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# FUZZY SLIDING MODE CONTROL OF A MULTI-DOF PARALLEL ROBOT IN REHABILITATION ENVIRONMENT \*

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Multi-DOF (degrees of freedom) parallel robot, due to its compact structure and high operation accuracy, is a promising candidate for medical rehabilitation devices. However, its controllability relating to the nonlinear characteristics challenges its interaction with human subjects during the rehabilitation process. In this paper, we investigated the control of a parallel robot system using fuzzy sliding mode control (FSMC) for constructing a simple controller in practical rehabilitation, where a fuzzy logic system was used as the additional compensator to the sliding mode controller (SMC) for performance enhancement and chattering elimination. The system stability is guaranteed by the Lyapunov stability theorem. Experiments were conducted on a lower limb rehabilitation robot, which was built based on kinematics and dynamics analysis of the 6-DOF Stewart platform. The experimental results showed that the position tracking precision of the proposed FSMC is sufficient in practical applications, while the velocity chattering had been effectively reduced in comparison with the conventional FSMC with parameters tuned by fuzzy systems.

*Keywords:* Multi-DOF parallel robot; fuzzy sliding mode control (FSMC); rehabilitation.

## 1. Introduction

It is evident that there is a strong trend that our society is fast-aging than expected. According to the official statistical data from the United Nations, the proportion of the world's population over 60 will be doubled from 11% to 22% between 2000 and 2050. Meanwhile, population of this age will increase from 605 million to 2 billion. Nowadays, 54% of the elderly in the world live in Asia and 22% in Europe, where many countries have gradually entered the aged society.<sup>1</sup> With the tendency of aged society, there is a considerable concern in the needs of health care and rehabilitation, especially for elderly and disabled people. Additionally, research on rehabilitation robots is an interdisciplinary area where medicine technologies are combined with the biomechanics, mechanics,

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electronics, and robotics, etc. It has become an outstanding field that attracts more and more attentions recently.<sup>2</sup>

Rehabilitation robots are a frequent topic of discussion when people try to find out an effective approach to assist the disabled or older ones. Especially in recent years, some institutions have developed various rehabilitation robots including those for both upper and lower limbs. Meanwhile, it has been widely acceptable that a robot-assisted system follows a basic principle of keeping its structure simple, lightweight, as well as easy to control.<sup>3</sup> However, current lower limb rehabilitation products in the market tend to have a relatively single function and limited application range.<sup>4</sup> Although the existing robots are able to provide patients with multi-DOF auxiliary, a majority of them are designed without guidance of medical theories. As a result, not all of the robots' movements can accurately follow the body motion law. Furthermore, most traditional rehabilitation robots are serial robots, wearable robots or the exoskeletons.<sup>5</sup> The inverse kinematic solutions of these robots are complicated, which are not easy to control. In addition, due to the inflexible mechanical mechanism, exoskeleton robots have fixed structure without movement range and expansibility. This problem is particularly evident in the lower extremity rehabilitation, because the movement of lower limb is more complicated. On the other hand, due to advantages of its superior stiffness, compact structure, and operation accuracy, the parallel platform has been widely used as the carrier of flight simulators and telescopes. It is reasonable that the parallel robots have full capability of producing a feasible solution for rehabilitation with simple structure and superior adaptability, enabling the training to be adjustable to different patients.

The Stewart platform is a kind of typical parallel robot with 6-DOF. As a multi-input multi-output (MIMO) and nonlinear system, the parallel robot's position and orientation are the results of six actuators' interaction and coordination.<sup>6</sup> In order to improve the trajectory tracking performance, many intelligent control methods for parallel robots have been proposed.<sup>7</sup> Sliding mode control (SMC) is one of the effective, nonlinear, robust control approaches. Since it is able to provide the system with an invariance property to uncertainties once the system is controlled in the sliding mode.<sup>8,9</sup> However, notice that the control law satisfying sliding condition has high frequency oscillations when across the sliding surface, which is defined as chattering. Several chattering reduction methods have been reported.<sup>10</sup> These approaches, however, provide no guarantee of convergence to the sliding mode and involve a tradeoff between chattering and robustness. Due to its inherent advantages of good robustness and nonlinear characteristics, fuzzy method is a potentially effective approach to control objectives with time-varying and nonlinear parameters. By combining the attractive features of fuzzy logic system with SMC, the controller can cope well with severe uncertainties without sacrificing robust performance.

During the training process, the rehabilitation robotic system may become unstable due to the uncertain variations of the system loads and dynamic parameters, which may lead to patient's secondary injury.<sup>11</sup> Therefore, it is of great significance to maintain the trajectory tracking accuracy and stability for medical purpose. Nowadays, a majority of research on parallel robot control algorithm is proposed based on theoretical analysis or

experimental simulation. In very few occasions, these research algorithms can be applied to the actual rehabilitation robots.<sup>12</sup> In this paper, based on the real-time feedback of position and velocity of the each robot joint, a closed-loop control strategy using fuzzy sliding mode algorithm for lower limb rehabilitation applications is proposed, which is realized on the control cabinet integrated with digital signal processors (DSPs).

This paper is organized as follows: Section 2 summarizes the related works. After studying the kinematics and dynamic models of 6-DOF parallel robot in Section 3, a conventional sliding mode controller for a 6-DOF parallel robot is designed in Section 4. To improve the tracking performance and avoid the chattering problem, a fuzzy sliding mode controller is proposed in Section 5, and its stability is verified via Lyapunov theorem. Practical experiments based on rehabilitation robot are conducted in Section 6. Finally, conclusion and future works are presented in Section 7.

