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A Pluralist Account of Non-Causal Explanations in Science and Mathematics

Marc Lange, *Because without Cause: Non-causal Explanation in Science and Mathematics*. Oxford: Oxford University Press, 2017, xxii + 489 pages, $ 74.00 HB

Juha Saatsi

J.T.Saatsi@leeds.ac.uk

School of Philosophy, Religion, and History of Science

University of Leeds, Leeds, UK

This is a tremendous book. It brings together and synthesises Marc Lange's highly original work over the past decade on non-causal explanation in science and mathematics. Like much of Lange's oeuvre, it represents naturalistic metaphysics of science that draws inspiration and support from a wealth of detailed, carefully researched examples from the sciences, going back to the early 19th century and beyond. Lange's rich set of examples features many intricate explanations that are bona fide scientific, but do not require any particular technical expertise in e.g. modern physics. The way in which these examples are coupled with open-minded — dare I say adventurous — metaphysics of modality makes for an exciting and thought-provoking read, inviting the reader to follow Lange down the rabbit hole into a world of subjunctive facts (familiar from Lange's (2009) *Laws and Lawmakers*).

By contemporary publishing standards this tome offers two-books-in-one: a book-length exploration of non-causal explanation in the empirical sciences (Parts 1 and 2) is followed by an almost equally substantive study of explanation internal to mathematics (Part 3) — a topic on which much less has been written. Lange finds fascinating connections and unifying threads running through all three parts, completely justifying the single-volume presentation of the wide-ranging material. The emerging theory of explanation is pluralistic, but while there is no single unifying account of non-causal explanation on offer — either with respect to science or maths — Lange brings out various 'family resemblances' between the different kinds of non-causal explanation that make them different 'species of the same genus'. Far from being a mere descriptive exercise in classifying and distinguishing between these different species, however, Lange's main aim is to provide a detailed philosophical theory of the very *explanatoriness* of these explanations — why and how they work qua explanations — that makes sense of how scientists and mathematicians have viewed, and argued about, the examples at stake.

In the rest of this review I will focus very selectively on Part 1 of the book, in which Lange develops his account of 'scientific explanations by constraint'. This account dominates Lange's discussion of non-causal explanation in science. (In Part II Lange identifies two much less prominent species: dimensional and 'really statistical' explanations.) Explanations by constraint form a very wide-ranging kind, covering many of the usual suspects in the flourishing literature on non-causal explanation, ranging from Koenigsberg's bridges to symmetry principles in physics. I will identify some challenges to Lange's account of at least some such explanations. While I also regard most of Lange's examples of 'explanation by constraint' non-causal, I am inclined to associate their explanatoriness to a rather different kind of modal information, involving an explanandum's counterfactual dependence on the explanans.

Explanations by constraint work "by describing how the explanandum arises from certain facts ("constraints") possessing some variety of necessity stronger than ordinary laws of nature possess" (10). Lange presents varied examples of such explanations, including explanations of conservation laws, of the parallelogram law of forces, and of the Lorentz transformations of special relativity, to name a few. One particularly important set of examples falls under the heading of *distinctly mathematical* explanations, which often turn on facts about the world that hold with mathematico-logical necessity. As a variety of necessity that is "stronger than ordinary laws of nature possess," mathematico-logical necessity seems relatively incontestable. (Lange's hierarchical theory of laws, with various degrees of modal strength amongst the contingent nomological facts themselves, is certainly much less orthodox.) It makes sense for Lange to begin the book by applying the notion of explanation by constraint to distinctly mathematical explanations, which offer a paradigmatic and relatively incontestable example of 'stronger than nomological' necessity. But, as I will argue, there are significant challenges to analysing the explanatoriness of distinctly mathematical explanations by reference to mathematical necessity.

