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Measuring mathematical affect in an international context: influences on pre-university attainment and degree choice

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Abstract

In many contexts, mathematical self-efficacy is known to be important, and distinct, predictor of mathematical attainment and the nature of further participation in the study of mathematics and mathematically-based disciplines. This study reports on the results of cross-sectional questionnaire-based study of over 500 former International Baccalaureate® students across 64 countries who had studied higher level pre-university mathematics. Two different sub-scales were employed in the online questionnaire measuring (i) mathematical self-efficacy on completion of the IB course (i.e. retrospectively), and (ii) mathematical self-concept at the time of completion of the survey (i.e. mainly during degree study). These scales were found to be essentially uni-dimensional, and to measure distinct but related constructs. In terms of predicting type of degree participation, important differences in patterns of influence were found relating to mathematical self-efficacy and attainment, although degree choice was not well-predicted in the modelling. Differences by gender and country are also reported, and the relationship between self-efficacy and self-concept is explored.

Keywords: self-efficacy, self-concept, attainment, degree choice

Word Count

6200

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Biographical note

Matt Homer is a former mathematician, mathematics teacher and now academic and applied statistician. His research interests are mostly related to assessment and psychometrics, often using modern quantitative methods. Substantive areas of interest include aspects of science and post-16 mathematics education, and medical education assessment.

Introduction

Research over the past 30 or so years has shown that students' self-efficacy in mathematics can be predictive of continuing participation and performance in further mathematically-related study, and is sometimes more predictive than is (prior) attainment in the subject itself (Betz & Hackett, 1983; Chen & Zimmerman, 2007; Hackett & Betz, 1989; Pajares & Miller, 1994, 1997). Whilst there are a number measures of self-efficacy in existence (Hackett & Betz, 1989; May, 2009; Pampaka, Kleanthous, Hutcheson, & Wake, 2011; Usher & Pajares, 2009), there is little recent evidence of their use internationally in measuring self-efficacy on entry into higher education, and, perhaps more importantly, on the extent to which views on mathematical self-efficacy together with achievement in mathematics relate to degree choice, and how this might vary across a range of demographic factors (e.g. country, gender and school type – e.g. state or privately funded).

Using data generated from a single administration of an online survey with over 500 respondents from across the globe, this study investigates the extent to which a particular measure of mathematical self-efficacy is valid in an international context. The respondents are International Baccalaureate Diploma alumni who had studied mathematics at a higher level at ages 16 to19 prior to going on to university study. This paper explores how self-efficacy relates to pre-university attainment and degree choice, and also compares mathematical self-efficacy with another important and related social-cognitive measure, that of mathematical self-concept.

The main research questions addressed in this study are the following:

1. To what extent can mathematical self-efficacy and mathematical self-concept be successfully measured in an international, relatively high-performing, group of students?
2. How do retrospective[1] ratings of mathematical self-efficacy relate to those of mathematical self-concept?
3. How does mathematical self-efficacy and mathematical attainment on entry to university (both retrospectively measured) relate to degree choice for this group?

The overall purpose of the paper is to add to the evidence with regard to influences on the study of mathematically-oriented and other degree studies in a global, high achieving context (see the next section, Research context, for more on this), and how this might vary across key factors including social-cognitive domains (self-efficacy, self-concept) and prior attainment in mathematics.

Research context

The international Baccalaureate (IB) Diploma programme (IBDP) is aimed at 16-19 year olds and consists of six subject groups including mathematics (as well as additional 'core' elements such as creativity and research study)[2]. All students doing the Baccalaureate have to study some mathematics, and this research focusses on students who have completed the Mathematics higher level (HL) course, which is designed to be demanding, challenging and rigorous, and is made up of a broad range of mathematical content areas[3] appropriate for preparation for progression into numerate disciplines in higher education.

Across the world, the Diploma programme has grown rapidly over the last few years so that as of November 2014 there were over 2,500 schools offering the IBDP in 139 countries (Bunnell, 2015). There is good general evidence that IBDP alumni are well-prepared for degree study and do well in their higher education studies compared to their peers who have followed different pre-university curricula (ACS International Schools, 2016; Bergerson, 2015; Conley, McGaughy, Davis-Molin, Farkas, & Fukuda, 2014; Higher Education Statistics Authority, 2011; Saavedra, 2014). However, little of this research has focussed on mathematical preparedness in particular, and one motivation for the wider study this work stems from, commissioned by the IB, was to generate some data specifically on student views on their preparedness in mathematics itself (Homer & Monaghan, 2016).

Mathematical self-efficacy

Reviews of the literature on the study of self-efficacy usually start with Bandura's (1977, 1985, 1997) work, where he defines self-efficacy as:

> *People's judgments of their capabilities to organize and execute courses of action* required to attain designated types of performances. (Bandura, 1985, p. 391)

This is sometimes referred to in the literature as task-specific self-confidence (Artino, 2012), so in mathematical context self-efficacy relates to individuals' beliefs about their own particular mathematical capabilities (for example, being confident that they are able to solve a linear equation). Bandura (1985) also states that students' individual beliefs about their ability impact on their motivations to study and learn – with more recent work focussed on mathematics particularly (Pampaka et al., 2011; Pampaka & Williams, 2010; Usher & Pajares, 2009; Zeldin, Britner, & Pajares, 2008).

