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# Is emission intensity or output U-shaped in the strictness of environmental policy?*

Bouwe R. Dijkstra†,‡ and Maria J. Gil-Moltó§

**Abstract.** We show that, in a range of market conditions, an ever stricter environmental policy does not always lead to ever cleaner production methods and ever lower production of polluting goods. We consider an integrated technology, where firms can reduce their emission intensities in a continuous fashion. Analogous to the previous literature we find that firms' emission intensities can be U-shaped in the strictness of policy, but we show that this applies only under low profitability conditions. Under high profitability conditions however, output levels are U-shaped in the strictness of the policy. The latter result is new in the literature. In the case where the U-shape arises in emission intensities, the minimum is reached where the Marginal Abatement Cost curves intersect.

**Keywords:** Abatement, environmental taxation, oligopoly, marginal abatement costs.

**JEL classification:** L13, Q55, Q58

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†School of Economics and GEP, University of Nottingham; CESifo Munich.

‡Corresponding author. School of Economics, University of Nottingham, Nottingham NG7 2RD, United Kingdom. Tel. +44 115 8467205, fax +44 115 9514159, email bouwe.dijkstra@nottingham.ac.uk.

§Department of Economics, University of Sheffield.

# 1 Introduction

Environmental policy gives polluting firms an incentive to find cleaner ways of producing. There is a large literature on the effect of environmental policy on innovation (see e.g. Requate (2005) for an overview) starting from Kneese and Schultze (1978). One question that has received relatively little attention until recently is: When environmental policy becomes stricter and stricter, will firms use ever cleaner production methods? Our immediate intuition might suggest that this should be the case. However, making production cleaner is only one of two ways in which a firm can respond to stricter environmental policy. The other way is to reduce output. This in turn may reduce firms' incentives to clean up production: If a firm will produce very little because of a very strict environmental policy, it also has little incentive to spend on abatement. This suggests that when output is decreasing in the strictness of environmental policy, the emissions-to-output ratio might be a U-shaped function of strictness. However, the effects might conceivably also be reversed: When an ever stricter environmental policy prompts a firm to spend more and more on cleaner production methods, production might eventually become so clean that it starts to increase again.

So far, the economic literature has established that the rate of clean technology adoption can be U-shaped in the strictness of environmental policy (Perino and Requate, 2012; Bréchet and Meunier, 2014), a result which is qualitatively similar (although at an aggregate level) to the first of the effects mentioned above. We show that the second effect may also arise: It is possible for the strictness of the policy to have a non-linear effect on output, instead of on the emission-to-output ratio. In this paper we also shed light on the circumstances under which each of these effects arise. We find that the first (second) effect is associated with low (high) profitability conditions. Our results suggest that the policy-maker should be aware of the potential interaction between environmental and industrial policies that could lead to lower abatement incentives in an industry.

The non-linear effects described above appear when firms have an integrated abatement technology at their disposal. This is a technology that allows firms to reduce the emission intensity of their production to a certain level. In our model, we will consider that firms have at their disposal this type of technology.[1] Examples of integrated technologies are modifications to the design of the combustion chamber and technologies that allow firms to use more environmentally friendly materials or to switch to a fuel with a lower emission factor (Frondel et al., 2007). For instance in the steel and iron industry, one of the largest industrial sources of $CO_2$ emissions, coal can be replaced by wood charcoal or even biomass (Carpenter, 2012).

Perino and Requate (2012) analyze the adoption of an integrated abatment technology in a market with a continuum of small firms. The firms choose between a clean and a dirty technology. In this setting, the number of firms adopting the cleaner technology is non-monotonic in the strictness of the environmental policy. Bréchet and Meunier (2014) consider the output market more explicitly than Perino and Requate (2012). Otherwise their model is similar, also finding that the share of firms adopting the clean technology is inverted U-shaped in the strictness of environmental policy.[2]

The crucial difference between our paper and the previous contributions lies in the type of choice available to firms. Perino and Requate (2012) and Bréchet and Meunier (2014) assume that firms face a discrete choice between a clear and a dirty technology. In contrast, in our model abatement is a continuous variable. In other words, rather than being a model of adoption of a clean technology, our model analyzes the decision by individual firms on how much to spend on abatement. It is important to notice that this is not merely a matter of interpretation of the model. In fact, considering a continuous decision in abatement yields a richer set of results than found in the previous literature. As mentioned above, the non-linear effects of the strictness of the environmental effect may not arise in emission intensities, they may appear in the levels of output instead.

As Perino and Requate (2012) have shown, the non-monotonic effect of the strictness of the environmental policy on the rate of adoption of cleaner technologies is linked to the

[1] By contrast, an end-of-pipe technology allows for the reduction of emissions by an absolute amount (e.g. Requate, 2005; Endres and Friehe, 2013; Lahiri and Symeonidis, 2017; Menezes and Pereira, 2017). Solving our model with an end-of-pipe technology, we find that both emission intensity and output are decreasing in the strictness of the policy. The analysis is available upon request.

[2] In a setting where consumers are environmentally aware and firms compete *à la* Cournot, Gil-Moltó and Varvarigos (2013) show that the benefits of adopting a clean technology are initially increasing and then decreasing in the emission tax, with the turning point occurring at lower tax rates when consumers' environmental awareness is stronger.

recent literature on pivoting Marginal Abatement Costs ($MAC$) curves. Traditionally it has been assumed that environmental innovation reduces $MAC$ at any level of emissions. In these models, a stricter environmental policy leads to more environmental innovation (e.g. Downing and White, 1986; Milliman and Prince, 1989; Jung *et al.*, 1996; Requate and Unold, 2003). A number of papers (Amir *et al.*, 2010; Baker *et al.*, 2008; Bauman *et al.*, 2008; Bréchet and Jouvet, 2008) have shown that an environmental innovation may not cause the whole $MAC$ curve to shift downward. Indeed, Amir *et al.* (2010), Bauman *et al.* (2008) and Bréchet and Jouvet (2008) show that a decrease in the marginal emission intensity of "dirty" inputs leads to a clockwise rotation or pivoting of the $MAC$ curve: it is lower for higher emission levels, but higher for lower emission levels.[3] While these three papers take the output response of a firm into account, they do so in a very simplified manner, as they only consider one firm faced with a constant output price. In addition to the constant price scenario, we will also consider the case of strategic interaction (Cournot oligopoly) and perfect competition.

Interestingly, there is also an earlier literature that explicitly takes the output market into account when considering firms' incentives to spend on abatement.[4] In a free-entry Cournot oligopoly model with constant marginal production cost and an integrated abatement technology, Ulph (1997) derives conditions under which an increase in the tax rate reduces output, finding that the effect of the tax rate on environmental R&D is ambiguous. Katsoulacos and Xepapadeas (1996) set up a Cournot oligopoly with technology spillovers, where the government taxes emissions and subsidizes environmental R&D. The authors' conclusion that the effect of the tax rate on output is ambiguous is in accordance with Ulph's (1997) findings. Katsoulacos and Xepapadeas (1996) further find that R&D spending is increasing in the tax rate.

The rest of this paper is organized as follows. In Section 2, we introduce our model. In Section 3, we solve the model under fairly general conditions. In fact, our solutions will encapsulate the cases of strategic interaction (Cournot oligopoly), constant price, as

[3] Endres and Friehe (2011) examine the effects of environmental liability law on the incentives to diffuse advanced abatement technology that reduces $MAC$ everywhere or pivots the $MAC$ curve clockwise.

[4] A related research strand focuses on abatement spending in the context of international trade, showing the ambiguous effect of domestic emission taxation (Simpson and Bradford, 1993; Ulph and Ulph, 1996; Feess and Muehlheusser, 2002; Greaker, 2003).

well as perfect competition (we relegate the analysis of the latter case to the appendix for expositional purposes). In Section 4, we will employ specific functional forms to identify under which circumstances the non-monotonic effects of the stringency of the environmental policy occur in either output levels or emission intensitities. In this section we will also derive and discuss the behaviour of the $MAC$ curves. In order to show that the non-monotonic effects of the strictness of the policy are not due to policy failure, the appendix applies the analysis from Sections 3 and 4 to the social optimum, where both abatement and output levels are set to maximize welfare. In Section 5, we discuss our results and how they fit with the literature. The concluding Section 6 discusses the implications for policy and empirical work.

## 2 The model

There are $n > 0$ identical firms producing a homogeneous good. Firm $i$, $i = 1, \cdots, n$, producing $q_i$ faces the inverse demand function $P(Q)$, where $P$ is the product price, $Q \equiv \sum_{i=1}^{n} q_i$, $P'(Q) \leq 0$ and

$$P'(Q) + P''(Q)q_i \leq 0 \tag{1}$$

Our analysis will encompass the scenarios of a constant product price ($P' = 0$), perfect competition ($P' < 0$ and a continuum of firms of mass $n$) and oligopolistic (Cournot) competition ($P' < 0$, small $n$). While the constant $P$ scenario may seem rather unrealistic, it is worth analyzing for the following reasons. First, it is the easiest to analyze, because there are no interactions between firms through the output market. Secondly, it maintains the standard assumption in the literature[5] that a firm's marginal abatement cost only depends on its own technology choice, and not on any variable chosen by other firms. The only way to maintain this assumption with integrated technology is to assume that $P$ is constant, as Amir *et al.* (2010), Bauman *et al.* (2008) and Bréchet and Jouvet (2008) have done explicitly. Finally, this scenario serves to highlight the crucial similarities and differences between our model and those in the previous literature. Perino and Requate (2012) do not model the output market explicitly, but assume that a firm's $MAC$ only

[5]For instance Downing and White (1986), Jung *et al.* (1996), Milliman and Prince (1989), Requate and Unold (2003), among many others.

depends on its own technology choice. Thus, they implicitly assume constant $P$. They also assume that the choice of technology is discrete. Bréchet and Meunier (2014) do model the output market explicitly, but assume a discrete choice of technology like Perino and Requate (2012). In our case, we consider a continuous choice of abatement options and both constant price, perfect competition and strategic interaction.

Production is polluting. Firm $i$'s total (net) emissions $e_i$ are given by:

$$e_i = \varepsilon_i q_i \tag{2}$$

where $\varepsilon_i \in [0,1]$ is the emissions-to-output ratio, which depends on the firm's abatement decision. We normalize $\varepsilon_i$ to one for the case where there is no abatement.

We shall assume that firms can use an *integrated abatement technology.* Following Ulph (1997), a firm wanting to reduce its emission-to-output ratio to $\varepsilon_i$ with an integrated technology has to spend an amount $F(\varepsilon_i)$. The firm's cost function is:

$$C(q_i, \varepsilon_i) = k(q_i) + F(\varepsilon_i) \tag{3}$$

with $k'(0) = 0$ and $k' > 0$ for $q_i > 0$; $k'' > 0$; $F'(1) = 0$ and $F'(\varepsilon_i) < 0$ for $\varepsilon_i \in [0,1)$; $F'' > 0$.

The environmental damage caused by pollution is given by $D(\beta, E)$, where $E \equiv \sum_{i=1}^{n} e_i$, $D(\beta,0) = D_E(\beta,0) = 0$, $D_E > 0$ and $D_{EE} > 0$ for $E > 0$; $D(0,E) = D_\beta(0,E) = 0$, $D_\beta > 0$ and $D_{\beta E} > 0$ for $E > 0$. Thus total and marginal damage are increasing in the environmental damage parameter $\beta$. This parameter measures the severity of the environmental problem or the strength of the policy maker's preference for the environment.[6]

The game we analyze in this paper is as follows. In stage one, the regulator sets the environmental tax rate $t$ that maximizes welfare. Welfare consists of consumer utility (the area below the inverse demand curve) minus firms' costs and environmental damage. We will only consider symmetric equilibria where $q_i = q$ and $\varepsilon_i = \varepsilon$. In a symmetric equilibrium, welfare is, from (2) and (3):

$$W(q, \varepsilon, \beta) = \int_0^{nq} P(Y)dY - n\left[k(q) + F(\varepsilon)\right] - D(\beta, n\varepsilon q) \tag{4}$$

[6] When analyzing emission taxation under imperfect competition, we will assume that $\beta$ is high enough to guarantee a positive tax rate ($t > 0$). For small $\beta$, the regulator would set $t < 0$, to induce the imperfectly competitive firms to produce more.

In stage two, each firm chooses its abatement and output levels simultaneously to maximize its profits.[7] Firm $i$'s profits are, from (2) and (3):

$$\Pi_i = \Pi(q_i, \varepsilon_i) = P(Q)q_i - k(q_i) - F(\varepsilon_i) - t\varepsilon_i q_i \tag{5}$$

We are ultimately interested in the effect of an increase in the environmental damage parameter $\beta$ via $t$ on output and abatement. Thus (as we shall see) the stricter environmental policy follows endogenously from an increase in $\beta$.

