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Managing Morning Commute with Parking Space Constraints in the Case of a Bi-modal Many-to-one Network

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Abstract

Recently, some studies examined how downtown parking space limitation re-shapes the morning commute in the case of a single origin-destination network. This paper further formulates and analyses the commuting equilibrium problem of both mode and departure time choices in a bi-modal (auto and public transit) many-to-one network. Several properties of the equilibrium under parking space constraints and the proposed parking reservation system are discussed. Procedures for computing the dynamic user equilibrium with a parking space constraint (either trading of reservations is allowed or not) have been developed. We show that parking reservation can help reduce deadweight loss due to parking competition and roadway congestion. We also found that assigning more reservations to travelers from a specific origin does not necessarily reduce total travel cost of them, while doing so might raise the total travel cost of travelers from other origins. When parking supply is less than the potential demand but is relatively large, it is socially preferred to retain some parking spaces open for competition. However, when the total parking supply is relatively small, all parking spaces should be reserved to travelers. Besides, we show trading of reservations among travelers would yield an efficiency loss. This loss can be fairly large thus trading should be prohibited.

Keywords: many-to-one network, parking space constraints, parking reservation, dynamic user equilibrium, reservation trading.

1. Introduction

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Parking limitation in downtown areas is a growing problem for both commuters and traffic managers in large cities. Finding a parking space often constitutes an appreciable fraction of the total travel time (Shoup, 2006). Glazer and Niskanen (1992) modelled the congestion caused by through-traffic and by traffic destined for the area where consumers park. Bifulco (1993) introduced parking fees and parking searching time in a stochastic user equilibrium assignment model for evaluation of different parking policies. Researchers have conducted a series of studies to analyze the interactions between cruising or searching for parking and traffic congestion (e.g., Arnott and Inci, 2010; Arnott and Rowse, 2009).

Arnott et al. (1991) embedded the parking problem in the morning commute model (Vickrey, 1969) and showed that parking fees alone can be efficient in increasing social welfare, and a combination of road tolls and parking fees can yield the system optimum that maximizes social welfare. Under the parking setup by Arnott et al. (1991), Zhang et al. (2008) derived the daily commuting pattern that combines both the morning and evening commutes, and investigated mechanisms and efficiencies of several road toll and parking fee regimes. Qian et al. (2012) investigated how parking fees and parking supply can be designed to help mitigate traffic congestion and reduce travel costs. Parking pricing has been advocated as an alternative of road pricing to help manage traffic (e.g., Fosgerau and de Palma, 2013). For a recent review of parking studies, one may refer to Inci (2015).

Recent studies (e.g., Qian et al., 2011; Habib et al., 2012) reported that parking availability can affect a commuter's trip plan, including departure time, mode and route choices. Zhang et al. (2011) showed that competition for limited parking spaces in the downtown would force travelers to depart from home earlier thus encounter larger schedule delay cost. Following the tradable travel credits proposed by Yang and Wang (2011), they introduced a parking permit distribution and trading scheme to reduce the inefficiency due to parking competition. More recently, a series of studies (e.g., Yang et al., 2013; Liu et al., 2014a) further investigated the morning commute problem with a binding parking space constraint, and proposed parking reservation to reduce total travel cost of travelers. While these studies provide some insights on impacts of competition for parking and develop strategies to improve traffic efficiency, they often focus on the problem in a simple dynamic network with one roadway bottleneck, and ignore the network-wide impacts of parking competition due to the parking space constraints.

This study looks at the commuting equilibrium problem of both mode and departure time choices with parking space constraints in a bi-modal many-to-one network. Under different network specifications, various aspects associated with parking have been analyzed in the literature, e.g., network with multiple parking facilities (Li, 2008; Lam et al., 2006; Qian and Rajagopal, 2015; He et al., 2015); parking information provision (Li et al., 2012; Qian and Rajagopal, 2014); park-and-ride service (Li et al., 2007; Liu et al., 2009; Liu et al., 2014c). In the current study, we focus on the bi-modal many-to-one network to explore the network-wide impacts of parking competition. Note that this network specification might be more suitable for those cities whose urban structure presents a largely radiocentric layout, as considered in, e.g., Badia et al. (2014).