One important conclusion of some of this work is that an emphasis merely on the teaching of mathematical content does not guarantee that students will learn, and that greater attention should be paid to affective factors in the successful learning of mathematics, particularly self-efficacy (May, 2009; Pampaka et al., 2011). The role of self-efficacy in the ‘choice’ to continue studying mathematics has long been acknowledged (Betz & Hackett, 1983; Hackett & Betz, 1989; Zeldin et al., 2008), whilst there is a growing awareness that any supposed ‘choice’ does not rest on one’s level of mathematical self-efficacy alone (Usher & Pajares, 2009), for example, the important role of parents/family and the wider ‘habitus’ (Bourdieu, 1977) has been confirmed in recent work (Kleanthous & Williams, 2013).

Measuring mathematical self-efficacy

Toland and Usher (2016) provide a recent overview of some of the extensive literature on the measurement of mathematical self-efficacy. Much of this work focuses on secondary education, rather than pre-university mathematics (Kung, 2009; Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014) where, in both of these studies, measures of mathematical self-efficacy from large scale international studies (PISA and TIMSS) are used for a within-country analysis (Taiwan and Australia respectively).

Work more relevant to this study includes the PhD study of May (2009) in the US that developed measures of mathematics self-efficacy and anxiety in college mathematics students with the aim of providing mathematics instructors with specific tools to measure these constructs in the hope of providing data that can lead to improvements in practice. Pampaka and colleagues in the UK have also developed a self-efficacy measure for pre-university students (Pampaka et al., 2011; Pampaka & Williams, 2010), with one of their focuses on valuing mathematical self-efficacy as an important outcome of learning in a mathematics course. The current study employs elements of the instruments developed by both Pampaka (and colleagues) and May and considers the different conceptions of mathematical self-efficacy (or more accurately, self-efficacy and self-concept respectively – these differences will be elaborated on later) that has underpinned the development of these tools.

Relationships between mathematical self-efficacy, attainment and future study choice

A range of research confirms that mathematical self-efficacy and mathematical attainment are closely linked (Pietsch, Walker, & Chapman, 2003; T. Williams & Williams, 2010), although, as commented on before, much of this work is centred on school mathematics rather than in the pre-university phase. A key study of college students, however, is that of Hackett and Betz (1989) who found in the US that mathematical self-efficacy was an important factor in degree choice, with for example, those with higher self-efficacy more likely to state that they would choose science-based degree programmes. They also found that perceptions of self-efficacy was a better predictor of educational outcomes and career choices than was mathematical attainment. To an extent, the current study aims to update the Hackett and Betz (1989) work, and to assess the generality of their findings in an international and relatively high-achieving sample. Other work relating academic choices to mathematical self-efficacy include that of Chen and Zimmerman (2007) and Pajares and Miller (1997), although school, rather than pre-university, mathematics was the focus of both of these latter studies.

Mathematical self-concept

The common definition of self-concept differs from that of self-efficacy in that the former is usually framed as a more general perception of oneself, rather than of what specific things one can do (Bong & Skaalvik, 2003; Kung, 2009). Hence, in a mathematical context, one might measure self-concept by asking questions about how individuals perceive themselves in relation to mathematics more generally, at the appropriate level (e.g. the extent to which one identifies as mathematician or regards oneself as a 'good' at maths). Often, self-concept is described as a more normative measure, relative to peers, in comparison with self-efficacy, which is more absolute measure (Parker et al., 2014). Again, evidence tends to suggest that mathematical self-concept is an important influence on academic achievement (Kung, 2009; Pajares & Miller, 1994; Parker et al., 2014, 2014). The relationship between mathematical self-concept and mathematical self-efficacy has been explored in a range of studies and contexts (Bong & Skaalvik, 2003; Kung, 2009; Pajares & Miller, 1994; Parker et al., 2014; Pietsch et al., 2003). Typically, the two constructs are found to be distinct but related (e.g. significantly correlated), but in some studies the nature of the causal relationship between them and the precise longitudinal impacts on mathematical achievement and progression into higher education are complex (Kung, 2009; Parker et al., 2014).

Methodology

The online survey

The online survey used in this study was developed using two pre-existing scales, one retrospectively measuring mathematical self-efficacy (Pampaka & Williams, 2010), and one measuring mathematical self-concept (May, 2009)[4]. Minor modifications to wording were made to make the scales appropriate to the particular research context - see the appendix for full details of the individual items. These two scales are quite different in nature, with the Pampaka and Williams assessing mathematical self-efficacy on completion of the IBDP (i.e. retrospectively) across a range of 10 particular topic areas (e.g. Calculating and estimating, Ratio and proportion and so on). The May-based scale consists of fourteen items assessing a more general self-concept in mathematics, asking for level of agreement (at the time of the survey) with statements such as 'I believe I am the type of person who can do mathematics'.