# 3 General analysis

In this section, we analyze the effect of changes in the environmental damage parameter and the tax rate on the equilibrium levels of output and abatement under Cournot oligopoly and the constant-price scenario. We will use Cournot as the benchmark case. The analysis of the constant price scenario can be obtained from the expressions in this section by setting to zero all terms containing $P'$ and $P''$. The case of perfect competition is relegated to Appendix A for expositional purposes. All three cases yield qualitatively similar results.

In stage two of the game presented in Section 2, firm $i$ sets the output level $q_i$ and emission intensity $\varepsilon_i$ to maximize its profits (5). The FOCs are, respectively:

$$P + P'(Q)q_i - k'(q_i) - t\varepsilon_i = 0 \tag{6}$$

$$-F'(\varepsilon_i) - tq_i = 0 \tag{7}$$

The Hessian matrix is:

$$\mathbf{\Pi}_{xx} \equiv \begin{pmatrix} 2P' + P''q_i - k'' & -t \\ -t & -F'' \end{pmatrix} \tag{8}$$

Note that $-F'' < 0$, and $2P' + P''q_i - k'' < 0$ by (1) and $k'' > 0$.[8] Therefore, in this case, $\mathbf{\Pi}_{xx}$ is negative definite as long as

$$(2P' + P''q_i - k'')(-F'') - t^2 > 0 \tag{9}$$

[7] One could argue that the abatement choice is a relatively long-term and irreversible decision. Therefore, alternative plausible timings could feature abatement being chosen before output or even before the regulator sets the tax rate. Under imperfect competition, firms would then choose their abatement levels strategically. The analysis of the effects of alternative timings falls outside the scope of the present paper but could constitute an interesting avenue for further research.

[8] While $k'' \geq 0$ combined with (1) is sufficient for $2P' + P''q_i - k'' < 0$ to hold under Cournot oligopoly, $k > 0$ is necessary for the constant-price scenario.

which is therefore sufficient SOC for maximization in this case.

We can characterize the effect of $t$ on the equilibrium levels of output and abatement by totally differentiating the FOCs with respect to $t$ and solving the system:

$$\frac{dq_i}{dt} = \frac{-\varepsilon_i F'' + q_i t}{\left[k'' - (n+1)P' - nP''q_i\right] F'' - t^2} \tag{10}$$

$$\frac{d\varepsilon_i}{dt} = \frac{q_i\left[(n+1)P' + nP''q_i - k''\right] + \varepsilon_i t}{\left[k'' - (n+1)P' - nP''q_i\right] F'' - t^2} \tag{11}$$

Note that the term in the denominator is positive given $F'' > 0$, (1) and (9). The signs of (10) and (11) are thus the signs of the numerators on the respective RHSs.

Interestingly, at $t = 0$, (10) and (11) yield:

$$\frac{dq_i}{dt} = \frac{-\varepsilon_i}{k'' - (n+1)P' - nP''q_i} < 0$$

$$\frac{d\varepsilon_i}{dt} = \frac{q_i\left[(n+1)P' + nP''q_i - k''\right]}{\left[k'' - (n+1)P' - nP''q_i\right] F''} < 0$$

Thus when environmental policy is very lenient (the emission tax rate is close to zero), an increase in the tax rate induces firms to reduce emissions by lowering their level of output as well as their emission-to-output ratio.

For higher values of $t$, the signs of (10) and (11) become ambiguous. However, we can establish that emissions are strictly decreasing in the tax rate. From (2), (10) and (11):

$$\frac{de_i}{dt} = \varepsilon_i \frac{dq_i}{dt} + q_i \frac{d\varepsilon_i}{dt} = \frac{-\varepsilon_i^2 F'' + 2\varepsilon_i q_i t + q_i^2\left[(n+1)P' + nP''q_i - k''\right]}{\left[k'' - (n+1)P' - nP''q_i\right] F'' - t^2} < 0 \tag{12}$$

To explain the inequality, note that as before, the denominator in (12) is positive. Using (8), the numerator can be written as:

$$\begin{pmatrix} q_i & \varepsilon_i \end{pmatrix} \mathbf{\Pi}_{xx} \begin{pmatrix} q_i \\ \varepsilon_i \end{pmatrix} + (n-1)q_i^2(P' + P''q_i) < 0$$

where the inequality follows from (1) and setting $\mathbf{h} = \begin{pmatrix} q_i & \varepsilon_i \end{pmatrix}$ in $\mathbf{h}\mathbf{\Pi}_{xx}\mathbf{h}' < 0$ because $\mathbf{\Pi}_{xx}$ is negative definite.

Now that we have established that emissions are decreasing in $t$, we can anticipate that when $t$ is sufficiently large (that is, when the environmental policy is sufficiently strict), emissions will be very low (that is, $e_i = \varepsilon_i q_i \to 0$).[9] This may be due to either $\varepsilon_i$ or $q_i$

[9] We assume that in this case, second order condition (9) still holds even with a large $t$. In Section 4, we will check whether this is indeed the case for the specific functional forms that we use.

approaching zero. Let us first consider the case when $\varepsilon_i$ approaches zero. From (10):

$$\left.\frac{dq_i}{dt}\right|_{\varepsilon_i \to 0} = \frac{q_i t}{\left[k'' - (n+1)P' - nP'' q_i\right] F'' - t^2} > 0$$

Thus, when $\varepsilon_i$ is decreasing towards zero for ever stricter environmental policy, $q_i$ is increasing in the emission tax rate.[10] Now let us consider the case where output approaches zero. In this case, (11) becomes:

$$\left.\frac{d\varepsilon_i}{dt}\right|_{q_i \to 0} = \frac{\varepsilon_i t}{\left[k'' - (n+1)P'\right] F'' - t^2} > 0$$

Thus, when $q_i$ is decreasing towards zero for ever stricter environmental policy, $\varepsilon_i$ is increasing. Indeed from (7) with $q_i \to 0$, $\varepsilon_i$ approaches unity again: The firm does not spend anything on reducing the emissions-to-output ratio when output is very low.

In stage 1, the regulator sets the emission tax rate $t$ that maximizes welfare. Writing welfare (4) as a function of $t$ and $\beta$, the first order condition with respect to $t$ is:

$$\frac{\partial W(t,\beta)}{\partial t} = P\frac{dQ}{dt} - nk'\frac{dq}{dt} - nF'\frac{d\varepsilon}{dt} - D_E\frac{dE}{dt} = 0 \qquad (13)$$

Using the implicit function theorem we find:

$$\frac{dt}{d\beta} = -\frac{\partial^2 W/\partial\beta\partial t}{\partial^2 W/\partial t^2} = D_{\beta E}\frac{dE/dt}{\partial^2 W/\partial t^2} > 0 \qquad (14)$$

The second equality follows from (13). The inequality follows from $D_{\beta E} > 0$, $dE/dt < 0$ by (12) and $\partial^2 W/\partial t^2 < 0$ as the SOC for welfare maximization. Thus an increase in $\beta$ will prompt the regulator to raise the tax rate. This means that the signs of $dq/d\beta$ and $d\varepsilon/d\beta$ are the signs of $dq/dt$ and $d\varepsilon/dt$ respectively.

Intuitively, one might think that a regulator who has become more concerned about environmental damage might be wary of increasing the emission tax rate, because it could result in more production or more polluting production methods. However, we show that the regulator will still increase the tax rate, because a higher tax rate reduces emissions in spite of any increase in output or emission intensity.

Summarizing our results, we can state the following:[11]

[10] Indeed from (6) with $\varepsilon_i \to 0$, $q_i$ approaches the output level without environmental policy.

[11] We have also worked with a more general cost function. We found that with emission taxation as well as in the social optimum, emissions $E$ are decreasing in the environmental damage parameter $\beta$. Furthermore, when $\beta$ is very low, both output $q$ and emission intensity $\varepsilon$ are decreasing in $\beta$. Details are available from the corresponding author upon request.

**Proposition 1** *In the constant-price and Cournot oligopoly scenarios, when firms have access to an integrated abatement technology with a continuous range of emission intensities $\varepsilon$:*

1. *The tax rate $t$ is increasing in the environmental damage parameter $\beta$;*
2. *Emissions $E$ are decreasing in $t$, and consequently in $\beta$;*
3. *When $t$ is very low, both output $q$ and emission intensity $\varepsilon$ are decreasing in $t$ and in $\beta$;*
4. *When $t$ is very high so that $E$ is close to zero:*
   (a) *When $\varepsilon$ is close to zero, $q$ is increasing in $t$ and in $\beta$.*
   (b) *When $q$ is close to zero, $\varepsilon$ is increasing in $t$ and in $\beta$.*

This proposition implies that, when we consider a continuous abatement technology, the effect of the strictness of the policy (here captured by the emissions tax rate), or indeed the underlying environmental damage parameter, is non-monotonic in either the level of output or in abatement under the different types of competition considered in this paper. That is, considering a continuous abatement technology (instead of a discrete technology choice as in Perino and Requate (2012) and Bréchet and Meunier (2014)) allows us to obtain a richer set of results, in the sense that the non-linearities in the effects of the stringency of the environmental policy may arise in output levels instead of emission intensity levels, as part 4 of the proposition states.[12] These results apply under the three types of competition considered in our paper. In order to shed more light on the circumstances under which each of the two scenarios (non-monotonicities in output vs non-monotonicities in emission intensities) will materialise, we need to place more restrictions on the cost functions. We will undertake this analysis in the next section.

[12] The non-monotonic effect of strictness on output levels is also in contrast with Ulph's (1997) findings for integrated technologies. We discuss the reason for this divergence in Appendix C.

# 4 Example

We know from the previous Section that either output $q$ or the emission-to-output ratio $\varepsilon$ must be non-monotonic in the tax rate. In order to shed some light on the conditions under which either scenario prevails, we have to assume specific functional forms for the cost function and the demand function.

In this section, we will assume that each firm $i$'s cost function is given by the following specification of the general integrated-technology cost function (3):

$$C(q_i, \varepsilon_i) = \frac{c}{2}q_i^2 + \frac{\gamma}{2}(1 - \varepsilon_i)^2, \qquad c, \gamma > 0 \tag{15}$$

First, we shall solve the game with constant product price $P$ in subsection 4.1. In subsection 4.2, we analyze the case where firms interact strategically in the output market (Cournot oligopoly),[13] where price decreases in total production $Q$ according to:

$$P = a - Q \tag{16}$$

with $a > t$ for positive output.[14] Our specific functions here satisfy all the restrictions imposed upon them in Section 2. The case of perfect competition with specific functional forms is relegated to Appendix A.2.

## 4.1 Constant product price

In this subsection we consider the case where the product price is constant, denoting this scenario by the subscript $p$. Since there are no interactions between firms, we can focus on the behaviour of a single firm that sets output $q$ and emission intensity $\varepsilon$ facing the constant product price $P$ and an emission tax rate $t$. Its emissions are given by (2) and its cost function by (15). The firm maximizes its profits $\Pi$, consisting of operating profits $\pi(q, \varepsilon)$ minus the tax bill:

$$\max\ \Pi = \pi(q, \varepsilon) - te = Pq - \frac{c}{2}q^2 - \frac{\gamma}{2}(1 - \varepsilon)^2 - t\varepsilon q \tag{17}$$

[13] We provide the general solution to the two scenarios, along with the formal proof, in Appendix B.

[14] The analogous condition for the case with constant price is $P > t$.

The first order conditions are, with respect to $q$ and $\varepsilon$ respectively:

$$P - cq - t\varepsilon = 0 \tag{18}$$

$$\gamma(1-\varepsilon) - tq = 0 \tag{19}$$

Solving the above system for $q$ and $\varepsilon$ yields:[15]

$$q = \frac{\gamma(P-t)}{c\gamma - t^2}, \qquad \varepsilon = \frac{c\gamma - Pt}{c\gamma - t^2}, \qquad e = \frac{\gamma(P-t)(c\gamma - Pt)}{(c\gamma - t^2)^2} \tag{20}$$

When $t = 0$, the firm's output, emissions and operating profits are given by:

$$q = \bar{q}_p \equiv \frac{P}{c}, \qquad e = \bar{q}_p, \qquad \pi = \bar{\pi}_p \equiv \frac{P^2}{2c} \tag{21}$$

Applying Lemma 1 and Proposition 3 from Appendix B, we see that when $\gamma < P^2/c$, emission intensity is monotonically decreasing in $t$ and output is U-shaped in $t$, with the turning point at $\varepsilon = \frac{1}{2}$. Intuitively, when production is very clean (to be precise: when the emissions-to-output ratio is below half the no-regulation level), output levels can increase again with the tax rate while becoming ever cleaner.

