Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **401,** 607–612 (2010) doi:10.1111/j.1365-2966.2009.15683.x

The structure of the hydrodynamic plasma flow near the heliopause stagnation point

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Accepted 2009 September 8. Received 2009 September 4; in original form 2009 March 14

ABSTRACT

The plasma flow in the vicinity of the heliopause stagnation point in the presence of the H atom flow is studied. The plasma at both sides of the heliopause is considered to be a single fluid. The back reaction of the plasma flow on the H atom flow is neglected, and the density, temperature and velocity of the H atom flow are taken to be constant. The solution describing the plasma flow is obtained in the form of power series expansions with respect to the radial distance from the symmetry axis. The main conclusion made on the basis of the obtained solution is that the heliopause is not the surface of discontinuity anymore. Rather, it is the surface separating the flows of the solar wind and interstellar medium with all plasma parameters continuous at this surface.

Key words: hydrodynamics – plasmas – solar wind – Solar system: general.

1 INTRODUCTION

The Solar system is embraced by a mixture of charged and neutral particles called the local interstellar cloud (LIC). The Sun is moving with a supersonic velocity with respect to LIC, so that there is a supersonic flow of the interstellar medium in the solar reference frame. The interaction of this supersonic flow with the supersonic solar wind results in creating an interaction region called the heliospheric interface. It consists of the termination shock at which the solar wind is decelerated, the bow shock at which the interstellar medium flow is decelerated and the heliopause separating the two decelerated flows. The model of the heliospheric interface with two shocks was first developed by Baranov, Krasnobaev & Kulikovski (1970). It has then been improved by Baranov, Krasnobaev & Ruderman (1976) and Baranov, Lebedev & Ruderman (1979).

In the first models of the heliospheric interface, only the interaction of the solar wind with the plasma component of LIC was considered. The neutral component (which mainly consists of the H atoms) was completely eliminated from the analysis. The reason for this was that, while the free path of charged particles in LIC is much smaller than the characteristic size of the heliospheric interface (which can be taken to be equal to the distance between the heliopause and termination shock along the symmetry axis), the free path of neutrals is of the order of or even larger than this characteristic size. However, in spite of this, the neutrals do not travel freely through the Solar system. Instead, they interact with the charged particles through the charge exchange. This interaction seriously affects the structure of the heliospheric interface. After the importance of the charge exchange was realized, Baranov, Ermakov & Lebedev (1981) carried out the numerical study of the structure of the heliospheric interface taking the neutral particles into account. In their model, Baranov et al. used the two-fluid description of the interstellar medium, one fluid consisting of charged and the other of neutral particles. This model has been then extended to the multifluid description (see e.g. Pauls, Zank & Williams 1995; Zank & Pauls 1996; Zank et al. 1996; Fahr, Kausch & Scherer 2000).

Although the multifluid models advanced the study of the heliospheric interface structure, they still do not provide its adequate description. The reason is that the fluid description of particle motion is based on the assumption that the particle distribution function only slightly deviates from Maxwellian. However, due to the fact that the mean-free path of neutrals is of the order of or even larger than the characteristic size of the heliospheric interface, the distribution function of neutrals is strongly non-Maxwellian. To overcome this problem Baranov & Malama (1993) developed a model with the mixed description, hydrodynamic for the solar wind and the plasma component, and kinetic for the interstellar neutral atoms. Note that the idea of such mixed description was also suggested by Osterbart & Fahr (1992). The kinetic–hydrodynamics model was then further developed by the effort of Moscow school (see review by Baranov 2009).

