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Plasticity Solutions of Undrained Cavity Contraction for Prediction of Soil Behaviour around Tunnels

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ABSTRACT: Cavity contraction method has been used for decades for the design of tunneling and the prediction of ground settlement, by modelling the cavity unloading process from the in-situ stress state. Analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) are used in this paper to predict the soil behaviour around tunnels. CASM is a critical state soil model with two additional material constants, which has the ability to capture the overall behaviour of clay and sand under both drained and undrained loading conditions. Large-strain and effective stress analyses of cavity contraction provide the distributions of stress/strain within elastic and plastic regions around tunnels. The effects of ground condition and soil model parameters are investigated from the results of stress paths and cavity contraction curves. Comparisons are also provided between the predicted and measured behaviour of tunneling, using data of centrifuge tunnel test in clay. To account for the effect of free ground surface, Loganathan & Poulos's method using undrained gap parameter is incorporated to estimate tunneling induced ground surface settlement.

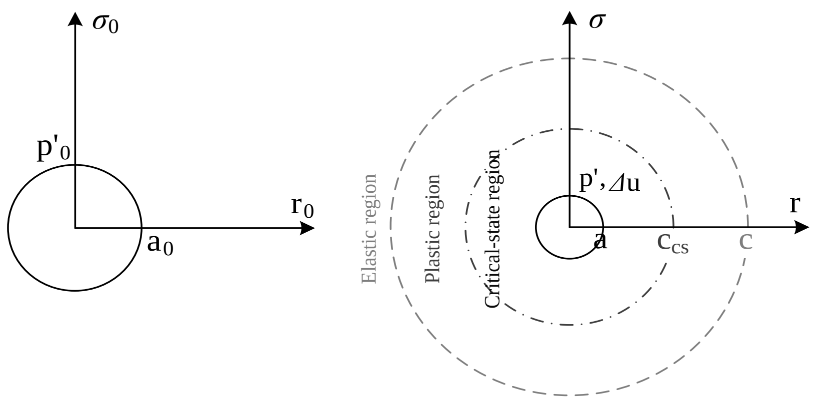
INTRODUCTION

Cavity expansion theory, concerning stress/displacement fields around cavities, has been developed and applied to a variety of geotechnical problems, as described in Yu (2000). By modelling the cavity unloading process from the in-situ stress state, cavity contraction method has been used for decades for the design of tunneling and the prediction of ground settlement (e.g. Hoek & Brown, 1980; Mair & Taylor, 1993). In the past two decades, critical state solutions were increasingly proposed to account for the dependence of soil strength with deformation history (e.g. Collins & Yu, 1996; Yu & Rowe, 1999). Undrained solutions of cavity expansion are recently developed using a unified state parameter model for clay and sand (CASM), which has the ability of capturing the overall behaviour of clay and sand (Mo & Yu 2016).

This paper provides analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) to predict the soil behaviour around tunnels. Taking account of the effect of stress history by an effective stress analysis, the predictions of stress fields and soil displacement are compared with previous analytical results and centrifuge data. In addition, to account for the effect of free ground surface, undrained gap parameter by Loganathan & Poulos (1998) is suggested to be incorporated for estimation of tunneling induced ground surface settlement.

PROBLEM DEFINITION

The contraction of a spherical/cylindrical cavity with initial radius a_0 in an infinite soil under undrained condition is concerned in this paper. The geometry and kinematics of cavity contraction are illustrated schematically in Fig. 1. Parameter 'm' is used to integrate both spherical $(m=2)$ and cylindrical $(m=1)$ scenarios. The preconsolidation pressure is referred to as p'_{y0} and $R_0 = p'_{y0} / p'_0$ represents the isotropic overconsolidation ratio in terms of the mean effective stress. To accommodate the effect of large deformation in cavity contraction process, large strain analysis is adopted for both elastic and plastic regions by using logarithmic strains. Note that a compression positive notation is used in this paper.