There were two main reasons for choosing to employ both scales: firstly, to measure specific areas of relative mathematical self-efficacy retrospectively at the end of IBDP (hence Pampaka and Williams), and then a more general mathematical self-concept at the time of the survey (hence May) – when the majority of respondents would not necessarily be studying mathematics per se. Secondly, there is methodological and theoretical interest in comparing and contrasting the constructs of self-efficacy and self-concept, and the issues around their measurement,

Additional items in the survey covered respondent demographics, views on IBDP HL mathematics, and ratings of preparedness for university study. It was designed by the research team in conjunction with the IB and, given the constraints of the timing of the project, it was not piloted before being administered. The survey was developed and delivered using Bristol online surveys[5], and an email invitation to respond was sent in September 2015 to 3,196 participants carefully selected from the IBDP HL mathematics alumni database (which itself contains around 6% of all mathematics HL alumni). This selection was made at random, but with the proviso that participants were not already being surveyed by the IB for other research purposes. The survey was open for three weeks, and there were 566 responses by the cut-off date (corresponding to a 17.7% response rate). As judged by the open text comments in the survey (not reported on in this work), the respondents demonstrated good engagement with the survey – for example the number responding with meaningful written comments on what in particular they liked about IBDP mathematics HL was 480 out of 566 (i.e. 85%).

Data analysis

Descriptive summaries are provided for demographic variables, and individual Likert scale items are summarised graphically using stacked bars charts.

Scale dimensionality for mathematical self-efficacy and self-concept scales is assessed using exploratory factor analysis, and scale internal consistency reliability using Cronbach's alpha. Correlations between scale mean scores and factor analysis scores are very high – of the order of r=0.99. One clear advantage of mean scores over factor scores is that the former has directly interpretable meaning being on the same numeric scale as the original individual Likert scale items. Hence, the mean score across sets of scale items is used as the overall scale score and will be treated as a continuous variable (Carifio & Perla, 2008; Norman, 2010) [6]. Differences in these mean scale scores by student characteristics will be illustrated using error bars.

The main modelling approach for continuous outcomes (self-efficacy/self-concept measures and attainment) is the general linear model – i.e. ordinary regression but allowing for the control of the main effects for categorical demographic predictors gender, country of study (USA, Canada, UK, India, Other) and school type (state or privately funded). The strength of demographic influences are compared using partial eta squared as the effect size measure in these models (Field, 2013, p. 501).

For predicting degree participation, nominal logistic regression is employed (Field, 2013, sec. 19.9) to model a discrete outcome with more than two levels (e.g. fields of study including mathematical sciences, natural sciences, and professions – see Table 1 for more on what subjects these include). Specific additional details of this analysis are provided at the relevant point in the article.

Pre-existing data provided by the IB is used to assess the representativeness of the responses to the survey across a range of characteristics (e.g. gender, country of study, private versus state schooling, and attainment level in mathematics HL).

Ethical considerations

The study was approved by the appropriate Faculty Research Ethics Committee at the University of Leeds (reference 106274) in June 2015.

Results

Representativeness of respondents

The respondents were 61% male, typically born in 1995 so around 20 years old when completing the survey (i.e. typically two years after completing the IBDP). The most commonly occurring countries of residence during IBDP study were USA (30%), Canada (8%) the UK (7%) and India (6%), with 64 countries in total represented in the sample. Respondents were evenly split between state-funded and private schooling (49% and 50% respectively, with the small proportion remaining unsure on the nature of their school in this regard).

Compared to pre-existing data on IBDP mathematics HL alumni, those responding to the survey were broadly representative by gender and country of this wider group, but state educated students were over-represented amongst survey respondents (49% in the survey vs. 43% in the pre-existing data).

The paper now reports in turn results for mathematical attainment and self-efficacy (both at the end of the IBDP), and then on degree study, and finally on mathematical self-concept at the time of the survey (and its relationship with self-efficacy).

Mathematical attainment at end of IBDP

Grading of subjects in the IB diploma takes the form of a number from 1 (lowest grade) to 7 (highest, nominally equivalent to A* at UK A-level). Most survey respondents reported scoring quite highly in mathematics - with a median grade of 5, and a mean 5.2.

Mean mathematical attainment in the sample was slightly higher than that of the full population of potential respondents (5.2 vs. 4.5, the latter derived from separate attainment data supplied by the IB) implying that those responding to the survey were a little more highly attaining than the average IBDP mathematics HL alumni.

Variation in mathematical attainment by demographic factors

A regression model predicting mathematical attainment using gender, school type (private vs. state) and country in five categories (Canada, India, UK, USA and other) had a low adjusted r-squared vale (0.07), and accounts for relatively little of the variance in attainment outcomes. The model indicates that males had higher levels of reported attainment (5.3 vs 5.1, $p=0.07$, partial eta squared=0.01), there was little difference between private and stated-educated respondents, and that the UK had the highest level of attainment and USA the lowest (5.6 vs. 4.7, $p<0.01$; partial eta squared=0.05)., Whilst these results indicate that there is some variation in attainment across demographic sub-groups, these effects are quite small overall and IBDP mathematical attainment is not well-predicted by this set of variables.