All the sophisticated models of the heliospheric interface that have been already developed constitute only the first step in studying the heliospheric interface structure. The next step is studying stability of the obtained solutions. Up to now the majority of studies concentrated on the heliopause stability. In the absence of magnetic field and any other stabilizing effects, the heliopause is subject to the

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Kelvin–Helmholtz instability. To our knowledge, Fahr et al. (1986) were the first who addressed the heliopause stability problem. They considered the stability of near flanks of the heliopause where the plasma flow can be assumed to be approximately incompressible. The investigation was restricted to the local analysis with respect to perturbations with the wavelengths much smaller than the characteristic size of the heliospheric interface. This restriction enabled Fahr et al. to consider the heliopause as a planar tangential discontinuity. This analysis has then been extended by Baranov, Fahr & Ruderman (1992) to the far flanks of the heliopause where the plasma compressibility plays an important role, and by Ruderman & Fahr (1993, 1995) and Ruderman & Brevdo (2006) to include the effect of the magnetic field. Chalov (1996) studied the effect of curvature on the heliopause stability. Belov & Myasnikov (1999) and Ruderman, Bredo & Erdélyi (2004) investigated the absolute and convective instabilities of the heliopause. Their conclusion was that the heliopause is only convectively unstable.

While the flow near the heliopause flanks can be assumed to be approximately one-dimensional when short-wavelength perturbations are considered, this approximation is not applicable to the region near the stagnation point. In this region, the flow near the heliopause is essentially at least two-dimensional. The stability of a model two-dimensional flow near the heliopause stagnation point was studied by Belov (1997a,b) in both the planar and cylindrical geometry. Reviews of studies on the heliopause stability are given by Ruderman (2000) and Baranov (2009).

In all papers on the heliopause stability mentioned up to now, only the plasma flow near the heliopause has been considered, the neutrals having been completely disregarded. To our knowledge, the first attempt to take the effect of neutral particles on the heliopause stability into account was made in the numerical study by Liewer, Karmesin & Brackbill (1996). However, later this paper was criticized by Pauls & Zank (1997) (see also the answer on this criticism by Liewer et al. 1997). Recently, the effect of neutral particles on the heliopause stability was studied by Florinsky, Zank & Pogorelov (2005). They presented both the numerical investigation of the global stability and analytical analysis of the stability of the flow near the heliopause stagnation point. We do not comment on the numerical results obtained by Florinsky et al. (2005); however, their analytical analysis is based on incorrect assumptions, the most important of those being the assumption that the flow in the vicinity of the stagnation point is one-dimensional. It immediately follows from this assumption that the plasma density at the stagnation point is infinite, which is physically meaningless.

To carry out the correct analytical study of the flow stability in the vicinity of the heliopause stagnation point in the presence of neutral particles, one needs first to obtain an approximate analytical solution describing this flow. Recently, this problem was addressed by Belov (2009). He obtained the approximate analytical solution under a simplifying assumption that the charge exchange frequency is constant. However, the comparison with the numerical kinetic–hydrodynamic solution shows that the charge exchange frequency substantially varies in the vicinity of the stagnation point (Baranov & Malama 1993; Baranov 2009). This paper aims to improve the analysis carried out by Belov (2009) and takes the dependence of the charge exchange frequency on the plasmas parameters into account. The paper is organized as follows. In the next section, we give the mathematical formulation of the problem. In Section 3, we obtain the solution describing the plasma flow in the vicinity of the heliopause in the form of power series expansions with respect to the radial distance from the symmetry axis. Section 4 contains the summary of the obtained results and our conclusions.

2 PROBLEM FORMULATION

The most accurate description of the flow in the heliospheric interface can be obtained by using the kinetic–hydrodynamic description (Baranov & Malama 1993; Baranov 2009). However, the kinetic description of neutrals makes the system of equations used in this description very complicated. As a result, it is difficult to believe that any analytical progress can be made using this description. For the sake of mathematical tractability, we turn to the most basic model that still accounts for the presence of neutral particles, which is the model suggested by Baranov et al. (1981). Although, as we have already mentioned, this model as well as other multifluid models does not provide an accurate description of the flow in the heliospheric interface, we believe that it can provide a reasonably accurate description of the flow in the vicinity of the stagnation point. It is assumed in that model that there is only one sort of neutral particles, the H atoms. It is also assumed that the flow of H atoms has the constant velocity, density and temperature. While the H atom flow affects the plasma flow, the back reaction of the plasma flow on the H atom flow is neglected. The plasma is considered to be a single fluid consisting of electrons and protons with equal temperatures. Its motion is described by the system of equations

