Proceedings Paper:
Peng, C, Wu, W and Yu, HS (2015) Hypoplastic constitutive model in SPH. In: Computer Methods and Recent Advances in Geomechanics. Computer Methods and Recent Advances in Geomechanics - Proceedings of the 14th Int. Conference of International Association for Computer Methods and Recent Advances in Geomechanics, IACMAG 2014, 24-25 Sep 2014, Kyoto; Japan. Taylor & Francis Group , pp. 1863-1867. ISBN 978-1-138-00148-0

Hypoplastic constitutive model in SPH

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ABSTRACT: The Smoothed Particle Hydrodynamics (SPH) is a pure Lagrangian method mainly used in Astrophysics and Computational Fluid Dynamics (CFD). Owing to features it poses, SPH is an attractive method for analyzing large deformation problems in Geotechnical Engineering if the physical process can be modeled correctly, which calls for appropriate constitutive models. In this paper, the feasibility of implementing hypoplastic model in SPH is investigated. A simple hypoplastic model is employed to simulate the non-linear stress-strain behaviour of geomaterials. Instead of using equation of state, in this study stresses are computed directly through hypoplastic constitutive equation, thus giving rise to decoupling of density and stress computation. Strain rate needed by the constitutive model is evaluated using SPH integral interpolation. The discretized governing equations are solved marching forward in time by Verlet integration. The problem of a column of sand subjected to gravity is simulated to check the capability of the proposed SPH application in geomechanics.

1 INTRODUCTION

Geo-hazards like landslide and debris flow lead to a huge loss of human lives and properties. Numerical simulation of these phenomenons help us understand their physical processes and the design of protection and prevention system. However, these phenomenons usually involve large deformation, which is still a challenge for the most widely used finite element method (FEM), because of the distortion of computational mesh. Smoothed Particle Hydrodynamics (SPH) is a pure Lagrangian method, in which material is represented by a set of irregular particles carrying field variables (i.e., position, mass, density, velocity, stress and strain) and moving with material velocity (Monaghan 1994, Gomez-Gesteira, Rogers, Dalrymple, & Crespo 2010, Liu & Liu 2010). Since there is no mesh, by an updated Lagrangian scheme, SPH can handle large deformation conveniently. Free surface and material interface can also be tracked naturally in SPH. Therefore, SPH provides an useful tool for the numerical simulation of problems involving large deformation.

To achieve reasonable results, appropriate constitutive models which describe the mechanical properties of geomaterials should be implemented. Conventionally, SPH computes pressure from an equation of state based on density variation (Monaghan 1994). Although this yield good results for fluid, it is obviously oversimplified for soils. In previous researches, simplified models like depth integration (Pastor, Haddad, Sorbino, Cuomo, & Drempetic 2009) or Bingham fluid model (Zhu, Martys, Ferraris, & Kee 2010, Hérault, Bilotta, Vicari, Rustico, & Del Negro 2011) were used to simulate the movement of debris flow and lava. Bui and his coworkers (Bui, Fukagawa, Sako, & Ohno 2008, Bui, Fukagawa, Sako, & Wells 2010) are the first to implement real soil constitutive model within the framework of SPH. The Drucker-Prager elastoplastic model with associated and non-associated flow rule is adapted to model the large deformation in soil flow. The same approach is adopted in the frictional contact algorithm proposed by Wang (Wang & Chan 2013) to compute stresses in soil.

Hypoplastic constitutive models are based on non-linear tensorial functions. Unlike other complex models in geomechanics, hypoplastic models do not have complicated concepts such as yield surface, plastic potential, flow rule or strain decomposition, thus its theory and implementation in numerical methods are simpler. Despite the simplicity, hypoplastic models are capable of capturing the salient behaviours of soil. Due to the method used to discretize governing equations, impementation of constitutive models in SPH is not as straightforward as in FEM. Therefore, simpler models are more attractive to the application of SPH.

To this end, in this paper a simple hypoplastic constitutive model (Wu & Bauer 1994) is implemented in SPH code to solve problems in geomechanics.

