able to reduce the fraction of a core that accretes on to the system. On the other hand, a more massive core is also more bound and so more difficult to disperse.

An interesting possibility is that the CSE depends not just on the mass of the system, but also on the mass ratio of that system. It is possible that an unequal-mass system (say  $1.8\text{--}0.2\,M_{\odot}$ ) will produce far more feedback than an equal-mass system (say  $1\text{--}1\,M_{\odot}$ ) with the same system mass. Therefore, a larger core is required to form the first system. Also, as pointed out by Myers (2008), the CSE should also depend on the core density.

#### 2.3 From systems to stars

In Fig. 2, we present the IMFs that result from systems with a binary fraction of unity for a selection of very different choices for the MRD. We have fine-tuned the SMFs in order to obtain good fits to the canonical IMF. What is clear from Fig. 2 is that very different choices for the MRD are *all* able to produce the canonical IMF.

A potential problem is that we have introduced a fine-tuning element in that the SMF has been chosen to give the best fit to the canonical IMF for a given MRD. However, this fine-tuning is not significant. As indicated in the captions of Fig. 2, the SMFs do not vary very much, with means of  $\mu_{\rm log_{10}M}=-0.3$ , -0.4, -0.3 and -0.35 and variances of  $\sigma_{\rm log_{10}M}=0.4$ , 0.55, 0.4 and 0.5, respectively, for the different models.

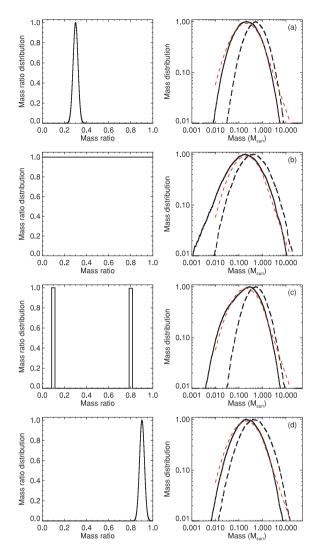
It would be extremely difficult, if not impossible, to distinguish observationally differences in the mean of the SMF between 0.4 and 0.5  $\rm M_{\odot}$ , and differences in the  $1\sigma$  widths of 0.1–0.2  $\rm M_{\odot}$  at one end, and 1.25–1.40  $\rm M_{\odot}$  at the other. Indeed, the models presented in Figs 2(a) and (c) have identical SMFs despite having completely different MRDs (a single peak at  $q \sim 0.3$ , compared to peaks at q = 0.1 and 0.8) and yet both give good fits to the canonical IMF.

All of the models presented in Fig. 2 have assumed that *all* systems produce a binary (as shown in Fig. 3). Clearly, the canonical IMF can also be recovered from an SMF of entirely single stars if the SMF is exactly the same as the IMF. However, we know that a significant fraction of systems in the field are multiples, and that presumably many more were multiples at birth as multiples are destroyed, but not created (Kroupa 1995a,b; Parker et al. 2009).

Goodwin et al. (2008) found that – assuming a constant CSE and a uniform MRD – a model with a binary fraction of unity was a better fit to the canonical IMF than one in which binarity declined with system (i.e. primary) mass as is observed in the field (see Lada 2006).

In Fig. 4, we show the best fits to the canonical IMF for the same set of MRDs as in Fig. 2, but for a binary fraction that varies with system mass as shown in Fig. 3 – unity for a system mass >1  $\rm M_{\odot}$ , linear for  $0.1 < M_{\rm sys}/\rm\,M_{\odot} < 1$  and 0.1 for  $M_{\rm sys} < 0.1\,\rm\,M_{\odot}$ . In order to fit the canonical IMF for each MRD, the best SMFs are  $\mu_{\rm log_{10}M} = -0.6$  and  $\sigma_{\rm log_{10}M} = 0.6$  for all of our models. The peaks of the SMFs ( $\sim 0.25\,\rm\,M_{\odot}$ ) are lower than for a binary fraction of unity as the reduced number of low-mass companions means that the low-mass end of the IMF must be made up largely of single stars directly from the SMF. The increased width is then required in order to allow sufficient numbers of higher mass stars to form.

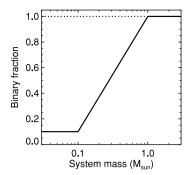
<sup>&</sup>lt;sup>2</sup> Assuming a main-sequence-like relationship, the mass-luminosity relationship would go as  $L \propto M^{3.5}$ , however the mass-luminosity relationship for pre-main sequence stars is highly complex and age- and mass-dependent.



**Figure 2.** In the right-hand panel, we show the IMFs (solid black lines) formed from a given SMF (dashed line) with an MRD as shown in the left-hand panel assuming a binary fraction of unity. The IMFs are compared to a canonical Chabrier (2003) IMF (light red dashed lines). The means and variances of the SMFs that give the best fits are (from top to bottom)  $\mu_{\log_{10}\mathrm{M}} = -0.3, -0.4, -0.3$  and -0.35 and  $\sigma_{\log_{10}\mathrm{M}} = 0.4, 0.55, 0.4$  and 0.5, respectively.