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{1}$$

$$\rho \partial_t \boldsymbol{v} + \rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + \nabla p = \boldsymbol{F}, \tag{2}$$

$$\partial_t p + \boldsymbol{v} \cdot \nabla p + \gamma p \nabla \cdot \boldsymbol{v} = Q. \tag{3}$$

Here, $\boldsymbol{v}, \rho$ and p are the velocity, density and pressure of the plasma, respectively, and γ is the ratio of specific heats. The symbol ∂_t denotes the partial derivative with respect to time. $\boldsymbol{F}$ is the force imposed by the H atoms on the plasma, and Q is the function of energy losses times $-(\gamma - 1)$ (e.g. Priest 1984). These quantities are given by

$$\boldsymbol{F} = -\nu\rho(\boldsymbol{v} - \boldsymbol{V}), \tag{4}$$

$$Q = \frac{\nu\rho}{2}\left[(\gamma - 1)(\boldsymbol{v} - \boldsymbol{V})^2 + \frac{2kT}{m} - \frac{p}{\rho}\right]. \tag{5}$$

Here, $\boldsymbol{V}$ and T are the velocity and temperature of the H atoms, k is the Boltzmann constant, m is the proton mass and ν is the charge exchange frequency given by (e.g. Holzer 1972)

$$\nu = n_{\rm H}\sigma U^*, \quad U^{*2} = (\boldsymbol{v} - \boldsymbol{V})^2 + \lambda\left(\frac{2kT}{m} + \frac{p}{\rho}\right), \tag{6}$$

where $n_{\rm H}$ is the concentration of the neutral atoms, σ is the effective cross-section of charge exchange and λ is a dimensionless constant of the order of unity. For the H atoms, $\lambda = 64/9\pi$. In what follows, we assume that $\boldsymbol{V}, T$ and $n_{\rm H}$ are constant. However, in contrast to Belov (2009), we do not assume that ν is constant.

The system of equations (1)–(6) has to be supplemented with the boundary conditions at the heliopause. To write down the boundary conditions, we introduce cylindrical coordinates r, φ, z. In these coordinates, the heliopause is defined by the equation $z = 0$, the solar wind flow is in the region $z < 0$ and the interstellar medium flow is in the region $z > 0$. The H atom velocity is antiparallel to the z-axis, $\boldsymbol{V} = -V\boldsymbol{e}_z$, where $\boldsymbol{e}_z$ is the unit vector of the z-axis. The flow near the heliopause stagnation point is schematically shown in Fig. 1.

Now, the boundary conditions at the heliopause, i.e. at $z = 0$, are

$$w = 0, \quad p_s = p_i, \tag{7}$$

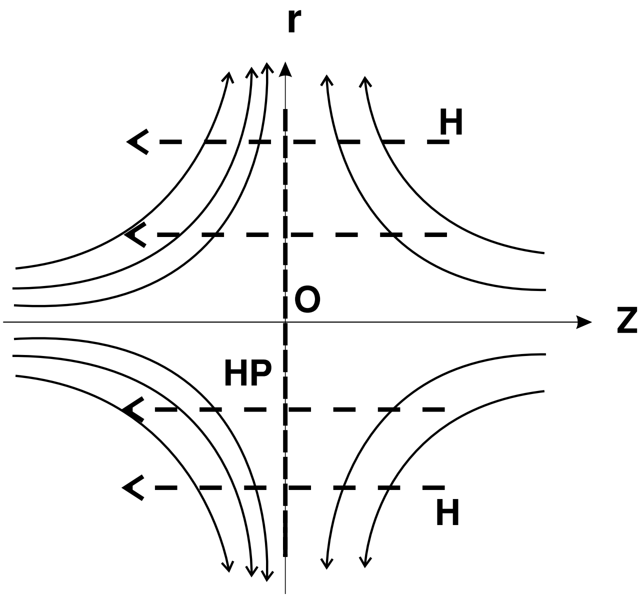

Figure 1. Schematic picture of the flow near the heliopause stagnation point (O). The heliopause (HP) is defined by the equation $z = 0$. The broken arrows show the H atom flow velocity.

where w is the z-component of the plasma velocity, and the subscripts 's' and 'i' indicate quantities in the solar wind and interstellar medium, respectively.