2 SPH FORMULATIONS

2.1 *SPH integral interpolation*

In SPH, problem domain is discretized by a set of particles, which carry physical properties and move with material. Physical variables on each particle are computed by an integral interpolation process over its neighbouring particles. By integral interpolation, a field function $f(\boldsymbol{x})$ is approximated by

$$f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')W(\boldsymbol{x}-\boldsymbol{x}', h)\mathrm{d}v \tag{1}$$

where W is a weighting function called smoothing kernel and h the smoothing length. Ω is the integration domain whose size is dependent on h, which determines a spherical support domain surrounding $\boldsymbol{x}$. A variety of weighting functions are proposed in literature. Here the most popular cubic spline function is applied, which has the following form

$$W = \alpha_D \begin{cases} 1 - 1.5q^2 + 0.75q^3 & 0 \le q < 1 \\ 0.25(1-q)^3 & 1 \le q < 2 \\ 0 & q \ge 2 \end{cases} \tag{2}$$

where α_D is a normalization factor, whose value is $10/(7\pi h^2)$ in 2D and $1/(\pi h^3)$ in 3D; q is the normalized distance defined as $q = r/h$ and r is the distance between $\boldsymbol{x}$ and $\boldsymbol{x}'$. It can be seen that the smoothing kernel is compactly supported, thus outside of the support domain the value of the weighting function is zero. Summing up the continuous integration over particles in the support domain, Eq. (1) can be written out in the following discretization form:

$$\langle f(\boldsymbol{x})\rangle = \sum_{j=1}^{n} f(\boldsymbol{x}_j)W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)m_j/\rho_j \tag{3}$$

where n is the number of particles in the support domain of kernel W_{ij} centered at $\boldsymbol{x}_i$, and m_j/ρ_j is the element volume of particle j. Following the same way, the differentiable interpolation of the field function $f(\boldsymbol{x})$ can be constructed by

$$\langle \nabla f(\boldsymbol{x}_i)\rangle = \sum_{j=1}^{n} f(\boldsymbol{x}_j)\nabla W_{ij} m_j/\rho_j \tag{4}$$

where ∇W_{ij} is the gradient of smoothing kernel W_{ij} at particle $\boldsymbol{x}_i$. Monaghan (Monaghan 1992) recommended to use the following form of gradient computation instead the above one

$$\langle \nabla f(\boldsymbol{x}_i)\rangle = \sum_{j=1}^{n} (f(\boldsymbol{x}_j) - f(\boldsymbol{x}_i))\nabla W_{ij} m_j/\rho_j \tag{5}$$

which gives more accurate results.

2.2 *Governing eqautions in SPH*

By the above integral interpolation the governing equations of soil movement can be expressed within the SPH framework. The mass and momentum conservation equations are as follows:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \mathrm{div}(\boldsymbol{v}) \tag{6}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{1}{\rho}\nabla\boldsymbol{\sigma} + \boldsymbol{f} \tag{7}$$

where ρ is the soil density; $\boldsymbol{v}$ stands for the velocity and $\boldsymbol{\sigma}$ the stress tensor of soil particle; $\boldsymbol{f}$ is the acceleration caused by external force, usually it is gravity in geomechanics. Compression is defined as positive in this paper.

Applying Eq (5), the velocity gradient of a particle i can be written as

$$\nabla \boldsymbol{v}_i = \sum_{j=1}^{n} (\boldsymbol{v}_j - \boldsymbol{v}_i)\nabla W_{ij} m_j/\rho_j \tag{8}$$

Applying Eq (3), (5) and (8) and employing some mathematical transformations, the SPH discretized governing equations can be derived as follows:

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \sum_{j=1}^{n} m_j(\boldsymbol{v}_i - \boldsymbol{v}_j)\cdot\nabla W_{ij} \tag{9}$$

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \sum_{j=1}^{n} m_j\left(\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} - \Pi_{ij}\mathbf{I}\right)\nabla W_{ij} \tag{10}$$

where Π_{ij} is a dissipative term introduced to avoid large unphysical oscillations and shocks, $\mathbf{I}$ is a identity matrix. The most widely used method is adding a artificial viscosity term, which helps to damp out oscillation and improve stability

$$\Pi_{ij} = \begin{cases} -\dfrac{\alpha h \bar{c}_{ij}}{\bar{\rho}_{ij}} \dfrac{\boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij}}{\boldsymbol{r}_{ij}^2 + \eta^2} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{11}$$

In the above equation, α is a user defined constant; $\overline{c}_{ij} = (c_i + c_j)/2$ is the average speed of sound in soil, which lies in the range 450-600 m/s; $\overline{\rho}_{ij} = (\rho_i + \rho_j)/2$ is the average density; and $\boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j$ is the relative velocity between particle i and j; η is a small value introduced to avoid singularity.