If $\gamma > P^2/c$, output is monotonically decreasing in $t$ and emission intensity is U-shaped in $t$, with the turning point at:

$$\tilde{q}_p \equiv \frac{\bar{q}_p}{2} = \frac{P}{2c} \tag{22}$$

Thus $\varepsilon$ decreases until the point where output is so low that it is no longer worthwhile to deploy cleaner production methods. From (22), this point is where output is at half its no-regulation level of $\bar{q}_p$, defined in (21).[16]

Intuitively, when $\gamma < P^2/c$, production and abatement costs are low relative to the product price (there are high profitability conditions). As $t$ increases, the firm is keen to take advantage of its low emission intensity to let output increase again. This means that it has to keep reducing its emission intensity as $t$ rises, but the firm is happy to do so as abatement is relatively cheap. When $\gamma > P^2/c$, production and abatement costs are relatively high compared to the product price (there are low profitability conditions).

[15] The numerators on the RHS of the $q$ and $\varepsilon$ equations are the equivalent of second order condition (9). They are positive by Proposition 3.3 in Appendix B.

[16] Due to space constraints, we omit discussion of the knife-edge case (here: $\gamma = P^2/c$) here and in the next subsection. We briefly discuss the knife-edge case in Appendix B.

Then the firm does not want to produce too much or spend too much on abatement. Thus as $t$ keeps increasing, the firm keeps reducing emissions by decreasing output rather than by reducing its emission intensity. When output has fallen below $\tilde{q}_p$ in (22), the firm can allow itself to increase its emission intensity again.

Let us now interpret this result in terms of Marginal Abatement Costs ($MAC$). We follow the approach by Amir *et al.* (2010), Bauman *et al.* (2008) and Bréchet and Jouvet (2008) who build upon McKitrick's (1999) definition of the $MAC$ function for a given level of emission intensity ($\varepsilon$ in our model). Substituting (2) into (17), operating profits can be written as a function of emissions and emission intensity:

$$\pi(e,\varepsilon) = \frac{Pe}{\varepsilon} - \frac{c}{2}\left(\frac{e}{\varepsilon}\right)^2 - \frac{\gamma}{2}(1-\varepsilon)^2$$

Marginal abatement costs, defined for a given level of $\varepsilon$, are then:

$$MAC(e,\varepsilon) \equiv \frac{\partial\pi(e,\varepsilon)}{\partial e} = \frac{P}{\varepsilon} - \frac{ce}{\varepsilon^2} \tag{23}$$

The firm sets $MAC = t$. When $t = 0$, the firm sets $MAC = 0$, so that $q = \bar{q}_p$ and $\pi = \bar{\pi}_p - \frac{\gamma}{2}(1-\varepsilon)^2$, with $\bar{q}_p$ and $\bar{\pi}_p$ given by (21). On the other hand, unless $\varepsilon = 0$, zero emissions ($e = 0$) can only be achieved by setting $q = 0$ which implies $\pi = -\frac{\gamma}{2}(1-\varepsilon)^2$. We can now determine the effect of decreasing $\varepsilon$ on the $MAC$ curve. A decrease in $\varepsilon$ shifts the horizontal intercept $\bar{e}_p = \varepsilon\bar{q}_p$ to the left. The area under the $MAC$ curve must remain the same, because it is the difference $\bar{\pi}_p$ in profits between $MAC = 0$ and $e = q = 0$ (Bauman *et al.*, 2008, p. 513). This means that the vertical intercept $MAC(0,\varepsilon)$ has to move up according to:

$$MAC(0,\varepsilon) = \frac{2\bar{\pi}_p}{\bar{e}_p} = \frac{P}{\varepsilon}$$

Figure 1 shows $MAC$ curves for different levels of $\varepsilon$ when $P = c = 1$.[17] The lower is $\varepsilon$, the closer to the origin is the point where $MAC = 0$, the higher is the vertical intercept and the steeper is the curve. This means that any $MAC$ curve for a given $\varepsilon$ value intersects every other $MAC$ curve.

[17] The Figure only shows the $MAC$ curves for $MAC \leq P = 1$, since by Proposition 3 and (B2), $e = 0$ is achieved for $MAC = t = \gamma c/P = \gamma$ when $\gamma < P^2/c = 1$ and for $MAC = t = P = 1$ when $\gamma > P^2/c = 1$. For the same reason, Figures 4 and 6 only show the vertical axis between 0 and 1.

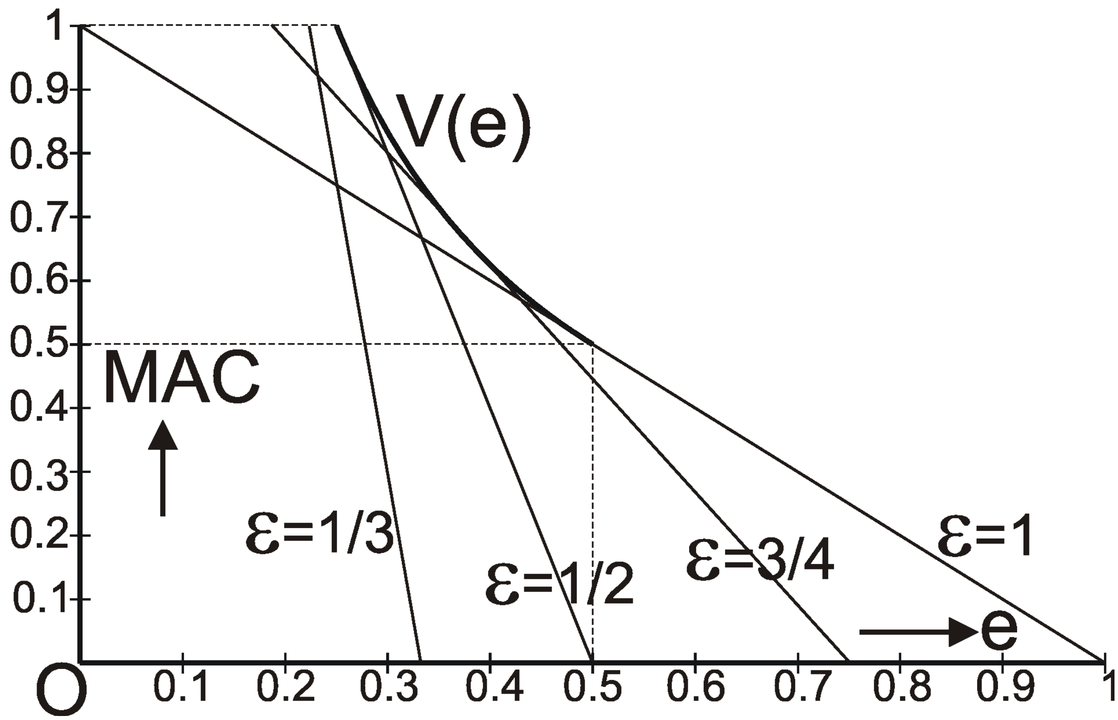


Figure 1: Marginal Abatement Cost ($MAC$) curves for different values of emission intensity $\varepsilon$ ($P = c = 1$).

Intuitively, a decrease in emission intensity $\varepsilon$ has two effects on marginal abatement costs $MAC$ as a function of emissions $e$. First, a lower $\varepsilon$ means that output $q$ has to be reduced further to achieve a given emission reduction. This effect raises $MAC$ and is dominant for low levels of $e$. Secondly, a lower $\varepsilon$ means that a given level of $e$ is achieved with a higher $q$. With increasing marginal production costs, the profit margin on the last unit of output, which has to be given up in order to reduce $e$, is lower when $\varepsilon$ is lower and $q$ is higher. This second effect reduces $MAC$ and is dominant for high levels of $e$.

When $\varepsilon$ falls marginally, the $MAC$ curve pivots clockwise around its middle point, so that the area underneath remains constant at $\bar{\pi}_p$ given by (21).[18] Since $MAC = 0$ at $e = \bar{e}_p$, the pivot point is at:

$$e = \tilde{e}_p \equiv \frac{P\varepsilon}{2c} = \frac{1}{2}\bar{e}_p = \varepsilon\tilde{q}_p \tag{24}$$

The pivot point is thus at $q = \tilde{q}_p$, defined in (22) as the output level where $\varepsilon$ reaches the bottom of its U-shaped curve. Substituting (24) back into (23) to eliminate $\varepsilon$, we can

[18]Formally, the pivot point is found by setting $\partial MAC/\partial\varepsilon = 0$ in (23).

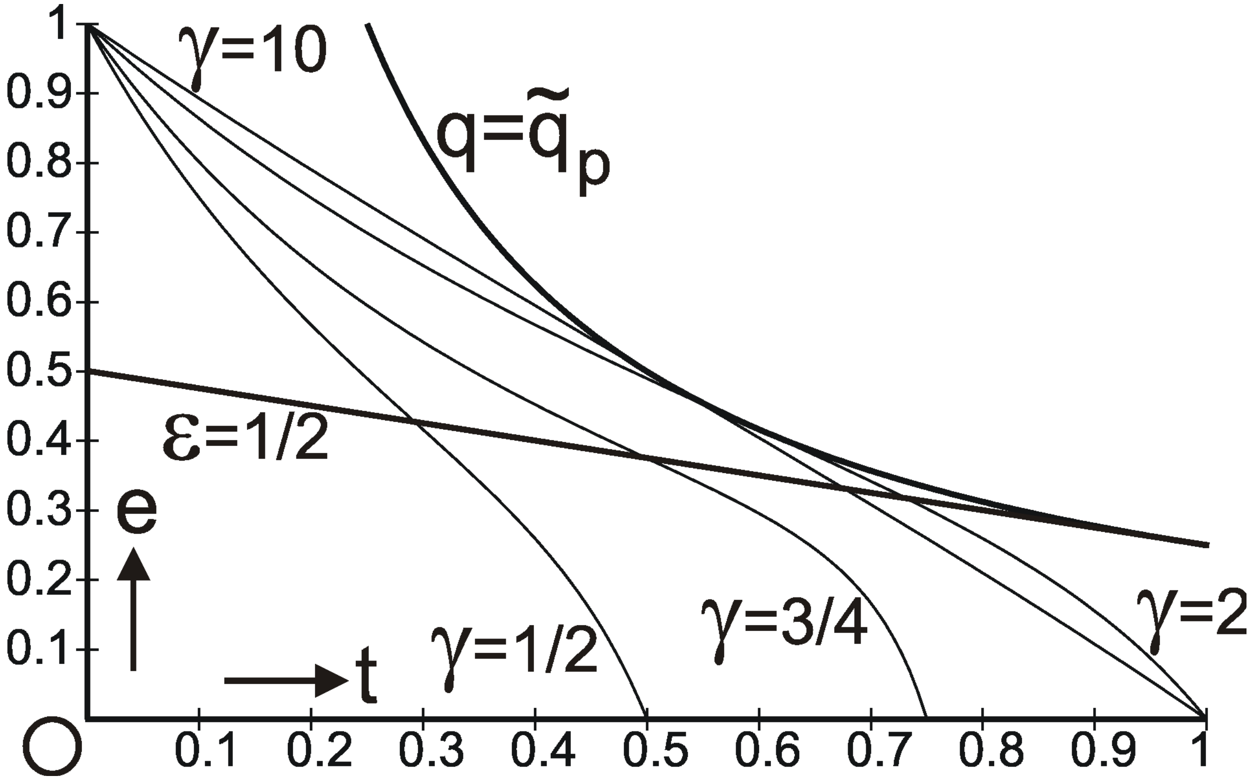


Figure 2: Emissions as a function of the tax rate for different values of $\gamma$ ($P = c = 1$).

find the curve that connects all these pivot points, which is the envelope curve $V(e)$ that gives the maximum value of $MAC$ for a given level of $e$:

$$V(e) = \frac{P^2}{4ce} \tag{25}$$

Figure 1 shows the envelope curve $V(e)$ for $P = c = 1$.

Figure 2 shows emissions $e$ in (20) as a function of the tax rate for $P = c = 1$ and different values of $\gamma$ (note that the axes are interchanged compared to Figure 1). When $\gamma > P^2/c = 1$ (that is, under low profitability conditions), $q$ is monotonically decreasing in $t$ and $\varepsilon$ is U-shaped in $t$, reaching its minimum at $q = \tilde{q}_p$ given by (22). In Figure 2, the point where $\varepsilon$ reaches its minimum is where the emissions curve touches the $q = \tilde{q}_p$ curve. This curve is the inverse of the $V(e)$ curve in (25) and Figure 1.

