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A Performance Evaluation Method to Compare the Multi-View Point Cloud Data Registration Based on ICP Algorithm and Reference Marker

Feng Li, Carl Hitchens, David Stoddart

Nuclear Advanced Manufacturing Research Centre (Nuclear AMRC), The University of Sheffield,
Advanced Manufacturing Park,

Brunel Way, Rotherham, S60 5WG, United Kingdom

Corresponding author: feng.li@sheffield.ac.uk

Abstract

Registration of range images of surfaces is a fundamental problem in three dimensional modelling. This process is performed by finding a rotation matrix and translation vector between two sets of data points requiring registration. Many techniques have been developed to solve the registration problem. Therefore, it is important to understand the accuracy of various registration techniques when we decide which technique will be selected to perform registration task. This paper presents a new approach to test and compare registration techniques in terms of accuracy. Among various registration methods, iterative closest point based algorithms and reference marker methods are two types of commonly applied methods which are used to accomplish this task because they are easy to implement and relatively low cost. These two methods have been selected to perform a comprehensively quantitative evaluation by using the proposed method and the registration results are verified using the calibrated NPL freeform standard.

Key words: optical metrology; registration; point clouds; iterative closest point; reference markers

1. Introduction

The applications of three dimensional (3D) shape measurements are increasingly required in the fields of quality control, reverse engineering, medical field and computer vision. There are many sorts of non-contact optical have been developed to address this demand, such as time-of-flight Schmidt and Jähne (1), computed tomography Lifton et al. (2), laser scanning Wang and Feng (3), photogrammetry Dong et al. (4) and pattern projection Zhang et al. (5). Each technique has its own characteristics and application. But in all the cases, a complete 3D model is constructed by acquiring its surface from multiple viewpoints due to occlusions and the limited field of view of the optical sensor. These multi-view scans are represented in their own local coordinate system, and then aligned into a common coordinate system. This issue is referred to the registration problem.

1.1. Mathematical model of 3D registration

When scanning the object surface, each scan has its own coordinate system therefore the coordinate position of the same point on the surface is different in a multiple measuring process. This change is equivalent to the 3D coordinate system transform. Therefore the original data registration problem can be converted to the coordinate transform problem.

Suppose one set of point cloud data has its 3D Cartesian coordinate $o-xyz$ and another set of point cloud data which is needed to be registered has its coordinate $o_D-x_Dy_Dz_D$, as the data registration

problem is only involving rotation and translation, the relationship between two coordinates can be described by the equation (1):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_D = R \begin{bmatrix} x \\ y \\ z \end{bmatrix}_O + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} \quad (1)$$

where $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_O^T$ is the point in coordinate $o - xyz$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_D^T$ is the point in coordinate $o_D - x_D y_D z_D$; $\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_O^T$ is translation vector T and R is rotation matrix. Then the point cloud data registration problem can be converted to finding optimal solutions of rotation matrix R and translation vector T .

1.2. Motivation

Many devices have been exploited to overcome the registration problem, which are basically based on calibrated mechanics to compute the geometry between the views such as rotating tables Li et al. (6) and robot arms Larsson and Kjellander (7), or auxiliary devices e.g. laser trackers Wan et al. (8). This type of methods is usually expensive and it is limited due to the fact that the object to be measured must be located inside the device or close to the working area. Another solution is that applying reference markers (RM) on the surfaces of object and registration of scans by computing the centres of these markers Ahn et al. (9). In addition, the registration problem can also be solved by exploiting clues involved in the range images or point clouds themselves. This type of techniques usually follows two basic steps: first a coarse registration and then fine registration. The main goal of coarse registration is to find an initial estimation of the rigid motion between two sets of point clouds using correspondences between both surfaces. The fine registration algorithm utilises an iterative optimisation process to converge to obtain a more accurate solution, for example the Iterative Closest Point (ICP) algorithms were first proposed by Besl and McKay (10) and Chen and Medioni (11).

