



This is a repository copy of *A hybrid stabilization technique for simulating water wave – structure interaction by Incompressible Smoothed Particle Hydrodynamics (ISPH) method.*

White Rose Research Online URL for this paper:  
<http://eprints.whiterose.ac.uk/123543/>

Version: Supplemental Material

---

**Article:**

Zhang, N., Zheng, X., Ma, Q. et al. (4 more authors) (2018) A hybrid stabilization technique for simulating water wave – structure interaction by Incompressible Smoothed Particle Hydrodynamics (ISPH) method. *Journal of Hydro-environment Research*, 18. pp. 77-94. ISSN 1570-6443

<https://doi.org/10.1016/j.jher.2017.11.003>

---

Article available under the terms of the CC-BY-NC-ND licence  
(<https://creativecommons.org/licenses/by-nc-nd/4.0/>).

**Reuse**

This article is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs (CC BY-NC-ND) licence. This licence only allows you to download this work and share it with others as long as you credit the authors, but you can't change the article in any way or use it commercially. More information and the full terms of the licence here: <https://creativecommons.org/licenses/>

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.



[eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk)  
<https://eprints.whiterose.ac.uk/>

**Tab.1**

Summary of ISPH methodology with key references.

Schemes	Formulations	References
Governing equations and pressure solution	$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \nu\nabla^2\mathbf{u}$ $\nabla \cdot \mathbf{u} = 0$ $\nabla^2 P_{t+\Delta t} = \alpha \frac{\rho - \rho^*}{\Delta t^2} + (\alpha - 1) \frac{\rho \nabla \cdot \mathbf{u}_*}{\Delta t}$	Shao and Lo (2003) Zheng et al. (2014)
Particle approximations	$\nabla \cdot \mathbf{u}_i = -\frac{1}{\rho_i} \sum_{j=1}^N m_j (\mathbf{u}_i - \mathbf{u}_j) \cdot \nabla_i W(\mathbf{r}_{ij})$ $\nabla P_i = \sum_{j=1, j \neq i}^N \frac{n_{i,x_m} B_{ij,x_m} - n_{i,xy} B_{ij,x_k}}{n_{i,x} n_{i,y} - n_{i,xy}^2} (P_j - P_i)$ $\nabla \cdot (\nu_i \nabla \mathbf{u}_i) = \sum_{j=1}^N 8m_j \left( \frac{\nu_i + \nu_j}{\rho_i + \rho_j} \frac{\mathbf{u}_{ij} \cdot \mathbf{r}_{ij}}{r_{ij}^2 + \eta^2} \right) \cdot \nabla_i W(\mathbf{r}_{ij})$ $\nabla \cdot \left( \frac{1}{\rho^*} \nabla P_{t+1} \right) = \sum_{j=1}^N m_j \frac{8}{(\rho_i + \rho_j)^2} \frac{P_{ij} \cdot \mathbf{r}_{ij}}{r_{ij}^2 + \eta^2} \cdot \nabla_i W(\mathbf{r}_{ij})$	Monaghan (1994) Ma (2008) Cleary and Monaghan (1999)
Solid boundary	$\mathbf{u} \cdot \mathbf{n} = \mathbf{U} \cdot \mathbf{n}$ $\mathbf{u} \cdot \nabla P = \rho(\mathbf{n} \cdot \mathbf{g} - \mathbf{n} \cdot \dot{\mathbf{U}})$	Liu et al. (2013) Ma and Zhou (2009)
Free surface	$P = 0$	Zheng et al. (2012)

Note:  $\rho$  is the fluid density;  $\mathbf{u}$  is the particle velocity;  $t$  is the time;  $P$  is the particle pressure;  $\mathbf{g}$  is the gravitational acceleration;  $\nu$  is the kinematic viscosity;  $\mathbf{u}_*$  is the intermediate particle velocity;  $\rho^*$  is the particle density at the intermediate time step;  $\alpha$  is the weighting coefficient in PPE source term;  $\Delta t$  is the time step;  $m$  is the particle mass;  $\eta$  is the small number to prevent singularity;  $i$  and  $j$  indicate the reference particle and its neighbours;  $P_{ij} = P_i - P_j$ ,  $\mathbf{u}_{ij} = \mathbf{u}_i - \mathbf{u}_j$ ,  $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$  are defined;  $\mathbf{n}$  is the unit normal vector of the solid boundary;  $\mathbf{U}$  and  $\dot{\mathbf{U}}$  are the velocity and acceleration of the solid boundary; and  $W(\mathbf{r}_{ij})$  is the SPH kernel function (cubic B-spline kernel).  $n_{i,xy} = \sum_{j=1, j \neq i}^N (\mathbf{r}_{j,x_m} - \mathbf{r}_{i,x_m})(\mathbf{r}_{j,x_k} - \mathbf{r}_{i,x_k}) / |\mathbf{r}_j - \mathbf{r}_i|^2 W_{ij}$ ,  $n_{i,x_m} = \sum_{j=1, j \neq i}^N (\mathbf{r}_{j,x_m} - \mathbf{r}_{i,x_m})^2 / |\mathbf{r}_j - \mathbf{r}_i|^2 W_{ij}$ ; and  $B_{ij,x_m} = (\mathbf{r}_{j,x_m} - \mathbf{r}_{i,x_m}) / |\mathbf{r}_j - \mathbf{r}_i|^2 W_{ij}$ . More details are in Zheng et al. (2014).