

This is a repository copy of *The evolution of host defence to parasitism in fluctuating environments*.

White Rose Research Online URL for this paper: http://eprints.whiterose.ac.uk/124875/

Version: Accepted Version

Article:

Ferris, C. and Best, A. (2018) The evolution of host defence to parasitism in fluctuating environments. Journal of Theoretical Biology, 440. pp. 58-65. ISSN 0022-5193

https://doi.org/10.1016/j.jtbi.2017.12.006

Article available under the terms of the CC-BY-NC-ND licence (https://creativecommons.org/licenses/by-nc-nd/4.0/).

Reuse

This article is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs (CC BY-NC-ND) licence. This licence only allows you to download this work and share it with others as long as you credit the authors, but you can't change the article in any way or use it commercially. More information and the full terms of the licence here: https://creativecommons.org/licenses/

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



eprints@whiterose.ac.uk https://eprints.whiterose.ac.uk/ ¹ Title: The Evolution of Host Defence To Parasitism in Fluctuating Environments.

² Authors: Charlotte Ferris¹, Alex Best¹

³ ¹ School of Mathematics and Statistics, University of Sheffield, Sheffield, S3 7RH, UK

4 Corresponding Author: Charlotte Ferris, School of Mathematics and Statistics, University
 5 of Sheffield, S3 7RH, UK. Email: ceferris1@sheffield.ac.uk

6 Keywords: Adaptive dynamics, Host-parasite, Host evolution, Seasonality

7 Abstract:

Given rapidly changing environments, it is important for us to understand how the evolution of 8 host defence responds to fluctuating environments. Here we present the first theoretical study 9 of evolution of host resistance to parasitism in a classic epidemiological model where the host 10 birth rate varies seasonally. We show that this form of seasonality has clear qualitative and 11 quantitative impacts on the evolution of resistance. When the host can recover from infection, 12 it evolves a lower level of defence when the amplitude is high. However, when recovery is absent, 13 the host increases its defence for higher amplitudes. Between these different behaviours we find 14 a region of parameter space that allows evolutionary bistability. When this occurs, the level 15 of defence the host evolves depends on initial conditions, and in some cases a switch between 16 attractors can lead to different periods in the population dynamics at each of the evolutionary 17 stable strategies. Crucially, we find that evolutionary behaviour found in a constant environment 18 for this model doesn't always hold for hosts with highly variable birth rates. Hence we argue 19 that seasonality must be taken into account if we want to make predictions about evolutionary 20 trends in real-world host-parasite systems. 21

22 1. Introduction

Given the ubiquity of infectious diseases in natural systems there is strong selection pressure 23 on host organisms to evolve costly defence mechanisms. A wide range of theoretical work has 24 been developed to understand the evolution of host defence against parasitism, with much of 25 this work focused on the ecological/epidemiological feedbacks that drive selection of quantitative 26 host defence (van Baalen, 1998; Boots & Haraguchi, 1999; Boots & Bowers, 1999, 2004; Restif 27 & Koella, 2003; Miller et al., 2005, 2007; Bonds, 2006; Best et al., 2008, 2009; Carval & 28 Ferriere, 2010). These studies have explored how long-term, stable investment in host defence 29 varies with ecological/epidemiological parameters, as well as determining the conditions that 30 can lead to coexistence of strains through evolutionary branching. However, the vast majority 31 of these studies assume that the populations live in a temporally static environment. In reality, 32 almost all natural systems are subject to some degree of temporal environmental heterogeneity, 33 in particular fluctuations caused by seasonality. For example, many natural species exhibit 34 seasonal reproductive strategies driven by regular environmental fluctuations (Rowan, 1938; 35 Stawski et al., 2014; Ketterson et al., 2015; Furness, 2016). It is therefore essential that we 36 consider the impact of fluctuating environmental conditions on the evolution of host defences. 37

It is well established that variable climates affect ecological systems (Ewing et al., 2016), in-38 cluding the spread and impact of diseases (Fine & Clarkson, 1982; Finkenstädt & Grenfell, 39 2000; Altizer et al., 2006). Many theoretical studies have considered the effects of seasonality 40 in purely epidemiological models (i.e., non-evolutionary), often through a periodic transmission 41 rate (Schwartz & Smith, 1983; Aron & Schwartz, 1984; Olsen & Schaffer, 1990). Increasing the 42 amplitude of the transmission rate can generate sub-harmonic oscillations or cause the popula-43 tion dynamics to move through a series of period-doubling bifurcations, eventually leading to 44 chaotic dynamics (Grossman, 1980; Schwartz & Smith, 1983; Greenman et al., 2004; Grassly 45 & Fraser, 2006; Childs & Boots, 2010). Small perturbations in these seasonal models can also 46 trigger the system to switch between distinct attractors, often due to resonance, potentially 47 leading to significant changes in the population dynamics and different patterns of outbreaks 48 (Smith, 1983; Schwartz, 1985; Keeling et al., 2001; Kamo & Sasaki, 2002; Greenman et al., 49 2004). These complex dynamics have been found to exist less frequently when seasonality is 50 assumed to occur in the host birth rate rather than transmission (White et al., 1996; Begon et 51 al., 2009; Duke-Sylvester et al., 2011; Dorélien et al., 2013; Peel et al., 2014). Predictions about 52

the impact of a disease are likely to be more accurate when either of these types of seasonality
are included in the model (White et al., 1996; Kamo & Sasaki, 2002).

There is an increasing appreciation of the importance of temporal heterogeneity in host-enemy 55 interactions within the experimental evolution literature (Blanford et al., 2003; Friman & Laakso, 56 2011; Hiltunen et al., 2012; Harrison et al., 2013), for example showing that rapidly fluctuat-57 ing environments constrain co-evolutionary arms races in a bacteria-phage system (Harrison et 58 al., 2013). Theoretically, however, evolution and seasonality have rarely been studied together 59 in a host-parasite context. The few studies that do exist have either investigated evolution 60 of only the parasite (Koelle et al., 2005; Sorrell et al., 2009; Donnelly et al., 2013), or used 61 a genetic-based, rather than ecology-driven, model for evolution of the host (Nuismer et al., 62 2003; Mostowy & Engelstädter, 2011; but see Poisot et al., 2012). Seasonality in the host's 63 birth rate does not affect the evolution of the parasite's transmission/virulence strategy un-64 less a density-dependence is applied to virulence (parasite-induced mortality) (Donnelly et al., 65 2013). This occurs because the average susceptible density, and therefore the parasite fitness, 66 doesn't depend on the seasonal parameters unless this density-dependence is included. Else-67 where, step-wise environmental variation implemented through a dynamic resource was found 68 to change the coevolutionary outcomes in a gene-for-gene based host-parasite system (Poisot et 69 al., 2012). In particular, they found that both the host and parasite invest more in resistance 70 and infectivity respectively for higher amplitudes in the seasonality. However, we currently have 71 no theory specifically addressing the impact that seasonality has on the evolution of host defence 72 to parasitism. 73

Here we investigate the impact of a continuous seasonal birth rate on the evolution of quantitative host avoidance through small mutation steps using an evolutionary invasion (adaptive dynamics) method. We use a classic SIS (Susceptible-Infected-Susceptible) model, and focus on how the amplitude and period of the implemented seasonality impacts the ecological/epidemiological dynamics, and therefore the evolution of the host.