When $\gamma < P^2/c = 1$ (that is, under high profitability conditions), $\varepsilon$ is monotonically decreasing in $t$ and $q$ is U-shaped in $t$, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for $\gamma$ and substituting this into the expression for $e$ in (20), we find that the point where

$q$ reaches its minimum is given by $e = (2P - t)/4c$. This is the line "$\varepsilon = \frac{1}{2}$" in Figure 2. The emission curves for $\gamma < P^2/c = 1$ feature decreasing $q$ above the $\varepsilon = \frac{1}{2}$ line and increasing $q$ below it.[19]

One might worry that when a cleaner technology shifts the $MAC$ curve upwards for low levels of emissions (let us call this the "perverse response" range of emissions), the regulator would respond to the arrival of a cleaner technology by allowing the firm to increase emissions (Bauman *et al.*, 2008, p. 517). Figure 2 illustrates why this does not happen in our model where technology is endogenous.[20] When $\gamma < P^2/c$, the emission path in Figure 2 never reaches the $q = \tilde{q}_p$ curve where the $MAC$ curves cross. This means that emissions never reach the perverse response range. In this case, emission intensity keeps decreasing in the strictness of environmental policy, and output is U-shaped. When $\gamma > P^2/c$ instead, emissions do reach the perverse response range. However, in this case the industry's response to stricter environmental policy is to use a more polluting technology and to reduce output. In both cases, a stricter environmental policy always results in lower emissions (Proposition 1.2).

## 4.2 Cournot oligopoly

We now consider the case of a Cournot oligopoly, denoted by subscript $l$. There are $n$ firms facing inverse demand function (16). From (2), (15) and (16), profits are:

$$\Pi_i = \pi(q_i, \varepsilon_i) - te_i = (a - Q)q_i - \frac{c}{2}q_i^2 - \frac{\gamma}{2}(1 - \varepsilon_i)^2 - t\varepsilon_i q_i \tag{26}$$

The FOCs for profit maximization are, with respect to $q_i$ and $\varepsilon_i$ respectively:

$$a - (2 + c)q_i - Q_{-i} - t\varepsilon_i = 0 \tag{27}$$

$$\gamma(1 - \varepsilon_i) - tq_i = 0 \tag{28}$$

Solving (27) and (28), the symmetric equilibrium solutions ($q_i = q$, $\varepsilon_i = \varepsilon$) are:

$$q = \frac{\gamma(a - t)}{\gamma(1 + c + n) - t^2}, \quad \varepsilon = \frac{\gamma(1 + c + n) - ta}{\gamma(1 + c + n) - t^2}, \quad E = \frac{n\gamma(a - t)\left[\gamma(1 + c + n) - ta\right]}{\left[\gamma(1 + c + n) - t^2\right]^2} \tag{29}$$

[19] The emission curves for $\gamma > 1$ also intersect the $\varepsilon = \frac{1}{2}$ line, but this is irrelevant for $\gamma > 1$.
[20] Similar arguments can be made for Cournot oligopoly in the next subsection.

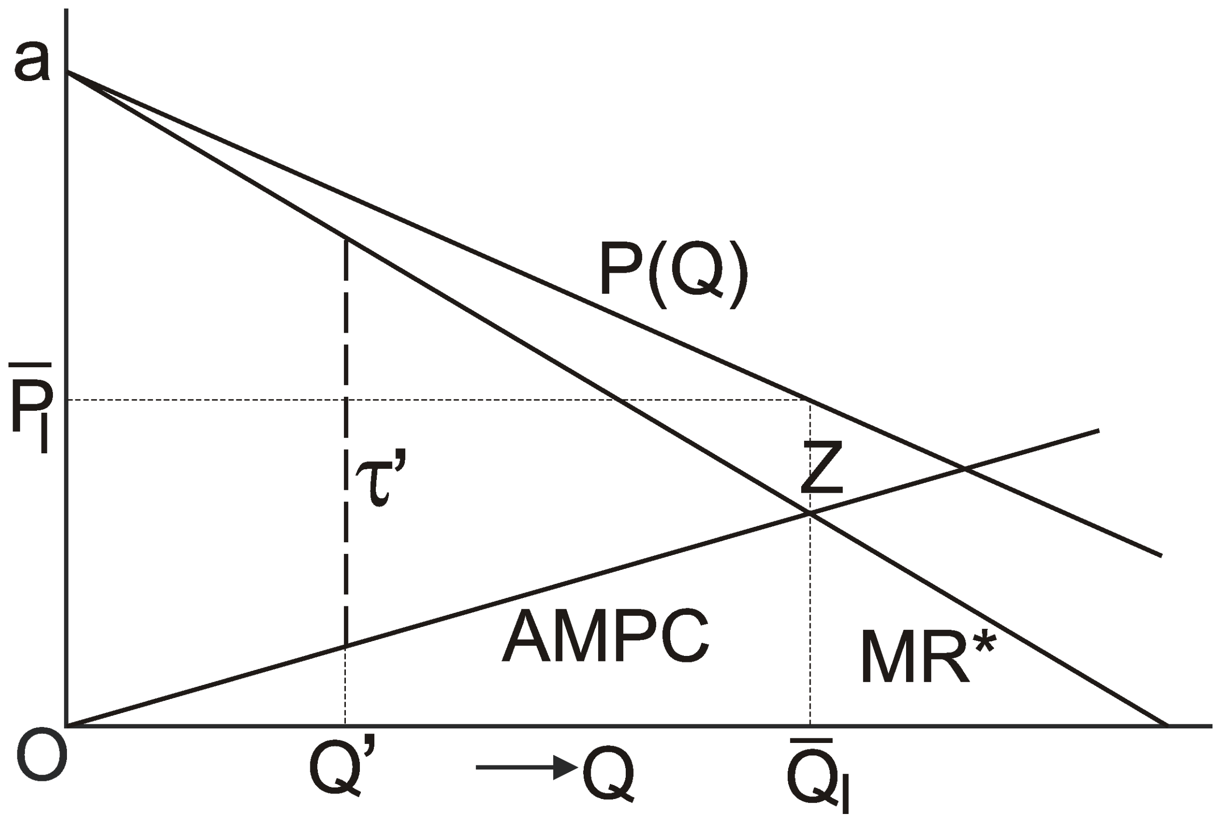


Figure 3: Symmetric market equilibrium under Cournot oligopoly

The numerators on the RHS are positive by Proposition 3.3 in Appendix B. However, as we shall see in our numerical examples, the equivalent of second order condition (9):

$$\gamma(2+c)-t^2>0 \tag{30}$$

does not hold for all values of $\gamma,\ c$ and $t$.

Without environmental policy, $t=0$ so that:

$$q=\bar{q}_l\equiv\frac{a}{1+c+n},\qquad e=\bar{q}_l,\qquad \pi=\bar{\pi}_l\equiv\frac{a^2\,(2+c)}{2\,(1+c+n)^2} \tag{31}$$

Figure 3 illustrates the symmetric equilibrium, where (27) becomes:

$$a-\frac{(n+1)Q}{n}=\frac{cQ}{n}+t\varepsilon \tag{32}$$

The LHS is each firm's equilibrium marginal revenue ($MR^*$ in Figure 3) where all firms produce the same amount $q=Q/n$. The RHS of (32) is the sum of the industry's aggregate marginal production costs $cq$ ($AMPC$ in Figure 3) and the effective tax rate

$\tau \equiv t\varepsilon$ on output. When $t = 0$, (32) holds at point $Z$ in Figure 3 so that each firm sets $q = \bar{q}_l$ as given by (31), total production is $\bar{Q}_l = n\bar{q}_l$ and the product price is $\bar{P}_l = P(\bar{Q}_l)$ by (16). Let us assume that for a given emission intensity level $\varepsilon'$ (not shown in the figure), the regulator sets the emission tax rate at $t'$ so that the effective tax rate on output is $\tau' \equiv t'\varepsilon'$ and the industry produces $Q'$ as shown in Figure 3. Thus $\tau$ creates a wedge between $MR^*$ and $AMPC$. As $\tau$ rises continuously from zero to $a$ to reduce output per firm from $\bar{q}_t$ to zero, the continuum of wedges fills the whole area $OaZ = \frac{1}{2}a\bar{Q}_l$ between $MR^*$ and $AMPC$.

Applying Lemma 1 and Proposition 3 from Appendix B and defining:

$$\gamma_l \equiv \frac{a^2}{1+c+n} \tag{33}$$

we see that when $\gamma < \gamma_l$, emission intensity is monotonically decreasing in $t$ and output is U-shaped in $t$, with the turning point at $\varepsilon = \frac{1}{2}$. Intuitively, when output is very clean (to be precise: when the emissions-to-output ratio is below half the no-regulation level), output can increase again with the tax rate while production is becoming ever cleaner. If $\gamma > \gamma_l$, output is monotonically decreasing in $t$ and emission intensity is U-shaped in $t$, with the turning point at:

$$\tilde{q}_l \equiv \frac{a}{2(1+c+n)} = \frac{\bar{q}_l}{2} \tag{34}$$

Thus, $\varepsilon$ decreases until the point where output is so low that it is no longer worthwhile to clean up production further. This occurs when output is at half its no-regulation level of $\bar{q}_l$, given by (31).

The intuition behind $\gamma_l$ in (33) being the critical value of $\gamma$ is similar to the intuition highlighted in the previous subsection regarding the profitability conditions associated to each scenario. When $\gamma < \gamma_l$, abatement costs $\gamma$, production costs $c$ and the number of firms $n$ are low relative to the size of the market $a$. These circumstances are associated with higher profitability conditions. Abatement costs are relatively low for two reasons. First, each firm's cost of reducing emission intensity to a certain level is low, because $\gamma$ is low. Secondly, since the number $n$ of firms is low, each firm has a relatively high production level. This raises the benefit of reducing the emission intensity of output. As $t$ keeps increasing, firms are keen to take advantage of their low emission intensity to let output increase again. This means that they have to keep reducing their emission intensity as $t$ rises, but they are happy to do so as abatement is relatively cheap. When $\gamma > \gamma_l$, abatement costs $\gamma$, production costs $c$ and the number of firms $n$ are high relative to the size of the market $a$. This implies that there are low profitability conditions. Then firms do not want to produce too much or spend too much on abatement. Thus as $t$ keeps increasing, firms keep decreasing their output. When output is getting very low, firms can increase their emission intensities again.

Let us now interpret these results in terms of marginal abatement cost ($MAC$) curves. Substituting (2) into (26), firm $i$'s profits can be written as a function of emissions, emission intensity and the aggregate output $Q_{-i}$ of all other firms:

$$\pi(e_i, \varepsilon_i, Q_{-i}) = \left(a - Q_{-i} - \frac{e_i}{\varepsilon_i}\right)\frac{e_i}{\varepsilon_i} - \frac{c}{2}\left(\frac{e_i}{\varepsilon_i}\right)^2 - \frac{\gamma}{2}(1-\varepsilon_i)^2$$

Firm $i$'s marginal abatement costs, defined for a given level of $\varepsilon_i$, are then:

$$MAC(e_i, \varepsilon_i, Q_{-i}) \equiv \frac{\partial \pi(e_i, \varepsilon_i, Q_{-i})}{\partial e_i} = \left(a - Q_{-i} - 2\frac{e_i}{\varepsilon_i}\right)\frac{1}{\varepsilon_i} - \frac{ce_i}{\varepsilon_i^2} \tag{35}$$

Unlike in (23) previously with constant product price, firm $i$'s $MAC$ now depends on the choice of $q_j$ by all other firms $j \neq i$. These $q_j$ are endogenous, because they depend on $t$. We will make use of the aggregate marginal abatement cost $AMAC$ for the whole industry in a symmetric equilibrium where $q_j = q$ and $\varepsilon_j = \varepsilon$ (and thus $e_j = e$ and $E = ne$) for all $j = 1, \cdots, n$. From (35):

$$AMAC(E, \varepsilon) \equiv nMAC\left(\frac{E}{n}, \varepsilon, \frac{(n-1)E}{n\varepsilon}\right) = \frac{a}{\varepsilon} - \frac{(1+c+n)E}{n\varepsilon^2} \tag{36}$$

When $t = 0$, each firm sets $AMAC = 0$, so that $Q = \bar{Q}_l \equiv n\bar{q}_l$ as defined by (31). Unless $\varepsilon = 0$, $e = 0$ can only be achieved by setting $Q = 0$. A decrease in $\varepsilon$ shifts the point $\bar{E}_l \equiv \varepsilon\bar{Q}_l$ where $AMAC = 0$ to the left. The area under the $AMAC$ curve must remain the same, because it is the area $OaZ = \frac{1}{2}a\bar{Q}_l$ filled by the wedges of $\tau$ in Figure 3, as discussed above. This means that $AMAC(0, \varepsilon)$ has to move up according to:

$$AMAC(0, \varepsilon) = \frac{a}{\varepsilon}$$

Figure 4 shows $AMAC$ curves for different levels of $\varepsilon$ when $P = c = 1,\ n = 4$, so that $\gamma_l$ in (33) equals $\frac{1}{6}$.