In what follows, we assume that the flow is axisymmetric with all quantities independent of φ and the φ-component of the velocity equal to zero. To facilitate the analysis, we introduce the dimensionless variables

$$\tilde{r} = \frac{r}{L}, \quad \tilde{z} = \frac{z}{L}, \quad \tilde{u} = \frac{u}{V},$$
$$\tilde{w} = \frac{w}{V}, \quad \tilde{p} = \frac{p}{p_0}, \quad \tilde{\rho} = \frac{V^2\rho}{p_0}, \tag{8}$$
$$\tilde{\nu} = \frac{\nu}{\nu_0}, \quad \tilde{T} = \frac{2kT}{mV^2}, \quad \tilde{Q} = \frac{Q}{\nu_0 p_0},$$

where u is the radial component of the plasma velocity, p_0 is the value of p at the stagnation point, L is the characteristic scale of the heliospheric interface (e.g. the distance between the heliopause stagnation point and the termination shock) and $\nu_0 = V/L$. Note that the value of p_0 at the stagnation point is the same at both the sides of the heliopause. Now, dropping the tildes, we rewrite equations (1)–(6) for a stationary flow ($\partial_t = 0$) in the form

$$\partial_r(\rho u) + \rho u/r + \partial_z(\rho w) = 0, \tag{9}$$

$$\rho(u\partial_r u + w\partial_z u) + \partial_r p = -\nu\rho u, \tag{10}$$

$$\rho(u\partial_r w + w\partial_z w) + \partial_z p = -\nu\rho(w + 1), \tag{11}$$

$$u\partial_r p + w\partial_z p + \gamma p(\partial_r u + u/r + \partial_z w) = Q, \tag{12}$$

$$Q = \frac{\nu\rho}{2}\left\{(\gamma - 1)\left[u^2 + (w + 1)^2\right] + T - \frac{p}{\rho}\right\}, \tag{13}$$

$$\nu = \nu^* \sqrt{u^2 + (w + 1)^2 + \lambda(T + p/\rho)}, \tag{14}$$

where $\nu^* = n_{\rm H}\sigma V/\nu_0$, and the symbols ∂_r and ∂_z denote the partial derivatives with respect to r and z, respectively. The boundary conditions at the heliopause written in the dimensionless variables are given by the same equation (7). In the next section, we use equations (9)–(14) and the boundary conditions (7) to obtain an approximate analytical solution describing the plasma flow near the stagnation point.

3 THE APPROXIMATE ANALYTICAL SOLUTION

In this section, we obtain the approximate solution describing the plasma flow in the vicinity of the stagnation point. The solution is obtained in the form of power series expansions with respect to r.

3.1 Power series expansions and derivation of the base system of equations

We look for the solution to equations (9)–(14) near the symmetry axis in the form of expansions with respect to r. It immediately follows from the assumption that the flow is axisymmetric that the expansion for u starts from a term proportional to r. Then, it is not difficult to show that the second terms in the expansions of ρ, p and w are proportional to r^2, while the second term in the expansion of u is proportional to r^3. Hence, we write the expansions in the form

$$\begin{aligned} \rho &= R(z) + r^2 R_1(z) + \cdots, \\ p &= P(z) + r^2 P_1(z) + \cdots, \\ u &= rU(z) + r^3 U_1(z) + \cdots, \\ w &= W(z) + r^2 W_1(z) + \cdots. \end{aligned} \tag{15}$$