79 2. Methods

The population is modelled using an SIS (susceptible-infected-susceptible) framework with the following set of ordinary differential equations:

$$\frac{dS}{dt} = a(1-qN)S - bS - \beta SI + \gamma I,$$
(1)

 $\frac{dI}{dt} = \beta SI - (b + \alpha + \gamma)I, \qquad (2)$

where S and I are the susceptible and infected population sizes respectively, and N = S + I is 83 the total population size (Anderson & May, 1981). All offspring are born susceptible at rate a, 84 and only susceptible hosts are able to reproduce, i.e. the parasite renders the host (temporarily) 85 sterile. The births are limited by density with crowding coefficient q, so that birth rate is low 86 when competition is high. All hosts die at baseline mortality rate b, with an additional infected 87 death rate α . The parasite is transmitted to susceptible hosts at rate βI due to contact with 88 infected individuals. Hosts recover from the parasite at rate γ and return to the susceptible 89 class with no acquired immunity. Default parameter values are given in table 1. 90

We assume that seasonality occurs on the ecological timescale, so to incorporate this we let the birth rate depend periodically on time t:

$$a = a(t) = a_0(1 + \delta \sin(2\pi t/\epsilon)), \tag{3}$$

⁹³ where a_0 is the average birth rate, $\delta \in [0, 1]$ is the amplitude and $\epsilon > 0$ is the period of the ⁹⁴ forcing. Periodic birth rates have been observed in a large number of species (Rowan, 1938; ⁹⁵ Ketterson et al., 2015), and this type of function has been used many times to model a time-⁹⁶ varying birth rate (He & Earn, 2007; Donnelly et al., 2013; Dorélien et al., 2013) or transmission ⁹⁷ rate (Schwartz & Smith, 1983; Grassly & Fraser, 2006; Childs & Boots, 2010). For our default

Parameter	Definition	Default Value
$\hat{a_0}$	Trade-off coefficient in the average birth rate	108
p	Trade-off coefficient in the average birth rate	103.75
c	Trade-off coefficient in the average birth rate	1.5
β	Transmission coefficient	Varies
β_{\min}	Minimum transmission coefficient	0.5
β_{\max}	Maximum transmission coefficient	10
δ	Amplitude of the birth rate forcing	Varies
ϵ	Period of the birth rate forcing	1
q	Crowding coefficient acting on births	0.1
b	Baseline mortality rate	1
γ	Recovery Rate	Varies
α	Virulence/additional death rate due to parasite	1

Table 1: Parameter definitions and default values.

82

⁹⁸ parameter values, the period ϵ is the same as the average lifespan b (1 year), but see section 3.4 ⁹⁹ for varying ϵ or Appendix F for alternative b.

We assume that the host evolves defence through the transmission coefficient (avoidance) β . We let the average birth rate depend on this as a trade-off so that there is a cost to resisting the parasite, as there is experimental support for such a relation to exist (Boots & Begon, 1993). We use the following trade-off function based on that used by White et al. (2006):

$$a_0 = a_0(\beta) = \hat{a_0} - p \frac{\left(1 + \frac{\beta - \beta_{\min}}{\beta_{\max} - \beta_{\min}}\right)}{\left(1 + c \frac{\beta - \beta_{\min}}{\beta_{\max} - \beta_{\min}}\right)},\tag{4}$$

where $\hat{a}_0 > 0$, 0 , <math>c > 1 and $\beta \in [\beta_{\min}, \beta_{\max}]$. $a_0(\beta)$ has minimum $\hat{a}_0 - p$, and 104 parameters p, c determine the gradient and curvature of the trade-off, which needs to have 105 positive gradient: as the host invests in defence against the parasite (β decreases), less can be 106 invested in reproduction $(a_0(\beta) \text{ decreases})$ (Boots & Haraguchi, 1999; Geritz et al., 2007). The 107 constraints on the trade-off parameters give accelerating costs of defence, so that it is more costly 108 to invest in resistance when defence is already high $\left(\frac{d^2a_0(\beta)}{d\beta^2} < 0\right)$, see figure A.1 in Appendix 109 A. Accelerating trade-offs generally lead to evolutionary attractors (Hoyle et al., 2008), which 110 will be our focus here. 111

We use the adaptive dynamics method to study evolution of the host in the transmission coef-112 ficient β . The method involves adding a rare mutant with susceptible and infected population 113 sizes $S_{\rm m}$, $I_{\rm m}$ and transmission coefficient $\beta_{\rm m}$ very close to the resident transmission coefficient β . 114 We assume that mutants occur infrequently so that the resident population reaches the dynamic 115 attractor of the population dynamics (generally a limit cycle here) before the next mutant is 116 introduced (Geritz et al., 1998). When a new mutant arises, it is rare compared to the current 117 population, so we assume that the resident remains at its limit cycle as long as the mutant 118 population is small (Geritz et al., 1998). To analyse how the host evolves, we consider the mu-119 tant's fitness, defined to be the long-term exponential growth rate of the mutant in the current 120 environment (Metz et al., 1992). 121

In the case where $\gamma = 0$, the fitness is relatively simple to find. We no longer have infected mutants (they are absorbed into I), and we can read off the time-varying growth rate r(t) of the mutant host from the linearisation of the equation for the susceptible mutant $(dS_m/dt = r(t)S_m,$ see Appendix B). Following the method from Donnelly et al. (2013), we can then take the average ¹²⁶ of this over one period to find the mutant fitness:

$$r = \frac{1}{T} \int_{P_0}^{P_1} r(t) dt = \frac{a_0(\beta_{\rm m})}{T} \int_{P_0}^{P_1} \left\{ \left[1 + \delta \sin\left(\frac{2\pi t}{\epsilon}\right) \right] \left[1 - qN(t) \right] \right\} dt - b - \frac{\beta_{\rm m}}{T} \int_{P_0}^{P_1} I(t) dt \,, \quad (5)$$

where T is the period of the system, P_0 is an arbitrary time after the resident dynamics have reached a limit cycle, and $P_1 = P_0 + T$.