## 2. Related Works

It has been proven by many medical theories and clinical rehabilitation practices that appropriate training strategies after stroke or surgery would be of important significance to recover the nervous system, as well as prevent the muscle atrophy. Rehabilitation robots not only be able to precisely control the motion of patients following a desired trajectory, but also record the patients' information during the treatment process, which can be used to get a clear evaluation of health conditions. Nowadays, there exist a wide variety of medical robots that differ in mechanical structures, control strategies, or even the target users. Especially in recent years, various rehabilitation robots including those for both upper and lower limbs have been developed by many institutions.<sup>13</sup> In terms of lower limb rehabilitation, the Lokomat is a typical training robot, which is designed in Switzerland to help the spine damaged or stroke patients recover their lower limbs' function by using the weight support and repeated training methods.<sup>14</sup> Moreover, the Rutgers Ankle developed by Rutgers University is an ankle rehabilitation system based on the Stewart platform.<sup>15</sup> The system consists of cylinders, position sensors and a force sensor, where an embedded computer is used as the controller. Another example of lower limb rehabilitation robot is the HAL in Japan that can implement active control from patients,<sup>16</sup> which is able to drive the actuator to assist limb movement by mapping the EMGs to lower joint angles. Harbin Institute of Technology in China also develops a rehabilitation robot for lower extremity to stimulate the normal walking trajectories.<sup>17</sup>

The mechanical design of the robot is the basis of robot-assisted rehabilitation system. In general, rehabilitation robots can be divided into two categories: exoskeletons and end-effector robots.<sup>18</sup> The exoskeleton robots usually have to be fixed to the patient's body, and pose force/torque on different parts of the body simultaneously. However, such robots would be not necessarily good for functional rehabilitation of patients, due to their drawbacks of inferior adaptability to diverse patients, and the design of exoskeletons also is very expensive and time-consuming. On the contrary, the end-effector robots usually contact with the patient body at a certain point, such as the foot, making this kind of robot easy to design. Since there is no restriction on the movement redundancy, end-effector

robots are more adaptable to different patients. For example, typical exoskeletons include Lokomat, HAL, and the end-effectors consist of MIT-Manus, MIME, Gentle/s, etc.<sup>19</sup>

Parallel robot, a typical class of end-effector robots, is a promising candidate for medical rehabilitation devices. Actually, due to the compact structure and high operation accuracy, there has been a growing trend that parallel robots can be used for lower limb rehabilitation. Xie et al. proposed a 4-axis redundant parallel robot based on the modeling and kinematics of ankle anatomy.<sup>20</sup> Hesse et al. designed a virtual reality based 6-DOF foot motion simulation called Haptic Walker, which allowed the foot to follow an arbitrary trajectory and orientation.<sup>21</sup> Zhu et al. proposed a new parallel rehabilitation robot based on the workspace and kinematical simulation. The University of Auckland designed a 6-DOF flexible parallel robot to perform the long bone surgery.<sup>22</sup> Moreover, Rutgers Ankle was a Stewart platform designed for lower limb rehabilitation, which had been used in the real environment allowing the patient's ankle move in six degrees of freedom and throughout the ankle's full range of motion.<sup>23</sup>

When robot interacts with patients in a rehabilitation environment, load disturbances are the main external disturbances of the parallel robots, and always impact on system performance greatly. In order to improve the trajectory tracking performance of parallel robots, many intelligent control methods have been proposed. Neural network methods have features of fault tolerance, adaptability and self-learning ability, and the controller does not require accurate mathematical model of the robot.<sup>24</sup> The current neural networks tend to be realized based on simplified robot model, and reduce the coupling effects between branches by combining them with fuzzy or adaptive control methods.<sup>25</sup> However, it is difficult to establish the precise fuzzy rules of the whole control system, especially the complex system such as parallel robots. And furthermore, fuzzy method must be combined with other control strategies in order to overcome the shortcomings of its own. Wu et al. established a dynamic model of the Stewart platform, and designed a new type fuzzy adaptive controller based on computed torque method, to overcome the adverse effects caused by parameter variations and disturbances.<sup>26</sup> Since these methods will be greatly impacted by the control response and the robot modeling independent factors, so there is a certain restriction when putting them into practical applications.

Sliding mode control (SMC) is one of the effective nonlinear robust control methods. Since it is able to provide the system with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode surface.<sup>27</sup> Kim and Lee proposed a sliding mode method to achieve high-speed trajectory tracking control of the Stewart platform manipulator, and utilized the DSPs to solve the dynamics online computation problems.<sup>28</sup> However, the complete dynamics model of 6-DOF parallel robot is very complicated, so the real-time computation is difficult to achieve even using high speed DSPs. Wu presented a nonlinear adaptive sliding mode controller for a six DOF flight simulator,<sup>29</sup> where the adaptive controller was utilized to identify the constant parameters while the sliding mode unit is used to deal with time-varying uncertain parameters. Many experimental results have shown that SMC has significantly improved the accuracy and achieved better performances comparing with the traditional controllers.<sup>30</sup> However, it is

noticed that the control law satisfying sliding condition has high frequency oscillations when across the sliding surface, which is defined as chattering. Chattering is an undesired phenomenon, and one of those chattering reduction approaches places a boundary layer around the surface such that the relay control is replaced by a saturation function.

The utilization of saturation function can reduce the chattering to some extent, but this method brings obvious drawbacks. On one hand, it is difficult to determine the parameters of saturation functions; on the other hand, this approach will inevitably affect the accuracy and stability of sliding mode controller. Gao et al. proposed a chattering reduction method by introducing the concept of reaching law method. Wang et al. further developed this theory by using discrete reaching law, and presented a practical discrete variable structure controller for parallel robot trajectory control.<sup>31</sup> Pi et al. proposed a sliding mode controller based on successive approximation method and fuzzy rules, to solve the trajectory tracking control problem of 6-DOF parallel robot under conditions of load disturbances.<sup>32</sup> These approaches, however, provide no guarantee of convergence to the sliding mode surface and involve a tradeoff between chattering and robustness. To tackle these difficulties, fuzzy SMC (FSMC) has also been used for this purpose, which is shown to be quite effective. Fuzzy adaptive method has been an active research topic in automation and control for its inherent advantages of being independent on mathematical model, as well as possessing good robustness and nonlinear characteristics.<sup>33</sup>

Kaynak et al.<sup>34</sup> presented a thorough survey on how computationally intelligent systems can be incorporated in the SMCs to improve their performance in practical implementations. The use of sliding mode system in design and stability analysis of fuzzy controllers was also discussed. The integration of the fuzzy logic system in an SMC can be found in many examples. Amer et al.<sup>8</sup> adopted an adaptive fuzzy sliding mode control approach with a PID sliding surface in the simulation of a 3-DOF robot manipulator, where the output gain of the fuzzy sliding mode control was tuned by a supervisory fuzzy system. An adaptive terminal sliding mode controller for a robotic system using fuzzy wavelet networks was presented by Lin et al.<sup>35</sup>, in which the parameters of dilation and translation of fuzzy wavelet basis functions and weights must also be on-line tuned. The combination of fuzzy control and SMC approach has gained great interests and much research has been done. However, among researches on adaptive fuzzy sliding mode control, only a few consider the computational requirement and real-time characteristics of the control algorithm. To address this problem, Qi et al.<sup>36</sup> designed a fuzzy sliding mode controller to reduce the chattering in trajectory tracking of a 4-DOF parallel robot, one of the main contributions of which is making the design simple and less fuzzy rules required. Furthermore, a model-free reference adaptive fuzzy sliding mode controller was proposed to control a 5-DOF robot by Chiou et al.<sup>37</sup>, where a boundary layer function was introduced to eliminate the chattering and the rules number of FLC is reduced by using the filter errors instead of the state variables.