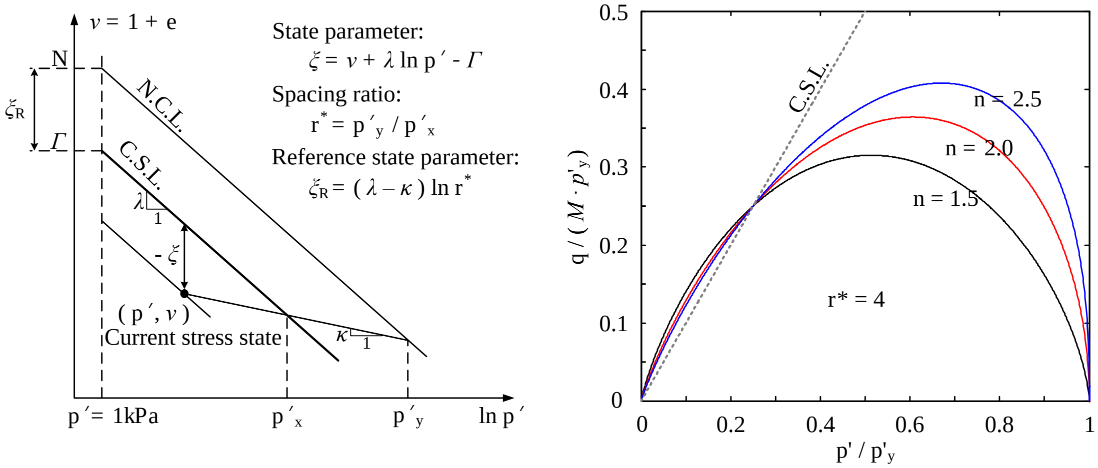
(a) Initial cavity before contraction (b) Cavity after contraction

FIG. 1. Geometry and kinematics of cavity contraction.

With the benefits of the concept of state parameter, Yu (1998) proposed a unified state parameter model for clay and sand, which is referred to as CASM. It is a simple constitutive model with two additional material constants introduced to the standard Cam-clay model, whereas the overall behaviour of clay and sand can be satisfactorily modelled by CASM under both drained and undrained loading conditions. The state boundary surface of CASM (Fig. 2b) is described as:

$$\left(\frac{\eta}{M}\right)^n = 1 - \frac{\xi}{\xi_R} = -\frac{\ln\left(p' / p'_y\right)}{\ln r^*} \tag{1}$$

where $\eta = q / p'$ is known as stress ratio; n is the stress-state coefficient; $\xi_R = (\lambda - \kappa) \cdot \ln r^*$, is the reference state parameter; and r^* is the spacing ratio, defined as p'_y / p'_x (see Fig. 2a). In addition, a non-associated flow rule based on Rowe's stress-dilatancy relation is adopted to better describing the deformation of sands and other granular media.



(a) Schematic of state parameter

(b) State boundary surfaces normalised by preconsolidation pressure

FIG. 2. Unified state parameter model for clay and sand (CASM).

PLASTICITY SOLUTIONS

Plasticity solutions are presented in this section, for a cavity contracted from a_0 to a until the soil around the cavity reaches the critical state (i.e. soil medium is deformed to have elastic, plastic and critical-state regions). 'c' is the radius of the elastic-plastic boundary, and c_{cs} is the radius where the soil starts to be in critical state. Thus, for $r > c$, soil is in elastic region; whereas for $c_{cs} < r < c$, soil is in plastic region, and critical-state zone is for soil at $a < r < c_{cs}$ (see Fig. 1b). In terms of undrained condition, the soil volume within an arbitrary radius (r) can be assumed as constant, and the relation can be written as:

$$r_0^{\,m+1} - r^{m+1} = a_0^{\,m+1} - a^{m+1} = T \tag{2}$$

The following subsections describe the solutions in elastic and plastic regions, while the details of derivations can be found in authors' personal notes (i.e. Mo & Yu, 2016; As these notes have not been published yet, readers are welcome to ask for relevant document).