¹²⁹ Unfortunately we cannot use this averaging method when $\gamma > 0$. Instead, we have to find the ¹³⁰ Lyapunov exponents or Floquet multipliers numerically (Metz et al., 1992; Klausmeier, 2008). ¹³¹ We do this by letting the linearly independent solutions of the linearised mutant equations be ¹³² of the form $\mathbf{X}_i(t) = e^{\mu_i t} \mathbf{p}_i(t)$ for $i \in 1, 2$ (Grimshaw, 1990), and then take the largest μ_i as the ¹³³ mutant fitness. A full discussion of the method is given in Appendix B. We also ran stochastic ¹³⁴ simulations which relax the separation of timescales assumption, and these confirm our key ¹³⁵ results, for examples see figure 2 and Appendix D.

136 3. Results

137 3.1. Population dynamics

To explore how the population dynamics shape selection, we first consider the nature of the attractors of equations (1) - (2). For most parameter sets, the period of the population dynamics is equal to that of the forcing in the birth rate, i.e. $T = \epsilon$. However, there are parameter regions where the population undergoes a period-doubling bifurcation with resulting cycles of period $T = \lambda \epsilon$ for some positive integer λ . We can also find cases of multiple attractors, often with different periods. After finding this period, we can write down the average size of each class as follows (method in Appendix C):

$$\hat{S} = \frac{1}{T} \int_{P_0}^{P_1} S(t) dt = \frac{\alpha + b + \gamma}{\beta}$$
(6)

145

$$\hat{I} = \frac{1}{T} \int_{P_0}^{P_1} I(t) dt = \frac{\beta}{(\alpha + b + \gamma)T} \int_{P_0}^{P_1} SI dt.$$
(7)

Immediately we can see that the average susceptible population \hat{S} does not depend on either of the seasonal parameters. However, this is not the case for the average infected population \hat{I} ,

which we have to evaluate numerically for $\delta > 0$. For the default parameter values in table 1, 148 we find that I increases with the amplitude of seasonality δ , and hence the average prevalence 149 $\left(\frac{1}{T}\int_{P_0}^{P_1}\frac{I(t)}{N(t)}dt\right)$ of the parasite also increases. When we vary the period ϵ , \hat{I} increases to a peak 150 at $\epsilon \approx 1.5$ due to resonance with the unforced system, then decreases as ϵ continues to increase. 151 This is discussed further in section 3.4. Considering the fitness expression in (5), it is clear that 152 the effect of seasonality on these population averages will have crucial impacts on host evolution 153 for all recovery values, unlike with parasite evolution (Donnelly et al., 2013). We can therefore 154 use these averages to explain how the host evolves in response to changes in parameters. 155

156 3.2. Evolution for $\gamma = 0$

¹⁵⁷ When we set $\gamma = 0$, we revert back to the simpler SI model. As stated in section 2, we can write ¹⁵⁸ down the fitness of the host in this case for all $\delta \in [0, 1]$ in equation (5). Here we only consider ¹⁵⁹ continuously stable strategies (CSSs) unless stated otherwise, i.e. singular points that are both ¹⁶⁰ evolutionarily stable (ES) and convergence stable (CS) as defined by Geritz et al. (1998) which ¹⁶¹ lead to long-term evolutionary attractors. This behaviour was confirmed using pairwise-invasion ¹⁶² plots (PIPs) and simulations over a range of parameters, for an example see Appendix D.

When δ is increased from 0, we find that the average infected population increases and so does the investment in defence (i.e. β^* decreases & higher defence), see figure 1(a),(b). This is what we would naively expect: as the average infected population increases, the host has to invest more in resistance against the parasite to reduce the proportion of infected individuals (Boots & Haraguchi, 1999; Boots et al., 2009).

In section 3.1 we mentioned that for particular parameter sets, period-doubling bifurcations and 168 bistability between different attractors in the population dynamics can occur. Figure 1(c), (d) 169 shows an example of this phenomenon together with host selection. As we increase δ , there 170 is a point at which the 1-year solution undergoes a period-doubling bifurcation. The resulting 171 2-year solution then goes through two folds, after which a stable solution exists, see Appendix E. 172 Bistability between different solutions for $\delta \in (0.57, 0.63)$ causes overlap of the singular points 173 given by each cycle, giving a discrete change in the CSS resistance β^* and average infected 174 population, figure 1(c), (d). Note that due to the basins of attraction for each CSS within 175 the bistability region, the host can only evolve towards the T = 2 singular point for initial 176 transmission coefficient β_0 greater than the lower bound of the bistablility region, see Appendix 177

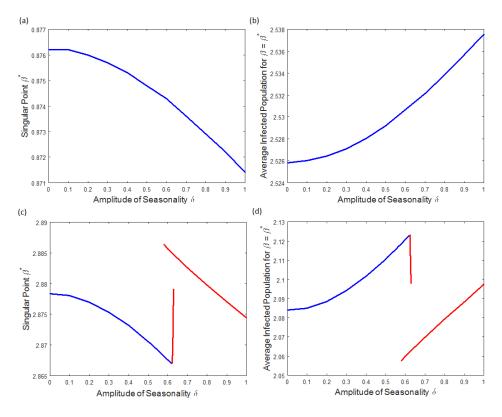


Figure 1: Change in (a),(c) the singular point β^* and (b),(d) the average infected population for $\beta = \beta^*$ as the amplitude of seasonality δ varies for $\gamma = 0$. Default parameters were used in (a),(b), with $\hat{a_0} = 104$ in (c),(d). In (c),(d), on the left only the 1-year solution is stable, and on the right only the 2-year solution. In the centre there is bistability between the 1 and 2-year cycles or between the two different 2-year cycles. Blue - period T = 1; Red - period T = 2.

E. This jump in the average infected population and singular point occurs whenever a perioddoubling bifurcation and bistability between attractors exists for $\gamma = 0$.

Overall the impact of the amplitude of seasonality δ on the singular point for $\gamma = 0$ is weak for a wide range of parameters as seen in figure 1. Seasonality has a much stronger effect for higher recovery rates, as discussed below.