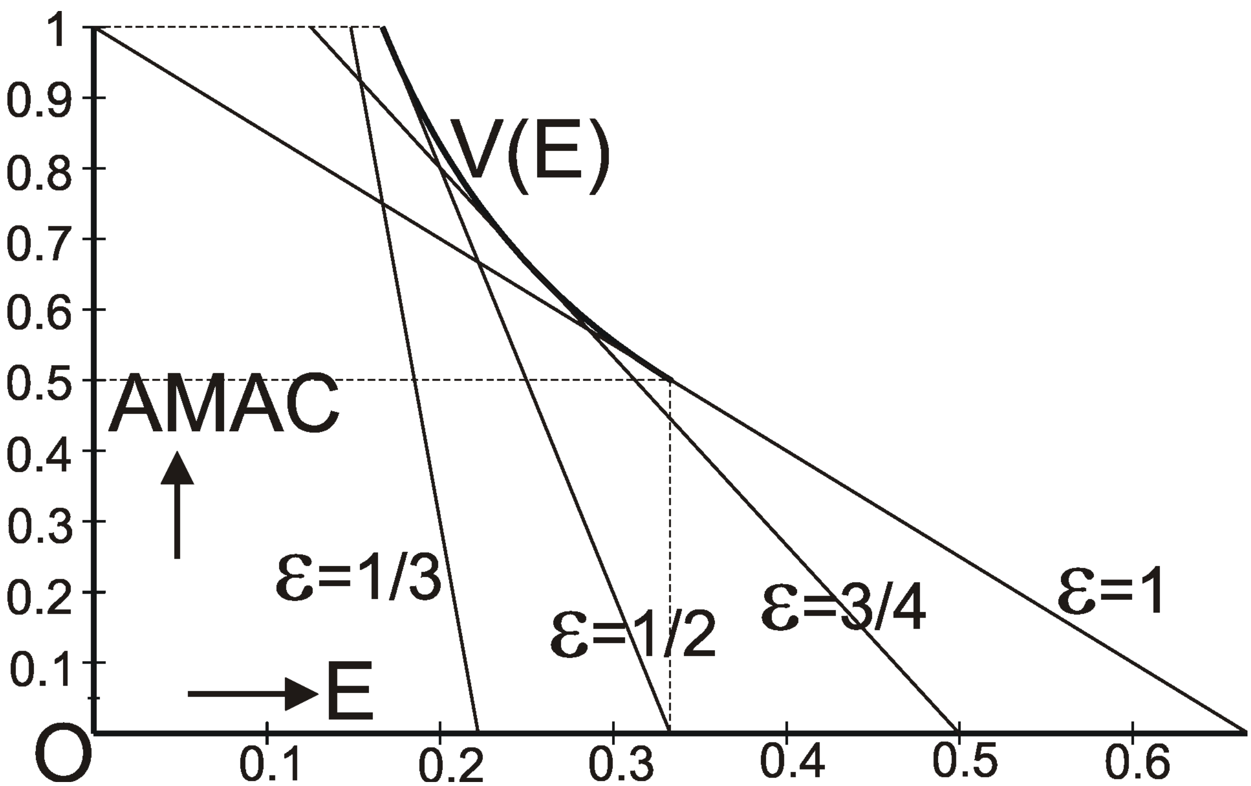


Figure 4: Aggregate Marginal Abatement Cost ($AMAC$) curves for different values of emission intensity $\varepsilon$ ($a = c = 1,\ n = 4$).

When $\varepsilon$ falls marginally, the $AMAC$ curve pivots clockwise around its middle point, so that the area underneath remains constant at $\frac{1}{2}a\bar{Q}_l$. Since $AMAC = 0$ at $E = \bar{E}_l$, the pivot point is at:

$$E = \tilde{E}_l \equiv \frac{na\varepsilon}{2(1+c+n)} = \frac{1}{2}\bar{E}_l = \varepsilon\tilde{Q}_l \tag{37}$$

The pivot point is thus where $Q = \tilde{Q}_l = n\tilde{q}_l$ as defined by (34). Substituting (37) back into (36) to eliminate $\varepsilon$, the curve that connects all these pivot points is the envelope curve $V(E)$ that gives the maximum value of $AMAC$ for a given level of $E$:

$$V(E) = \frac{na^2}{4E(1+c+n)^2} \tag{38}$$

Figure 4 shows the envelope curve $V(E)$ for $P = c = 1,\ n = 4$.

Figure 5 shows total emissions $E$ from (29) as a function of the tax rate for $P = c = 1,\ n = 4$ and different values of $\gamma$ (with the axes interchanged compared to Figure 4). For the $\gamma$ values of $\frac{1}{10}$ and $\frac{1}{4}$, second order condition (30) for profit maximization does not

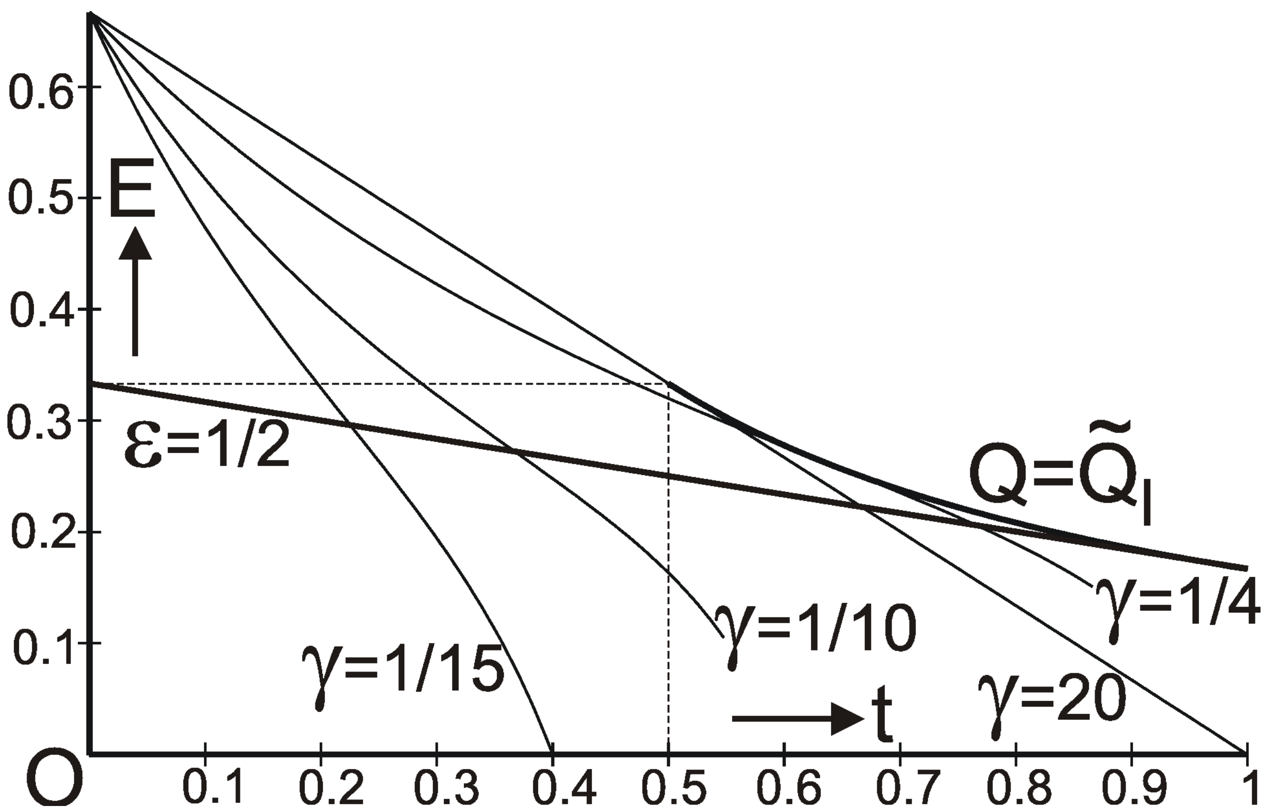


Figure 5: Emissions as a function of the tax rate for different $\gamma$ values ($a = c = 1,\ n = 4$).

hold when the tax rate is very high, so that (29) results in negative profits. The curves for these $\gamma$ values are only shown for $t$ values where profits are positive. For the $\gamma$ values of $\frac{1}{15}$ and 20 however, profits are positive throughout.

When $\gamma > \gamma_l = \frac{1}{6}$ by (33) (that is, under low profitability conditions), $q$ is monotonically decreasing in $t$ and $\varepsilon$ is U-shaped in $t$, reaching its minimum at $q = \tilde{q}_l$ given by (34). As we have seen above, this is where the $AMAC$ curves cross. In Figure 5, the point where $\varepsilon$ reaches its minimum is where the emissions curve touches the $Q = \tilde{Q}_l$ curve. The $Q = \tilde{Q}_l$ curve is the inverse of the $V(E)$ curve in (38) and Figure 4.

When $\gamma < \gamma_l = \frac{1}{6}$ by (33) (that is, under high profitability conditions), $\varepsilon$ is monotonically decreasing in $t$ and $q$ is U-shaped in $t$, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for $\gamma$ and substituting this into the expression for $E$ in (29), we find that the point where $q$ reaches its minimum is given by:

$$E = \frac{n(2a - t)}{4(1 + c + n)}$$

This is the line "$\varepsilon = \frac{1}{2}$" in Figure 5. The emission curves for $\gamma < \gamma_l$ feature decreasing $q$ above the $\varepsilon = \frac{1}{2}$ line and increasing $q$ below it.

# 5 Discussion

In Section 3 we saw that with integrated technologies when the environmental policy is very lenient, firms respond to increases in the emission tax rate by both reducing output and using cleaner production methods. However, after a certain level of policy strictness, either output or emission intensity will increase following an increase in the emission tax rate. After imposing more structure to the model in Section 4, we have been able to shed light on the circumstances under which either of these will apply. In particular, we have shown that with low profitability conditions, output is decreasing in the strictness of the policy while emission intensity is U-shaped, while the opposite occurs when there are high profitability conditions. This applies both when the product price is constant and when firms interact strategically in the output market. Higher profitability conditions imply lower abatement and production costs, as well as higher output price for the case without strategic interaction or larger market sizes and lower number of firms for the case of strategic interaction.

In the context of technology adoption, Perino and Requate (2012) find that the share of firms adopting a clean technology is inverted U-shaped in the strictness of environmental policy. In their paper, firms face a discrete choice of technology and a constant product price. In our model, keeping the constant product price assumption as in Perino and Requate (2012), we show that individual emission intensities may be decreasing throughout in the strictness of the policy, in which case output is U-shaped. Hence, we can state that this result is not due to our explicit modelling of the output market, but because we consider the case where the abatement choice is continuous, while they assume a discrete choice between two alternative technologies. Qualitatively similar results to those in Perino and Requate (2012) can be found in Bréchet and Meunier (2014). In the latter paper, firms also face a discrete choice of integrated technology, but the output market is explicitly modelled. The comparison to Bréchet and Meunier (2014) serves to further make our point. The scenario where output is U-shaped while emission-to-output ratios are decreasing in the strictness of the policy which emerges in our paper is not the result of the modelling of the output market; rather, it is due to the abatement choice being continuous rather than discrete.

The reader may wonder whether the non-monotonic effects of the strictness of the policy emerge because of policy failure. We address this issue in Appendix A where we analyze the social optimum. We see that the effects of environmental policy strictness are qualitatively the same as with emission taxation (whether under strategic interaction or constant price). Again there are two scenarios, one in which the environmental damage parameter $\beta$ has a non-monotonic effect on output but emission-to-output ratios are strictly decreasing in $\beta$ and a second scenario where the opposite applies. The analysis presented in the appendix also allows us to show that emission taxation alone can implement the welfare optimum under both perfect competition and constant price. Under Cournot oligopoly, however, a single instrument is not sufficient to deal with the two market failures of negative externalities and underproduction. Our analysis in Appendix A therefore shows that the non-monotonic effects of the strictness of the policy are not the result of policy failure.