Although many techniques have been developed to register 3D surfaces by determining the motion between the different views, there exists only limited literature summarising and comparing the different techniques. Rusinkiewicz and Levoy (12) classified and compared several ICP variants and especially discussed the effect each has on convergence speed. However, the paper is incomplete because only point cloud pairs have been considered and multi-view registration is neglected. Furthermore, only synthetically-generated data was used to compare the different techniques without considering some of the problems involved using real point data. Dalley and Flynn (13) quantitatively evaluated the output of several ICP variants on real-world data. Their work focused on registration of partially overlapping range image pairs. Salvi et al. (14) presented a more complete survey and extended previous works; it analysed the different techniques in both pair-wise and multi-view registration and evaluated these algorithms based on real point cloud data.

In general, all above literatures focus on comparing similar technologies, for example ICP algorithm and its variants, by calculating rotation error, translation error and root mean square (RMS), based on calculation of distance between point-correspondences. The point-to-point RMS is calculated as a by-product when the ICP algorithm is searching the temporal point correspondences. Therefore, this evaluation method is not suitable for the evaluation the different type of techniques. In fact, this kind of comparison is usually difficult to implement and even not available as each type of techniques has its own characteristics and working principle. To the best of the authors' knowledge, there is no relevant work that quantitatively evaluates different type of techniques. Therefore, we present a new approach to test and compare the registration techniques, using two types of common used registration methods - ICP algorithms and RM method, based on evaluation of normal vectors of surfaces (which

we refer to as normal-vector-based RMS); then compare their performances using the same fringe projection scanner - GOM ATOS III Triple Scan.

The remainder of this paper is organised as follows: Section 2 describes the pros and cons and theory knowledge of ICP algorithms and RM methods and their corresponding accuracy evaluation methods; Section 3 provides an introduction on test setup and proposed method; Section 4 introduces the evaluation methods used in this paper. The experiment-based evaluation is discussed and analysed in Section 5. Finally, the conclusions of this piece of work are presented in Section 6.

2. ICP algorithm versus RM method

ICP algorithms and RM methods are popular approaches to register range images; they are very economical and easy to implement in comparison to other registration methods. In general, the main drawback for those two methods is that they cannot cope with non-overlapping regions when a lack of correspondences exists in the data sets. Their features and theory are elaborated and described below.

2.1. Pros and cons of ICP algorithms & RM methods

The biggest advantage for ICP algorithms is that it does not need any preliminary work and auxiliary facilities and only exploits the common features between both point data sets to complete the task. However its disadvantages are also very prominent: ICP method greatly depends on a proper initial guess or a rough registration to converge and obtain the global optimal solution. If the initial estimation is not accurate enough then the convergence is not guaranteed or just convergence to local minima - resulting in converging to an incorrect solution. Another drawback of this technique is the large number of overlapping sampled points usually required to assure sufficient accuracy. In addition, it also does not work for plane, cylinder and objects with repeated features.

The RM method is usually fast and reliable if in the individual measurements at least three reference points from preceding measurements are captured by the cameras. However, except for the time-consuming preparation work before the measurement, the drawbacks of this strategy are that the areas covered by the markers cannot be digitised reliably and usually holes will be left on the surface after registration. This problem is especially outstanding for workpiece with complex and rough surface. Moreover, adhering markers on the surface to be measured is even prohibited in some cases.

2.2. Theory aspects of ICP algorithm & RM method

2.2.1. Theory of ICP method

The goal of registration using ICP methods is to obtain the most accurate solution as possible. When an initial estimation is known or estimated, all the points are transformed to a reference system by finding and minimising the correspondences between clouds of points, which can be based on points, curves, surfaces and directional vectors. The process is repeated and iterated until convergence when distances between corresponding points decrease below a threshold or achieving the pre-set number of iterations.

The approach based on ICP and introduced by Li et al. (15) (Li's method) is exploited to register point clouds without any known information. $k-d$ trees Simon (16) is used to speed up the algorithm in order to improve the efficiency of searching speed of neighbour points. Then the singular value decomposition (SVD) method Arun et al. (17) is exploited to find the least-squares solution of rotation matrix R and translation vector T .