183 3.3. Evolution for $\gamma > 0$

Unlike in the SI model above, when $\gamma > 0$ we use a numerical approximation to find the host fitness. When γ is relatively close to zero, we find one singular point which decreases as δ increases, as seen in section 3.2. However for positive but small values of γ , this behaviour changes direction. We start to see both the singular point β^* and the average infected population increasing, in contrast to $\gamma = 0$ where the trends go in opposite directions. As recovery increases, selection for defence is weakened, and so at this small recovery maintaining a large population size through births becomes more important than resistance to the parasite, causing the change

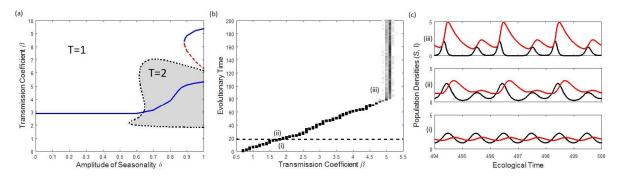


Figure 2: (a) Change in the singular points as δ varies for $\hat{a}_0 = 104$, $\gamma = 0.005$. Blue lines indicate the CSS points, red dashed lines the repeller point and black dotted lines the switch between attractors. The period of the population dynamics is 2 in the shaded region and 1 (ϵ) elsewhere. (b) Simulation example corresponding to (a) with initial transmission coefficient $\beta_0 = 0.7$ and $\delta = 0.9$, which evolves towards the lowest CSS $\beta_L^* = 5.067$. Darker squares indicate a higher proportion of the population with the corresponding transmission coefficient β , and the dashed line marks the point where evolution drives the population to switch to an attractor with period T = 2. (i)-(iii) correspond to sample population dynamics of the resident strain shown in (c), with black for S and red for I at evolutionary times (i) 10, (ii) 20 and (iii) 100.

As we continue to increase the recovery rate, we reach a region of γ values where three singular 192 points exist, two CSSs with a repeller between them, for an example see figure 2(a). Here we 193 have evolutionary bistability between two CSSs, and for certain parameter sets the CSSs have 194 different cycle lengths due to the stability of the attractors in the population dynamics, as in 195 the example shown. In this case the host could start in a 1-year cycle, but evolution would 196 drive it into a 2-year regime, i.e. evolution can drive changes in the population dynamics, see 197 figure 2(b), (c). We can also have the situation where all three singular points give period two 198 population dynamics (not shown). Figure 3 shows two-dimensional contour plots for two CSS 199 points in the parameter regions where they occur. Both CSS points increase with δ , as argued 200 above, but they go in opposite directions as γ increases. This occurs because at high levels of 201 defence (low β^* , figure 3(a)), selection for even higher defence weakens as recovery increases, 202 and so the host decreases its resistance. However, when the host has a low level of defence (high 203 β^* , figure 3(b)), the susceptible hosts become infected more quickly and an increase in recovery 204 raises the infected population further, hence there is strong selection for defence and the host 205 invests more in resistance. Recovery therefore has a much more complicated effect on evolution 206 when seasonality is included in the model, since most of these bistability regions occur for large 207 amplitudes. 208

If we increase γ further, the size of the interval of δ values where bistability occurs decreases to zero. For all γ values above this point, we find only one singular point β^* that increases with δ ,

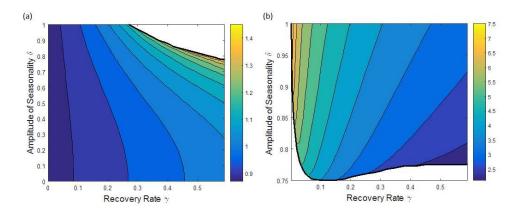


Figure 3: Two-dimensional contour plots showing the change in the two CSS points that occur as γ and δ vary for default parameters. (a) β_L^* , the smallest CSS point; (b) β_H^* , the highest CSS point. White areas indicate where each singular point does not exist.

figure 4(a), for the same reasons as explained above.

Figure 4(a) shows a two-dimensional contour plot for the singular point β^* as δ and γ vary 212 in the region where one singular point exists. For the majority of amplitudes, the average 213 infected population decreases with increasing recovery, and hence the host invests less in defence. 214 However, we have slightly more complicated behaviour for high δ . Initially we find that the host 215 increases defence (decreases β^*), then at an intermediate recovery the trend turns and the host 216 decreases its defence (increases β^*). This behaviour is due to changes in the average infected 217 population, which peaks for intermediate γ since initially the increase in susceptible individuals 218 available to be infected outweight the loss from recovery. 219

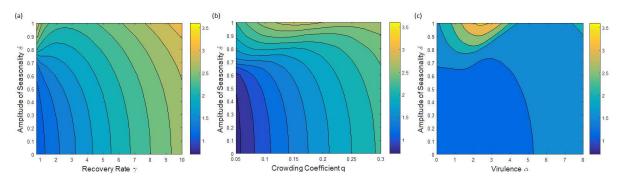


Figure 4: Two-dimensional contour plots showing the value of the singular point β^* as amplitude of seasonality δ and (a) recovery rate γ , (b) crowding factor q and (c) virulence α vary. Other parameters were fixed at default values from table 1 with $\gamma = 1$.

Alterations to other model parameters also causes variation in the host's evolution. Figure 4(b) shows the change in the singular point β^* as δ and the crowding coefficient q are varied. As above, we see that β^* increases with δ for all values of q. As we increase q for fixed δ , the infected population size decreases. We therefore expect the host to invest less in defence as q increases,

i.e. β^* to increase, which is exactly what we find for most values of δ . However, for very high 224 amplitudes we find that the level of defence has a more complicated relationship with q, and 225 in particular that defence is minimal (β^* maximum) for intermediate and very high values of 226 q. For low q, the average infected population decreases as q increases, hence the host invests 227 less in defence as for lower δ . However, there comes a point where the susceptible population is 228 relatively low due to the decreased resistance, and so the host invests more in defence rather than 229 births to increase the average susceptible population. As q continues to increase, the average 230 infected population becomes small enough that selection for defence is weakened, and so the 231 host returns to its previous behaviour and invests less in defence (β^* increases) for very high q. 232 We find similar results when the virulence α varies, figure 4(c). As α increases, the average 233 infected population decreases and the host can afford to invest less in defence, which is exactly 234 what we find for δ up to intermediate values. However, as for varying q, the trend becomes 235 more complicated for highly seasonal birth rates. In this region, we now have a large peak in β^* 236 for an intermediate value of α , followed by a trough and a small increase in β^* for high α . For 237 small and very large α , this behaviour is due to the average infected population decreasing and 238 therefore the host can afford to invest less in defence. However, the initial behaviour causes the 239 total population to decrease, and there is a region of α values where the host needs to evolve 240 in such a way that the population size increases. Therefore the host has to balance changes in 241 the infected and total population sizes, giving the more complicated evolutionary behaviour for 242 high amplitudes. 243

The results discussed above are for a parameter set where the host lifespan is equal to the period of forcing (one year). The effects seen are dampened for longer lived hosts (smaller b), and there can be no difference in the evolutionary behaviour with γ , q or α for different amplitudes (see Appendix F). Hence the effect of the amplitude on the host's evolutionary behaviour with other parameters depends on context, and in particular we cannot rely on the behaviour remaining the same as the amplitude of the birth rate increases for short-lived hosts.