In the considered network with multiple bi-modal traffic corridors, commuters live in several different residential areas (i.e., "many" origins), and every morning they travel to the same city center (i.e., "one" destination) through a corridor. Every traveler can choose to either drive and park in the downtown or take public transit without parking consideration. For commuters from different origins, they have competition for parking spaces in the downtown area, but do not share the same roadway and public transit thus there is no direct flow interaction among them. For commuters from the same origin, they not only compete for parking in the city center, but also share the same roadway and public transit. This treatment helps us to focus on the impacts of parking competition and allows for tractability, while it simplifies the modeling of traffic interaction. We then examine how these interactions among all travelers through parking competition, and the interactions among travelers from the same origin through shared highway and transit would re-shape the bi-modal commuting equilibrium in the context of multiple traffic corridors. Also, we will examine how the allocation of parking reservations among commuters from different origins would affect the morning commute equilibrium and help reduce the inefficiency caused by competition for parking and roadway congestion.

The rest of the paper is organized as follows. Next section presents the basic model formulation, and revisits the case of a one-to-one network that incorporates parking space constraints and parking reservations. In Section 3, the bi-modal many-to-one network with parking space constraints is introduced, and the commuting equilibrium without or with parking reservations in such a network is formulated and analyzed. Section 4 discusses the efficiency loss of the parking reservation system due to trading of reservations among commuters. Numerical studies are presented in Section 5 for illustration of the results. Finally, Section 6 concludes the paper.

2. Revisit the Bi-modal equilibrium in a one-to-one network

In this section, we will firstly present the formulation of travel cost functions of auto and transit modes. Then, the bi-modal commuting equilibrium without and with parking space constraints in a simple network with single origin-destination (O-D) will be revisited.

2.1. Travel costs formulation

Vickrey (1969) introduced the first bottleneck model of congestion dynamics. Smith (1984) and Daganzo (1985) further established existence and uniqueness of the time-dependent equilibrium distribution of arrivals at a single bottleneck respectively. Thanks to its analytical tractability, the bottleneck model has been adopted to study various issues (e.g., Arnott et al., 1990; Laih, 1994; Liu et al., 2015a,b). For recent comprehensive reviews of the bottleneck model, one may refer to, e.g., Small (2015).

In the bottleneck model, travel cost by auto, including travel time cost and schedule delay cost, departing at time t can be expressed as

$$c_a(t) = \alpha \cdot T(t) + \beta \cdot \max\left\{0, t^* - t - T(t)\right\} + \gamma \cdot \max\left\{0, t + T(t) - t^*\right\} \tag{1}$$

where $T(t)$ is the travel time at departure time t, α is the value of unit travel time, and β and γ are the schedule penalty for a unit time of early arrival and late arrival respectively. It is assumed that $\gamma > \alpha > \beta > 0$, and denote $\delta = \beta\gamma/(\beta+\gamma)$. $T(t)$ contains free flow travel time t_f and the queuing time at the bottleneck whose service capacity is constantly equal to s. Namely, $T(t) = t_f + q(t)/s$, where $q(t)$ is the queue length experienced by the traveler departing at time t. And

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = \begin{cases} r(t) - s, \; r(t) > s \text{ or } q(t) > 0 \\ 0, \; r(t) \le s \text{ and } q(t) = 0 \end{cases} \tag{2}$$

where $r(t)$ is the departure rate from home at time t. When there is no parking space constraint, given the total number of auto commuters N_a, the equilibrium auto travel cost will be

$$P_a(N_a) = \alpha \cdot t_f + \delta \cdot \frac{N_a}{s}, \tag{3}$$

which is an increasing function of the number of auto commuters, N_a.