In practical applications, the controllers have to deal with complex systems, which may have multiple variables and parameters with nonlinear coupling. The conventional approaches based on analytical and emulational techniques can prove to be inadequate,

which is perhaps too inflexible or too complicated to cope with the special requirements of the real world systems<sup>34</sup>. Chou et al.<sup>38</sup> proposed a digital signal processor (DSP)-based complementary sliding mode control with fuzzy neural network compensator for the control of a dual linear motor servo system. In Ref. 9, a sliding mode control with discontinuous adaptation law was proposed to improve the tracking performance of a 6-DOF hydraulic parallel robot, where the load disturbances are directly measured by force sensors. When dealing with such systems, it has to face a certain degree of tolerate imprecision, because in this context, trying to increase precision can be very costly. For instance, a cascade-control algorithm based on sliding mode was proposed by Guo et al.<sup>27</sup> to realize the trajectory tracking control of a hydraulically driven 6-DOF parallel robot manipulator, and both mechanical and hydraulic dynamics of the manipulator were taken into account. However, the control loops were composed of several parts with heavy computation burden and thus there is an obvious time delay in the position control results. In addition, Erbatur et al.<sup>39</sup> proposed the combination of adaptive fuzzy systems with SMCs to solve the chattering problem, where on-line tuning of parameters by fuzzy rules was carried out for the SMC, and the method is tested on a direct-drive robot. A fuzzy logic system can utilize the qualitative knowledge for designing a practical controller. By combining the attractive features of fuzzy control with SMC, the controller can cope well with severe uncertainties in rehabilitation without sacrificing the robust performance.

### 3. Modeling of the Robot Kinematics and Dynamics

The Stewart platform is a class of parallel robots. The model of a 6-DOF parallel robot is shown in Fig. 1, which consists of a mobile upper platform, a fixed base platform, and several actuators. There are two schemes for controlling such parallel robots: one is to design a control in the Cartesian space and the other is in the joint space. The control based on the workspace coordinates needs a 6-DOF sensor to measure the displacement or velocity of the mobile plate. Otherwise, it needs forward kinematics commands which rely on the numerical method or the observer design to estimate the plate movement. It is well known that the forward kinematics of a parallel manipulator have always been a difficult and challenging problem.<sup>40</sup> On the contrary, the joint space control is relatively conventional. It is a kind of tracking control to follow the desired link length computed from the position command by inverse kinematics. Hence it is relatively easier than control in Cartesian space. The kinematics problems of the parallel robot can be divided into the inverse kinematics and the forward kinematics. Given the desired translational and rotational motion of the upper platform, i.e., the effector, the inverse kinematics could be used to transform the desired trajectory into the displacement of individual actuators. Thus we can control the respective joints instead of the moving platform. The geometric model of the Stewart platform and its vector diagram can be seen in Fig. 1(b), where the radius of the upper platform is defined as  $r_b$ , and the angle is  $\theta_2$ , likewise, the parameters of the fixed platform are defined as  $r_a$  and  $\theta_1$ , respectively.

Fig. 1. Model of a 6-DOF parallel robot: (a) Mechanical structure (b) Geometric model (c) Vector diagram.

Fig. 1(c) shows the coordinates of the platform. Positions of the base and the upper platform are uniquely defined by the coordinates of the six joints. The vector  $B_i$  describes the positions of 6 vertices with respect to frame of  $\{B\}$  which is fixed on the upper platform. Vector  $A_i$  describes the positions of 6 vertices with respect to the frame of  $\{A\}$  which is fixed on the base platform. Define the vector  $[x, y, z, \alpha, \beta, \gamma]$  as the six variables to describe the translation and orientation of the upper platform. And let  $R$  and  $C_p$  represent the rotational transformation matrix and the translation vector, respectively.

$$R = \begin{bmatrix} \cos \beta \cos \gamma & -\cos \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma \\ \cos \beta \sin \gamma & \cos \alpha \cos \gamma + \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma \\ -\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta \end{bmatrix}. \quad (1)$$

$$C_p = [x \quad y \quad z]^T. \quad (2)$$

The vector  $L_i$  and the length  $l_i$  which correspond to leg  $i$  can be derived as:

$$L_i = C_p + RB_i - A_i \quad (i = 1, 2, \dots, 6), \quad l_i = \|L_i\|, (i = 1, 2, \dots, 6). \quad (3)$$

The dynamic model of a parallel robot can be regarded as the motion equation of the robotic mechanical system. It represents the relationship between the driving force or torque and the variables of each actuator. The dynamic equations can be established in a variety of ways,<sup>41</sup> whereas the Lagrange method only needs to calculate kinetic and potential energy of the robotics system. Assuming the moving platform is a rigid body, and ignoring the frictions of joints, the Lagrange method is used to establish a dynamic model of the parallel platform. The dynamics of the platform is obtained as follows.<sup>42</sup>

$$M(X)\ddot{X} + C(X, \dot{X})\dot{X} + G(X) = F - F_e, \quad M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau - \tau_e. \quad (4)$$

Where  $X$  represents the Cartesian coordinate vector,  $q$  the joint space vector,  $\tau$  the force vector in Joint space,  $F$  the force vector in task space,  $M(q)$  the  $n \times n$  inertia matrix,  $C(q, \dot{q})$  the  $n \times n$  Coriolis/centrifugal coefficient matrix, and  $G(q)$  the  $n \times 1$  gravity vector. The relationship between the force in workspace and the joint space torque can be established by using Jacobi matrix. Thus the dynamic equation of this parallel platform can also be derived as:

$$F = J^{-T}[M(q)\ddot{q}(t) + C(q, \dot{q})\dot{q}(t) + G(q)]. \quad (5)$$

Similarly, on the basis of dynamic equations of platform, the acceleration vector ( $\ddot{q}(t)$ ) can be obtained through the forward dynamics. Subsequently the velocity and position of the actuator can be obtained via integration operation. The forward dynamic problem has a clear physical meaning, which can be stated as: given the input force/torque of each joint, find the corresponding motion in terms of position, velocity and acceleration for some initial position and velocity conditions.

$$\ddot{q}(t) = M^{-1}(q)[J^T \tau - C(q, \dot{q})\dot{q} - G(q)]. \quad (6)$$

In order to design a controller in joint space coordinates, the inverse kinematic model is utilized to transform the trajectory in task space to that in the joint space, and then it is



able to drive each joint of the robot. If the lengths of the six legs are well controlled, the upper moving platform of parallel manipulator can move with the six desired degrees of freedom. In general, dynamics analysis is utilized to build the relationship between the input force or torque of the six branches and the desired trajectories. Dynamics modeling of parallel robot is very important for robot motion control. On one hand, the model can be utilized for parallel robot's simulation analysis and design; on the other hand, the dynamics are especially indispensable for model-based control methods, such as the computed torque controller, sliding mode controller, or other model reference algorithms. Therefore, kinematics and dynamics modeling is the basis of SMC controller design and robot control in this paper, and through the kinematics and dynamics analysis of the parallel robot, we can achieve a more effective control performance.