Mathematical self-efficacy at the end of the IBDP

On a scale from 1 ('not confident at all') to 4 ('very confident'), respondents were asked to retrospectively rate their mathematical self-efficacy on completion of the IBDP across a range of ten topic areas (Pampaka, Kleanthous, Hutcheson, & Wake, 2011; Pampaka & Williams, 2010) - see the Appendix for the individual items. Figure 1 shows summaries of levels of confidence across each of these items, and is ordered from most to least confident. Figure 1 shows that reported levels of self-efficacy were typically quite high, with the highest level of confidence in 'Manipulating algebraic expressions' and the lowest for 'Proofs/proving'.

FIGURE 1 HERE

Exploratory factor analysis indicates that these 10 items can be considered to measure a single underlying latent trait (first factor accounting for 43% of variance), which one could label as 'mathematical self-efficacy'. The scale has good internal consistency reliability (Cronbach's alpha=0.84, with no items detracting from this). The mean score across the 10 items shows good discrimination across the sample, with some negative skew (mean=3.2, i.e. typically 'confident'; standard deviation =0.52; skewness=-0.75, 5^{th} percentile=2.3, 95^{th} percentile=4.0). In the analysis, this mean score will be used as the measure of mathematical self-efficacy in respondents.

Variation in mathematical self-efficacy by demographic factors

A regression model predicting retrospective ratings of mathematical self-efficacy using gender, school type (private vs. state) and country in five categories (Canada, India, UK, USA and Other) gives a relatively poor model (adjusted r-squared=0.02) but indicates that males reported slightly higher levels of mathematical self-efficacy (3.2 vs 3.1, p=0.03, partial eta squared=0.01), privately educated students also had higher levels (3.2 vs 3.1, p=0.06, partial eta squared=0.01), and that Canada had the highest level of mathematical self-efficacy and India the lowest (3.4 vs. 2.9, p=0.01, partial eta squared=0.03). Whilst sometimes achieving statistical significance at the 5% level, all of these effects are quite small, implying that whilst there are some minor demographic differences in retrospective ratings of mathematical self-efficacy as measured by this scale, the variation in responses is not well-accounted for using gender, school type and country as predictors.

Relationship between mathematical self-efficacy and attainment at the end of the IBDP

The Pearson correlation between retrospective ratings of mathematical self-efficacy and mathematical attainment in the IBDP was r=0.45 (n=561, p<0.01). Modeling self-efficacy based on attainment alongside gender, school type and country gives a reasonable model (adjusted r-squared=0.22) and by far the strongest (positive) predictor of self-efficacy is attainment (p<0.01, partial eta squared=0.21). Of the other predictors, only country was statistically significant (p<0.01; partial eta squared=0.01) but had a much weaker influence on ratings of self-efficacy than did attainment.

Field of degree study

When asked what stage respondents were at in terms of their degree study, the majority reported that they were currently studying for a university degree (74.9%), with a further 13.4% having completed their degree and 10.1% stating that they would be starting their degree soon. The remaining 10 respondents (1.6%) gave 'Other' as their response and will not be included in the rest of the analysis.

Survey participants were also asked about what degree they were doing or intending to do in five broad categories of degree type. The results are summarised in Table 1, which shows that the Professions (37%) and the Natural sciences (24%) dominate, with 12% of the sample identifying their degree choice as mathematical in nature.

TABLE 1 HERE

Variation in mathematical self-efficacy by field of degree study

Comparing retrospective ratings of mathematical self-efficacy by field of study, Figure 2 shows that self-efficacy is reportedly very similar in Natural sciences, Mathematical sciences and Professions, and is a little higher than in the other fields of study. However, these differences are not particularly large (approximately 0.3 on a scale from 1 to 4; p=0.002; r-squared=0.035).

FIGURE 2 HERE

Variation in mathematical attainment by field of degree study

An analysis of the variation in mathematics attainment in the IBDP across field of degree study shows that those attaining most highly were more likely to go onto mathematical disciplines – see Figure 3. Again – these differences are not particularly large (largest difference in mean grade approximately 1 unit on a scale of grades from 1 to 7; p=0.002; r-squared=0.034).

FIGURE 3 HERE

Predicting field of degree study based on mathematical self-efficacy and attainment

In order to best model field of degree study as an outcome based on retrospective ratings of mathematical self-efficacy and attainment in IBDP mathematics HL (and other demographics – gender, school type and country), it was necessary to collapse the three smaller degree type categories (Humanities, Social sciences and Other) into a single category (referred to from now on as HSO). Field of degree study can then be predicted in the four remaining categories relative to this HSO group, using mathematical self-efficacy and attainment as continuous covariate predictors, and gender, school type and country as categorical factors (with reference groups male, state school and USA respectively).