In the bi-modal setting, travelers can either drive their car (auto mode) or take transit (transit mode). For simplicity, it is assumed that the cost of taking transit is an increasing function of the number of transit users, i.e.,

$$P_t(N_t) = c_t(N_t), \tag{4}$$

where N_t is the number of transit users. Note that, $c_t(N_t)$ can be regarded as a reduced form of the transit cost function for a model in which transit users are subject to schedule delay costs, and have a time-of-use decision to make (Kraus and Yoshida, 2002; Kraus, 2003). More complicated situations, e.g., when transit operator is responsive to parking supply or roadway capacity expansion (Zhang et al., 2014), might be considered in further study.

2.2. Bi-modal equilibrium without and with parking space constraints

Denote the total number of commuters by N. Now we look at the case when parking supply in the city center is sufficient such that there is no parking space constraint. By assuming an interior equilibrium, with travel cost functions given by Eq.(3) and Eq.(4), we immediately have $P_a(\bar{N}_a) = P_t(\bar{N}_t)$ and $\bar{N}_a + \bar{N}_t = N$, where $\bar{N}_a$ and $\bar{N}_t$ are the numbers of auto and transit commuters at the bi-modal equilibrium without parking space constraint respectively. This $\bar{N}_a$ is indeed the potential demand for auto mode, and for parking as well.

Now we will revisit the morning commute problem with binding parking space constraints for a network with single origin-destination (O-D) discussed by Yang et al. (2013). Specifically, we consider the bi-modal equilibrium when the parking capacity M in the city center is less than the potential demand $\bar{N}_a$. Following Yang et al. (2013), let M^r and M^u denote the numbers of parking spaces for reservation and for competition respectively, where $M^r + M^u = M$. As the parking space constraint is binding, at equilibrium, the numbers of auto commuters with and without reservation will be equal to M^r and M^u, respectively. For commuters with reservations, their choices of departure time (from home) are not directly affected by parking availability; while for commuters without reservations, they have to either depart from home early enough to secure a parking space or take public transit to avoid parking competition. Hereinafter, we denote the commuters with reservation by r-commuter, the commuters without reservation but choosing auto mode (they have to depart from home early enough to secure parking spaces) by u-commuter, and the commuters taking transit by transit commuter. At the bi-modal equilibrium, travel cost of u-commuter will be identical to transit commuters, which is equal to $P_t = c_t(N - M)$.

Define the following critical number for a given parking capacity M :

$$m = m(M) = \frac{s}{\beta} \cdot \left(c_t(N - M) - \alpha \cdot t_f\right). \tag{5}$$

As $M < \bar{N}_a$, it can be verified that $m > \frac{\gamma}{\beta+\gamma} \bar{N}_a$. Note that Eq.(5) reduces to that defined in Yang et al. (2013) if we consider $t_f = 0$. As shown in Yang et al. (2013), different scenarios can appear at commuting equilibrium depending on the values of M^r, M^u and M, and their relations to $m = m(M)$. For specific auto commuting equilibrium scenarios, one may refer to Yang et al. (2013), where three general scenarios and two critical scenarios are discussed in more details. In this paper, we classify those scenarios into two categories: Category I when $M^r \geq \frac{\gamma}{\delta} \cdot (M - m)$ and Category II when $M^r < \frac{\gamma}{\delta} \cdot (M - m)$, where m is defined by Eq.(5) for given M. For Category I, the arrivals (at destination) of commuters with reservations (r-commuter) and auto commuters without reservations (u-commuter) are completely separated. For Category II, some commuters with reservations (r-commuter) have to queue after the last commuters without reservations (u-commuter), thus the arrivals of this two kinds of commuters are not fully separated.

As mentioned, given the parking capacity, $M\left(<\bar{N}_a\right)$, the travel cost of u-commuter will be identical to that of transit commuters, i.e., $P_a^u = P_t = c_t\left(N-M\right)$. And the travel cost of commuters with a reservation, P_a^r, is

$$P_a^r = \begin{cases} \alpha \cdot t_f + \delta \cdot \dfrac{M^r}{s}, & \text{Category I} \\ \alpha \cdot t_f + \gamma \cdot \dfrac{M-m}{s}, & \text{Category II} \end{cases}. \tag{6}$$

For Category I, P_a^r is increasing in M^r, while for Category II, P_a^r is constant. It can be shown that $P_a^r \le \alpha t_f + \delta M/s$ holds. Therefore, $P_a^r < \alpha t_f + \delta \bar{N}_a/s < P_a^u = P_t = c_t\left(N-M\right)$, indicating the commuters with reservation (r-commuter) can enjoy a lower cost than those without reservations (u-commuter). This also means that travelers are willing to pay a price to obtain a parking reservation, which is also the motivation of our discussion on trading of reservations later in Section 4.