Solution in Elastic Region

The effective stresses, total stresses and strains in elastic region are expressed in Equation (3).

$$\begin{cases} \sigma'_r = p'_0 - mA(r) \\ \sigma'_\theta = p'_0 + A(r) \end{cases} \qquad \begin{cases} \sigma_r = p_0 + 2G_0 mB(r) \\ \sigma_\theta = p_0 + 2G_0 mB(r) + (m+1)A(r) \end{cases} \qquad \begin{cases} \varepsilon_r = \dfrac{-m}{2G_0} A(r) \\ \varepsilon_\theta = \dfrac{1}{2G_0} A(r) \end{cases}$$

where

$$A(r) = \frac{2G_0}{m+1} \ln\left(\frac{r^{m+1} + T}{r^{m+1}}\right) \text{ and } B(r) = \frac{1}{m+1} \sum_{k=1}^{\infty} \frac{\left(-T / r^{m+1}\right)^k}{k^2} \tag{3}$$

The elastic-plastic boundary ‘ c ’ can be written as:

$$c = \left\{ \frac{-T}{1-\exp\left[\left(\frac{\ln R_0}{\ln r^*}\right)^{\frac{1}{n}} \times \frac{Mp'_0}{2G_0}\right]} \right\}^{\frac{1}{m+1}} \tag{4}$$

Solution in Plastic Region:

Elastic volumetric strain and plastic volumetric strain:

$$\varepsilon_p^e = \frac{\kappa}{\upsilon}\ln\left(\frac{p'}{p'_0}\right) \qquad \varepsilon_p^p = \frac{\lambda-\kappa}{\upsilon}\ln\left(\frac{p'_y}{p'_{y0}}\right) \tag{5}$$

Elastic deviatoric strain:

$$\varepsilon_q^e = \frac{-(m+1)}{2G_0}A(c) - \frac{[1+(m-1)\mu]\kappa M}{(m+1)(1-2\mu)\upsilon}\left\{\frac{n}{(1+n)A_2}[A_1 + A_2 \times \ln p']^{\frac{1}{n}+1} + \right.$$

$$\left.[A_1 + A_2 \times \ln p']^{\frac{1}{n}} - \frac{n}{(1+n)A_2}[A_1 + A_2 \times \ln p'_0]^{\frac{1}{n}+1} - [A_1 + A_2 \times \ln p'_0]^{\frac{1}{n}}\right\}$$

where

$$A_1 = \frac{\ln R_0 + \Lambda^{-1}\ln p'_0}{\ln r^*} \text{ and } A_2 = -\frac{\Lambda^{-1}}{\ln r^*} \tag{6}$$

Plastic deviatoric strain:

$$\varepsilon_q^p = \frac{\kappa n(m+1)}{9\upsilon A_2 M^n m}\left\{\frac{2M}{n}[\eta^n - \eta_c^n] + (9+3M-2M^2)\int_{\eta_c}^{\eta}\frac{\eta^{n-1}}{M-\eta}d\eta\right\}$$

$$\int\frac{\eta^{n-1}}{M-\eta}d\eta = \begin{cases} 0 & (\eta_c = M) \\ \frac{\eta^n}{M}\sum_{k=0}^{\infty}\left[\frac{1}{n+k}\times\left(\frac{\eta}{M}\right)^k\right] & (\eta_c < M) \\ \sum_{k=0}^{\infty}\left[-M^k\frac{\eta^{n-1-k}}{n-1-k}\right] & (\eta_c > M) \end{cases} \tag{7}$$

Total stresses can be calculated by numerical integration from Equation (8):

$$\int\partial\sigma_r = -m\int\frac{q}{r}dr \tag{8}$$

RESULTS AND DISCUSSION

Comparisons with Results of Solutions by Yu & Rowe (1999)

In this section, the results of soil behaviour around tunnels are presented by using the provided plasticity solutions of cavity contraction in undrained condition. As the yield criterion of the original Cam-clay model can be recovered from CASM by selecting the material constants: $n = 1.0$ and $r^* = 2.7183$, the validation of the solutions is carried out by comparing the results of original Cam-clay model with the results of solutions by Yu & Rowe (1999). The values of the critical state parameters, chosen to be relevant for London clay, are identical to Yu & Rowe (1999). It needs to be noted that ambient pore pressure is not included in the results of total stresses (i.e. $\sigma = \sigma' + \Delta u$).