A constitutive equation is imperative to close the governing equations Eq. (9) and (10), that is, we need a method to compute the stress tensor in Eq. (10). When applied in CFD the stress tensor $\boldsymbol{\sigma}$ is usually divided into two parts: an hydrostatic pressure p and a deviatoric shear stress $\boldsymbol{\sigma}^\star$. Conventionally, hydrostatic pressure p is computed by equation of state, which is an observed relationship between pressure and density in fluid (Monaghan 1994); while the effect of deviatoric shear stress $\boldsymbol{\sigma}^\star$ is simulated either by artificial viscosity (Monaghan 1994) or fluid viscosity (Takeda, Miyama, & Sekiya 1994). Such approach is also used in some SPH application to solid (Libersky & Petschek 1991). In (Bui, Fukagawa, Sako, & Ohno 2008) Bui computed the stress tensor directly from a elastoplastic model, which generates more accurate and realistic results for soil computation. In this work the simpler hypoplastic constitutive model is used to compute stress rate from velocity field directly.

3 HYPOPLASTIC CONSTITUTIVE MODEL IN SPH

A simple hypoplastic model proposed by Wu (Wu & Bauer 1994) is employed in this paper to describe the nonlinear soil behaviour. The formulation of this model is simple thus it can be implemented in SPH easily. The formulations and it's implementation in SPH will be presented in this section.

3.1 *Hypoplastic constitutive equation*

Let the motion of a soil body be described by $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X}, t)$, then the material velocity can be expressed as $\boldsymbol{v} = \dot{\boldsymbol{x}}$. The strain rate and spin tensors are computed through velocity gradient as follows:

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}[(\nabla \boldsymbol{v}) + (\nabla \boldsymbol{v})^{\mathrm{T}}] \tag{12}$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{2}[(\nabla \boldsymbol{v}) - (\nabla \boldsymbol{v})^{\mathrm{T}}] \tag{13}$$

where the superscript T denotes a transposition. The Jaumann stress rate which is invariant with respect to rigid body rotation, is employed:

$$\mathring{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}\boldsymbol{\sigma} \tag{14}$$

The hypoplastic constitutive model is defined as a tensorial function $\mathbf{H}$:

$$\mathring{\boldsymbol{\sigma}} = \mathbf{H}(\boldsymbol{\sigma}, \dot{\boldsymbol{\varepsilon}}) \tag{15}$$

It is required that function $\mathbf{H}$ is not differentiable in and only in $\dot{\boldsymbol{\varepsilon}} = 0$. The constitutive function is assumed to be positively homogeneous of the first order in $\dot{\boldsymbol{\varepsilon}}$ and homogeneous in $\boldsymbol{\sigma}$, according to general principals of continuum mechanics and experimental observations. The function should also fulfill the condition of objectivity:

$$\mathbf{H}(\boldsymbol{Q}\boldsymbol{\sigma}\boldsymbol{Q}^{\mathrm{T}}, \boldsymbol{Q}\dot{\boldsymbol{\varepsilon}}\boldsymbol{Q}) = \boldsymbol{Q}\mathbf{H}(\boldsymbol{\sigma}, \dot{\boldsymbol{\varepsilon}})\boldsymbol{Q}^{\mathrm{T}} \tag{16}$$

in which $\boldsymbol{Q}$ is an orthogonal tensor. If the constitutive function $\mathbf{H}$ is constructed by choosing terms from representation theorems for isotropic tensorial functions, the requirement of objectivity can be fulfilled automatically.

Following these restrictions, a specific version of hypoplastic constitutive equation is proposed by Wu (Wu & Bauer 1994):

$$\mathring{\boldsymbol{\sigma}} = c_1(\mathrm{tr}\boldsymbol{\sigma})\dot{\boldsymbol{\varepsilon}} + c_2\frac{\mathrm{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\varepsilon}})}{\mathrm{tr}\boldsymbol{\sigma}}\boldsymbol{\sigma} + (c_3\frac{\boldsymbol{\sigma}^2}{\mathrm{tr}\boldsymbol{\sigma}} + c_4\frac{(\boldsymbol{\sigma}^\star)^2}{\mathrm{tr}\boldsymbol{\sigma}}) \parallel \dot{\boldsymbol{\varepsilon}} \parallel \tag{17}$$

where c_i $(i = 1, \cdots, 4)$ are dimensionless material parameters. The $\boldsymbol{\sigma}^\star$ is the deviatoric stress tensor and $\parallel \dot{\boldsymbol{\varepsilon}} \parallel = \sqrt{\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^2)}$ stands for the Euclidean norm.