# 6 Conclusion

Does an increasingly strict environmental policy spur on the polluting industry to employ ever cleaner production methods and lower production? The answer might appear obvious at first sight but it is not, as it depends on the type of technology available to firms. We have shown that with an integrated abatement technology where firms face a continuous choice in terms of emission intensity, making environmental policy stricter leads to non-monotonic responses by firms in either output levels or emission intensities. Tightening environmental policy induces firms to reduce both their levels of output and emission levels when the the policy is not very stringent to start with. However, when the policy is sufficiently strict, either output or the emissions-to-output ratio will turn increasing in the strictness of the policy. This applies both when emissions are taxed in a range of market settings (whether firms interact strategically or not), and in the welfare optimum. Thus if we see polluting output increasing or production methods becoming less clean as environmental policy becomes stricter, this is not necessarily a sign that the policy is ineffective (or even counterproductive) or misguided.

With an integrated technology, a cleaner production method pivots the Marginal Abatement Cost ($MAC$) curve clockwise, with the new $MAC$ curve intersecting the old one. We find that when emission intensity is U-shaped, the turning point occurs where the $MAC$ curves cross. When the product price is constant, the definition of $MAC$ is relatively straightforward: It is a firm's decrease in profits from reducing emissions by reducing output. When the product price is decreasing in total output, however, a firm's profits and thus its $MAC$ depend on the output decisions of the other firms. In this setting, we define the industry's Aggregate $MAC$ for the case where all firms set the same emission intensity and output levels. For the welfare-maximizing outcome, the relevant concept is the Social $MAC$, which includes the changes in the industry's profits and in consumer surplus.

One might worry that when a cleaner technology shifts the $MAC$ curve upwards for low levels of emissions, the regulator's response to the arrival of a cleaner technology would perversely result in higher emissions (Bauman *et al.*, 2008, p. 517). This does not happen in our model where technology is endogenous to environmental policy. First, it could be that the emission path never reaches the "perverse response" range. In this case, emission intensity keeps decreasing in the strictness of environmental policy, and output is U-shaped. The other possibility is that emissions do reach the perverse response range. In this case the industry's response to stricter environmental policy is to use a more polluting technology and to reduce output.

Emission intensity is more likely to be U-shaped if the profitability conditions are low; that is, if production and abatement costs and the number of firms are large relatively to the size of the market. This dichotomy draws our attention to the potential effects of the interaction between environmental and industrial policy. For example, policies aimed at reducing entry barriers, combined with a stricter environmental policy, could result in lower abatement incentives. An increase in the number of firms and a simultaneous increase in the strictness of environmental policy could imply a switch from our U-shape in output scenario to the U-shape in emission-to-output ratio scenario, resulting in an increase in emission intensities across the industry. This could have ambiguous effects on welfare.

Endogenous entry is best understood in a model of heterogeneous firms, such as the Melitz (2003) model of monopolistic competition. Using this model, Cao et al. (2016) find there can be an inverted U-shaped relation between firm productivity and abatement investment under certain conditions. They also find empirical support for this relation. In our model, firm heterogeneity could imply that some firms face a U-shape in output while others have U-shaped emissions intensity. Further research on these issues considering endogenous entry and firm heterogeneity would therefore be welcome.

It should be noted that we have used a partial equilibrium framework to derive our result that output (emission intensity) is U-shaped in strictness under high (low) profitability conditions. In a general equilibrium framework, a reduction in the consumption of one polluting good will increase the demand for substitute goods. Since an increase in demand will raise the profitability of producing these substitute goods, they are then more likely to exhibit a U-shape in output rather than in emission intensity.

Although it may be optimal for environmental policy, especially climate change policy, to become ever stricter over time, it is likely that policy makers are unable to credibly commit to this. An alternative could be to stimulate environmental R&D, reducing the future cost of stricter environmental policy by reducing marginal abatement costs (Abrego and Perroni, 2002; Golombek *et al.*, 2010). However, as we have seen, higher investment into integrated technologies does not reduce the $MAC$ curve for all emission levels, but pivots it clockwise. Indeed, it is not clear that ever cleaner production methods are needed for further and further reductions in emissions. This highlights a potential limitation of environmental R&D subsidies in overcoming the commitment problem.

Ulph and Ulph (2013) found that an environmental R&D subsidy for an integrated technology can be useful in dealing with a different commitment problem. The authors show that a government that faces uncertainty about the environmental preferences of a future government may want the firm to adopt a cleaner production technology. Applying the analysis of the present paper, we know that a cleaner production technology comes with a steeper $MAC$ curve. Thus with the cleaner technology in place, the future government will implement an emission level that is closer to the current government's preferred level.

Our findings also have implications for empirical research. We find that the cleanliness of production is a far from perfect indicator of the strictness of the environmental policy. It may well be the case that stricter environmental policy will lead to less clean production. This has implications for empirical studies which have used the emission-to-output ratio as a proxy for the stringency of environmental policy. For instance, List and Co (2000) use the ratio of pollution abatement operating expenditures to value added as one of the measures of US state environmental regulation. Ederington *et al.* (2005) take the ratio of pollution abatement costs to total costs of materials as their measure of stringency of US (federal) environmental regulation.

In future empirical work, it would be interesting to examine whether abatement technology for a specific pollutant and industry can be described as an integrated technology. In this case, further investigation could reveal whether stricter environmental policy would lead, or perhaps has already led, to a U-shaped response in output or emission intensity. Another avenue for potential research would be to study the interaction between industrial policy and environmental policy instruments, as discussed above.

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# A Appendix A: Welfare optimum and perfect competition

The objective of this section is twofold. First, to show that the non-monotonic behavior of the emission-to-output ratio or output is not due to market or policy failure. To this end, we will solve the welfare optimum $w$, where output and abatement are set to maximize welfare. Second, to analyze the case of perfect competition, where we will see that our qualitative results from previous sections stand. In the case of perfect competition, we will also show that emission taxation alone implements the welfare optimum (the same applies to the case of constant price). As firms are symmetric, we focus on the symmetric outcome where $q_i = q$ and $\varepsilon_i = \varepsilon$ for all $i = 1, \cdots, n$.

## A.1 General analysis

With perfect competition, there is a continuum of firms of mass $n$. Each firm takes the product price $P$ as given. In stage two of the game presented in Section 2, firm $i$ sets the output level $q_i$ and emission intensity $\varepsilon_i$ to maximize its profits (5). The FOCs are, respectively:

$$
\begin{aligned}
P - k'(q_i) - t\varepsilon_i &= 0 && \text{(A1)} \\
-F'(\varepsilon_i) - tq_i &= 0 && \text{(A2)}
\end{aligned}
$$

The Hessian matrix is:

$$\mathbf{\Pi}_{xx} = \begin{pmatrix} -k'' & -t \\ -t & -F'' \end{pmatrix}$$

Note that $k'' > 0, F'' > 0$. Therefore, in this case, $\mathbf{\Pi}_{xx}$ is negative definite as long as:

$$k''F'' - t^2 > 0 \tag{A3}$$

which is therefore the sufficient SOC for maximization in this case.

Now let us move to welfare maximization. The first order conditions for maximizing welfare (4) are:

$$
\begin{aligned}
W_q &= n\left[P - k'(q_i) - \varepsilon D_E\right] = 0 && \text{(A4)} \\
W_\varepsilon &= -n\left[F'(\varepsilon_i) + qD_E\right] = 0 && \text{(A5)}
\end{aligned}
$$

A comparison of (A4) and (A5) with (6), (7), (A1) and (A2) shows that with constant $P$ and with perfect competition, the regulator can implement the welfare optimum by setting the emission tax rate $t = D_E$. Under Cournot oligopoly, however, setting $t = D_E$ does not implement the welfare optimum, because of the underproduction characterizing imperfectly competitive markets. The regulator would therefore need an additional policy instrument to implement the welfare optimum in this case.

Going back to the social optimum, if there is no environmental damage ($\beta = 0$ so that $D_E = 0$), then from (A4) and (A5) $C_\varepsilon = 0$ so that $\varepsilon = 1$ and the welfare-maximizing output level is $\bar{q}_w$, implicitly defined by:

$$P(n\bar{q}_w) - k'(\bar{q}_w) = 0 \tag{A6}$$

The sufficient SOC for welfare maximization is that the matrix

$$\mathbf{W}_{xx} = n \begin{pmatrix} nP' - k'' - n\varepsilon^2 D_{EE} & -D_E - n\varepsilon q D_{EE} \\ -D_E - n\varepsilon q D_{EE} & -F'' - nq^2 D_{EE} \end{pmatrix} \tag{A7}$$

is negative definite.[21] This implies that $\mathbf{h}\mathbf{W}_{xx}\mathbf{h}' < 0$ for all vectors $\mathbf{h}$ and the determinant is positive, so that:

$$\begin{aligned} \Delta_w &\equiv \frac{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2}{n^2} \\ &= -\left[nP' - k'' - n\varepsilon^2 D_{EE}\right]\left[F'' + nq^2 D_{EE}\right] - \left(D_E + n\varepsilon q D_{EE}\right)^2 > 0 \end{aligned} \tag{A8}$$

Totally differentiating (A4) and (A5) with respect to $\beta$ yields, using (A7):

$$\frac{dq}{d\beta} = \frac{nD_{\beta E}\left[\varepsilon W_{\varepsilon\varepsilon} - qW_{q\varepsilon}\right]}{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2} = \frac{D_{\beta E}}{\Delta_w}\left[qD_E - \varepsilon F''\right] \tag{A9}$$

$$\frac{d\varepsilon}{d\beta} = \frac{nD_{\beta E}\left[qW_{qq} - \varepsilon W_{q\varepsilon}\right]}{W_{qq}W_{\varepsilon\varepsilon} - W_{q\varepsilon}^2} = \frac{D_{\beta E}}{\Delta_w}\left[q\left(nP' - k''\right) + \varepsilon D_E\right] \tag{A10}$$

Note that $D_{\beta E} > 0$, and $\Delta_w > 0$ by (A8). Thus the sign of (A9) and (A10) as well as of (A11) and (A12) below is the sign of the term in square brackets. We cannot sign (A9) and (A10) unambiguously, but we can do so for the case where environmental damage is very high, so that emissions are very low: $E = n\varepsilon q \to 0$. This means that either $\varepsilon$ or $q$ or both must be close to zero.

[21] Since $P' \le 0$ and $D_{EE} > 0$, the condition $k'' \ge 0$ would be sufficient for $nP' - k'' - n\varepsilon^2 D_{EE} < 0$.

When $\varepsilon$ is close to zero, the term in square brackets on the RHS of (A9) is positive, so that output is increasing in the severity of environmental damage as $\varepsilon$ falls to a very low level. Indeed, from (A4) with $\varepsilon \to 0$, $q$ approaches the output level $\bar{q}_w$ without environmental damage, as defined by (A6).

When $q$ is close to zero, the term in square brackets on the RHS of (A10) is positive, so that the emissions-to-output ratio is increasing in the severity of environmental damage as output falls to a very low level. Indeed, from (A5) with $q \to 0$, $\varepsilon$ approaches unity again: the firm does not spend anything on reducing its emission intensity.

We can also use (A9) and (A10) along with (2) to show that emissions are decreasing in $\beta$:

$$\frac{dE}{d\beta} = n\varepsilon\frac{dq}{d\beta} + nq\frac{d\varepsilon}{d\beta} = \frac{D_{\beta E}}{\Delta_w}\left[q^2 W_{qq} - 2\varepsilon q W_{q\varepsilon} + \varepsilon^2 W_{\varepsilon\varepsilon}\right] < 0 \tag{A11}$$

The term in square brackets is negative from setting $\mathbf{h} = \begin{pmatrix} q & -\varepsilon \end{pmatrix}$ in $\mathbf{h}\mathbf{W}_{xx}\mathbf{h}' < 0$ which holds by negative definiteness of $\mathbf{W}_{xx}$ in (A7).

We can also show that marginal environmental damage $MD \equiv D_E$ is increasing in $\beta$:

$$\frac{dMD}{d\beta} = D_{\beta E} + D_{EE}\frac{dE}{d\beta} = \frac{D_{\beta E}}{\Delta_w}\left[\left\{k''F'' - D_E^2\right\} - nP'F''\right] > 0 \tag{A12}$$

The second equality follows from (A8) and (A11). The term in square brackets is positive, because $P' \leq 0$, $k'' > 0$ and the term in curly brackets is positive. The latter term is second order condition (A3) for perfect competition which implements the welfare optimum with $t = D_E$.

Finally, let us return to perfect competition. Inequality (A12) together with our finding that setting $t = MD$ implements the welfare optimum means that under perfect competition, $dq/dt$ and $d\varepsilon/dt$ have the same signs as $dq/d\beta$ in (A9) and $d\varepsilon/dt$ in (A10) respectively, and $dE/dt < 0$ by (A11).