Figures 3 and 4 present the results of soil behaviour around tunnels using cylindrical and spherical scenarios, with the overconsolidation ratio of $R_0 = 1.001$. The final contraction for both cylindrical and spherical tests is $a_0 / a =$ 1.95 and 1.12, respectively. The results are found to be comparable with data from Yu & Rowe (1999) when using non-associated flow rule, while identical results are shown for tests using associated flow rule.

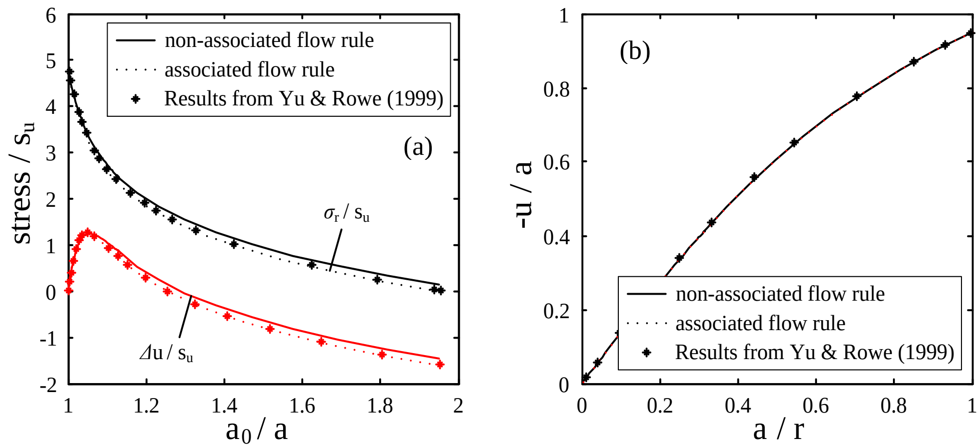


FIG. 3. Soil behaviour around tunnels using cylindrical cavity model.

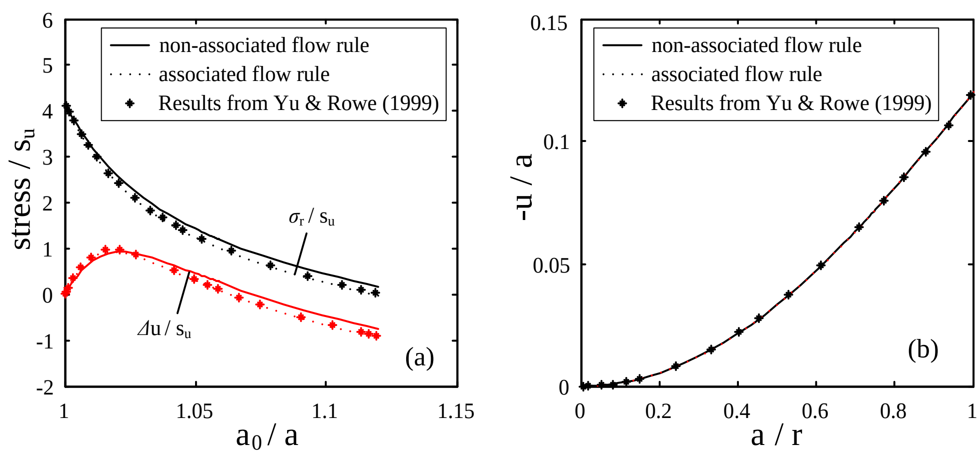


FIG. 4. Soil behaviour around tunnels using spherical cavity model.