It can be observed that the constitutive function in Eq. (17) consists of linear part and nonlinear part. The first two terms are linear in $\dot{\boldsymbol{\varepsilon}}$, and the last term is nonlinear, due to the non-differentiable $\parallel \dot{\boldsymbol{\varepsilon}} \parallel$. Together the constitutive equation is incrementally nonlinear, and can be used to describe the behaviour of soil. Loading and unloading are not explicitly defined in Eq. (17), they are implied by the constitutive equation. Concepts in elastoplastic theory such as yield surface and flow rule are not predefined in hypoplastic model, they can be derived from Eq. (17) as by-products. Furthermore, there is no need to decompose deformation into elastic and plastic parts, which makes its implementation in numerical methods much simpler.

The four parameters in the hypoplastic constitutive model can be identified with a single triaxial compression test under confining pressure.

3.2 *Implementation in SPH*

The implementation of constitutive equation Eq. (17) in SPH is quite straightforward. For a given particle i its stress-strain relationship can be written as

$$\frac{\mathrm{d}\boldsymbol{\sigma}_i}{\mathrm{d}t} = \dot{\boldsymbol{\omega}}_i \boldsymbol{\sigma}_i - \boldsymbol{\sigma}_i \dot{\boldsymbol{\omega}}_i + c_1 (\mathrm{tr}\boldsymbol{\sigma}_i)\dot{\boldsymbol{\varepsilon}}_i + c_2 \frac{\mathrm{tr}(\boldsymbol{\sigma}_i \dot{\boldsymbol{\varepsilon}}_i)\boldsymbol{\sigma}_i}{\mathrm{tr}\boldsymbol{\sigma}_i} + (c_3 \frac{\boldsymbol{\sigma}_i^2}{\mathrm{tr}\boldsymbol{\sigma}_i} + c_4 \frac{(\boldsymbol{\sigma}_i^\star)^2}{\mathrm{tr}\boldsymbol{\sigma}_i}) \parallel \dot{\boldsymbol{\varepsilon}}_i \parallel \quad (18)$$

It is seen from Eq. (18) that the stress rate at particle i is only dependent on its current stress state $\boldsymbol{\sigma}_i$ and deformation rate $\dot{\boldsymbol{\varepsilon}}_i$ and $\dot{\boldsymbol{\omega}}_i$. The strain $\dot{\boldsymbol{\varepsilon}}_i$ and spin rate tensor $\dot{\boldsymbol{\omega}}_i$ can be obtained through SPH discretization, by applying Eq. (8).

Eq. (9), (10) and (18) consist the complete system of equations for numerical simulation. Since in the constitutive model employed here density is not taken into consideration, the density and stress computation are decoupled. Therefore, Eq. (9) is not always necessary and can be omitted if density change in soil is not our concern.

To solve the system of equations, Verlet integration scheme is adapted to march forward in time. This scheme is used to discretize Eq. (9), (10) and (18) in time. In general, variables at particle i are computed according to

$$\begin{cases} \boldsymbol{v}_i^{n+1} = \boldsymbol{v}_i^{n-1} + 2\Delta t \boldsymbol{A}_i^n \\ \boldsymbol{\rho}_i^{n+1} = \rho_i^{n-1} + 2\Delta t D_i^n \\ \boldsymbol{x}_i^{n+1} = \boldsymbol{x}_i^n + \Delta t \boldsymbol{v}_i^n + 0.5\Delta t^2 \boldsymbol{A}_i^n \\ \boldsymbol{\sigma}_i^{n+1} = \boldsymbol{\sigma}_i^{n-1} + 2\Delta t \boldsymbol{S}_i^n \end{cases} \quad (19)$$

where n, $n-1$, $n+1$ indicate the time step; $\boldsymbol{A}$, D and $\boldsymbol{S}$ are the acceleration, density rate and stress rate, respectively. Variable time step (Gomez-Gesteira, Rogers, Dalrymple, & Crespo 2010) is employed in this study, with a fixed speed of sound $c = 600\mathrm{m/s}$ in soil.