The resulting multinomial regression analysis had an adequate, but perhaps low, pseudo-R-square value (Nagelkerke=0.14), and no evidence of misfit (Pearson chi-square goodness of fit p-value=0.58). Details of the resulting parameter estimates are shown in Table 2 with statistically significant predictors shaded.

TABLE 2 HERE

The key parameters (exp(β), 6th column) in Table 2 are odds ratios (OR)[7]. For a unit increase in a predictor, these tell us the corresponding change in the odds of choosing the particular field of study (relative to HSO), whilst holding other predictors constant. Results for each field of study are described in turn:

- For Natural sciences, the OR for mathematical self-efficacy is 1.91. So higher levels of this predictor are associated with greater likelihood of choosing the Natural sciences over HSO. The only other significant predictor for Natural sciences is gender, where females are less likely to choose the subject (over HSO) compared to males (OR for females=0.59<1).

- For Mathematical sciences, higher attainment predicts participation (OR=1.58), as does gender (OR for females 0.39, so again females are less likely to choose this field of study over HSO), and respondents from Other countries (OR=0.037, so these respondents less likely to choose this field of study relative to those respondents from the USA).

- For Professions, Table 2 indicates that predicting participation in Professions, mathematical self-efficacy is important (OR=1.60). Again, females are less likely to choose this field relative to HSO (OR=0.41), as are students from the UK (OR=0.31, compared to those from the USA).

Mathematical self-concept at time of survey

Respondents were asked to rate their mathematical self-concept at the time of the survey across a range of fourteen Likert scale items (1='never' to 5='usually'). These items were originally devised by May (2009) and are detailed in the Appendix. The responses are summarised graphically in Figure 4.

FIGURE 4 HERE

Self-concept was generally highest in the item related to 'understanding the content in a mathematics course' ', whereas it was lowest in the item relating to 'thinking like a mathematician' Exploratory factor analysis indicates that these items form a single uni-dimensional scale (i.e. ‘mathematical self-concept) with this single factor accounting for 62% of the variance in responses. The scale has good internal consistency reliability (Cronbach’s alpha=0.95, with no items substantially detracting from this) and shows good discrimination across the sample. For the purposes of this study, the mean score across the fourteen items is the measure of mathematical self-concept at the time surveyed (mean=4.0, i.e. ‘often’; SD=0.73, skewness=-0.66, 5^{th} percentile=2.8, 95^{th} percentile=5.0).

Variation in mathematical self-concept across fields of study

Figure 5 shows that there was some variation in average ratings of mathematical self-concept when comparing by field of degree study.

FIGURE 5 HERE

Although the overall effects size for these differences is quite small (r-squared=0.06, $p<0.001$), respondents studying mathematical subjects do rate their self-concept more highly (4.3), and those in the humanities the lowest (3.6).

The relationship between retrospective ratings of mathematical self-efficacy prior to university and mathematical self-concept once in higher education

The Pearson correlation between the two social-cognitive measures, mathematical self-efficacy and mathematical self-concept, is r=0.60 ($p<0.001$, n=563). To develop the analysis of this relationship further mathematical self-concept as an outcome is modelled in terms of mathematical self-efficacy, mathematical attainment (both at end of IBDP study), as well as gender, school type, country of study (during IBDP) and field of study in HE.

The resulting model is statistically efficacious (r-squared=0.43) with the strongest influence on self-concept being self-efficacy (β=0.68, $p<0.01$, partial eta squared=0.24) followed by IBDP mathematical attainment (β=0.12, $p<0.01$, partial eta squared=0.06). All other factors are considerably weaker predictors, but there are some statistically significant effects including:

- Country of IB study ($p<0.01$, partial eta squared=0.03) - with USA having the highest levels of mathematical self-concept.
- Field of study ($p=0.01$; partial eta squared=0.02) - with respondents studying Mathematical sciences having the highest levels of mathematical self-concept.

Discussion

The discussion takes each research question in turn and situates the key findings in the wider literature. The paper finishes with consideration of the limitations of the study, and concluding remarks.

Measuring mathematical self-efficacy and self-concept (RQ1)

Both socio-cognitive scales appear to work relatively well from a technical measurement perspective, although in the original development of the Pampaka scale (Pampaka & Williams, 2010), Rasch analysis (Bond & Fox, 2007) was used as a key component of the validation. The Rasch analysis of the scales has not been reported in this paper, as this was found to be problematic – for example, there was significant overall misfit to the Rasch model for both scales (Tennant & Conaghan, 2007), and misfit for some of the individual items. It could be that the international nature of the current study is generating differential item functioning by country that has not yet been identified (Bond & Fox, 2007, Chapter 5). Clearly, more work and better data is needed to investigate the problems with validation of these scales using the Rasch measurement model where a wide range of countries are involved, and perhaps other approaches that provide additional validity evidence are required (Kane, 2006; Pepper, Hodgen, Lamesoo, Kõiv, & Tolboom, 2016).