Substituting these expansions in equations (13) and (14), we obtain

$$Q = \frac{\mu}{2}(RG - P) + r^2 Q_1(z) + \cdots, \tag{16}$$

$$\nu = \mu(z) + r^2\nu_1(z) + \cdots, \tag{17}$$

where

$$\mu = \nu^* \sqrt{(1 + W)^2 + \lambda(T + P/R)}, \tag{18}$$

$$G = (\gamma - 1)(W + 1)^2 + T. \tag{19}$$

Substituting (15) in equations (9)–(12), collecting terms of the lowest order with respect to r and using equations (16) and (17), we arrive at

$$WR' + R(2U + W') = 0, \tag{20}$$

$$R(U^2 + WU') + 2P_1 = -\mu RU, \tag{21}$$

$$P' + RWW' = -\mu R(W + 1), \tag{22}$$

$$WP' + \gamma P(2U + W') = \frac{\mu}{2}(RG - P), \tag{23}$$

where the prime denotes the derivative with respect to z. This system of four equations contains five unknown functions, R, P, U, W and P_1, so that it is not closed. If we continue to any higher order approximation, we at each step obtain a non-closed system of equations. The situation here is similar to that with the Chapman–Enskog theory (Chapman & Cowling 1953) for deriving hydrodynamic equations from the Boltzmann–Maxwell kinetic equation for the distribution function.

In order to close the system of equations (20)–(23), we need to express P_1 in terms of R, P, U and W. Up to now we have only used the fact that we are looking for the solution in the vicinity of the symmetry axis. However, in fact, we are only interested in the solution near the stagnation point, where $|z|$ is much smaller than the characteristic spatial scale in the heliospheric interface. This condition enables us to write $U(z)$ in the form

$$U = U_0 + f(z), \tag{24}$$

where $U_0 = U(0)$ and $f(z) \to 0$ as $z \to 0$. Hence, $|f(z)| \ll |U_0|$ when $|z|$ is small. Then, we consider equation (21) as an equation relating P_1 and $f(z)$, while we substitute U_0 for $U(z)$ in equations (20), (22) and (23). As a result, we obtain the system of equation for R, P and W with one unknown parameter, U_0. Resolving this system with respect to the derivatives and introducing the new variable $S = P/R$ instead of P yields

$$W(\gamma S - W^2)R' = R\left[2U_0W^2 - \mu W(W+1)] - \frac{\mu}{2}(G - S)\right], \tag{25}$$

$$(\gamma S - W^2)S' = (\gamma - 1)SW[2U_0W - \mu(W+1)] + \frac{\mu}{2}(G - S)(S - W^2), \tag{26}$$

$$(\gamma S - W^2)W' = -2\gamma U_0 S + \mu W(W+1) + \frac{\mu}{2}(G - S). \tag{27}$$

Substituting the expansions (15) in the boundary conditions (7), we obtain in the first-order approximation

$$W(0) = 0, \quad R_0 S_0 = 1, \tag{28}$$

where $R_0 = R(0)$ and $S_0 = S(0)$. In the second-order approximation, it follows from the second boundary condition in (7) that $P_{s1} = P_{i1}$. Using equation (21), we rewrite this relation as

$$R_{s0}U_{s0}(U_{s0} + \mu_{s0}) = R_{i0}U_{i0}(U_{i0} + \mu_{i0}), \tag{29}$$

where $\mu_0 = \mu(0)$. When deriving equation (29), we have assumed that

$$W(z) f'(z) \to 0 \quad \text{as} \quad z \to 0. \tag{30}$$

Since $f(z) \to 0$ as $z \to 0$, it follows that $z f'(z) \to 0$ as $z \to 0$. Hence, the condition (30) is satisfied if $W(z) = \mathcal{O}(z)$ as $z \to 0$.