Rigid and non-slip boundary condition is used in the numerical simulation. In computation only at soil particle Eq. (9), (10) and (18) is evaluated. If in the influence domain of soil particle i there are boundary particles, then the velocities and stresses of boundary particle are approximated by the velocity and stress of particle i. The stresses at the boundary particles is assigned the same stress at soil particle i, while the velocity is computed by

$$\boldsymbol{v}_j^{\mathrm{B}} = (1-\beta)\boldsymbol{v}_i + \beta \boldsymbol{v}^{\mathrm{B}} \quad (20)$$

In the above equation $\boldsymbol{v}_j^{\mathrm{B}}$ is the artificial velocity at boundary particle j, which is only used to evaluate Eq. (9), (10) and (18) at particle i; $\boldsymbol{v}^{\mathrm{B}}$ is the rigid velocity of the boundary; β is a coefficient computed by the relative location of the soil particle i and the boundary particle j (Bui, Fukagawa, Sako, & Ohno 2008).

4 NUMERICAL RESULTS

A very simple numerical simulation is performed to preliminarily verify the implementation of hypoplastic constitutive model in SPH. A column of sand subjected to gravity is simulated. Plain-strain condition is considered. The sand is loose Karlsrule sand, which has a density of $1.85\mathrm{g/cm^3}$. The hypoplastic constitutive model introduced in section 3.1 is used, with constitutive parameters $c_1 = -69.4$, $c_2 = -673.1$, $c_3 = -655.9$, $c_2 = 699.6$. The width and height of the column of sand is 0.5m and 1.0m, respectively. The bottom, left and right side of the column is subjected to rigid non-slip boundary, which is modeled by three layers of boundary particles.

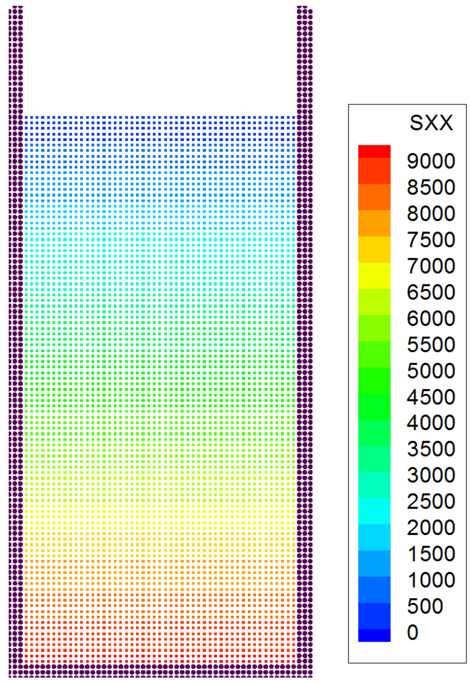

Figure 1: The computed distribution of horizontal stress σ_{xx} in the column of sand.

Initially the soil particles are assigned a vertical stress by $\sigma_{zz} = \rho g h$, while other stress components remain zero, which will be computed by the proposed SPH method. Figure 1 gives the computed distribution of horizontal stress. Along the center of the column the vertical and horizontal stresses at different height are given in Figure 2.

The ratio between σ_{xx} and σ_{zz}, which is soil pressure coefficient K_0 at rest, is estimated having a value of 0.51 according to Figure 2. In the paper by Wu (Wu & Bauer 1994), a method to compute K_0 through hypoplastic constitutive parameters c_1 to c_4, was proposed

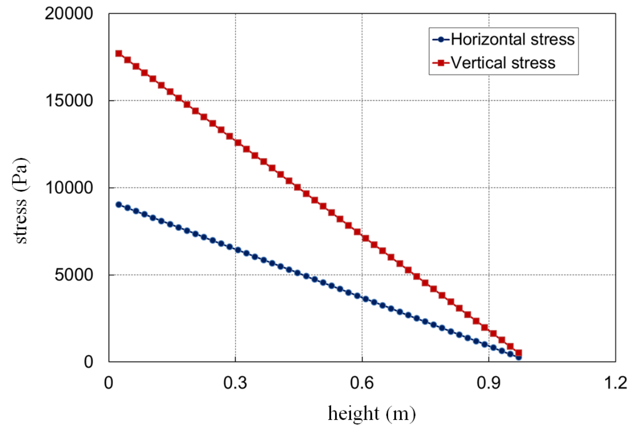

Figure 2: The variation of vertical and horizontal stress at different height.