A more 'classical analysis' (i.e. using classical test theory and factor analysis) indicates that both measures are functioning relatively well. The May self-concept scale (2009) has slightly better psychometric properties compared to the Pampaka self-efficacy scale (2010), certainly in this context – it is more uni-dimensional (i.e. more variance explained in the first factor), and has higher internal consistency reliability. Both measures discriminate well between respondents. Given the very high value of alpha for the May scale (α=0.95), one might argue there is redundancy in it, and that it could be easily shortened without much loss of reliability/validity. However, the comparison between the psychometric properties of the two scales should be treated with some caution, as the Pampaka scale was used retrospectively which might be adding 'noise' to the measurement if respondent recall is not entirely accurate (Nimon, 2014).

In terms of actual levels of mathematical self-efficacy, respondents generally rated this highly when looking back to the time when they finished their IBDP studies as one might expect in this group (Bergerson, 2015; Higher Education Statistics Authority, 2011).. The mean score in the study on mathematical self-concept was higher than that in the original May (2009) study (4.0 vs 3.2), which employed a very similar set of items on 109 non-mathematics students in the US taking a pre-calculus college course. This is further evidence of the validity of this measure, as one would expect the IBDP sample to outscore May's sample. However, given the normative nature of measures of self-concept (Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014), differences between groups might not always be large.

Neither scale showed much variation by student characteristics (gender, country and school type), which contrasts to an extent with existing research on other populations (Pampaka et al., 2011). This provides important emergent evidence that levels of mathematical affect in particular contexts might vary more within sub-groups compared to across them. In turn, this suggests that pedagogical interventions aimed at improving students' self-perceptions with regards to mathematics might sometimes need to be focussed at the individual level rather than at easily identified sub-groups.

The relationship between mathematical self-efficacy and self-concept (RQ2)

The two measures employed in the study clearly have different conceptions of the traits they are measuring, with the self-efficacy measure (Pampaka & Williams, 2010) topic-based ('generalised mathematical competences' in the words of Williams, Wake and Jervis (1999)) , whereas the self-concept measure (May, 2009) is more obviously affective and holistic – in asking respondents about how they view themselves in relation to mathematics generally, and to being a mathematician. May's theoretical framework is based on a general-expectancy value model (Azjen & Fishbein, 2005) where individuals' views and beliefs impact on their motivation, in this case, to 'do' mathematics.

These two socio-cognitive measures have a moderately strong correlation (r=0.60), which is very similar to that found in the Parker et. al (2014) study (r=0.65), so the current study confirms empirically that the two traits are strongly related but measure distinct constructs (Bong & Skaalvik, 2003; Kung, 2009).

The modelling of self-concept in terms of retrospective ratings of self-efficacy (and pre-university mathematical attainment and demographic factors) confirms that there are many additional un-measured factors contributing to variation in the self-concept scores (r-squared=0.43) above and beyond mathematical self-efficacy. More work is therefore needed to develop understanding of this unexplained variance, perhaps using fine-grained individual-focussed qualitative methods such as those advocated recently by Pepper and colleagues (2016), although this would obviously be challenging to do at scale. It is also important to consider the nature of the direction of causality between these two measures of mathematical affect, and with attainment (Kung, 2009; Marsh & Martin, 2011).This is an important area for additional research, and for the development of better theoretical understanding.

Influences on field of study (RQ3)

Whilst higher levels of mathematical self-efficacy were generally found for respondents in the Natural and Mathematical sciences (Figure 2), as one might expect for a valid measure (Kane, 2006), the absolute differences between fields of study are quite small. Many mathematically self-efficacious IBDP alumni are choosing to go into a range of degree courses, not just those that are mathematical in nature. Similar findings were discussed in a school mathematics context by Usher and Pajares (2009), and also in Hackett and Betz's (1989) work and that of Parker et al (2014). Although measures of mathematical self-efficacy were statistically significant predictors of science majors in the Hackett and Betz work, there remained considerable unexplained variance in degree choice. In the current context, one could argue that these are generally a high-achieving sample who have many options in terms of which degrees to study, and therefore that it might be a surprise if their choice in this regard were simply predicted based on mathematical self-efficacy/attainment and demographic factors alone.

Moving from mathematical self-efficacy to attainment, a similar effect size (i.e. small) was found for the variation in attainment by field of degree study (Figure 3). Again, it is clear that attainment is not strongly predicting field of study. However, the ordering of fields is as one might expect, giving some additional validity evidence for the scale – the highest attaining in mathematics are more likely to choose to continue studying the subject in HE.

Mathematical self-concept at the time of the survey (Figure 5) showed slightly more variation by field of study compared to the self-efficacy measure (Figure 2), and a different ordering when comparing fields of study. This could in part be due to the experiences respondents were actually having on their degree courses, which might tend to increase any 'baseline' differences – so that students no longer studying any mathematics might be expected to have lower levels of self-concept compared to those still studying mathematics all other things being equal. If this is indeed the case, then one might say that this is tentative additional evidence for the validity of the self-concept measure in particular (Kane, 2006).