The system of equations (25)–(27) is valid only in the vicinity of the stagnation point. Since $W(0) = 0$, it follows that $W(z)$ is small in this vicinity. This observation enables us to simplify equations (25)–(27) by neglecting the terms proportional to W^2. As a result, equations (25)–(27) reduce to

$$WSR' = -\frac{\mu}{2\gamma}R(2\gamma W + G_0 - S), \tag{31}$$

$$WS' = \frac{\mu}{2\gamma}(G_0 - S), \tag{32}$$

$$SW' = -2U_0S + \frac{\mu}{2\gamma}(2\gamma W + G_0 - S), \tag{33}$$

where, in accordance with equation (19), $G_0 = G(0)$ is given by

$$G_0 = \gamma - 1 + T. \tag{34}$$

Equation (18) reduces to

$$\mu = \nu^* \sqrt{1 + 2W + \lambda(T + S)}. \tag{35}$$

3.2 Singular points and the asymptotic solution

Equations (32) and (33) constitute an autonomous system of ordinary differential equations for S and W. When the solution to this system is obtained, R can be found by the integration of equation (31). Hence, the solution of the problem is essentially reduced to studying the integral curves of the system (32), (33). This system can be reduced to the first-order equation

$$\frac{dS}{dW} = \frac{\mu S(S - G_0)}{W[4\gamma U_0 S + \mu(S - G_0 - 2\gamma W)]}. \tag{36}$$

The straight line $W = 0$ is an integral curve of this equation. However, it is easy to see that there is no solution of the system (32), (33) with $W = 0$, so that we can eliminate the line $W = 0$ from our analysis. There are three singular points of equations (36) with $W = 0$. They are (0, 0), (0, ∞) and (0, G_0). Since $W(0) = 0$, the integral curve corresponding to the solution describing the flow in the vicinity of the stagnation point has to pass through one of these three singular points. At the first point, which is a node, $S = 0$. This corresponds to an unphysical state with $R = \infty$. At the next point, which is a saddle, $S = \infty$, so that $R = 0$. This is once again unphysical. Hence, we conclude that the integral curve corresponding to the solution describing the flow has to pass through the third singular point. It is straightforward to show that the integral curves in the vicinity of this point are given by

$$S = G_0(1 + C|W|^\alpha), \tag{37}$$

where C is an arbitrary constant of integration, and α is given by

$$\alpha = \frac{\mu_0}{4\gamma U_0}, \quad \mu_0 = \nu^* \sqrt{1 + \lambda(T + G_0)}. \tag{38}$$

Under a viable assumption that $U_0 > 0$, we have $\alpha > 0$, which implies that the singular point (0, G_0) is a node. Equation (37) determines the relation between S and W in the vicinity of the stagnation point. The constant C can be found by matching the solution in the vicinity of the stagnation point with the external solution.

Now, we can find the expressions for the other variables in the vicinity of the stagnation point and determine the function $W(z)$. Substituting (37) in (35), we obtain the approximate expression

$$\mu = \mu_0 \left[1 + \frac{1}{2\mu_0^2}(2W + \lambda G_0 C|W|^\alpha)\right]. \tag{39}$$

Substituting (37) in (33) yields

$$W' = -2U_0(1 - 2\beta W + \alpha C|W|^\alpha), \quad \beta = \frac{\mu_0}{4G_0U_0}. \tag{40}$$

Finally, substituting (37) in (31) and using (40), we arrive at

$$\frac{dR}{dW} = R(2\beta - \alpha C|W|^{\alpha-1}). \tag{41}$$

Integrating this equation, we obtain

$$R = R_0 \exp(2\beta W - C|W|^\alpha) \approx R_0(1 + 2\beta W - C|W|^\alpha), \tag{42}$$

where $R_0 = 1/G_0$. Now, when we have the approximate expressions for S and R, we can find the approximate expression for P as

$$P = 1 + 2\beta W. \tag{43}$$

It follows from equation (40) and the condition $W(0) = 0$ that

$$W = -2U_0 z + \mathcal{O}(z^2) + \mathcal{O}(|z|^{\alpha+1}). \tag{44}$$

We see that $W = \mathcal{O}(z)$, so that the condition (30) is satisfied.