Summarizing our findings, we have:

**Proposition 2** *When firms have access to an integrated abatement technology with a continuous range of emission intensities $\varepsilon$:*

1. *In the constant-price and perfect competition scenarios, setting the emission tax rate equal to marginal damage $MD \equiv D_E$ implements the welfare optimum. An emission tax alone cannot implement the social optimum in Cournot oligopoly.*

*In the welfare optimum:*

2. *Emissions $E$ are decreasing and $MD$ is increasing in the environmental damage parameter $\beta$;*
3. *When $\beta$ is very low, both output $q$ and emission intensity $\varepsilon$ are decreasing in $\beta$;*
4. *When $\beta$ is very high so that $E$ is close to zero:*
   - *(a) When $\varepsilon$ is close to zero, $q$ is increasing in $\beta$.*
   - *(b) When $q$ is close to zero, $\varepsilon$ is increasing in $\beta$.*

As before, we will conduct further analysis on specific integrated technology to establish the behavior of output and emission intensity for any level of environmental damage, and the conditions under which either is non-monotonic.

## A.2 Example

Here we investigate the welfare optimum $w$ with the specific functions (15) for cost and (16) for demand. By Proposition 2.1, the analysis also applies to perfect competition.

In a symmetric outcome, welfare (4) is $W = nw$ where, from (15) and (16):

$$w = \left(a - \frac{n}{2}q\right)q - \frac{c}{2}q^2 - \frac{1}{2}\gamma(1-\varepsilon)^2 - \frac{D(\beta, E)}{n} \tag{A13}$$

The first order conditions are:

$$\frac{\partial w}{\partial q} = a - nq - cq - \varepsilon MD = 0 \tag{A14}$$

$$\frac{\partial w}{\partial \varepsilon} = \gamma(1-\varepsilon) - qMD = 0 \tag{A15}$$

with marginal damage $MD \equiv D_E(\beta, E)$.

Without environmental damage ($\beta = 0$ so $D = MD = 0$), the welfare optimum is:

$$Q = \overline{Q}_w \equiv \frac{na}{n+c}, \qquad E = \overline{Q}_w, \qquad W = \overline{W} \equiv \frac{na^2}{2(n+c)} \tag{A16}$$

Note that it suffices to solve for $Q$ and $\varepsilon$ as functions of $MD$ rather than $\beta$, because we know from Proposition 2.2 that $MD$ is increasing in $\beta$. This also implies we do not need to use a specific functional form for the environmental damage function $D(\beta, E)$. Solving (A14) and (A15) for $Q$, $\varepsilon$ and $E$ as functions of $MD$ yields:[22]

$$Q = \frac{n\gamma\,(a - MD)}{\gamma(n+c) - MD^2}, \qquad \varepsilon = \frac{\gamma(n+c) - aMD}{\gamma(n+c) - MD^2}, \qquad E = \frac{n\gamma\,[a - MD]\,[\gamma(n+c) - aMD]}{[\gamma(n+c) - MD^2]^2} \tag{A17}$$

Applying Lemma 1 and Proposition 3 from Appendix B, we see that when $\gamma < a^2/(n+c)$, emission intensity is monotonically decreasing in $MD$ and output is U-shaped in $MD$, with the turning point at $\varepsilon = \frac{1}{2}$. Intuitively, output is initially declining in the marginal damage. When production is very clean however (to be precise: when the emissions-to-output ratio is below half the no-regulation level), the level of output can increase again with marginal damage while production is becoming ever cleaner.

When $\gamma > a^2/(n+c)$, output is monotonically decreasing in $MD$ and emission intensity is U-shaped in $MD$, with the turning point at

$$\tilde{Q}_w \equiv \frac{\overline{Q}_w}{2} = \frac{na}{2(n+c)} \tag{A18}$$

Thus, $\varepsilon$ decreases until the point where output is so low that it is no longer worthwhile to further clean up production. This point is where output is at half its no-regulation level of $\overline{Q}_w$ given in (A16).

The significance of the comparison between $\gamma$ and $a^2/(n+c)$ can be explained as follows. When $a$ is high, demand is high, so that the regulator does not want to reduce output by too much and is anxious to increase it again if possible. When $\gamma$ and $n$ are high, the cost of reducing emission intensity per firm $\gamma$ and for all firms $n$ is high. Then the regulator does not want to spend too much on reducing emission intensity and is happy to

[22] The numerators on the RHS of the $Q$ and $\varepsilon$ equations are the equivalent of second order condition (A3) for perfect competition. They are positive by Proposition 3.3 in Appendix B.

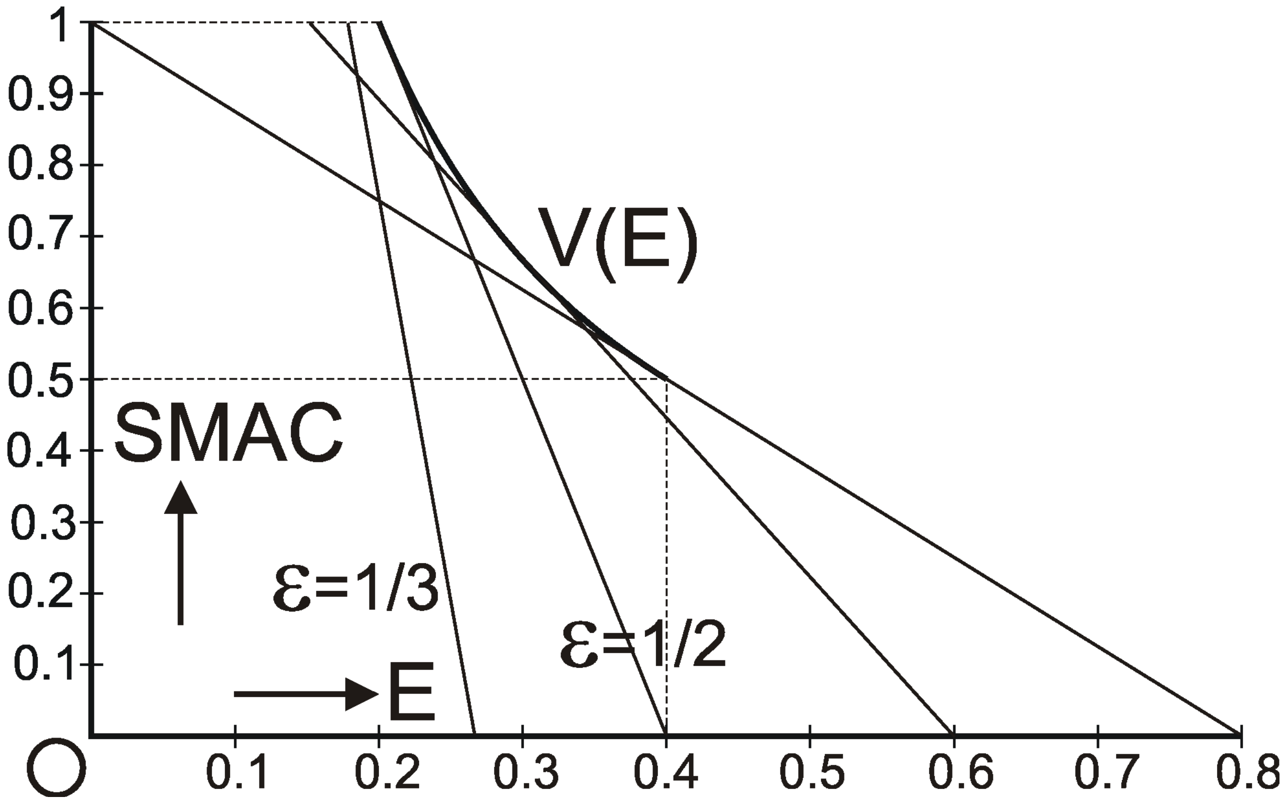


Figure 6: Social Marginal Abatement Cost (SMAC) curves for different emission intensities $\varepsilon$ $(a = c = 1, n = 4)$.

increase emission intensity again if possible. Finally when $c$ is high, production is costly, again making emission reduction more socially efficient than increasing output.

The interpretation in terms of marginal abatement cost depends on whether we apply the analysis to the welfare optimum directly or to perfect competition. With perfect competition, we can follow the discussion and analysis in subsection 4.1 to write a firm's marginal abatement costs as (23). By (2) and (16), this becomes:

$$MAC(e, \varepsilon) = \frac{P}{\varepsilon} - \frac{ce}{\varepsilon^2} = \frac{a}{\varepsilon} - \frac{(n + c)e}{\varepsilon}$$

We can then interpret the outcome as each firm setting $MAC = t = MD$.

Returning to the welfare optimum, it is useful to define the social marginal abatement cost ($SMAC$) as a function of total emissions $E = ne$ divided equally among all firms. Substituting (2) into (A13), welfare $W = nw$ can be written as a function of $E$ and $\varepsilon$:

$$W(E, \varepsilon) = PB - \frac{n\gamma}{2}(1 - \varepsilon)^2 - D(\beta, E) \tag{A19}$$

with pollution benefits $PB$ the difference between the utility and the production cost of output:

$$PB \equiv \left(a - \frac{E}{2\varepsilon}\right)\frac{E}{\varepsilon} - \frac{c}{2n}\left(\frac{E}{\varepsilon}\right)^2 \tag{A20}$$

Maximizing (A19) with respect to $E$ shows that social marginal abatement cost[23] ($SMAC$) should equal marginal damage ($MD$):

$$SMAC \equiv \frac{dPB}{dE} = \frac{a}{\varepsilon} - \frac{(n+c)E}{n\varepsilon^2} = MD \tag{A21}$$

When there is no environmental damage ($\beta = 0$ so that $MD = 0$), the welfare optimum has $SMAC = 0$ in (A21), so that $Q = \overline{Q}_w,\ E = \overline{E}_w \equiv \varepsilon\overline{Q}_w$ and $PB = \overline{PB} \equiv \overline{W}$, with $\overline{Q}_w$ and $\overline{W}$ given by (A16) and $PB$ by (A20). Unless $\varepsilon = 0$, $E = 0$ can only be achieved by setting $Q = 0$ which implies $PB = 0$. A decrease in $\varepsilon$ shifts the point $\overline{E}_w \equiv \varepsilon\overline{Q}_w$ where $SMAC = 0$ to the left. The area under the $SMAC$ curve must remain the same, because it is the difference $\overline{PB}$ in pollution benefits between $SMAC = 0$ and $E = Q = 0$. This means that by (A16), $SMAC(0)$ has to move up according to:

$$SMAC(0) = \frac{2\overline{W}}{\overline{E}_w} = \frac{a}{\varepsilon}$$

Figure 6 shows $SMAC$ curves for different levels of $\varepsilon$ when $a = c = 1,\ n = 4$.

When $\varepsilon$ falls marginally, the $SMAC$ curve pivots clockwise around its middle point, so that the area underneath remains constant at $\overline{PB}$. Since $SMAC = 0$ at $E = \bar{E}_w$, the pivot point is at:

$$E = \tilde{E}_w \equiv \frac{na\varepsilon}{2(c+n)} = \frac{1}{2}\bar{E}_w = \varepsilon\tilde{Q}_w \tag{A22}$$

The pivot point is thus where $Q = \tilde{Q}_w$ defined by (A16). Substituting (A22) back into (A21) to eliminate $\varepsilon$, the curve that connects all these pivot points for different $\varepsilon$ values is the envelope curve $V(E)$ that gives the maximum value of $SMAC$ for a given $E$:

$$V(E) = \frac{na^2}{4E\,(c+n)} \tag{A23}$$

Figure 6 shows the envelope curve $V(E)$ for $a = c = 1,\ n = 4$.

Figure 7 shows emissions (A17) as a function of marginal damage $MD$ in the optimum for $a = c = 1, n = 4$ and different values of $\gamma$ (note that the axes are interchanged

[23] Whereas marginal abatement costs are usually defined in terms of a single firm's profits, our definition of social marginal abatement costs emcompasses all the firms' profits as well as the consumer surplus.