The logistic modelling results, predicting field of study in terms of self-efficacy, attainment and demographic factors (Table 2), show that across the Natural sciences, Mathematical sciences and Professions, females are less likely to choose these fields relative to the Humanities, Social sciences and Other fields of study – this echoes the findings of Hackett and Betz (1989) and those of Parker et al. (2014), where both studies found that females were less likely to state a preference or chose science-based majors.

Other important differences are apparent when comparing between fields of study. It is mathematical attainment, rather than retrospective ratings of mathematical self-efficacy, that best predicts progression to the Mathematical sciences, whereas for the Natural sciences, the relative influence of these two factors is reversed. In Parker et al's (2014) study, STEM subjects in higher education were grouped together in the analysis which, according to the results, might lead to important differences in these effects to an extent cancelling each other out.

Overall, however, the relatively low explained variance in the modelling results (pseudo-rsquared=0.14) again shows that unmeasured factors are playing important roles in determining field of study, and that the measured factors are not particularly good on their own in predicting what students will go on to study. By contrast, previous research indicates that self-efficacy levels have high predictive power for degree choice (Hackett & Betz, 1989; Parker et al., 2014). One could speculate that the respondents in the current study are generally high achieving, and so have a wider range of options when it comes to degree choice in comparison to a more typical sample of university entrants.

Study limitations

There are a number of limitations to this quantitative study, mostly related to the socio-cognitive measures employed and the cross-sectional nature of the design. The retrospective use of the mathematical self-efficacy scale to measure this construct at the time of completion of the IBDP is problematic in that respondent recall of particular aspects of self-efficacy could be inaccurate given the time delay involved, typically two years (Nimon, 2014). A better design would have involved using this scale longitudinally: once at the actual time of completion of the IBDP, and once during degree study, but for obvious reasons this was not logistically possible given the time-frame of the study.

It was also not possible to employ diagrammatic examples in this study as part of the self-efficacy measures as originally intended by Pampaka and colleagues (Pampaka et al., 2011; Pampaka & Williams, 2010). This goes against the argument in the literature (Pajares & Miller, 1995) that ratings of mathematical self-efficacy should relate to specific mathematical problems, rather than being based on general topic-level statements. However, the analysis in this study generally indicates that psychometric quality of this self-efficacy measure is good.

Finally, the analysis by field of study is to an extent overly-simplistic – these are broad and varied categories of academic disciplines, and with a larger sample size a more fine-grained analysis that recognises these subtleties would have been possible. There is a similar limitation with the analysis by country. One might expect the individual countries, perhaps with their different mathematical cultures (Larvor, 2016), would require additional country-level analysis that simply wasn't possible with the data generated in this project.

Conclusion

There are ontological and epistemological assumptions behind the use of Likert-scale items/scales to measure subtle, affective traits such as mathematical self-efficacy and self-concept (Maul, 2017; Waring, 2012, Chapter 3). For example, to what extent are such traits stable and distinct over time, and how can these be measured practically at scale for research purposes across international contexts? In a large scale study, this study has shown that discriminating, valid quantitative measures of such socio-cognitive constructs can be employed in a relatively high-performing cohort of students who had taken a well-regarded pre-university mathematics course. This research indicates that the choice to study in particular academic fields for such a group of students is influenced by a range factors of which mathematical attainment and mathematical self-efficacy are merely two aspects: there are a number of additional demographic and other (unmeasured/unmeasurable) factors that are also important. More research is therefore required to develop greater understanding of the interplay between all these factors, and also of how the differences between mathematical self-efficacy and mathematical self-concept might be best conceived, measured and, finally, how these might influence future study and career behaviours.

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Appendix

Mathematical self-efficacy items

TABLE A1 HERE

These are based on Pampaka & Williams' (2010) study.

Mathematical self-concept items

TABLE A2 HERE

These are based on May's (2009) study.

Table with captions

Field of study of degree	**Frequency**	**Percentage**
Professions (e.g. Medicine, Law, Engineering)	206	37.1
Natural sciences (e.g. Biology, Chemistry, Physics)	133	23.9
Social sciences (e.g. Sociology, Political science, Economics)	96	17.3
Mathematical sciences (e.g. Mathematics, Statistics)	65	11.7
Humanities (e.g. Arts, History)	18	3.2
Other	38	6.8
Total	**556**	**100**