2.2.2. Theory of RM method

The core techniques for RM method is reference object detection and geometric transformation method Mortenson (18). The 2D circular markers are the most popular reference objects as they are low costs and easy to apply. A typical flowchart of this type of approaches can be described in the following steps, as shown in Fig. 1.

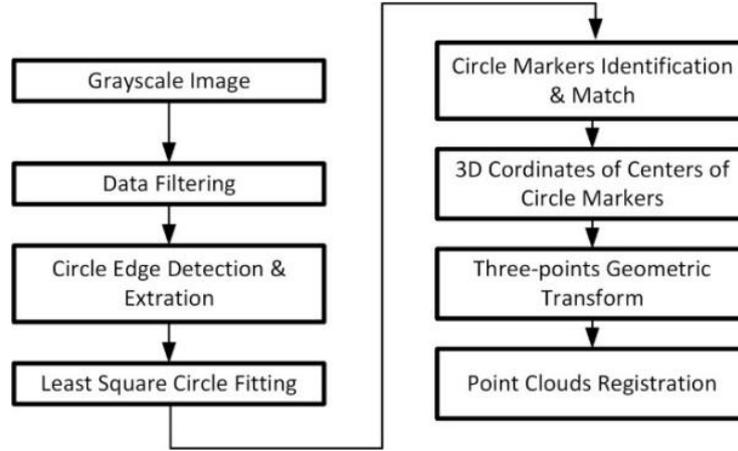


Fig. 1. The architecture of the RM method

First of all, grayscale images are filtered to remove the noise and the Edge Detection method (for example the Canny approach Canny (19)) can be used to generate the single-pixel-width edges, which is essential to accurately detect and locate circle markers. Then least square algorithm can be used to calculate the centre information for each edge pixel cluster (usually an ellipse in the 2D image due to markers' lack of parallelism to the Camera charge-coupled device (CCD)). All these potential circle markers need to be further validated and identified and conjugated reference pairs in different images also need to be correctly matched. Finally, the centres of reference points can be used to calculate the rotation and translation matrix of the 3D datasets to be registered. Two pieces of point clouds can be oriented to a common coordinate system when at least three conjugated datum points, which are also called common reference points, can be identified in both data sets.

In this paper, the commercial software ATOS Professional which is provided by GOM can automatically recognise and register all scans after applying the reference point markers (ATOS method).

2.3. Accuracy evaluation of ICP algorithm & RM method

Most of the ICP algorithms register the point clouds by finding and minimising the correspondences between points. The correspondences can be point-pairs and they are needed to be identified in the registration process, then the RMS (root mean squares) can be used for the evaluation of registration result and it can be calculated via following equations Besl and McKay (10):

$$RMS = \sqrt{\frac{\sum_{i=1}^n [d(p_i, p_i')]^2}{n}} \quad (2)$$

where p_i is one point in the first point cloud and p_i' is its corresponding point in the second point cloud which needs to be registered. $d(p_i, p_i')$ is the distance between p_i and p_i' , n is the number of those conjugate point pairs.

While for the reference marker methods, the core technique is that the detection and identification the centres of reference markers. The centres are used as datum points and best-fitted, for example exploiting least-square methods. Then the residuals of the registration can be calculated in the x , y and z direction after alignment of the datum points Li et al. (20).

In general, above two registration methods have different working principles and evaluation methods of the registration accuracy. To the authors' best knowledge, there are no existing techniques/relevant publications to compare these two methods. Therefore, we proposed a new method in the paper to compare these two methods and the results are verified by the calibrated freeform standard – NPL-WP-150.

3. Test setup

3.1. Elements of the test

In this paper, the commercially available optical scanner - GOM ATOS III Triple Scan (see Fig. 2 (a)) is used for image capture. The main system configurations for GOM ATOS III Triple Scan are shown in Table 1 (21). The software platform ATOS Professional V7.5 SR2 software is used for data acquisition and pre-processing.