$$(36c_1 - 4c_4)K_0^3 + (36c_1 + 9c_3 + 9c_4)K_0^2 + (9c_1 - 9c_3 - 6c_4)K_0 + c_4 = 0 \quad (21)$$

By this equation, it is obtained that $K_0 = 0.5102$, which is in good agreement with the result obtained by SPH.

5 CONCLUSIONS

In this paper, the feasibility of implementing hypoplastic constitutive model in Smoothed Particle Hydrodynamics is investigated. A simple hypoplastic model proposed by Wu (Wu & Bauer 1994) is adapted within the framework of SPH. The discretization of soil's governing equation is demonstrated. The hypoplastic model is used to compute stress rate directly from the velocity field. Owing to the simplicity of hypoplastic model, complicated concepts like yield surface, decomposition of deformation, are not needed. No additional state variable is needed to record load history. These conveniences make the programming easier and the computation less time consuming.

A numerical example is carried out to verify the proposed method. Good agreement between numerical results and model prediction is shown. Considering the potential advantages of SPH in the simulation of large deformation problems, the proposed method provides an attractive approach to the numerical analyses in geomechanics.

REFERENCES

Bui, H., R. Fukagawa, K. Sako, & J. Wells (2010). Slope stability analysis and discontinuous slope failure simulation by elasto-plastic smoothed particle hydrodynamics (sph). *Geotechnique 61*(7), 565–574.

Bui, H. H., R. Fukagawa, K. Sako, & S. Ohno (2008). Lagrangian meshfree particles method (sph) for large deformation and failure flows of geomaterial using elastic–plastic soil constitutive model. *International Journal for Numerical and Analytical Methods in Geomechanics 32*(12), 1537–1570.

Gomez-Gesteira, M., B. D. Rogers, R. A. Dalrymple, & A. J. Crespo (2010). State-of-the-art of classical sph for free-surface flows. *Journal of Hydraulic Research 48*(S1), 6–27.

Hérault, A., G. Bilotta, A. Vicari, E. Rustico, & C. Del Negro (2011). Numerical simulation of lava flow using a gpu sph model. *Annals of Geophysics 54*(5).

Libersky, L. D. & A. Petschek (1991). Smooth particle hydrodynamics with strength of materials. In *Advances in the free-Lagrange method including contributions on adaptive gridding and the smooth particle hydrodynamics method*, pp. 248–257. Springer.

Liu, M. & G. Liu (2010). Smoothed particle hydrodynamics (sph): an overview and recent developments. *Archives of computational methods in engineering 17*(1), 25–76.

Monaghan, J. J. (1992). Smoothed particle hydrodynamics. *Annual review of astronomy and astrophysics 30*, 543–574.

Monaghan, J. J. (1994). Simulating free surface flows with sph. *Journal of computational physics 110*(2), 399–406.

Pastor, M., B. Haddad, G. Sorbino, S. Cuomo, & V. Drempetic (2009). A depth-integrated, coupled sph model for flow-like landslides and related phenomena. *International Journal for Numerical and Analytical Methods in Geomechanics 33*(2), 143–172.

Takeda, H., S. M. Miyama, & M. Sekiya (1994). Numerical simulation of viscous flow by smoothed particle hydrodynamics. *Progress of Theoretical Physics 92*(5), 939–960.

Wang, J. & D. Chan (2013). Frictional contact algorithms in sph for the simulation of soil–structure interaction. *International Journal for Numerical and Analytical Methods in Geomechanics*.

Wu, W. & E. Bauer (1994). A simple hypoplastic constitutive model for sand. *International Journal for Numerical and Analytical Methods in Geomechanics 18*(12), 833–862.

Zhu, H., N. S. Martys, C. Ferraris, & D. D. Kee (2010). A numerical study of the flow of bingham-like fluids in two-dimensional vane and cylinder rheometers using a smoothed particle hydrodynamics (sph) based method. *Journal of Non-Newtonian Fluid Mechanics 165*(7), 362–375.