250 3.4. Varying the Period of the Forcing ϵ

The population dynamics have period determined by that of the forcing ϵ , as discussed in section 3.1. We can investigate how changing this period over a wide range of values affects the evolution of the host, figure 5(a) (although in many systems a 1-year cycle ($\epsilon = 1$) may be the

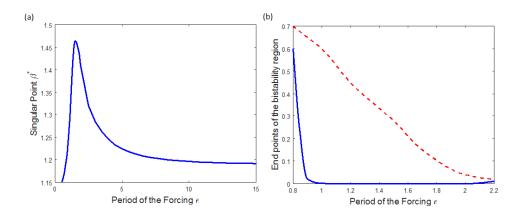


Figure 5: (a) Change in the CSS singular point β^* as ϵ varies for default parameters with $\delta = 0.5$ & $\gamma = 1$. (b) Change in the size of the bistability region in recovery rate γ as ϵ varies. Blue: γ value where bistability starts; Red dashed: γ value where bistability ends.

most appropriate). We found that there is a large peak in both the average infected population 254 and the singular point β^* caused by resonance with the natural timescale of the model, after 255 which they decrease slowly as ϵ is increased further. Hence for rapidly changing environments 256 $(\epsilon \text{ low})$, any alteration to the period would have a significant impact on the host's evolution. In 25 comparison, for slowly varying environments any change in the period barely alters the host's 258 evolution. This behaviour with ϵ stays roughly the same for all parameters tested. Similarly, 259 when both the period and other parameters are varied simultaneously, the period doesn't affect 260 the evolutionary behaviour we find as other parameters change and vice versa. 261

The bistability region studied in section 3.3 changes in size for varying period ϵ . Figure 5(b) shows this, indicating that the bistability region is largest (in γ) for $\epsilon \approx 1$, slightly lower than the peak seen in figure 5(a). Above and below this value the bistability region decreases in size and quickly disappears. The period of the seasonality therefore has a large impact on whether or not these bistability regions occur.

267 4. Discussion

We have shown that seasonality in the ecological dynamics, specifically the birth rate, has a clear quantitative and qualitative effect on the evolution of host resistance against a parasite in our model. The relative size and nature of the impact depends crucially on the underlying epidemiological model, and particularly on the potential for recovery from infection. We found regions of parameter space where there is bistability between two distinct evolutionary strategies (CSS points), which can occur alongside a switch between attractors in the population dynamics. In these regions, evolution could drive the population to a different attractor, fundamentally altering the population dynamics the host experiences. Crucially, we also found that well known patterns for the host's evolutionary strategy in a constant environment don't necessarily hold for variable birth rates, particularly when the amplitude of fluctuations is high.

We found that the amplitude of the seasonality and the recovery rate are key processes affecting 278 the evolution of the host's defence for a seasonal birth rate in our model. When recovery is 279 absent, the host invests more in defence as the amplitude of seasonality increases as this leads 280 to an increase in the average infected population and thus selection for increased defence. The 281 trends observed were weak, but are consistent with existing theory on the evolution of avoidance 282 in the absence of recovery (Boots & Haraguchi, 1999; Donnelly et al., 2015). When the host 283 can recover from the parasite, the evolutionary dynamics become more complicated. The trend 284 of host investment with the amplitude of seasonality switches direction at a low recovery rate, 285 above which the host decreases its defence as the amplitude increases, since the host is now 286 balancing reduced transmission against the increased contribution to fitness made by infected 287 hosts through recovery. These results emphasise the importance of recovery in host-parasite 288 infections as they prevent the parasite from being a 'functional predator' (Boots, 2004; Donnelly 289 et al., 2015; Best et al., 2017). We also note that our results with recovery for host evolution are 290 similar to the findings of Donnelly et al. (2013) for parasite evolution, where the parasite invests 291 more in infectivity as the amplitude of seasonality increases. This suggests a robust result that 292 in many systems increased seasonal amplitude will lead to higher transmission, though a full 293 coevolutionary study that includes recovery would be needed to confirm this. 294

There has been a lack of attention to how seasonality might affect host evolution in theoretical 295 studies, even though it has been shown that epidemiological dynamics can be greatly impacted 296 by a variable environment (Altizer et al., 2006; Grassly & Fraser, 2006). In addition, it is well 297 known that a wide range of species reproduce seasonally due to environmental fluctuations, for 298 example in bats (Stawski et al., 2014), killifish (Furness, 2016) and birds (Ketterson et al., 2015). 299 The theorectical studies that do consider seasonality are generally co-evolutionary with a gene-300 for-gene based infection interaction (Nuismer et al., 2003; Mostowy & Engelstädter, 2011; Poisot 301 et al., 2012). Of particular relevance to our study, Poisot et al. (2012) include explicit ecological 302 dynamics in their model, using an additional resource variable with discrete fluctuations to 303 implement seasonality, as well as a partial gene-for-gene infection mechanism. Despite these 304

different underlying assumptions, they too find that the host invests more in defence when the 305 amplitude of the seasonality is high and there is no recovery. Moreover, in an experimental study, 306 Blanford et al. (2003) showed that pea aphids, Acyrthosiphon pisum, evolved higher resistance 307 against a fungal pathogen, *Erynia neoaphidis*, when periodically exposed to higher temperatures. 308 Since the fecundity of aphids varies with temperature (Ramalho et al., 2015) and aphids lack 309 many of the genes associated with immune response to microbes (Gerardo et al., 2010), these 310 results agree with the theoretical results found here and by Poisot et al. (2012), that increased 311 seasonality leads to increased resistance in the absence of recovery. 312

Interestingly, we found that evolutionary bistability can exist between two convergence stable 313 strategies for small recovery rates. When the amplitude of the birth rate is high, the host may 314 evolve towards either of two levels of defence depending on initial conditions. This bistability 315 only occurs for a finite range of amplitudes, meaning that a small change in the amplitude could 316 lead to a large change in the level of defence the host evolves. Furthermore, the bistability 317 can occur in conjunction with a switch between attractors with different cycle lengths, with the 318 higher level of defence (lower transmission) giving a regime of two-year cycles in the population 319 dynamics, whereas the lower defence (higher transmission) is in a one-year regime, meaning that 320 evolution can in fact drive the population dynamics into a cycle with a different period. This 321 effect of evolution moving host-parasite systems into regions of qualitatively different population 322 dynamics has also been shown in systems which assume a constant environment but population 323 cycles occur naturally (Hoyle et al., 2011; Best et al., 2013). These results emphasize that 324 ecology/epidemiology and evolution are involved in a two-way feedback, as not only does ecology 325 drive selection, but evolution can determine the nature of the population dynamics. 326