Table 1. The configurations of GOM ATOS III Triple Scan

Camera Pixels	8 Megapixel (each) \times 2
Measuring Volume	$38 \times 29 \times 15 - 320 \times 240 \times 240$ mm ³
Point Spacing	0.01 - 0.61 mm
Operating Temperature	5 - 40 °C

The artefact used for test is FreeForm reference standard WP-150 (Fig. 2 (d)) which is developed by National Physical Laboratory (NPL). This artefact is manufactured with high accuracy and has been calibrated by NPL using a high precision CMM with maximum permissible measurement error of (MPE = $1.3 + L/400$) μ m (L in mm, ISO10360-2:2009 (22)), using a suitable diameter ball ended stylus with a 0.05 N measuring force and probing dynamic of 50%. The characteristics of this standard can be found in Table 2 and more information please refer (23).

Table 2. NPL-WP-150 characteristics

Design	National FreeForm Centre, NPL
Material	6082-T6 – Aluminium Dural
Coefficient of Expansion	$22.5 \mu\text{m m}^{-1} \text{K}^{-1}$
Mass	< 5 kg

This artefact is manufactured with high accuracy and bears several geometrical forms that are blended to form a single surface, therefore is an ideal object to evaluate the instrument performance and accuracy after registration.

3.2. Test setup

Both equipment and artefact are soaked in a temperature-controlled metrology room for at least 24 hours, with the environmental temperature controlled at 20 ± 0.5 °C. All sensors have also been running more than 15 minutes to warm up before execute calibration and scanning. For GOM Triple

Scan, two laser pointers are used to adjust the optimal distance between cameras and objects (see Fig. 2 (b)).

Firstly, the artefact WP-150 is horizontally placed on the granite measuring table and the scanner is aligned perpendicular to the table surface. The artefact is moved accordingly in the order 1→2→3→4 (see Fig. 2 (c)). It is important to note that the scanner is always fixed when the cameras are capturing images. All movements have been strictly controlled and the sequence has been repeated five times. More than 50% overlapping area will be applied between two scans to ensure enough correspondences points are available to obtain better registration accuracy. The movement information on the granite table can be exploited as an initial estimate and then the point clouds can be registered using ICP algorithm.

Secondly, we apply the reference markers (supplied by GOM) with 3 mm diameter on four gauge blocks positioned up against the sides of the artefact, as shown in Fig. 2 (d). The movements of artefact follow the same routes as above. The gauge block is manufactured with high accuracy and has very good flatness. With this method more accurate circle centres can be achieved and a better registration results are obtained. The ATOS Professional software will recognise the reference markers in 2D images and automatically register the multi-view point clouds.