#### 4. Sliding Mode Controller

The purpose of the sliding mode control law is to ensure that the nonlinear plant's state trajectory reaches a specified surface within a finite time and then remains on this surface in the subsequent time once intercepted.<sup>43</sup> In general, the sliding mode control design has two-step process: choose a switching surface which is chosen by desired behavior; and define a switching control such that the trajectory of the system converges to the sliding surface, and then stays on the sliding surface. In this context, given the vector  $q_d, \dot{q}_d, \ddot{q}_d$  representing the desired position, velocity, and acceleration of the robot respectively, we want to find the right torque vector of each joint to drive the robot. So it is able to get the actual movement vectors  $q, \dot{q}, \ddot{q}$  of the robot to track the desired trajectory. The sliding mode controller is proposed based on the dynamic model of the parallel robot:

$$\tau = J^T f = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q). \quad (7)$$

Define the tracking error vector  $e$  and its differential  $\dot{e}$  as following:

$$e = q - q_d, \quad \dot{e} = \dot{q} - \dot{q}_d. \quad (8)$$

where  $q_d$  and  $q$  is the desired and actual position vector of the joint, respectively. In order to make the system track  $q \equiv q_d$ , let us define a sliding surface  $s$  as:

$$s = \dot{e} + \lambda e = \dot{q} - \dot{q}_r. \quad (9)$$

here the reference velocity vector  $\dot{q}_r = \dot{q}_d - \lambda e$  is formed by modifying the desired velocities  $\dot{q}_d$  according to the error  $e$ . Considering the necessary condition for reaching the sliding surface  $s = 0$ , we have the following form as the control law of SMC:

$$u = u_{eq} + \Delta u. \quad (10)$$

The controller includes two parts: One part of the controller is the equivalent control component  $u_{eq}$ , and the other part is the chattering control component  $\Delta u$ .

For the 6-DOF parallel robot, controller output is the joint torque  $\tau = \tau_{eq} + \Delta \tau$

Based on the dynamic model of the parallel robot, we have:

$$\tau_{eq} = \hat{M}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{G}. \quad (11)$$

Here, the symbol  $\hat{\cdot}$  denotes the estimated value, that is:

$$\hat{M} = M(q); \hat{C} = C(q, \dot{q}); \hat{G} = G(q); \quad (12)$$

Chattering is undesired phenomenon, and it is better to be reduced or eliminated for the controller to perform properly. This objective can be achieved by smoothing the control discontinuity in a thin boundary layer near the switching surface. We have:

$$\Delta \tau = -ksat(s / \Phi) . \quad (13)$$

Where  $\Phi$  is the boundary layer thickness and  $sat(\cdot)$  is saturation function, thus:

$$\tau = \tau_{eq} + \Delta \tau = \hat{M}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{G} - ksat(s / \Phi) . \quad (14)$$

The designed sliding mode controller for parallel robot is shown in Fig. 2.<sup>44</sup>

Fig. 2. The sliding mode controller for parallel robot.

## 5. Fuzzy Sliding Mode Controller

In medical applications, related to lower limb rehabilitation, when the upper platform moves within its workspace and contacts with the environment, the plate will be confined by the environment. Therefore, it is hard to determine the uncertainties when contacting with the patient's limb, such as the inertia conditions or external forces.<sup>45</sup> Traditional SMC controllers need to obtain the mathematical model of the controlled object, making it difficult to provide perfect performance in controlling system with uncertain conditions such as the rehabilitation robots. In practical applications for lower limb rehabilitation, when the phase reaches the switching surface, the errors usually result in chattering due to discontinuity in the switching control. To avoid this problem, a robust control based on fuzzy sliding mode controller (FSMC) is proposed.

### 5.1. FSMC for robot with human interaction

In medical practice, regarding the lower limb rehabilitation robot in our laboratory, when the upper platform moves, the contact force with human interaction including inertia conditions and external disturbance forces. To eliminate the effect of the external force, we used a sliding mode controller with force feedback, which can be the torque measured by the force sensor represented by  $\tau_d$ , as illustrated in Fig. 3. Considering the parallel robot in a rehabilitation environment, the dynamic model should be modified with external torque feedback  $\tau_d$ , caused by the lower limb disturbance, yields:

$$\tau = J^T f = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \tau_d . \quad (15)$$

Fig. 3. Robot kinetic model with human interaction in rehabilitation environment.

Applying fuzzy inference to adjust the controller parameters can effectively alleviate the chattering without the sacrifice of robustness.<sup>46</sup> The introduction of fuzzy logic to sliding mode controller can greatly improve the system performance in two ways: one is

to shift the control object from the tracking errors to the sliding mode function; the other is that the fuzzy sliding mode controller can simplify the complexity of traditional system.<sup>47</sup> For example, for a system with order  $n > 2$ , the input of the traditional system is  $n$ -dimensional, that is  $\begin{bmatrix} e, \dot{e}, \ddot{e}, \dots, e^{(n-1)} \end{bmatrix}$ , while the fuzzy sliding mode controller is able to reduce the dimensionality, making the input to be two-dimensional, which is  $(s, \dot{s})$ . In the parallel robot control system, fuzzy rules can correct the controller output even in the absence of precise model of the controlled object. Many fuzzy sliding mode controllers in literatures are too complicated and have too much computation to be applied in practice. The basic structure and integrating of the fuzzy logic systems with SMC in practical implementations are used to be fulfilled in two ways. One is to use fuzzy systems for SMC parameters tuning<sup>34</sup>, such as in Refs. (35, 37, 39). In these particular SMC schemes,  $u(t)$  is a function of  $s$  and  $\dot{s}$ , the gain  $k$  (as in Eq. (13)) balances the chattering and the error in the system. The fuzzy system tunes the parameters in such a way as to get the best tracking performance without chattering. Another one is to use fuzzy logic system controller complementary to SMC<sup>34</sup>: Such approaches as seen in the papers (8, 36, 38, 48) that include fuzzy logic systems as additional compensators to the SMC outputs for performance enhancement and chattering elimination. Here the SMC combined with fuzzy tuning is used to compensate for the influence of external disturbances and reduce chattering while maintaining sliding behavior at the same time.