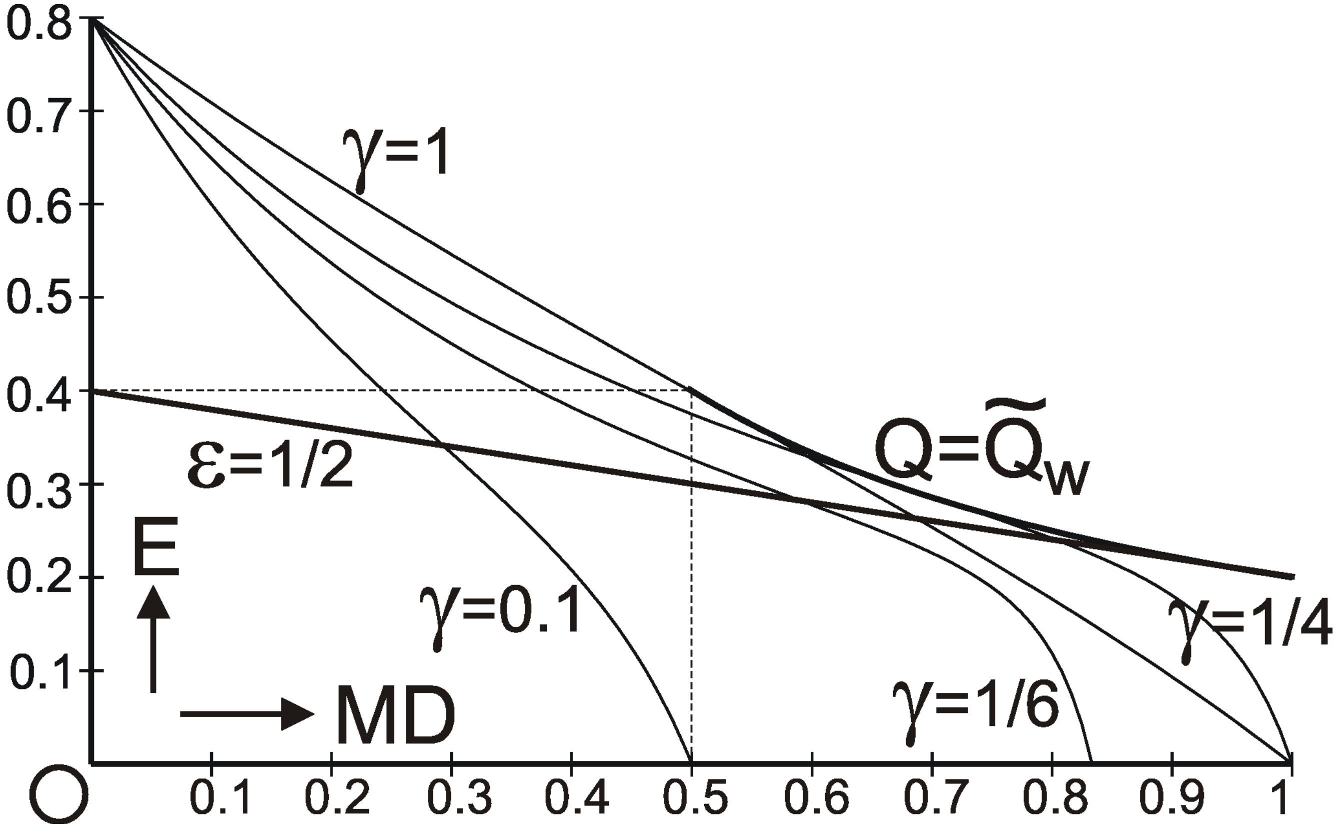


Figure 7: Industry emissions as a function of marginal damage in the welfare optimum for different values of $\gamma$ ($a = c = 1, n = 4$).

compared to Figure 6). When $\gamma > a^2/(n+c) = 1/5$, $Q$ is monotonically decreasing in $MD$ and $\varepsilon$ is U-shaped in $MD$, reaching its minimum at $Q = \tilde{Q}_w$ given by (22). As we have seen above, this is where the $SMAC$ curves cross. In Figure 7, the point where $\varepsilon$ reaches its minimum is where the emissions curve touches the $Q = \tilde{Q}_w$ curve. This curve is the inverse of the $V(E)$ curve in (A23) and Figure 6.

When $\gamma < a^2/(n+c) = 1/5$, $\varepsilon$ is monotonically decreasing in $MD$ and $Q$ is U-shaped in $MD$, reaching its minimum at $\varepsilon = \frac{1}{2}$. Solving $\varepsilon = \frac{1}{2}$ for $\gamma$ and substituting this into the expression for $E$ in (A17), the point where $Q$ reaches its minimum is given by:

$$E = \frac{n\,(2a - MD)}{4(n+c)}$$

This is the line "$\varepsilon = \frac{1}{2}$" in Figure 7. The emission curves for $\gamma < a^2/(n+c) = 1/5$ feature decreasing $Q$ above the $\varepsilon = \frac{1}{2}$ line and increasing $Q$ below it.

# B Appendix B: Example

**Lemma 1** *Let each firm's cost function be given by (15) and its emissions by (2). Then under emission taxation with a constant product price (scenario p) and with Cournot oligopoly (scenario l) as well as in the welfare optimum w, the solution has the form:*

$$q = \frac{\gamma(\Lambda - T)}{\gamma\Theta - T^2}, \qquad \varepsilon = \frac{\gamma\Theta - \Lambda T}{\gamma\Theta - T^2}, \qquad E = \frac{n\gamma(\Lambda - T)(\gamma\Theta - \Lambda T)}{(\gamma\Theta - T^2)^2} \tag{B1}$$

*where:*

$$T_p = t, \qquad \Lambda_p = P, \qquad \Theta_p = c \tag{B2}$$

$$T_l = t, \qquad \Lambda_l = a, \qquad \Theta_l = 1 + n + c \tag{B3}$$

$$T_w = MD, \qquad \Lambda_w = a, \qquad \Theta_w = n + c \tag{B4}$$

*so that $T = 0$ implies:*

$$q = \bar{q} \equiv \frac{\Lambda}{\Theta} \tag{B5}$$

**Proof.** Equation (B1) follows from substituting (B2) into (20) in scenario $p$, (B3) into (29) in scenario $l$, and (B4) into (A17) in scenario $w$. ■

We can now state:[24]

**Proposition 3** *In the general solution:*

1. *If $\gamma < \Lambda^2/\Theta$, then $d\varepsilon/dT < 0$ for all $T, E > 0$; and $dq/dT \underset{>}{\overset{<}{=}} 0$ for $\varepsilon \underset{<}{\overset{>}{=}} \frac{1}{2}$. When $T = \gamma\Theta/\Lambda$, $E = 0$ with $\varepsilon = 0$ and $q = \bar{q}$ given by (B5).*

2. *If $\gamma > \Lambda^2/\Theta$, then $dq/dT < 0$ for all $T, E > 0$; and $d\varepsilon/dT \underset{>}{\overset{<}{=}} 0$ for $q \underset{<}{\overset{>}{=}} \tilde{q}$ with:*

$$\tilde{q} \equiv \frac{\Lambda}{2\Theta} \tag{B6}$$

*When $T = \Lambda$, $E = 0$ with $q = 0$ and $\varepsilon = 1$.*

3. *Whether $\gamma < \Lambda^2/\Theta$ or $\gamma > \Lambda^2/\Theta$:*

$$\gamma\Theta - T^2 > 0 \tag{B7}$$

[24] Due to space constraints, we omit the formal analysis of the knife-edge case $\gamma = \Lambda^2/\Theta$. In this case, $dq/dT < 0$ and $d\varepsilon/dT < 0$ for low $T$ values until $\varepsilon = \frac{1}{2}$ and $q = \tilde{q}$ given by (B6). For higher $T$ values there are two solutions, one with $dq/dT < 0$ and $d\varepsilon/dT > 0$, and one with $dq/dT > 0$ and $d\varepsilon/dT < 0$.

**Proof.** Differentiating $q$ and $\varepsilon$ in (B1) with respect to $T$ yields:

$$\frac{dq}{dT} = \frac{\gamma\left[T(2\Lambda - T) - \gamma\Theta\right]}{\left(\gamma\Theta - T^2\right)^2} = \frac{\gamma\left[1 - 2\varepsilon\right]}{\gamma\Theta - T^2} \tag{B8}$$

$$\frac{d\varepsilon}{dT} = \frac{2\Theta\gamma T - \Lambda(\gamma\Theta + T^2)}{\left(\gamma\Theta - T^2\right)^2} = \frac{2\Theta\left(\tilde{q} - q\right)}{\gamma\Theta - T^2} \tag{B9}$$

The second equality in (B8) follows from (B1). The second equality in (B9) follows from (B1) and (B6).

Emissions drop to zero either because $q = 0$, which from (B1) happens at $T = \Lambda$, or because $\varepsilon = 0$, which from (B1) happens at $T = \gamma\Theta/\Lambda$.

1. If $\gamma < \Lambda^2/\Theta$, $E = 0$ when $T = \gamma\Theta/\Lambda$, so that by (B1), $\varepsilon = 0$ and $q = \bar{q}$ given by (B5). For the numerator of the fraction in the middle of (B9), $\Lambda^2 > \gamma\Theta$ implies:

   $$2\Theta\gamma T - \Lambda(\gamma\Theta + T^2) < -\sqrt{\Theta\gamma}\left(T - \sqrt{\Theta\gamma}\right)^2 < 0$$

   With $\varepsilon$ falling monotonically from 1 to zero, (B8) implies that $dq/dT \overset{<}{\underset{>}{=}} 0$ for $\varepsilon \overset{>}{\underset{<}{=}} \frac{1}{2}$.

2. If $\gamma > \Lambda^2/\Theta$, $E = 0$ when $T = \Lambda$, so that $q = 0$ and $\varepsilon = 1$ by (B1). For the term in square brackets in the middle of (B8), $\Lambda^2 < \gamma\Theta$ implies:

   $$T(2\Lambda - T) - \gamma\Theta < -(T - \Lambda)^2 < 0$$

   With $q$ in (B1) decreasing monotonically from $\bar{q} > \tilde{q}$ (by (B5) and (B6)) to zero, (B9) implies that $d\varepsilon/dT \overset{<}{\underset{>}{=}} 0$ for $q \overset{>}{\underset{<}{=}} \tilde{q}$.

3. Condition (B7) is always met for $\gamma < \Lambda^2/\Theta$ since

   $$\gamma\Theta - T^2 > \gamma\Theta - \left(\frac{\gamma\Theta}{\Lambda}\right)^2 = \gamma\Theta\left(1 - \frac{\gamma\Theta}{\Lambda^2}\right) > 0$$

   Condition (B7) is also met for $\gamma > \Lambda^2/\Theta$ since $\gamma\Theta > \Lambda^2 > T^2$.

$\blacksquare$

# C Appendix C: Comparison with Ulph (1997)

In this appendix we reconcile our findings in Section 3 with Ulph (1997). We find that with integrated technology, output $q$ is decreasing in the emission tax rate $t$ for low values of $t$ (Proposition 1.3) and can be increasing in $t$ for high $t$ (Proposition 1.4). By contrast, Ulph (1997, p. 49) lists integrated technology cost functions where $q$ is constant or increasing monotonically in $t$.

According to Ulph (1997), $q$ is constant when $\varepsilon(F) = \varepsilon_0 e^{-\alpha F}$, or inverting the function and normalizing $\varepsilon_0 = 1$:

$$F(\varepsilon) = -\frac{1}{\alpha}\ln\varepsilon, \qquad F'(\varepsilon) = -\frac{1}{\alpha\varepsilon} \tag{C1}$$

Substituting (C1) into (6) and (7), we find that the two FOCs are satisfied with equality if and only if $q$ is constant at $q^* < \bar{q}_l$ implicitly defined by:

$$P(nq^*) + P'(nq^*)q^* - k'(q^*) - \frac{1}{\alpha q^*} = 0 \tag{C2}$$

so that $\varepsilon$ is given by:

$$\frac{1}{\alpha\varepsilon} - tq^* = 0 \tag{C3}$$

However since $\varepsilon \leq 1$, (C2) and (C3) can only be satisfied for:

$$t \geq t^* \equiv \frac{1}{\alpha q^*} \tag{C4}$$

and $E \leq nq^*$. The regulator can achieve a total emission level $E \in (nq^*, n\bar{q}_l)$ by setting $t < t^*$, to which the firm will respond by not abating, so that $\varepsilon = 1$ and (C3) does not hold (McKitrick, 1999) and setting $q$ according to:

$$P + P'(Q)q - k'(q) - t = 0$$

We do not allow for integrated technology cost function (C1) in our model, because it features $F'(1) = -1/\alpha < 0$ which violates our assumption $F'(1) = 0$.

According to Ulph (1997), $q$ is increasing in $t$ when $\varepsilon(F) = \varepsilon_0(1 - \frac{1}{2}\alpha F)^2$, or inverting the function and normalizing $\varepsilon_0 = 1$:

$$F(\varepsilon) = \frac{2\,(1 - \sqrt{\varepsilon})}{\alpha}$$

This function also has $F'(1) = -1/\alpha < 0$, again violating our assumption $F'(1) = 0$.

Then for $0 < t < t^*$, with $t^*$ again defined by (C4) and (C2), the firm will respond to a higher tax rate by decreasing its output and keeping $\varepsilon$ at 1. For $t > t^*$, the firm will reduce $\varepsilon$ and raise $q$.