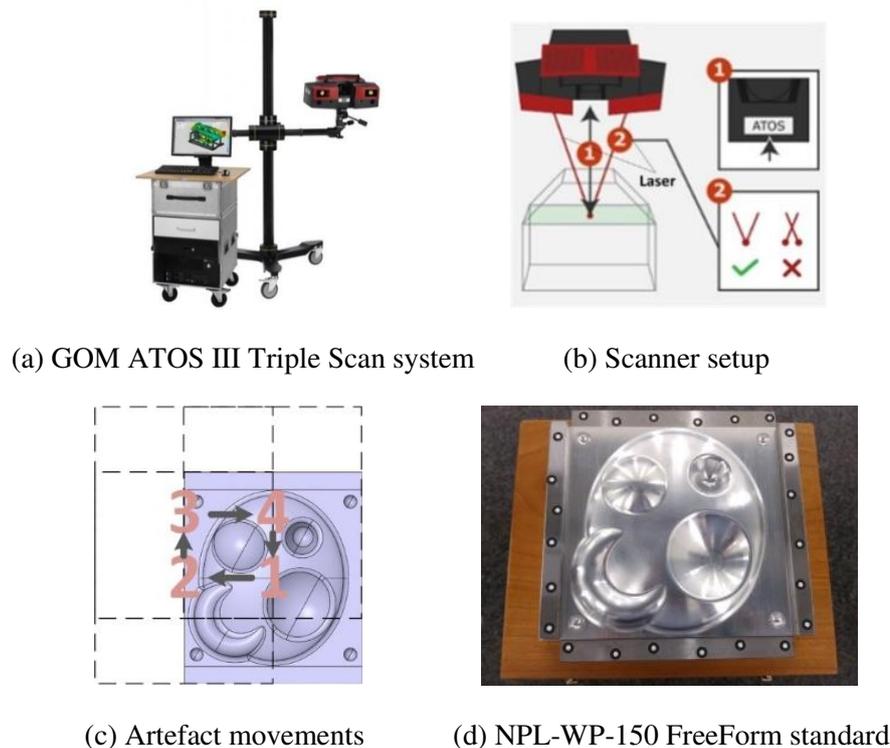


Fig. 2. Measuring equipment & setup

4. A new evaluation method

In practice, the actual CAD model of artefact may be difficult or even impossible to obtain. As the NPL-WP-150 reference standard has been calibrated and then its CAD model (Fig. 3 (a)), which is also provide by NPL, can be exploited as the reference surface and used to compare with the registered point cloud data. First, the artefact has been moved four times and four scans are obtained, as described in Section 3.2. The ATOS Professional is exploited to remove irrelevant points in the data sets. Then four point data sets are registered into one common coordinate system using ICP and

RM methods, separately. The operation and consecutive measurements procedure have been repeated five times and five data samples relative to each method can be obtained. Finally, the registration accuracy and repeatability are evaluated using proposed methods.

4.1. Registration accuracy evaluation

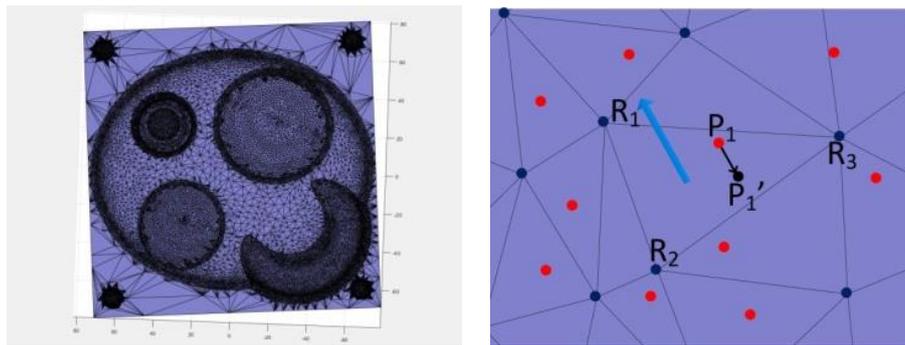
4.1.1. Projection theory

We suppose $P_1(x_o, y_o, z_o)$ is one of the measured points (red points in Fig. 3 (b)) and P_1' is its projection on reference surface and $P_1' \in r$, where $r = \vec{a}x + \vec{b}y + \vec{c}z + \vec{d}$ is a plane which is on the reference surface; R_1, R_2 and R_3 are three points on the plane r and $\vec{n}: (\vec{a}, \vec{b}, \vec{c})$ is the normal vector of plane r (blue arrow in Fig. 3 (b)). The facet normal \vec{n} can be obtained from CAD model or calculated from the vertices of triangle:

$$\vec{n} = \frac{(\vec{R}_2 - \vec{R}_1) \times (\vec{R}_3 - \vec{R}_1)}{|(\vec{R}_2 - \vec{R}_1) \times (\vec{R}_3 - \vec{R}_1)|} \quad (3)$$

Then the distance between P_1 and plane r can be calculated by

$$\|\vec{P}_1, \vec{P}_1'\| = \frac{|\vec{a}x_o + \vec{b}y_o + \vec{c}z_o + \vec{d}|}{\sqrt{\vec{a}^2 + \vec{b}^2 + \vec{c}^2}} \quad (4)$$