One of the main problems of the prior SMC parameters fuzzy tuning method is that the performance and robustness of the controller is affected by the selection of the gain matrix. An effective tuning mechanism for these SMC parameters is critical to the control performance. While the appropriate parameters need to be searched by a time-consuming trial-and-error procedure<sup>37</sup>, so the computational burden of such control scheme will be very heavy. Moreover, there is always a contradiction between the chattering and the robustness of the system, where high gain can easily cause chattering whereas small gain leads to degradation of the tracking performance. While the SMC fuzzy compensation way does not have such difficulties when realize the practical implementation of fuzzy controller. In this paper, we investigated the control of a parallel robot using FSMC for constructing a simple controller in practical implementation. In order to simplify the controller design in practical experiments, the fuzzy controller is only used to compensate the sliding mode controller and improve the performance of the system. Compared with other approaches in [8, 36, 38, 39], which used the similar structure and integrating way, fewer and simpler fuzzy rules are needed to realize the trajectory tracking in our system. The accurate system model is not required and the computational burden is lowered. Moreover, the system shows good robustness to disturbance and external uncertainties. It can improve the performance of such controller and reduce the chattering. Select the switching surface function  $s$  and its change rate  $\dot{s}$  as input signals of the fuzzy system, and the fuzzy reasoning component  $u_f$  as the output signal, which can greatly reduce the dependence of the sliding mode controller on the object model. That is:

$$u = u_{eq} + \Delta u + u_f. \quad (16)$$

where  $u_q$ ,  $\Delta u$  and  $u_f$  are the equivalent component, switching component and fuzzy inference component, respectively.  $u_f$  is the output of fuzzy system with a gain  $g$  adaptable to tracking performance, that is,  $u_f = g \cdot FSMC(s, \dot{s})$ . According to the fuzzy sliding mode model of the parallel robot, the proposed controller is illustrated in Fig. 4.

Fig. 4. Fuzzy sliding mode control model with external force compensation.

## 5.2. Fuzzy logic system design

It is well known that the robustness of the sliding mode control is achieved by driving the representative point towards a desired sliding regime. When the state trajectories are far from the sliding surface,  $|s|$  is large, so the switching gain should be correspondingly increased, and vice versa. In Ref. 8 a fuzzy tuning technique is used to translate the fuzzy operations into expressions for a continuous adjustment of the controller parameters.  $u_f$  is the output of the FSMC, which is determined by the normalized  $s$  and  $\dot{s}$ . The fuzzy rules table (Table 1) can be represented as the mapping of the input variables  $s$  and  $\dot{s}$  to the output variable  $u_f$  as described in [8]. For the fuzzy defuzzification, the minimum operation and center average process are selected to calculate the output variable. By using this technique, a compensational component  $u_f$  is added to the control output.

Table 1. The fuzzy inference rules for FSMC

$\begin{matrix} s \\ \dot{s} \end{matrix}$	<i>NB</i>	<i>NM</i>	<i>NS</i>	<i>ZO</i>	<i>PS</i>	<i>PM</i>	<i>PB</i>
<i>NB</i>	PB	PB	PM	PM	PS	ZO	ZO
<i>NM</i>	PB	PB	PM	PS	PS	ZO	NS
<i>NS</i>	PM	PM	PM	PS	ZO	NS	NS
<i>ZO</i>	PM	PM	PS	ZO	NS	NM	NM
<i>PS</i>	PS	PS	ZO	NS	NS	NM	NM
<i>PM</i>	PS	ZO	NS	NM	NM	NM	NM
<i>PB</i>	ZO	ZO	NM	NM	NM	NB	NB

The normalized membership functions of input variables  $s$  and  $\dot{s}$  and that of output variable  $u_f$  are shown in Figs. 5 and 6, respectively. They are all decomposed into several fuzzy subsets as “NB, NM, NS, ZO, PS, PM, PB”. In this fuzzy sliding mode controller, *Mamdani* inference method is adopted as the relationship expressing fuzzy conditional statement. Considering the impact of sliding surface errors on the compensational output, we set the membership functions extracted in such a way that the stability of the system can be satisfied and these rules contain the input and output relationships that define the proposed control strategy. For the fuzzy sliding mode controller, the input is no longer the position errors ( $e, \dot{e}$ ) but the sliding surfaces ( $s, \dot{s}$ ). According to the membership functions, we have the rules such as “if  $s$  is *PB* (positive big) and  $\dot{s}$  is *PB*, then  $u_f$  is *NB* (negative big)” to complete the respective parameters configuration of input and output

variables in their domains. The proposed method can adjust the value of  $u_f$  in real-time according to disturbances and uncertain parameters. Thus control errors that exceed the sliding surface can be minimized without destroying the sliding mode conditions.

Fig. 5. The normalized membership functions of input variables  $S$  and  $\dot{S}$ .

Fig. 6. The normalized membership function of output variable  $u_f$ .

### 5.3. Stability analysis

Since the dynamic process of the fuzzy sliding mode controller also mainly contains two phases: making the phase trajectory reach the sliding surface and maintaining it on the surface. The most important task is to design switched control that will drive the plant's state to the switching surface and maintain it on the surface once intercepted. Lyapunov method is usually used to determine the stability properties of an equilibrium point without solving the state equation.<sup>49</sup> In this paper, the Lyapunov stability theory is used to derive the gains that make the plant's state reach the sliding surface within finite time and remain on it for the subsequent time. Define the Lyapunov function as follows:

$$V = \frac{1}{2} s^T M s. \quad (17)$$

And the differential equation of Lyapunov function is:

$$\dot{V} = \frac{1}{2} \dot{s}^T M s + \frac{1}{2} s^T \dot{M} s + \frac{1}{2} s^T M \dot{s} = \frac{1}{2} s^T \dot{M} s + s^T M \dot{s}. \quad (18)$$

In rehabilitation practice, the dynamic model compensated with external force  $\tau_d$ , is:

$$\tau = J^T f = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \tau_d. \quad (19)$$