Equations (42)–(44), together with the expansions (15) and the approximate relation $U(z) \approx U_0$, completely determine the solution describing the plasma flow in the vicinity of the stagnation point. At each side of the heliopause, this solution depends on two arbitrary constants, U_0 and C. In general, C takes different value for $z < 0$ (solar wind) and $z > 0$ (interstellar medium). However, U_0 is the same at the two sides of the heliopause. To show this, we use equation (29) together with equation (38) and the relation $R_0 = 1/G_0$. As a result, we obtain

$$U_{s0}(U_{s0} + \mu_0) = U_{i0}(U_{i0} + \mu_0).$$

It follows from this equation and the condition $U_0 > 0$ that $U_{s0} = U_{i0}$. Hence, eventually, we conclude that the solution that we obtained in this section depends on three arbitrary constants, U_0, C_s and C_i. It is straightforward to see that this solution is continuous at the heliopause. Hence, in the presence of the H atom flow, the heliopause is not a surface of discontinuity anymore. It still separates the solar wind and interstellar medium flows; however, all the plasma parameters, including the velocity, are continuous at the heliopause.

Belov (2009) obtained the solution describing the plasma flow in the vicinity of the stagnation point in the case when there are no H atoms. We can obtain this solution from the solution presented in this paper by taking the limit $\nu^* \to 0$. In that case $\alpha \to 0$, and it follows from equation (37) that

$$S = \begin{cases} G_0(1 + C_s), & z < 0, \\ G_0(1 + C_i), & z > 0, \end{cases}$$

where, in general, $C_s \neq C_i$. We see that, in the absence of the H atoms, S, and, as a consequence, the plasma density, is discontinuous at the heliopause. In the presence of H atoms, the discontinuity is smeared out in the transition region where all plasma parameters vary continuously from their values in the solar wind to those in the interstellar plasma.

It is interesting to estimate the thickness of the transitional region. To do this, we take the H atom temperature and speed equal to 8000 K and 20 $\mathrm{km\,s^{-1}}$, respectively. We also take $\gamma = 5/3, L = 50$ au, and the plasma temperatures at the interstellar and solar wind boundaries of the transitional region equal to 2×10^4 K and 10^6 K, respectively. Then, we obtain

$$T \approx 0.32, \quad G_0 \approx 1, \quad S_i \approx 0.8, \quad S_s \approx 40,$$

where S_i and S_s are the values of S at the interstellar and solar wind boundaries of the transitional region, respectively.

Let us now estimate ν^*. We take $n_H \approx 2 \times 10^5\ \mathrm{m^{-3}}$. To estimate σ, we use the formula given by Mahrer & Tinsley (1977):

$$\sigma = 10^{-19}(13.2 - 0.695 \ln |\boldsymbol{v} - \boldsymbol{V}|)\ \mathrm{m}^2,$$

where the velocities are measured in $\mathrm{km\,s^{-1}}$. Note that in Mahrer & Tinsley (1977) σ is given in cm^2 and the velocities are measured in $\mathrm{cm\,s^{-1}}$. Using this expression for σ and taking $|\boldsymbol{v} - \boldsymbol{V}| \approx 20\ \mathrm{km\,s^{-1}}$, we obtain $\sigma \approx 6 \times 10^{-19}\ \mathrm{m}^2$. This gives $\nu^* \approx 1$. Then, $\mu_0 \approx 2\nu^* \approx 2$ and $\alpha \approx 0.3/U_0$.

Let us estimate U_0. This quantity is equal to the radial gradient of the radial velocity. In accordance with the continuity equation, it is of the order of the half of the gradient of the velocity z-component in the z-direction. This gradient is, in turn, of the order of variation of the velocity z-component in the heliosheath divided by the characteristic spatial scale. In the dimensional variables, it is of the order of $100\ \mathrm{km\,s^{-1}} : 50$ au at the solar wind side of the heliopause, and of the order of $20\ \mathrm{km\,s^{-1}} : 100$ au at the interstellar side of the heliopause. Hence, the estimate for U_0 depends very much on what quantities we use: those in the solar wind side or in the interstellar side. The comparison with the results obtained in kinetic fluid modelling by Baranov & Malama (1993) shows that using the quantities at the solar wind side gives much better estimate of the velocity gradient than using the quantities at the interstellar side. Hence, in what follows, we accept that the gradient of the velocity z-component in the z-direction near the stagnation point is equal to $100\ \mathrm{km\,s^{-1}} : 50$ au Then, in dimensionless variables, we obtain

$$U_0 \approx \frac{1}{2}\frac{L}{V}\frac{100\ \mathrm{km\,s^{-1}}}{50\ \mathrm{au}} = \frac{1}{2}\frac{50\ \mathrm{au}}{20\ \mathrm{km\,s^{-1}}}\frac{100\ \mathrm{km\,s^{-1}}}{50\ \mathrm{au}} = 2.5.$$