Comparisons with Results of Centrifuge Test by Mair (1979)

The proposed analytical solutions are related to soil behaviour around tunnels, with comparisons to centrifuge results by Mair (1979). The selected centrifuge test is 2DP with cover to diameter ratio: $H / D = 1.67$. The tunnel test in clay can be assumed to be undrained condition. According to Mair (1979) and Yu & Rowe (1999), soil properties are chosen as: $\Gamma = 3.92$, $\lambda = 0.3$, $\kappa = 0.05$, $M = 0.8$, $\mu = 0.3$, $s_u = 26\text{kPa}$.

The predictions of crown settlement (u_c) and mid-surface settlement (u_s) are shown in Fig. 5. The crown settlement in Fig. 5a shows comparable results with previous analytical results (Yu & Rowe, 1999) and centrifuge data (Mair, 1979). As shown in Fig. 5b and noted by Yu & Rowe (1999), the cavity solutions tend to underpredict the observed mid-surface settlement, probably owing to the shallow tunnel test of 2DP with the effect of free ground surface.

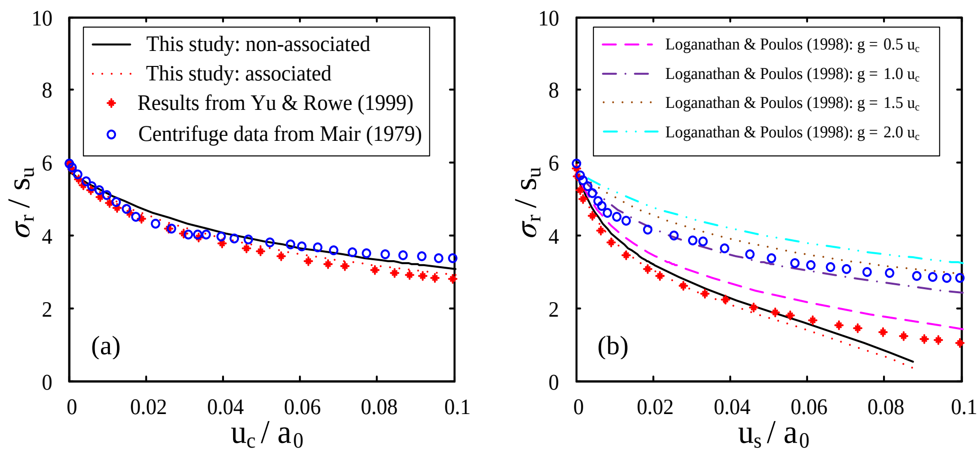


FIG. 5. Predicted and observed settlements for a centrifuge test in clay.

By using a virtual image technique suggested by Sagaseta (1987), Verruijt & Booker (1996) provided an analytical solution for a tunnel in a homogeneous elastic half-space (Fig. 6a). However, Verruijt & Booker's method results in wider settlement trough and larger horizontal movements, as reported by Loganathan & Poulos (1998). The settlements caused by tunnelling are often characterized by ground loss, which is defined as a percentage of the ratio of the surface settlement trough volume and the tunnel volume per unit length. The undrained ground loss was suggested to be defined based on the 'gap' parameter (referred to as 'equivalent ground loss', introduced by Lo & Rowe, 1982 and Rowe & Kack, 1983) by Loganathan & Poulos (1998), for an analytical prediction for tunnelling-induced ground movements in clays.

Regards to the gap parameter 'g' for the magnitude of the equivalent two-dimensional void formed around the tunnel, the ovalisation (Fig. 6b) is attributed to the combined effects of the three-dimensional elastoplastic ground deformation at the surface, overexcavation of soil around the periphery of the tunnel shield, and the physical gap that is related to the tunnelling machine, shield, and lining geometry. Rowe & Kack (1983) defined the undrained gap parameter 'g' as shown in $g = G_p + U_{3D}^{*} + \omega$, where G_p = physical gap ($G_p = 2\Delta + \delta$) that represents the geometric clearance between the outer skin of the shield and the lining; Δ = thickness of the tailpiece; δ = clearance required for erection of the lining; U_{3D}^{*} = equivalent 3D elasto-plastic deformation at the tunnel face; and ω = value that takes into account the quality of workmanship. The theoretical method for estimation of the gap parameter can be found in Lee et al. (1992).