Then,

$$M\ddot{q} = J^T f - C\dot{q} - G - \tau_d. \quad (20)$$

Since  $\dot{s} = \ddot{e} + \lambda\dot{e} = \ddot{q} - \ddot{q}_r$ , where  $\ddot{q}_r = \ddot{q}_d - \lambda\dot{e}$

$$\dot{V} = \frac{1}{2} s^T \dot{M} s + s^T (J^T f - C\dot{q} - G - \tau_d - M\ddot{q}_r). \quad (21)$$

Note that, with the particular property of parallel robot, the matrix  $\dot{M}(q) - 2C(q, \dot{q})$  is indeed skew-symmetric, and based on the skew-symmetry of the matrix, we have  $s^T [\dot{M} - 2C] s = 0$ . By using this property, the term of  $1/2 s^T \dot{M} s$  can be rewritten as:

$$\frac{1}{2} s^T \dot{M} s = s^T C s. \quad (22)$$

Then, substituting (22) into (21) yields:

$$\dot{V} = s^T (J^T f - M\ddot{q}_r - C\dot{q}_r - G - \tau_d). \quad (23)$$

In the designed FSMC with fuzzy compensational signal, the controller output is:

$$J^T f = \tau_{eq} + \Delta \tau + \tau_f = \hat{M}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{G} + \tau_{d'} - ksat(s / \Phi) + J^T FSMC(s, \dot{s}). \quad (24)$$

Substituting  $J^T f$  into (23) yields

$$\dot{V} = s^T ((\hat{M} - M)\ddot{q}_r + (\hat{C} - C)\dot{q}_r + (\hat{G} - G) + \dot{\tau}_{d'}) - s^T ksat(s / \Phi) + s^T \tau_f. \quad (25)$$

Let us define  $\Delta M = \hat{M} - M$ ,  $\Delta C = \hat{C} - C$ ,  $\Delta G = \hat{G} - G$ ,  $\Delta \tau_{d'} = \dot{\tau}_{d'} - \tau_{d'}$ .

And we have  $\hat{M} = M(q)$ ,  $M = M(q_s)$ ,  $\hat{C} = C(q, \dot{q})$ ,  $C = C(\dot{q}_d, \dot{q}_d)$ ,  $\Delta G = 0$ ,  $\Delta \tau_{d'} = 0$

So, the final Lyapunov function is expressed as:

$$\dot{V} = s^T (\Delta M\ddot{q}_r + \Delta C\dot{q}_r) - s^T ksat(s / \Phi) + s^T \tau_f. \quad (26)$$

In order to satisfy the stability condition of such control system as:  $\dot{V} \leq -\eta|s|$ , where the constant  $\eta$  is strictly positive.

The vector  $k$  can be chosen as following equation:

$$k \geq |\Delta M\ddot{q}_r + \Delta C\dot{q}_r| + |\tau_f| + \eta. \quad (27)$$

The output fuzzy sets are normalized in an interval, so  $|u_f = FSMC(s, \dot{s})| \leq \Omega$ .

Therefore, if the chosen control gain  $k$  is large enough to satisfy the above equation, one can conclude that the reaching condition  $V = 1/2 s^T Ms < 0$  is always satisfied. Thus the closed loop system is asymptotically stable and the error state trajectory converges to the sliding surface  $s(t) = 0$ . The sliding condition guarantees that the trajectory reaches sliding surface in a finite time. Once intercepted, the trajectories will remain on the surface, and thus drive robot following the desired trajectory.

#### 5.4. Controller design in practice

This article proposes a sliding mode controller with fuzzy compensation, further, the Lyapunov stability theory is utilized to determine the range of fuzzy inference output to ensure the stability of the whole system. In order to control the 6-DOF parallel robot for rehabilitation properly, its control features and model parameters must be considered. Because this kind of controller has a design that operates directly on the errors between the desired and measured position values, the robot in our lab is equipped with encoder sensors to measure the position and velocity of the platform.

As for the 6-DOF parallel robot in this experimental system, the radius of the upper moving plate is 180mm, while the radius of the fixed base plate is 270mm, and the angles of the platforms are 28° and 22°, respectively. The mass of the upper platform is 3.6kg, and the mass of the base platform and linkages are 17.9kg and 1.2kg, respectively. So inertia term matrix  $M(q)$  of the platform can thus be derived:

$$M(q) = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & M_{45} & M_{46} \\ 0 & 0 & 0 & M_{54} & M_{55} & 0 \\ 0 & 0 & 0 & M_{64} & 0 & M_{66} \end{bmatrix}, \text{ where } \begin{cases} M_{44} = I_x \cos^2 \beta \cos^2 \gamma + I_y \cos^2 \beta \sin^2 \gamma + I_z \sin^2 \beta \\ M_{45} = (I_x - I_y) \cos \beta \cos \gamma \sin \gamma \\ M_{46} = I_z \sin \beta \\ M_{55} = I_x \sin^2 \gamma + I_y \cos^2 \gamma \\ M_{54} = M_{45} \\ M_{64} = M_{46} \\ M_{66} = I_z \end{cases} \quad (28)$$

Here,  $I_x, I_y, I_z$  are rotational inertias of the platform around  $x$ -axis,  $y$ -axis,  $z$ -axis.

The Coriolis/centrifugal term matrix  $C(q, \dot{q})$  of the platform can be obtained:

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_1 \dot{\beta} - K_2 \dot{\gamma} & -K_1 \dot{\alpha} - K_3 \dot{\beta} + K_4 \dot{\gamma} & -K_2 \dot{\alpha} + K_4 \dot{\beta} \\ 0 & 0 & 0 & K_1 \dot{\alpha} + K_4 \dot{\gamma} & K_5 \dot{\gamma} & K_4 \dot{\alpha} + K_5 \dot{\beta} \\ 0 & 0 & 0 & K_2 \dot{\alpha} - K_4 \dot{\beta} & -K_4 \dot{\alpha} - K_5 \dot{\beta} & 0 \end{bmatrix}. \quad (29)$$

Here,  $K_1, \dots, K_5$  are the intermediate parameters.

The proposed fuzzy sliding mode controller includes three parts: the equivalent control input  $\tau_{eq}$  which is to achieve  $s = 0$ ; the chattering control input  $\Delta \tau$  which is used to satisfy the Lyapunov requirement, and the fuzzy inference output  $\tau_f$  which is used to compensate the SMC and reduce the chattering. It is expected that the fuzzy sliding mode controller can greatly improve the performance during motion control since it takes into consideration on disturbances of this system. According to the parameters of this parallel robot model, we choose operating bandwidth  $\lambda = 10$ . And the gain  $k$  can be calculated from Eq. (27). Typically, the arbitrary constant  $\eta$  is a small value compared to the value of  $k$ . In our experiments, parameter  $\eta$  is chosen to be 0.1.