Let us focus on the simple example that Lange begins with: why cannot Mother divide 23 strawberries evenly amongst her 3 children (without cutting any)? Because 23 is not divisible by 3 — a mathematical fact. Although not an example of scientific explanation, this is a nice exemplar of a mathematical explanation of an empirical fact: a 'because without cause'. One can feel the pull of the explanation-by-constraint idea by noticing that the causal features involved in any possible attempt to divide the strawberries, as per impossible, are irrelevant to the failure. Any particular set of causal trajectories, and even the causal laws involved, are either irrelevant, or presupposed by the why-question at stake (which takes it as read, for example, that strawberries have persistence as individuals, such that they do not undergo spontaneous fission). Change the causal features of the world however you like, the negative outcome is always determined ("constrained") by the fact that a set of 23 distinct individuals does not have equinumerous non-overlapping proper subsets. This is a necessary truth of a mathematico-logical sort, the necessity of which transcends those of ordinary laws of nature. Lange reasons that this simple example is distinctly mathematical by virtue of thus turning on the mathematico-logical necessity involved. Hence it is also non-causal, since no contingent causal fact is responsible for the negative outcome. Reflections broadly along these lines provide Lange the initial reason and impetus to regard some non-causal explanations as explanations by constraint. Having fleshed out this idea, Lange puts it to work in providing a unified account of various non-causal explanations that involve 'modally exalted' facts (relative to more contingent nomological and causal features). This philosophical work done by the explanation-by-constraint notion then provides further evidence for it, as well as indications how to refine it further.

I grant that there is mathematico-logical necessity involved in e.g. the strawberry example, and also independence of the explanandum from any actual or possible causal laws. So, plausibly we are indeed dealing with, in some sense, 'distinctly mathematical' explanation, which furthermore seem genuinely non-causal — in as far as it seems plausible that any causal explanation should turn on some contingent causal regularity. But granting all that, I do not yet see why we should think that what is *doing the explaining*, and *providing us the explanatory understanding*, crucially involves information about the necessity involved. On the contrary, I see at least a couple of distinct challenges to the idea that explanatory understanding here hangs on seeing how the explanandum arises from facts more necessary than ordinary laws.

One challenge, as I will explain below, is that information about the strong degree of necessity involved risks being *too cheap*: the exalted modal aspect of the explanandum can be communicated without doing much explaining, and it can be grasped without having much understanding. Another challenge relates to the way in which the idea of explanation by constraint envisions a substantial 'joint' in the nature of scientific explanations: non-causal explanations-by-constraint work by providing modal information about strong degree of necessity, which is a rather different kind of modal information from that provided by causal explanations regarding e.g. contingent difference-makers or dependence. In advocating such pluralism regarding how explanations work, Lange faces the challenge of pinning down the difference due to which explanations work so differently.

Let me elaborate, starting with the second challenge. One might think that in the case of distinctly mathematical explanations, at least, we can justify their sui generis character by reference to the aforementioned independence of the explanandum from *any* contingent nomological regularity. Given this *complete* independence, what else could possibly be doing the explaining, apart from the sheer necessity of the negative outcome, given the presumed mathematico-logical fact? Well, in the case of strawberries, for example, there is also all the information from basic arithmetic regarding *how things would be different* if Mother had a different number of strawberries or kids to play with. Ideologically this kind of information is nicely continuous with the prominent idea — familiar from causal accounts of explanation — that explanatory understanding is a matter of possessing *dependence information* regarding how the explanandum would be different if the explanans were different. Thus, a natural alternative to Lange's take on the strawberry example is to emphasise the continuity between this case and garden-variety causal explanations. From what we can call the *counterfactual-dependence perspective*, explanations, causal and non-causal alike, can explain by virtue of providing what-if-things-had-been-different information that captures a dependence relation between the explanandum and the explanans. We can even regard the specific number of strawberries as a causal feature of the set-up, as Lange does, and nevertheless deem the explanation at hand non-causal on the grounds that the explanatory connection between the number of strawberries and the failure of Mother's attempts is mediated by an explanatory connection that is more intimate and necessary than any causal law. Similar remarks apply to, e.g., Koenigsberg's bridges, for which graph-theory conveniently provides us analogous modal information (see Jansson and Saatsi (forthcoming); Woodward (forthcoming).)