There have been many studies considering the evolution of host defence against parasites that 327 did not include seasonality (van Baalen, 1998; Boots & Bowers, 1999; Boots & Haraguchi, 1999). 328 We have shown here that many classic results are likely to be true in a weakly seasonal system. 329 but may not hold for an increasingly variable birth rate. For example, as virulence varies, 330 investment in resistance decreases as found previously (Boots & Haraguchi, 1999; Best et al. 331 2017) for low amplitudes of seasonality, but at high amplitudes is maximized at either minimum 332 or relatively high virulence. We see similar behaviour for varying crowding factor, in that our 333 results agree with those found by Boots & Haraguchi (1999) for low amplitudes, but disagree for 334 high amplitudes. These differences are a result of complicated feedbacks between seasonality. 335

population sizes and selection which alter the costs/benefits of resistance and births. However, we have shown that this effect is dampened for hosts with longer lifespans, returning to the behaviours seen in previous work for all amplitudes of the seasonality (see Appendix F). It is clear that while many results found for constant environments remain true when the birth rate is variable in time, this may not be the case when the amplitude is particularly high, especially for short-lived hosts.

We also investigated the impact of changing the period of the forcing on the evolution of the 342 host's defence. We found that changing the period induces a peak in the infected density, caused 343 by resonance in the population dynamics with the unforced system. Naively we would expect 344 this to lead to a maximum level of investment in defence, however, as with varying amplitude 345 in the presence of recovery, the host evolves towards a minimum level of defence in order to 346 maintain a large overall population size through increased birth rate. Near the peak, small 347 alterations in the period will lead to relatively large changes in the evolutionary investment in 348 defence. Away from the peak, the curve is almost flat and so the host's evolution is barely 349 affected by changes in the period when it is already large. In an experimental study, Harrison 350 et al. (2013) found that resistance of P. fluorescens SBW25 to a phage was constrained most 351 strongly in rapidly fluctuating environments, while Duncan et al. (2017) showed that resistance 352 of the same bacteria evolved more quickly in rapidly fluctuating environments. It is unclear to 353 what extent our results agree with these experimental studies, in part due to these systems being 354 co-evolutionary with genetic specificity, and in part because it is difficult to ascertain which side 355 of the resonance peak these studies may be focusing on. It is clear, though, that the time-frame 356 of the fluctuations has important consequence to the evolutionary outcome. 357

Temporal heterogeneity, including seasonal fluctuations, are a fundamental aspect of all natural 358 ecological systems. However, both experimental and theoretical studies have rarely investigated 359 the impact of fluctuating environments on evolutionary patterns. Here we have shown that a 360 seasonal birth rate has a significant qualitative impact on the evolution of host defence in an 361 SIS model, which is highly dependent on the presence and size of recovery. It is clear that 362 key features of evolutionary dynamics may be missed by assuming a constant environment, and 363 therefore important for us to consider how seasonality may impact host-parasite evolution more 364 widely. There is clearly scope for further theoretical and experimental work to explore the 365 impacts of seasonality on host-parasite evolution. 366

Acknowledgements We thank Mike Brockhurst and Dylan Childs for comments on an earlier
 version of the manuscript. This work was funded by The Leverhulme Trust.

- [1] S. Altizer, A. Dobson, P. Hosseini, P. Hudson, M. Pascual, and P. Rohani. Seasonality and
 the dynamics of infectious diseases. J. Anim. Ecol., 9:467–484, 2006. doi: 10.1111/j.1461 0248.2005.00879.x.
- [2] R.M. Anderson and R.M. May. The population dynamics of microparasites and their invertebrate hosts. *Phil. Trans. R. Soc. B.*, 291:451–524, 1981. doi: 10.1098/rstb.1981.0005.
- [3] J.L. Aron and I.B. Schwartz. Seasonality and period-doubling bifurcations in an epidemic
 model. J. Theor. Biol., 110:665–679, 1984. doi: 10.1016/S0022-5193(84)80150-2.
- [4] M. Begon, S. Telfer, M.J. Smith, S. Burthe, S. Paterson, and X. Lambin. Seasonal host
 dynamics drive the timing of recurrent epidemics in a wildlife population. *Proc. R. Soc. B.*, 276:1603–1610, 2009. doi: 10.1098/rspb.2008.1732.
- [5] A. Best, H. Tidbury, A. White, and M. Boots. The evolutionary dynamics of withingeneration immune priming in invertebrate hosts. J. R. Soc. Interface, 10:20120887, 2013.
 doi: 10.1098/rsif.2012.0887.
- [6] A. Best, A. White, and M. Boots. Maintenance of host variation in tolerance
 to pathogens and parasites. *Proc. Natl. Acad. Sci.*, 105:20786–20791, 2008. doi:
 10.1073/pnas.0809558105.
- [7] A. Best, A. White, and M. Boots. The implications of coevolutionary dynamics to host parasite interactions. Am. Nat., 173:779–791, 2009. doi: 10.1086/598494.
- [8] A. Best, A. White, and M. Boots. The evolution of host defence when parasites impact
 reproduction. *EER*, 18:393–409, 2017.
- [9] S. Blanford, M.B. Thomas, C. Pugh, and J.K. Pell. Temperature checks the red queen?
 Resistance and virulence in a fluctuating environment. *Ecology Letters*, 6:2–5, 2003. doi:
 10.1046/j.1461-0248.2003.00387.x.
- [10] M. Boots and M. Begon. Trade-offs with resistance to a granulosis virus in the indian meal
 moth, examined by a laboratory evolution experiment. *Functional Ecology*, 7:528–534, 1993.
 doi: 10.2307/2390128.