So, in this FSMC system for parallel rehabilitation robot, we have

$$\tau_{eq} = \hat{M} \ddot{q}_r + \hat{C} \dot{q}_r + \hat{G} + \tau_d, \quad k = |\Delta M \ddot{q}_r + \Delta C \dot{q}_r| + |\tau_f| + \eta. \quad (30)$$

And finally the controller output yields

$$\tau = \tau_{eq} - k \text{sat}(s / \Phi) + \tau_f. \quad (31)$$

In the designed fuzzy sliding mode control system for our 6-DOF parallel robot, based on the fuzzy relationships established among tracking errors, sliding functions and the control outputs, the fuzzy compensation can achieve parameters self-tuning for SMC. According to the above fuzzy rules and designed parameters for FSMC, the controller for rehabilitation robot with external force in practice can be implemented as in Fig. 7.

Fig. 7. Implementation of FSMC for parallel robot in experiment.

## 6. Experimental Results and Discussions

The Stewart platform shown in Fig. 8 was designed by the authors' research group for the purpose of investigating lower limb rehabilitation.<sup>50</sup> Specifically, the system mainly includes an industrial PC, six motion controllers based on DSP boards of model TMS320 LF2407A, and Panasonic Minas servo drivers with output power of 400 W and rated speed of 3000 r/min, as well as the Stewart platform. The proposed FSMC with fuzzy compensator is realized in the DSPs using the C language. Furthermore, the incremental photoelectric encoders are mounted to provide position and velocity feedback. The robot control model contains six independent control loops, where each joint is separately driven by an AC servo motor. In the main program, the positions and velocities of the motors are read from the encoders. Next, the program calculates the errors between the feedback and the desired motion and generates the control efforts according to the proposed FSMC algorithm to realize the close-loop control of the system.

Fig. 8. The Stewart platform and control system designed in rehabilitation environment.

In this experimental system, the trajectory for upper platform is defined as following: the initial position of the platform center is (0, 0, 0), and move the plate to (90, 0, 90), here the unit is mm. And then the platform moves by following a counter-clockwise circle, the radius is 90mm, thus the planned path of the moving platform is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 90 \\ 0 \\ 90 \end{bmatrix} - R \begin{bmatrix} 90 \cos \varphi \\ 90 \sin \varphi \\ 0 \end{bmatrix}, \text{ where } \varphi \in [0 \ \pi], \ R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (32)$$

In the practical environment, the fuzzy sliding mode controller for parallel robot is implemented on the six DSP chips which control the six joints independently. The FSMC is realized based on fuzzy inference compensation to make the output signal compliant, and thus achieve the high-precision position and velocity tracking control. The proposed algorithm implemented on DSP is written in the software platform CCStudio 3.1, and the program is downloaded to the DSP board through emulator as the robot motion controller. The desired trajectory and the velocity of each joint can be achieved as shown in Fig. 9, according to the inverse kinematics of 6-DOF parallel robot.

Fig. 9. The planned trajectory for each joint in this experiment: (a) is the desired trajectory of each joint, and (b) is the desired velocity of each joint.

Trajectory parameters can be achieved by path planning based on the interpolation operation. Firstly, the experimental results of the traditional SMC fuzzy tuning controller and the proposed simple SMC fuzzy compensation controller are given to verify their effectiveness in controlling the 6-DOF parallel robot in a real environment. Six joints' position tracking errors of the traditional controller and the proposed SMC controller are shown in Fig. 10. Based on the analysis of experimental results, it is obvious that the performance of our method is close to the classical SMC fuzzy tuning method. It must be admitted that the classical FSMC strategy can follow the desired trajectory in a better



performance, but our approach can also effectively reduce the position tracking delay and dynamic deviation, and the robot can be operated in a smooth way.

Fig. 10. Joint's position tracking results of 6-DOF parallel robot: (a) is the joint's tracking errors of traditional SMC fuzzy tuning controller, and (b) is the tracking errors of proposed SMC fuzzy compensation controller.

The maximum error of six legs for the proposed controller is about  $1mm$ , in other words, the position tracking results of our method are relatively crude but still sufficient in the practical applications. However, although the classical FSMC controller can satisfy the trajectory tracking performance of the 6-DOF parallel robot, the velocity control results (as in Fig. 11(b)) cannot meet the requirement of robotics for medical purpose, since the chattering is distinct especially in high speed situations. The reason why velocity tracking performance is so concerned in this rehabilitation practice is conducted as follows. Lower limb rehabilitation robots have to be manipulated within a wide range of speeds. In practical implementations, velocity chattering is highly undesirable because it may impact the plant dynamics and thus result in unforeseen instabilities, which is especially destructive in the training process. However, among researches on fuzzy sliding mode control, only a few considered the chattering characteristics of the velocity tracking control. In papers (36-38), the systems all realized trajectory tracking in simulation or practical environment. A cascade-control algorithm based on sliding mode was proposed in Ref. 27 to realize the trajectory tracking control of a parallel robot. However, the system lacked direct measurements of the velocity signal, and the velocity observer was designed using the position error, which may introduce severe noises.

Only in Ref. 48, both position and velocity errors were considered in the controller. However, this method is time-consuming and cannot provide enough robustness when uncertainty and external disturbance exist. Inspired by previous work, a SMC with fuzzy compensation scheme is developed to attenuate the velocity chattering. Then an adaptive controller was added into the sliding mode controller for compensating the uncertainties and smoothing the control signal. In this paper, we choose  $s = \dot{e} + \lambda e$  as the input to the fuzzy logic system. By doing so, it can be noted that is a function of position and velocity errors, and the velocity tracking performance directly influences the controller output for each joint. Furthermore, to improve the performance of the proposed intelligent control approach, a DSP is adopted for the implementation of the proposed FSMC. The output of the fuzzy logic compensator is added to the control effort output of the SMC for the elimination of the velocity chattering. In the following, the behavior of the proposed fuzzy SMC compensatory controller is compared with classical fuzzy SMC methods.