Nothing in Lange's discussion indicates how this alternative perspective on distinctly mathematical explanations falls short of locating the real source of explanatoriness in a rather different kind of modal information also involved in the strawberry example. (To be fair, Lange notes in relation to some other specific explanations of constraint that the kind of modal information required by the counterfactual-dependence perspective does not seem to be available.) Furthermore, by comparing the explanatory import of the two kinds of modal information we can raise the first challenge. If we squeeze out, as it were, all the modal information regarding how Mother's predicament would differ as a function of the number of strawberries/kids, it looks that we are left with a very shallow explanation at best, *even if we fully retain* the information concerning the exalted modal status of the explanandum.

For instance, we could deductively prove, with logic alone, that Mother is bound to fail given her *specific* number of strawberries. This is because a mathematical explanation for any specific finite number of strawberries is nominalisable. That proof would presumably provide information of the exact sort that Lange identifies as doing the explanatory heavy lifting, but presumably an agent who received *only* that information — without any background arithmetical knowledge, perhaps — does not understand Mother's plight very well. Adding basic arithmetic into the picture does not increase or change the quality of the information about the strong degree of necessity *per se*, but it does add a lot in terms of how the failure *depends on* the specific numbers involved. To my mind considerations along these lines support the counterfactual-dependence perspective with respect to various distinctly mathematical explanations (see Jansson and Saatsi (forthcoming); Saatsi (2016)). At the very least, they suggest that we should not hang the analysis of explanatoriness entirely on the hook of modal 'constraint'.

An advocate of the counterfactual-dependence perspective need not deny that some explanations are worth identifying as 'distinctly mathematical'. From this perspective it is natural to hypothesise that a necessary condition for a distinctly mathematical explanation is that the explanatory connection between the explanandum and the explanans — the connection that underwrites the explanatory what-if-things-had-been-different information — holds with logico-mathematical necessity. Lange considers and rejects this idea. He argues that some explanations can turn on contingent law-like features of the world, while still being distinctly mathematical due to suitably involving both mathematics and an exalted degree of necessity. Lange demonstrates this with the following example. Why does a double pendulum have at least four equilibrium configurations? An answer of considerable explanatory generality employs the mathematical fact that the configuration space for *any* double pendulum has the same doughnut-like torus topology, given how it is parameterised by two angles α and β both ranging 360 degrees (see https://en.m.wikipedia.org/wiki/Double_pendulum). Lange argues that this mathematical fact guarantees that there will be at least four configurations for which the forces acting on the pendulum vanish.

Now, in order for a system to count as a pendulum there needs to be a force that acts upon the system, causing it to move unless it is in an equilibrium position. This can be specified by a potential energy function $U(\alpha, \beta)$. There also needs to be a law-like connection between the gradient of potential energy and acceleration: a special case of Newton's second law. Despite the fact that the double pendulum explanation absolutely requires appeal to such causal features concerning forces and accelerations, Lange regards it as distinctly mathematical, since "no aspect of the *particular* forces operating on or within the system (which would make a difference to [the system's potential energy] U(α, β) matters to this explanation" (27, my emphasis).

I do not think Lange has demonstrated an appropriate independence of the explanandum from the relevant causal features to show that this is an explanation by constraint, or even a distinctly mathematical explanation. While it is undoubtedly true that the explanation abstracts away from various *particular* aspects of the forces, or, equivalently, the potential energy function, involved, it is nevertheless the case that the torus topology of the configuration space only entails the minimum number of equilibrium configurations *in conjunction with* features of a potential energy function. Properly understanding how the configuration space topology is related to the number of equilibrium configurations still involves grasping how the former would be different if the potential energy function was different, as considered below. This again seems to involve what-if-things-had-been-different information — and in this case with respect to contingent nomological features of the system, perhaps even rendering it a causal explanation, albeit a fairly abstract one.

Consider, for instance, changing the potential energy function so that it does not pull uniformly down, as in the case of a standard gravitational pendulum that Lange probably has in mind, but instead pulls symmetrically up above the centre of the pendulum, and down below it, so that there is a plane running through the centre where the potential energy vanishes. With such forces acting upon the pendulum it will have at least 8 equilibrium configurations. Or consider a three-fold symmetrical situation, with three competing forces (e.g. identical magnets that are equidistant from the pendulum's centre, and 120 degrees apart). Such a system has at least 12 equilibrium configurations. Explaining why a double pendulum has a given number of equilibrium configurations thus indispensably involves not only the torus topology of its configuration space — a mathematical fact pertaining to all double pendulums — but also how features of this space in conjunction with contingent facts about forces entail different facts about locally vanishing potential energy gradients, viz. equilibrium configurations.