- [11] M. Boots, A. Best, M.R. Miller, and A. White. The role of ecological feedbacks in the
 evolution of host defence: what does the theory tell us? *Phil. Trans. R. Soc. B.*, 364:27–36,
 2009. doi: 10.1098/rstb.2008.0160.
- [12] M. Boots and R.G. Bowers. Three mechanisms of host resistance to microparasites avoidance, recovery and tolerance show different evolutionary dynamics. J. Theor. Biol., 201:13–
 23, 1999. doi: 10.1006/jtbi.1999.1009.
- [13] M. Boots and R.G. Bowers. The evolution of resistance through costly acquired immunity.
 Proc. R. Soc. Lond. B., 271:715–723, 2004. doi: 10.1098/rspb.2003.2655.
- [14] M. Boots and Y. Haraguchi. The evolution of costly resistance in host-parasite systems.
 Am. Nat., 153:359-370, 1999. doi:10.1086/303181.
- [15] D. Carval and R. Ferriere. A unified model for the coevolution of resistance, tolerance and
 virulence. *Evolution*, 64:2988–3009, 2010. doi: 10.1111/j.1558-5646.2010.01035.x.
- ⁴⁰⁷ [16] D.Z. Childs and M. Boots. The interaction of seasonal forcing and immunity and
 ⁴⁰⁸ the resonance dynamics of malaria. J. R. Soc. Interface, 7:309–319, 2010. doi:
 ⁴⁰⁹ 10.1098/rsif.2009.0178.
- [17] R. Donnelly, A. Best, A. White, and M. Boots. Seasonality selects for more acutely virulent
 parasites when virulence is density dependent. *Proc. R. Soc. B.*, 280:20122464, 2013. doi:
 10.1098/rspb.2012.2464.
- [18] R. Donnelly, A. White, and M. Boots. The epidemiological feedbacks critical to the evolution
 of host immunity. J. Evol. Biol., 28:2042–2053, 2015. doi: 10.1111/jeb.12719.
- [19] A.M. Dorélien, S. Ballesteros, and B.T. Grenfell. Impact of birth seasonality on dynamics
 of acute immunizing infections in sub-saharan africa. *PLoS ONE*, 8:e75806, 2013. doi:
 10.1371/journal.pone.0075806.
- [20] S.M. Duke-Sylvester, L. Bolzoni, and L.A. Real. Strong seasonality produces spatial asynchrony in the outbreak of infectious diseases. J. R. Soc. Interface, 8:817–825, 2011. doi:
 10.1098/rsif.2010.0475.
- 421 [21] A.B. Duncan, E. Dusi, F. Jacob, J. Ramsayer, M.E. Hochberg, and O. Kaltz. Hot spots

- become cold spots: coevolution in variable temperature environments. J. Evol. Biol., 30:55–
 65, 2017. doi: 10.1111/jeb.12985.
- ⁴²⁴ [22] D.A. Ewing, C.A. Cobbold, B.V. Purse, M.A. Nunn, and S.M. White. Modelling the effect
 ⁴²⁵ of temperature on the seasonal population dynamics of temperate mosquitoes. J. Theor.
 ⁴²⁶ Biol., 400:65–79, 2016. doi: 10.1016/j.jtbi.2016.04.008.
- ⁴²⁷ [23] P.E.M. Fine and J.A. Clarkson. Measles in england and walesi: an analysis of factors
 ⁴²⁸ underlying seasonal patterns. *Int. J. Epidemiol.*, 11:5–14, 1982. doi: 10.1093/ije/11.1.15.
- ⁴²⁹ [24] B.F. Finkenstädt and B.T. Grenfell. Time series modelling of childhood diseases: a dynamical systems approach. *Appl. Stat.*, 49:182–205, 2000. doi: 10.1111/1467-9876.00187.
- [25] V-P. Friman and J. Laakso. Pulsed-resource dynamics constrain the evolution of predatorprey interactions. Am. Nat., 177:334–345, 2011. doi: 10.1086/658364.
- [26] A.I. Furness. The evolution of an annual life cycle in killifish: adaptation to ephemeral aquatic environments through embryonic diapause. *Biol. Rev.*, 91:796–812, 2016. doi: 10.1111/brv.12194.
- ⁴³⁶ [27] N.M. Gerardo et al. Immunity and other defenses in pea aphids, acyrthosiphon pisum.
 ⁴³⁷ Genome Biology, 11:R21, 2010. doi: 10.1186/gb-2010-11-2-r21.
- [28] S.A.H. Geritz, E. Kisdi, G. Meszéna, and J.A.J. Metz. Evolutionarily singular strategies
 and the adaptive growth and branching of the evolutionary tree. *Evol. Ecol.*, 12:35–57,
 1998. doi: 10.1023/A:1006554906681.
- ⁴⁴¹ [29] S.A.H. Geritz, E. Kisdi, and P. Yan. Evolutionary branching and long-term coexistence
 of cycling predators: critical function analysis. *Theor. Pop. Biol.*, 71:424–435, 2007. doi:
 ⁴⁴³ 10.1016/j.tpb.2007.03.006.
- [30] N.C. Grassly and C. Fraser. Seasonal infectious disease epidemiology. Proc. R. Soc. B.,
 273:2541-2550, 2006. doi: 10.1098/rspb.2006.3604.
- 446 [31] J. Greenman, M. Kamo, and M. Boots. External forcing of ecological and epi447 demiological systems: a resonance approach. *Physica D.*, 190:136–151, 2004. doi:
 448 10.1016/j.physd.2003.08.008.

- [32] R. Grimshaw. Nonlinear Ordinary Differential Equations. Blackwell Scientific Publications,
 1990. ISBN 0-632-02708-8.
- [33] Z. Grossman. Oscillatory phenomena in a model of infectious diseases. *Theor. Population*Biol., 18:204–243, 1980.
- [34] E. Harrison, A-L. Laine, M. Hietala, and M.A. Brockhurst. Rapidly fluctuating environments constrain coevolutionary arms races by impeding selective sweeps. *Proc. R. Soc. B.*,
 280:20130937, 2013. doi: 10.1098/rspb.2013.0937.
- [35] D. He and D.J.D. Earn. Epidemiological effects of seasonal oscillations in birth rates. *Theor. Pop. Biol.*, 72:274–291, 2007. doi: 10.1016/j.tpb.2007.04.004.
- [36] T. Hiltunen, V-P. Friman, V. Kaitala, J. Mappes, and J. Laakso. Predation and resource
 fluctuations drive eco-evolutionary dynamics of a bacterial community. *Acta Oecol.*, 38:77–
 83, 2012. doi: 10.1016/J.Actao.2011.09.010.
- [37] A. Hoyle, R.G. Bowers, and A. White. Evolutionary behaviour, trade-offs and cyclic and
 chaotic population dynamics. *Bull. Math. Biol.*, 73:1154–1169, 2011. doi: 10.1007/s11538010-9567-7.
- [38] A. Hoyle, R.G. Bowers, A. White, and M. Boots. The influence of trade-off shape on
 evolutionary behaviour in classical ecological scenarios. J. Theor. Biol., 250:498–511, 2008.
 doi: 10.1016/j.jtbi.2007.10.009.
- [39] M. Kamo and A. Sasaki. The effect of cross-immunity and seasonal forcing in a multi-strain
 epidemic model. *Physica D.*, 165:228–241, 2002. doi: 10.1016/S0167-2789(02)00389-5.
- [40] M.J. Keeling, P. Rohani, and B.T. Grenfell. Seasonally forced disease dynamics explored
 as switching between attractors. *Physica D.*, 148:317–335, 2001. doi: 10.1016/S01672789(00)00187-1.
- [41] E.D. Ketterson, A.M. Fudickar, J.W. Atwell, and T.J. Greives. Seasonal timing and population divergence: when to breed, when to migrate. *Current Opinion in Behavioral Sciences*,
 6:50–58, 2015. doi: 10.1016/j.cobeha.2015.09.001.
- [42] C.A. Klausmeier. Floquet theory: a useful tool for understanding nonequilibrium dynamics.
 Theor. Ecol., 1:153–161, 2008. doi: 10.1007/s12080-008-0016-2.