Using these estimates yields $\alpha = \mu_0/(4\gamma U_0) = 0.12$.

In accordance with (44), equation (37) can be rewritten in the approximate form as

$$S_{s,i} = G_0\left(1 + C_{s,i}|5z|^{0.12}\right). \tag{45}$$

In the transitional region, $S_{s,i}$ changes from G_0 to $G_0(1 + C_{s,i})$. In accordance with equation (45), $S_{s,i} = G_0(1 + C_{si})$ at $z = 0.2$. However, equation (45) is valid only near the stagnation point. Far from stagnation point, it should be substituted by a more accurate expression that, probably, would give $S_{s,i}$ only asymptotically approaching $G_0(1 + C_{s,i})$ when formally $|z| \to \infty$ similar to what we have in the solution describing the structure of a shock. Hence, it looks reasonable to take only half of the value obtained before, which gives for the thickness of the transitional region at each side of the heliopause $|z| = 0.1$. Then, in dimensional units, we have for the thickness of the transitional region at each side of the heliopause $|z| = 5$ au.

In spite that the estimates for the thickness of the transition region are very crude, they clearly show that this thickness is a substantial fraction of the thickness of the heliospheric interface. Thus, it cannot be substituted by a discontinuity which would be possible if the transitional region would be very thin. This implies that the existence of the transitional region can seriously affect the stability properties of the plasma flow in the vicinity of the stagnation point.

4 SUMMARY AND CONCLUSIONS

In this paper, we have studied the plasma flow in the vicinity of the heliopause stagnation point in the presence of the H atom flow. We neglected the back reaction of the plasma flow on the H atom flow, so that the number density, temperature and velocity of the H atoms were assumed to be constant. Comparison with the numerical solution on the basis of the kinetic fluid model shows that this assumption is approximately satisfied in a vicinity of the stagnation point with the size of a few tens of au (Malama, private communication).

The solution describing the plasma flow is obtained in the form of power series expansions with respect to the radial distance from the symmetry axis. Only the first terms of these expansions are calculated. The main conclusion that follows from our analysis is that the heliopause is not a surface of discontinuity. It still remains the surface separating the solar wind and interstellar medium flows; however, all the plasma parameters at this surface are continuous.

The transition from the plasma parameters at the interstellar side of the heliopause to those at the solar wind side occurs at a transitional region embracing the heliopause. For typical parameters of the interstellar medium, the thickness of this region is of the order of 5 au from each side of the heliopause.

The fact that the heliopause is not a surface of discontinuity can sufficiently affect the stability properties of the flow near the heliopause stagnation point. It is especially important for studying the absolute and convective instabilities of the heliopause. To study the absolute and convective instabilities, we have to solve the initial value problem. This problem is ill-posed for a flow with tangential discontinuity because the instability increment grows unboundedly when the wavenumber of a normal mode is increased. Hence, it is impossible to study the absolute and convective instabilities of a flow with a tangential discontinuity.

When we have a transitional region instead of a discontinuity, no matter how narrow it is, the initial value problem becomes well-posed, and we can study the absolute and convective instabilities.

ACKNOWLEDGMENTS

We are thankful to our colleagues from the Laboratory of Physical Gasdynamics of the Institute for Problems in Mechanics and the Laboratory of Planetary Physics of the Space Research Institute for the fruitful discussions. Part of the work was performed under goskontrakt 02.740.11.5025 in the frame of FCP 'Kadry'. This work was also partly supported by RFBR grants 07-01-00291 and 06-02-72557.

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