(a) CAD model of NPL artefact (b) Normal-vector-based method

Fig. 3. CAD model & registration error evaluation method

4.1.2. Algorithm description

The specific algorithm to calculate the normal-vector-based RMS is described as follows:

(1) Find the projections of the measured points (after registration) on the reference surface;

Build a line along the normal direction that passes through measured point;

Compute the intersection point of the line and the reference surface; then the intersection is the projection of measured point on the reference surface;

In rare cases, the measured point may have more than one intersection on the reference surface. If multiple intersections have been found then choose the closest one to the measured point;

If no intersection point has been spotted then return a null value.

(2) Calculate the Normal-vector-based RMS (NRMS) between measured points and its corresponding projections on the reference surface using equation (5):

$$NRMS = \sqrt{\frac{\sum_{i=1}^n [d(\bar{P}_i, \bar{P}_i')]^2}{n}} \quad (5)$$

where \bar{P}_i is the scanned point after registration, \bar{P}_i' is its projected point on the reference surface (CAD model in this case). $d(\bar{P}_i, \bar{P}_i')$ is the distance between \bar{P}_i and \bar{P}_i' , and n is the number of measured points.

4.2. Repeatability evaluation

According to QS9000 (24), computation of the repeatability is one of the most effective methods to statistically analyse process or equipment. The repeatability of different registration methods can be estimated by:

$$Repeatability = \sqrt{\frac{\sum_{i=1}^m [NRMS_i - \overline{NRMS}]^2}{m}} \quad (6)$$

$$\overline{NRMS} = \frac{\sum_{i=1}^m NRMS_i}{m} \quad (7)$$

where m is the number of the measurements.

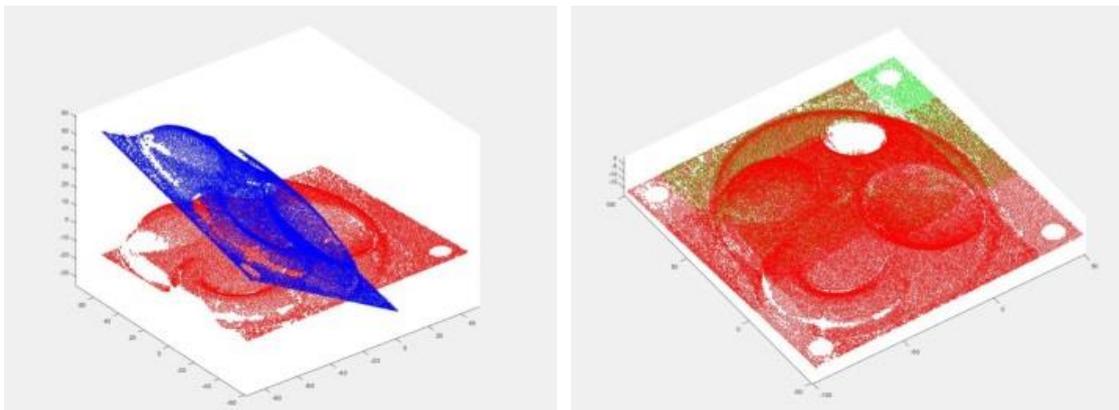
5. Experimental results analysis

In this paper, Matlab R2015a is implemented to program and perform the evaluation of the registration results. The registered point clouds have been aligned to the reference CAD model by minimising Euclidean transformation based on least squares criterion. After alignment, the registration results using ICP algorithm (Li's ICP method) and RM method (ATOS RM method) can be compared by proposed approach.

5.1. Experimental results

5.1.1. ICP algorithm - Li's method

The point clouds before registration is shown below in Fig. 4 (a). The point clouds after registration is shown in Fig. 4 (b).



(a) Point data before registration

(b) Point data after registration

Fig. 4. Point clouds before & after registration

The NRMS of registration using ICP algorithm and the number of points for each scan sequence is shown in Table 3.