- [43] K. Koelle, M. Pascual, and M. Yunus. Pathogen adaptation to seasonal forcing and climate
 change. Proc. R. Soc. B., 272:971–977, 2005. doi: 10.1098/rspb.2004.3043.
- [44] Boots M. Modelling insect diseases as functional predators. *Physiological Entomology*,
 29:237-239, 2004. doi: 10.1111/j.0307-6962.2004.00403.x.
- [45] J.A.J. Metz, R.M. Nisbet, and S.A.H. Geritz. How should we define 'fitness' for general
 ecological scenarios? *TREE*, 7:198–202, 1992. doi: 10.1016/0169-5347(92)90073-K.
- [46] Bonds M.H. Host life-history strategy explains pathogen-induced sterility. Am. Nat.,
 168:281–293, 2006. doi: 10.1086/506922.
- [47] M.R. Miller, A. White, and M. Boots. The evolution of host resistance: Tolerance and control as distinct strategies. J. Theor. Biol., 236:198–207, 2005. doi:
 10.1016/j.jtbi.2005.03.005.
- [48] M.R. Miller, A. White, and M. Boots. Host life span and the evolution of resistance
 characteristics. *Evolution*, 61:2–14, 2007. doi: 10.1111/j.1558-5646.2007.00001.x.
- [49] R. Mostowy and J. Engelstädter. The impact of environmental change on host-parasite coevolutionary dynamics. *Proc. R. Soc. B.*, 278:2283–2292, 2011. doi: 10.1098/rspb.2010.2359.
- ⁴⁹² [50] S.L. Nuismer, R. Gomulkiewicz, and M.T. Morgan. Coevolution in temporally variable
 ⁴⁹³ environments. Am. Nat., 162:195–204, 2003. doi: 10.1086/376582.
- L.F. Olsen and W.M. Schaffer. Chaos versus noisy periodicity: alternative hypotheses for
 childhood epidemics. *Science*, 249:499–504, 1990. doi: 10.1126/science.2382131.
- ⁴⁹⁶ [52] A.J. Peel, J.R.C. Pulliam, A.D. Luis, R.K. Plowright, T.J. OShea, D.T.S. Hayman, J.L.N.
 ⁴⁹⁷ Wood, C.T. Webb, and O. Restif. The effect of seasonal birth pulses on pathogen
 ⁴⁹⁸ persistence in wild mammal populations. *Proc. R. Soc. B.*, 281:20132962, 2014. doi:
 ⁴⁹⁹ 10.1098/rspb.2013.2962.
- [53] T. Poisot, P.H. Thrall, and M.E. Hochberg. Trophic network structure emerges through
 antagonistic coevolution in temporally varying environments. *Proc. R. Soc. B.*, 279:299–
 308, 2012. doi: 10.1098/rspb.2011.0826.
- ⁵⁰³ [54] F.S. Ramalho, J.B. Malaquias, A.C.S. Lira, F.Q. Oliveira, J.C. Zanuncio, and F.S. Fernan ⁵⁰⁴ des. Temperature-dependent fecundity and life table of the fennel aphid hyadaphis foeniculi

10

- ⁵⁰⁵ (passerini) (hemiptera: Aphididae). *PLoS ONE*, 10:e0122490, 2015. doi: 10.1371/jour ⁵⁰⁶ nal.pone.0122490.
- ⁵⁰⁷ [55] O. Restif and J.C. Koella. Shared control of epidemiological traits in a coevolutionary ⁵⁰⁸ model of host-parasite interactions. *Am. Nat.*, 161:827–836, 2003. doi: 10.1086/375171.
- ⁵⁰⁹ [56] W. Rowan. Light and seasonal reproduction in animals. *Biol. Rev.*, 13:374–401, 1938. doi:
 ⁵¹⁰ 10.1111/j.1469-185X.1938.tb00523.x.
- ⁵¹¹ [57] I.B. Schwartz. Multiple stable recurrent outbreaks and predictability in seasonally forced
 ⁵¹² nonlinear epidemic models. J. Math. Biol., 21:347–361, 1985. doi: 10.1007/BF00276232.
- ⁵¹³ [58] I.B. Schwartz and H.L. Smith. Infinite subharmonic bifurcation in an seir epidemic model.
 ⁵¹⁴ J. Math. Biol., 18:233–253, 1983. doi: 10.1007/BF00276090.
- ⁵¹⁵ [59] H.L. Smith. Multiple stable subharmonics for a periodic epidemic model. J. Math. Biol.,
 ⁵¹⁶ 17:179–190, 1983. doi: 10.1007/BF00305758.
- ⁵¹⁷ [60] I. Sorrell, A. White, A.B. Pedersen, R.S. Hails, and M. Boots. The evolution of covert,
 ⁵¹⁸ silent infection as a parasite strategy. *Proc. R. Soc. B.*, 276:2217–2226, 2009. doi:
 ⁵¹⁹ 10.1098/rspb.2008.1915.
- [61] C. Stawski, C.K.R. Willis, and F. Geiser. The importance of temporal heterothermy in
 bats. J. Zool., 292:86–100, 2014. doi: 10.1111/jzo.12105.
- ⁵²² [62] M. van Baalen. Coevolution of recovery ability and virulence. Proc. R. Soc. Lond. B.,
 ⁵²³ 265:317–325, 1998. doi: 10.1098/rspb.1998.0298.
- [63] A. White, J.V. Greenman, T.G. Benton, and M. Boots. Evolutionary behaviour in ecological
 systems with trade-offs and non-equilibrium population dynamics. *EER*, 8:387–398, 2006.
- [64] K.A.J. White, B.T. Grenfell, R.J. Hendry, O. Lejeune, and J.D. Murray. Effect of seasonal
 host reproduction on hostmacroparasite dynamics. *Math. Biosci.*, 137:79–99, 1996. doi:
 10.1016/S0025-5564(96)00061-2.