Table 1: Field of degree study

Field of study (reference group HS0)	Predictor	B	Std. Error	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
						Lower Bound	Upper Bound
Natural sciences	Intercept	-2.59	0.84	<0.01			
	Mathematical attainment	0.16	0.11	0.13	1.18	0.96	1.45
	Mathematical self-efficacy	0.65	0.27	0.02	1.91	1.12	3.27
	Female	-0.52	0.25	0.04	0.59	0.36	0.97
	Country - Canada	0.63	0.53	0.23	1.88	0.67	5.31
	Country - India	-0.08	0.68	0.90	0.92	0.24	3.51
	Country - Other	0.01	0.35	0.97	1.01	0.51	2.02
	Country - UK	0.29	0.49	0.56	1.33	0.51	3.50
	School type - Private school	-0.51	0.31	0.10	0.60	0.33	1.11
Mathematical sciences	Intercept	-2.71	1.05	0.01			
	Mathematical attainment	0.46	0.14	<0.01	1.58	1.20	2.07
	Mathematical self-efficacy	0.11	0.34	0.75	1.11	0.58	2.16
	Female	-0.94	0.33	0.01	0.39	0.20	0.75
	Country – Canada	-0.53	0.75	0.48	0.59	0.14	2.55
	Country - India	-1.52	0.92	0.10	0.22	0.04	1.32
	Country - Other	-0.99	0.44	0.03	0.37	0.16	0.89
	Country - UK	-1.07	0.66	0.11	0.34	0.09	1.27
	School type - Private school	0.27	0.41	0.51	1.31	0.59	2.91
Professions	Intercept	-1.60	0.75	0.03			
	Mathematical attainment	0.15	0.10	0.13	1.16	0.96	1.39
	Mathematical self-efficacy	0.47	0.25	0.06	1.60	0.99	2.59
	Female	-0.89	0.23	<0.01	0.41	0.26	0.65
	Country - Canada	0.83	0.50	0.10	2.28	0.86	6.03
	Country - India	-0.03	0.57	0.96	0.97	0.32	2.96
	Country - Other	0.01	0.33	0.97	1.01	0.53	1.93
	Country - UK	-1.18	0.58	0.04	0.31	0.10	0.96
	School type - Private school	0.10	0.29	0.73	1.11	0.63	1.96

Table 2: Parameter estimates for predicting field of degree study based on mathematical self-efficacy, attainment and demographics

Item no.	Topic area
When you had finished your DP mathematics HL, how confident were you with the following mathematical topics?	
1	Calculating and estimating
2	Ratio and proportion
3	Manipulating algebraic expressions
4	Proofs/ proving
5	Problem solving
6	Modelling real situations
7	Basic calculus (differentiation/integration)
8	Complex calculus (e.g. differential equations)
9	Statistics
10	Complex numbers
Response format: 1= Not confident at all, 2=Somewhat confident, 3=Confident, 4=Very confident, 5=Don't know	

Table A1: Mathematical self-efficacy items

Item no.	Topic area
Please answer these questions about how you think and feel about your mathematical ability.	
1	I feel confident enough to ask questions in a mathematics class
2	I believe I can do well on a mathematics test
3	I believe I can complete all of the assignments in a mathematics course
4	I believe I am the kind of person who is good at mathematics
5	I believe I will be able to use mathematics in my future career when needed
6	I believe I can understand the content in a mathematics course
7	I believe I can get the highest grade in a mathematics course
8	I believe I can learn well in a mathematics course
9	I feel confident when taking a mathematics test
10	I believe I am the type of person who can do mathematics
11	I feel that I will be able to do well in future mathematics courses
12	I believe I can do the mathematics in a mathematics course
13	I believe I can think like a mathematician
14	I feel confident when using mathematics outside of school/college/university.
Response format: 1= Never, 2=Seldom, 3=Sometimes, 4=Often, 5=Usually	

Table A2: Mathematical self-concept items

Figure captions

Figure 1: Summary of confidence in mathematical topics on completion of DP mathematics HL

Figure 2: Mean mathematical self-efficacy by field of degree study

Figure 3: Mean IBDP mathematics HL grade by field of degree study

Figure 4: Summary of mathematical self-concept items at time of survey

Figure 5: Mean mathematical self-concept by field of degree study

Notes

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[1] At the time of completion of the survey, most respondents (75%) were in the middle of their degrees. The self-efficacy measure is retrospective – looking back approximately two years to the end of their pre-university studies. More details of the precise approach will be given in the methodology section.

[2] See http://www.ibo.org/programmes/diploma-programme/curriculum/

[3] See http://www.ibo.org/programmes/diploma-programme/curriculum/mathematics/mathematics/

[4] May actually refers to her scale as measuring self-efficacy, but given the nature of the items, it is better described as measuring self-concept – see, for example, (Bong & Skaalvik, 2003) for more on the distinction between these two constructs.

[5] https://www.onlinesurveys.ac.uk/

[6] An attempt at Rasch analysis (Bond & Fox, 2007) was also made for the two scales in question but will not be reported on in detail as this proved problematic. This issue will be further discussed in the Limitations section of the Discussion.

[7] An odds ratio of 1 implies no effect of the predictor on field of study. For categorical predictors, an odds ratio greater than 1 implies that students from that group are more likely than the reference group to choose that particular field of study relative to HSO, whilst an odds ratio less than one means these students are less likely to choose it. For continuous predictors, an odds ratio greater than 1 implies higher levels of the predictor are associated with higher likelihood of choosing that field of study, and for odds ratios less than one the effect is reversed.