I have dwelled on this particular case because it brings out the second challenge I mentioned earlier. Why exactly should we think that the double pendulum, for instance, is explained via modally exalted constraints, while some rather similar looking explanations explain differently, by providing abstract causal information? Lange contrasts the double pendulum case with the abstract causal equilibrium explanation of why a ball is bound to end up at the bottom of a concave bowl (30), but I am unable to clearly see the difference. Given that the distinction between distinctly mathematical and abstract causal explanations can feel so thin and elusive, why think there is a clear difference in the way these explanations work? This question is pressing for Lange since many (most? all?) distinctly mathematical explanations that we are inclined to regard as genuinely explanatory seem to offer *also* the kind of modal information that the counterfactual, dependence-based perspective of explanation capitalises on.

Lange might respond by saying that an explanation's status as distinctly mathematical is a *contextual* matter (37), but this may only exacerbate the challenge at stake. By Lange's lights, the double pendulum explanation may count as causal in a context where we emphasise the dependence of the minimum number of equilibrium configurations on features of the potential energy function, and distinctly mathematical in a context in which we keep the forces more fixed and place emphasis on the topology of the configuration space. A specification of the potential energy function could of course be packed into the why-question: e.g., why does a double pendulum *in homogenous gravitational field* have at least four equilibrium configurations? If we do that, I can see how the connection between the explanandum and the presuppositions of the why question holds with logico-mathematical necessity. But now it is no longer clear to me how explanatory as opposed to demonstrative the relevant deduction is.

Lange's discussion implies, indeed, that any explanation involving applied mathematics can be turned into a distinctly mathematical explanation by incorporating all of the non-mathematical facts at stake into the why question (39). For instance, why is it that given that mass is additive, if A has the mass of 1kg, and B has the mass of 1kg, then the union A+B has the mass of 2kg? Because 1+1=2. This answer, and presumably a huge array of explanations akin to it, which can also involve complex mathematics, seem utterly shallow as explanations of empirical phenomena, perhaps even entirely non-explanatory. But what is it exactly that makes them so deficient as explanations? What do they lack? After all, they give the exact right kind of information about stronger-than-nomological necessity that in some other cases is identified as doing all (?) the explanatory work? One challenge for Lange is to provide an account of *explanatory power* that makes sense of these respective differences of explanatory goodness within the explanation-by-constraint framework. I am more optimistic for accounting for these differences from the perspective of the counterfactual-dependence account that I have touted as an alternative. It is an important desideratum for any philosophical account of explanatoriness to make sense of our largely shared judgements of explanations' relative virtues: one explanation being better, more powerful, or deeper, than another. After all, presumably these judgments quite reliably track differences in how well explanations work qua explanations.

There is a huge deal more to be said about the potential virtues and vices of Lange's explanation-by-constraint account, and I have only managed to discuss some very limited aspects of it. Significantly, Lange's account also aims to provide an ideologically unified theory behind many different kinds of explanations of this ilk, distinctly mathematical explanations being just one end of the spectrum. Very broadly applicable philosophical theories like Lange's cannot be fairly evaluated independently of the much bigger picture to which I have not been able to do any justice here. In particular, while I have suggested that there is an alternative perspective available to at least some distinctively mathematical explanations, and I have elsewhere argued that various other non-causal explanations can also be captured in these terms (e.g. Saatsi (2016); French and Saatsi (forthcoming)), it is admittedly very much an open question how far the counterfactual-dependence perspective can be pushed to accommodate the numerous non-causal explanations that Lange has brought to the table. So while my preferred way of thinking about distinctly mathematical explanations provides a more unified account in relation to causal explanations, it may lose out in conceptual unity elsewhere if it fails to capture many of the other explanations that motivate Lange's theory of explanations by constraint.

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