Table 3. NRMS using ICP algorithm - Li's method

Scan sequences	1	2	3	4	5
NRMS (mm)	0.0253	0.0253	0.0252	0.0249	0.0259
Number of points	53,514	53,427	53,536	53,241	53,387

5.1.2. RM method - ATOS method

After applying the reference markers, the ATOS Professional can recognise these markers in 2D images and align 3D points automatically. The registration results can be found in Table 4.

Table 4. NRMS using RM algorithm - ATOS method

Scan sequences	1	2	3	4	5
NRMS (mm)	0.0275	0.0285	0.0279	0.0267	0.0277
Number of points	46,123	46,216	45,763	45,982	46,673

5.2. Experimental analysis

The registration results using different methods for all five scan sequences are shown in Fig. 5.

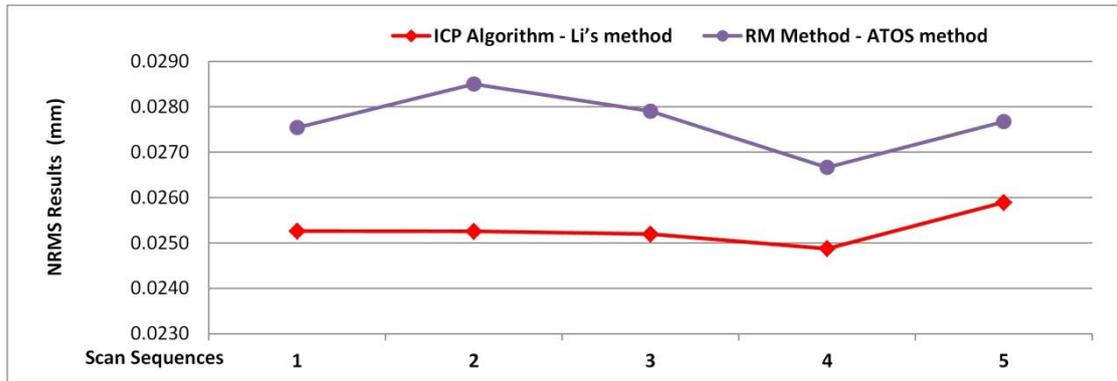


Fig. 5. Registration results using both methods

The repeatability of both methods can obtain using equation (6):

$$Repeatability_{ICP-Li's\ method} = 0.0003\ mm$$

$$Repeatability_{RM-ATOS\ method} = 0.0006\ mm$$

From Fig. 5 we can see that ICP approach (Li's method) provides a slightly better registration results than RM method (ATOS method), in this case. The result of the tests shows a repeatability of 0.0003 mm and 0.0006 mm, respectively.

The tested registration methods are used to prove the feasibility of the evaluation method proposed in this paper. In general, our approach can be used to compare the registration accuracy of various techniques under the same conditions. Divergent results may be obtained if we use a different measuring system or another registration method e.g. ICP algorithm which is developed by other authors.

6. Conclusions

Registration is one of the most important and decisive steps in computer vision. Different techniques have been developed to solve this issue and each technique has its own characteristics and working principle. It is important to evaluate the performance each type of techniques in terms of accuracy. In this paper, we introduce an approach which we refer to as NRMS method, to evaluate the performance of two commonly used registration techniques - ICP approach and RM method. The NPL FreeForm reference standard WP-150 is the artefact used to test the proposed method using a series of series real images and a state-of-the-art structured light 3D scanner GOM ATOS III Triple Scan is used to generate the point cloud data. In general, the ICP approach (Li's method) provides better registration results than the RM method (ATOS method) in this case.

As the evaluation method is based on evaluation of normal vectors of references surfaces, it is a general approach which can be used to evaluate the registration accuracy of other different methods as well, e.g. point clouds alignment using calibrated mechanics or auxiliary devices. Our approach can provide an important reference when we select registration methods to perform registration tasks. Moreover, this method can also be exploited to evaluate the measurement accuracy of different measuring systems.

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