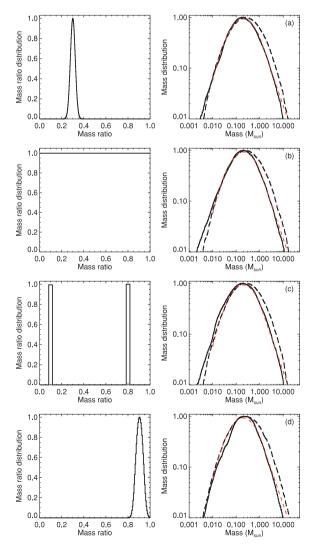
It might seem that this result is at odds with Goodwin et al. (2008). However, Goodwin et al. set the CMF with which to generate the SMF and then the IMF as being the Orion/Pipe Nebula CMF (with a mean of  $\sim 1\,M_{\odot}$  and variance of  $\sigma_{\rm log_{10}M}=0.55$ , see above) and a constant CSE. Given these constraints, it is impossible to form an SMF with a mean of  $0.25\,M_{\odot}$  and a variance of  $\sigma_{\rm log_{10}M}=0.6$  as required to produce the canonical IMF with a decreasing binary fraction. However, as we show here, if the CMF is different or if the CSE is such that the variance is increased and the mean lowered significantly, such a CMF can produce the canonical IMF with a varying binary fraction.

We have also examined the effect of using a CMF that is not lognormal (in particular, triangular and top-hat CMFs). Features in the CMF are always reflected in some way (even if distorted) in the SMF and the IMF no matter our choice of the CSE and MRD. Thus, the apparently smooth lognormal-like shape of the IMF would seem to be a natural consequence of the lognormal-like shape of the



**Figure 3.** The dependence of binary fraction on system mass used to generate the IMFs from the SMFs in Fig. 2 (dashed line) – where all systems are binaries – and Fig. 4 (solid-line) – where there is an increasing binary fraction with system mass.

CMF (cf. Elmegreen & Mathieu 1983; Zinnecker 1983). However, the lack of features tells us that the CMF that produces the SMF that produces the IMF is highly unlikely to contain any sharp features or discontinuities as these should be apparent in the IMF.



**Figure 4.** As Fig. 2, but for a binary fraction that increases with system mass as shown in Fig. 3 (solid line). In all cases, the canonical IMF can be fitted with an SMF of mean  $\mu_{\rm log_{10}M}=-0.6$  and variance  $\sigma_{\rm log_{10}M}=-0.6$ .

It is surprising that the IMF is so insensitive to the MRD of stars. The reason is that the IMF is so insensitive that it retains the lognormal-like shape of the SMF (which itself is retained from the CMF). As the SMF is distributed in a roughly lognormal way, each chosen mass ratio from the MRD retains this shape (e.g. for an MRD of two delta functions, the SMF will produce two stellar distributions, each a lognormal). Thus, the IMF is the sum of a number of lognormals thus retaining the lognormal shape.<sup>3</sup>

It is interesting to examine what form we might expect the MRD to have. We would expect that most binaries form via the fragmentation of discs (or massive disc-like objects) around young stars (see Goodwin et al. 2007 for a review). If discs fragment early, then the secondary will form whilst there is still a significant amount of material to accrete, and we might expect the MRD to favour more equal-mass systems. If the discs fragment late after the primary has accreted most of its mass, then there will be little material left to accrete and unequal-mass systems might predominate (as might also happen if the secondary formed at a large distance from the primary). Thus, the MRD might contain information on the formation time of secondaries (which might depend on the amount of angular momentum or strength of magnetic fields in the core?). If the MRD favours unequal-mass binaries, then the primary IMF will reflect the form of the SMF well, however if many binaries are more equal-mass then the primary IMF would not.

#### 3 DISCUSSION AND CONCLUSIONS

We have investigated the origin of the stellar IMF as produced by an SMF, which is itself formed from a particular CMF. The CMF is converted to the SMF via a CSE factor, and then turned into the IMF via an MRD. Our main findings are as follows.

- (i) Different CSEs can significantly change the width and shape of the SMF formed from a given CMF.
- (ii) Very different MRDs within systems can produce very similar (and canonical) IMFs from very similar SMFs.

Thus, we conclude that observing the same IMF in different regions does not necessarily mean that star formation was the same in those regions. For very different choices of the MRD in star-forming cores, the same (canonical) IMF may be found. In addition, the form of the IMF is surprisingly insensitive to the variation of the binary fraction with mass (modulo the position of the peak of the SMF).

This raises a question about the universality, or non-universality, of star formation based on observing the same IMF in different regions. If one star-forming region produced systems with generally equal-mass stars, whilst another produced systems with generally comparatively low-mass companions, it seems very difficult to claim that star formation in those regions was the same. However, the CMFs, SMFs and IMFs could all be very similar. Therefore, in order to claim that star formation is the same in different regions, the *birth* SMF and MRD of binaries, and the binary fraction, must also be the same.

It is important to note that it is the *birth* SMFs and MRDs that must be determined. Dynamical evolution can seriously alter the

<sup>3</sup> It is possible to produce other distributions. For example, an MRD which always produces a star with 99.9 per cent of the mass in a core and a planet with 0.1 per cent of the mass. In such a situation, the two lognormals would be so far apart that they would appear as two separate lognormals. However, observations show that typical mass ratios are in the range 0.1–1 (e.g. Duquennoy & Mayor 1991).

initial binary population in a cluster (e.g. Kroupa 1995a,b; Parker et al. 2009) and destroy much of the vital information on the birth properties.

It is important to note that the peak mass of the IMF which always seems to occur at  ${\sim}0.1\text{--}0.3\,M_{\bigodot}$  does constrain the star formation process. The initial CMFs must always have a similar peak mass which is then imposed on the SMF (regulated by the CSE) which is then imposed on the IMF (but regulated by the MRD including the binary fraction). If the binary fraction varies between different star formatting regions, the peak of the CMFs could change by a factor of 2 or 3, but no more. Thus, the underlying CMFs in all regions should probably have a peak mass of the order of  $1\,M_{\bigodot}$ .

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# L40 S. P. Goodwin and M. B. N. Kouwenhoven

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