In the actual robot platform, the velocity tracking results obtained by performing the same above trajectory are shown in Fig. 11, and we can compare the proposed FSMC with the experimental results of pure SMC and conventional SMC fuzzy tuning controller. From the experimental results, it is concluded that our proposed FSMC showed superior performance for each of the six joints. Comparing Figs. 10(a) and 11(b), it is clear that for the classical FSMC, the position tracking errors can be attenuated effectively by the introducing of boundary layer function. However, as the control law needs to be tuned up

appropriately to achieve a compromise between the tracking precision and the chattering results, the price is the velocity chattering increasing as shown in Fig. 11(b). Especially in first stage of Fig. 11, peak velocity errors occur, because reference velocity of Fig. 9(b) is so large for 0~12 sec. On the other hand, however, for the proposed SMC compensation controller, the velocity errors are much smaller, as the compensation values obtained by using the fuzzy logic observer are added to sliding mode values. The inherent chattering occurred in the conventional sliding mode controller is reduced by using the sliding mode controller with fuzzy logic compensator. Therefore, the proposed controller is able to provide superior performance over the conventional sliding mode controller.

Fig. 11. Joint's velocity tracking results of 6-DOF parallel robot: (a) is the tracking results of pure SMC, (b) is the tracking results of conventional FSMC with fuzzy tuning, (c) is the tracking results of the proposed FSMC with fuzzy compensation, (d) is the velocity tracking errors of pure SMC, (e) is the velocity tracking errors of conventional FSMC with fuzzy tuning, (f) is the tracking errors of proposed FSMC with fuzzy compensation.

Moreover, in order to have a quantitative comparison between control performance of the proposed control system and classical FSMC, the maximum tracking error (ME), the average tracking error (AE) and the standard deviation of the tracking errors (SD) for the trajectory and velocity tracking are selected as the criterion indices. In order to simplify the index table, we just analyze the experimental data of joint 1, joint 3 and joint 5, and it can be proven that other joints have the similar results. Table 2 gives the index values for the above mentioned methods. The results indicate that the performance of our method is better than the classical FSMC. The position tracking results of the SMC with parameters fuzzy tuning controller is very satisfied, but the velocity chattering phenomenon is not alleviated significantly. The main reason for this is that the use of fuzzy tuning law for SMC gains cannot achieve an excellent result both in chattering and error performances simultaneously, as the system is subject to achieve a compromise between the tracking error precision and the smooth control. However, for the proposed FSMC with fuzzy compensator, it is illustrated that velocity chattering of the six joints is considerably reduced than the conventional method dealt with in this paper. Experimental results show that the proposed FSMC has faster velocity tracking with smaller error values than both conventional SMC and traditional SMC with fuzzy tuning method.

Table 2. Performance comparison of the controllers for trajectory tracking control

Methods		maximum error (ME) / mm			average error (AE) / mm			standard deviation (SD)		
		Joint 1	Joint 3	Joint 5	Joint 1	Joint 3	Joint 5	Joint 1	Joint 3	Joint 5
Position tracking results	Pure SMC	1.266	1.820	1.881	0.279	0.359	0.372	0.289	0.458	0.473
	Conventional FSMC	0.387	0.548	0.552	0.083	0.110	0.114	0.088	0.141	0.145
	Proposed FSMC	0.654	0.948	0.949	0.148	0.189	0.194	0.150	0.233	0.240
Velocity tracking results	Pure SMC	3.233	4.705	4.754	0.178	0.243	0.243	0.619	0.988	1.022
	Conventional FSMC	2.235	3.807	2.280	0.113	0.159	0.145	0.394	0.647	0.421
	Proposed FSMC	1.445	2.235	2.375	0.026	0.017	0.010	0.059	0.091	0.129

It is observed from indexes that the proposed FSMC has the smallest velocity error among the other controllers, which prove the efficiency of the proposed controller. The most striking difference between the proposed FSMC and the conventional SMC fuzzy tuning performances is the overall reduction in velocity track chattering. This is a result of the fuzzy sliding mode controller canceling the non-linear components of the system and compensating the control output. The proposed simple FSMC possesses a strong self-adaptability and also have the capacity to control the external disturbances and uncertain factors in a timely and effective way. The fuzzy sliding mode controller in this paper utilizes the fuzzy rules to smooth the sliding mode output, and the FSMC is applied to the trajectory tracking of an actual 6-DOF parallel robot. Experimental results show that the improved fuzzy sliding mode control method can effectively improve the position and especially the velocity tracking performance of the system. In addition, the proposed FSMC approach based on fuzzy compensation is quite efficient, which can meet the requirements of real-time computing and data transmission for a 6-DOF parallel robot. The fuzzy sliding mode control algorithm for such parallel robot is implemented on DSP, and can achieve good trajectory tracking results. In other words, the proposed FSMC method can be used in rehabilitation applications for its ability to obtain efficient and reliable robot control as long as the predefined trajectory is within its constraints.

## **7. Conclusion**

This paper has introduced an effective method for controlling the Stewart robot used for lower limb rehabilitation. We have investigated the control of a parallel robot systems using FSMC for constructing a simple controller in practical rehabilitation. In order to simplify the controller design in practical experiments, the fuzzy logic system is used as additional compensator to the SMC outputs for performance enhancement and chattering elimination. Compared with other conventional FSMC approaches, fewer and simpler fuzzy rules are needed to realize the trajectory tracking in our system. Furthermore, the proposed FSMC method does not have to achieve a compromise between the position tracking precision and the velocity smoothing control, like most of the existing FSMC controllers with their parameters online tuned by fuzzy rules. The experimental results in the actual environment demonstrate that applying the proposed FSMC method to 6-DOF parallel robot is reasonable and valid. The position tracking results are relatively crude but sufficient in the practical applications, while the velocity chattering can be greatly reduced compared with the pure SMC as well as the conventional FSMC systems. The proposed FSMC with fuzzy compensator improves the convergence of the sliding mode system and solves the problem that it is easy to fall into the chattering. It not only approaches the control objective in a more effective way, but also responses more quickly, providing an effective method for the control of the nonlinear objects such as the parallel robots. In this way, a straightforward simple control algorithm and robust control can be achieved and easily realized in rehabilitation practice.

However, there are still a lot of research opportunities and potential problems needing to be tackled in this field. For example, the dynamic model of drive actuator was not

considered in great details. And additional efforts in the limb acting force modeling are also required. In the future, in order to improve the rehabilitation results, force feedback between the robot and the patient should be taken into consideration. Putting the position/force hybrid control and impedance control into practice is also important in this field. On the other hand, the introduction of modern techniques such as the functional electrical stimulation, bio-signals acquisition, including both the electromyographic signals (EMG) and the electroencephalogram (EEG) can also play an important role in active training and rehabilitation robot control. Finally, in order to implement the control strategies in the actual rehabilitation robot, more attentions should be paid to the extraction and processing of biomedical signals, the human-machine interface and the establishment and optimization of rehabilitation assessment strategies.

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