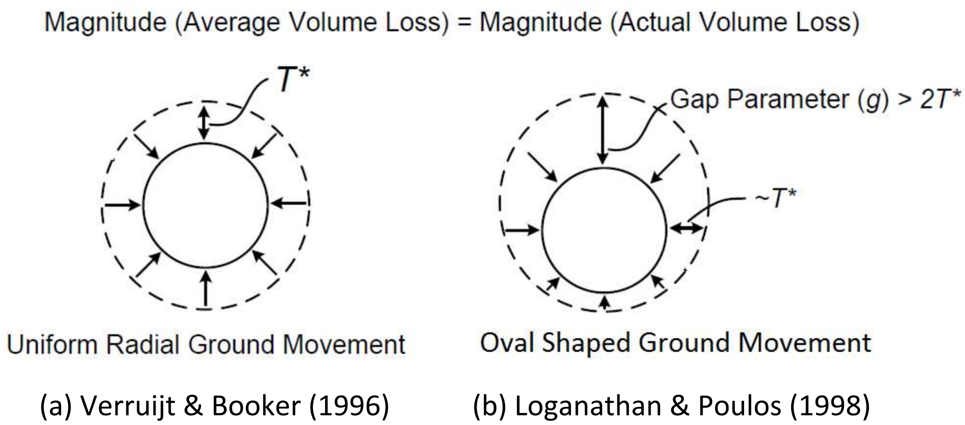


FIG. 6. Circular- and Oval-shaped ground deformation patterns around tunnels.

Loganathan & Poulos (1998) defined the equivalent undrained ground loss ε_0 with respect to the gap parameter: $\varepsilon_0 = \left(4gR + g^2\right)/\,4R^2 \times 100\%$, where R = radius of the tunnel. According to the ground deformation patterns and ground loss boundary conditions, Loganathan & Poulos (1998) derived the nonlinear ground movement around the tunnel soil interface, as shown in Equation (9).

$$\varepsilon_{x,z} = \frac{4gR + g^2}{4R^2} \exp\left\{ -\left[\frac{1.38x^2}{(H+2R)^2} + \frac{0.69z^2}{H^2} \right] \right\} \tag{9}$$

After incorporated into the closed form elastic solutions derived by Verruijt & Booker (1996), the modified formula for the prediction of surface settlement can be wright as shown in Equation (10).

$$u_{z=0} = 4(1-\upsilon)R^2 \frac{H+R}{(H+R)^2 + x^2} \frac{4gR + g^2}{4R^2} \exp\left[-\frac{1.38x^2}{(H+2R)^2} \right] \tag{10}$$

As shown in Fig. 5b, Loganathan & Poulos's method with $g = 0.5u_c \sim 2.0u_c$ is incorporated to estimate tunneling induced ground surface settlement, which gives better agreement with experimental observation, especially for $g = 1.0u_c \sim 1.5u_c$.

CONCLUSIONS

By modelling cavity unloading process, analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) were proposed in this paper to predict the soil behaviour around tunnels, including stress fields and crown/ground settlements. Taking the advantages of CASM with the ability of capturing overall behaviour of clay and sand, large-strain and effective stress analyses of cavity contraction provided the distributions of stress/strain within elastic and plastic regions around tunnels. The results of soil behaviour around tunnels using cylindrical and spherical scenarios showed identical results with previous analytical solutions using original Cam-clay model. Comparisons were also provided between the predicted and measured behaviour of tunneling, using data of centrifuge tunnel test in clay. To account for the effect of free ground surface, Loganathan & Poulos's method was suggested to be incorporated to estimate tunneling induced ground surface settlement, by using undrained gap parameter $g = 1.0 u_c \sim 1.5 u_c$.

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