Cosmic string network evolution in arbitrary Friedmann-Lemaître models

Carsten van de Bruck
Institut für Astrophysik und Extraterrestrische Forschung, Auf dem Hügel 71, 53121 Bonn, Germany

(Received 4 June 1997; published 18 December 1997)

We use the modified “one-scale” model by Martins and Shellard to investigate the evolution of a GUT long cosmic string network in general Friedmann-Lemaître models (with and without a cosmological constant). Four representative cosmological models are used to show that in general there is no scaling solution.

PACS number(s): 98.80.Cq, 11.27.+d

I. INTRODUCTION

Cosmic strings might be responsible for structure formation in the universe. To understand how cosmic strings could influence the cosmic medium we need to know how the string network evolves. In recent years the theory was applied mainly to the Einstein–de Sitter model and an open model. The cosmic strings become frozen in the cosmic expansion. Therefore cosmic strings become dominant over matter but could not influence the expansion of the universe (because \( \rho_{\text{vac}} = \Lambda / 8 \pi G \) is constant). The values of \( \log(R_0/R_{eq}) \) for the different models are summarized in Table I.

In Fig. 3 we plot the number of loops produced per Hubble volume and Hubble time. Because we assume \( \bar{c}_{\text{vac}} = \bar{c}_{\text{curv}} = \bar{c}_m \), we overestimate this number in the vacuum epoch. In this epoch the strings become frozen in the

II. NETWORK EVOLUTION

In the modified “one-scale” model the energy of a string (or the typical length scale \( L \), defined by \( \rho_s = \mu/L^2 \)) and the rms velocity of the string \( v_{\text{rms}} \) are treated as independent quantities, which describe the statistical properties of the network. The equations of motion of the string network can be found in [9]. The reader is referred to that paper or to [10] for more information and discussions on the modified “one-scale” model. The loop-chopping efficiency \( \bar{c} \), a phenomenological parameter which is included in order to describe the loop production, is chosen as in [9] for the different epochs. We know the values of \( \bar{c} \) in the radiation- (\( \bar{c}_r = 0.24 \)) and matter-dominated (\( \bar{c}_m = 0.17 \)) epochs [10], but not in the curvature- and vacuum-dominated epochs. However, the scaling properties of the string network do not crucially depend on the parameter \( \bar{c} \) [10,9]. Therefore, we assume a smooth transition between \( \bar{c}_r \) and \( \bar{c}_m \) between the radiation- and matter-dominated epochs and \( \bar{c} \) remains constant (\( \equiv \bar{c}_m \)) in the subsequent epochs. For the calculations we assume \( G \mu = 10^{-6} \), corresponding to the Grand Unified Theory (GUT) energy scale.

Our results are presented in Figs. 2–4. They are consistent with the results from Martins and Shellard [10] and Martins [9] for the Einstein–de Sitter model and for the open model. In the vacuum dominated epoch, the scale factor grows as \( R \propto \exp(Ht) \) \( (H = \dot{R}/R = \text{const}) \). One finds

\[
L \propto \exp(Ht) \propto R. \tag{2.1}
\]

The cosmic strings become frozen in the cosmic expansion. Therefore cosmic strings become dominant over matter but could not influence the expansion of the universe (because \( \rho_{\text{vac}} = \Lambda / 8 \pi G \) is constant). The values of \( \log(R_0/R_{eq}) \) for the different models are summarized in Table I.

In Fig. 3 we plot the number of loops produced per Hubble volume and Hubble time. Because we assume \( \bar{c}_{\text{vac}} = \bar{c}_{\text{curv}} = \bar{c}_m \), we overestimate this number in the vacuum epoch. In this epoch the strings become frozen in the

<table>
<thead>
<tr>
<th>Model</th>
<th>( K )</th>
<th>( \Omega_0 )</th>
<th>( \lambda_0 )</th>
<th>( H_0 )</th>
<th>( \log(R_0/R_{eq}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1</td>
<td>0.014</td>
<td>1.08</td>
<td>90</td>
<td>2.66</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.0</td>
<td>0.0</td>
<td>60</td>
<td>4.16</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>0.1</td>
<td>0.0</td>
<td>60</td>
<td>3.16</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.1</td>
<td>0.9</td>
<td>60</td>
<td>3.16</td>
</tr>
</tbody>
</table>

TABLE I. The four representative cosmological models. \( K \) is the curvature parameter, \( \Omega_0 \) is the matter density parameter, \( \lambda_0 \) is the cosmological term, \( H_0 \) is the Hubble parameter (in km s\(^{-1}\) Mpc\(^{-1}\)). \( R_0 \) and \( R_{eq} \) are the scale factors at the present time and at matter-radiation equality, respectively.
cosmic expansion and therefore the probability of loop production decreases. As a result, the parameter $\tilde{c}$ should also decrease. In the case of the loitering epoch in the closed model, the number is underestimated. The velocity of the strings increases (see Fig. 3) and so the probability of loop production. Therefore, the parameter $\tilde{c}$ should increase in the loitering epoch. In contrast with open models or to the Einstein–de Sitter case, the energy loss (due to loop production) of the string network at the present epoch is not as efficient as in models with a cosmological constant.

One would expect that the string network approaches scaling in the radiation-dominated epoch, after a transition the network approaches scaling in the matter-dominated epoch and than the network becomes frozen in the cosmic expansion in the vacuum-dominated epoch. However, from Fig. 2 one can see that the scaling in the matter-dominated epoch is not reached by the network. The reason for this is that in the Einstein–de Sitter model the transition between the scaling behavior in the radiation- and the matter-dominated epoch is much longer than previously estimated [10]. In the other models the matter-dominated epoch is too short for the network to reach scaling. The same holds for the open model, where the universe becomes curvature dominated [9].

In Fig. 4 we plot the transition from the matter to the vacuum epoch for the models with a cosmological constant. In the case of the closed model there is a loitering epoch between the matter-dominated and the vacuum-dominated epochs. The transition between these two regimes is a very slow process.

We point out that the results do not depend strongly on the values of $\chi$ in the ansatz for $k$, which is a parameter related to the small scale structure of the strings [10].

The main difference between the network evolution in open models and in models with a cosmological constant is that the energy loss of the network at the present epoch is larger in open models. This has impact on the gravitational wave background and/or high energy particle fluxes from cosmic strings. It will also affect the structure formation theory with cosmic strings.
ACKNOWLEDGMENTS

I thank C.J.A.P. Martins for helpful discussions on the velocity-dependent ‘‘one-scale’’ model. The careful reading of the manuscript by W. Priester and M. Soika are gratefully acknowledged. This work was supported by the Deutsche Forschungsgemeinschaft (DFG).