In addition, at equilibrium, the first and the last u-commuter will arrive at destination at time

$$t_{u,s} = t^* - \frac{m\left(M^r + M^u\right)}{s};\; t_{u,e} = t_{u,s} + \frac{M^u}{s}, \tag{7}$$

where M^u is the number of u-commuter. Let $t_{u,e}\left(x\right)$, $x \in \left(0, \bar{N}_a\right)$ denote the latest arrival time at the destination of the u-commuters. With Eq.(7), we have

$$t_{u,e}\left(M^u\right) = t^* - \frac{m\left(M^r + M^u\right)}{s} + \frac{M^u}{s}. \tag{8}$$

When the available parking space is 0^+ (a positive value approaching 0), the latest arrival time of u-commuters at the destination is $t_{u,e}\left(0\right)$. When the number of public parking spaces is set to be the possible maximum $\bar{N}_a - M^r$, the latest arrival time of u-commuters at the destination is $t_{u,e}\left(\bar{N}_a - M^r\right)$. Let $t_{u,e}^{-1}\left(t\right)$ denote the inverse function of $t_{u,e}\left(M^u\right)$ which represents the number of u-commuters with respect to the latest arrival time of u-commuters at the destination. Since the latest arrival time of u-commuters at the destination is exactly the time when the public parking space for competition is used up, therefore $t_{u,e}^{-1}\left(t\right)$ can also be regarded as a function of the auto demand with respect to the ending time of public parking spaces. Clearly there is a positive auto demand corresponding to each parking ending time $t \in \left(t_{u,e}\left(0\right), t_{u,e}\left(\bar{N}_a - M^r\right)\right)$. If no parking space is available before time $t_{u,e}\left(0\right)$, the number of u-commuters will be zero. If parking spaces are still available after time $t_{u,e}\left(\bar{N}_a - M^r\right)$, there is indeed no parking space constraint. In this case, whether reserving a space or not makes no difference, and $M^u = \bar{N}_a - M^r$. After expanding the feasible region of the public parking space ending time, the number of u-commuters can be rewritten as follows:

$$\pi(t) = \begin{cases} 0 & t \le t_{u,e}(0) \\ t_{u,e}^{-1}(t) & t \in \left(t_{u,e}(0), t_{u,e}\left(\bar{N}_a - M^r\right)\right) \\ \bar{N}_a - M^r & t \ge t_{u,e}\left(\bar{N}_a - M^r\right) \end{cases}. \tag{9}$$

Function $\pi(t)$ constructs a mapping between public parking space ending time and number of u-commuters, through the internal Wardropian bi-modal equilibrium. The construction of function $\pi(t)$ reduces to that in Zhang et al. (2011) if we consider the special case of our model here that all parking spaces are not for reservation but open for competition, i.e., $M^r = 0$. Evidently $\pi(t)$ is an increasing (but not strictly increasing) function. This function will be used in Section 3 for computing the equilibrium traffic pattern in the bi-modal many-to-one network.

3. The bi-modal many-to-one network with parking space constraints

Now we consider a bi-modal many-to-one network with n origins (i.e., “many”) and the same destination D (i.e., “one”) as shown in Figure 1. Each origin-destination (O-D) pair is connected by a highway with bottleneck and a parallel public transit line with dedicated right-of-way. Note that there are some studies exploring the optimal dynamic traffic assignment and commuting equilibrium in queuing networks, e.g., Yang and Meng (1998), Zhang and Zhang (2010). In the bi-modal many-to-one network, commuters are living in different residential areas and they travel to the same city center every morning either by driving or taking transit. For commuters from different residential areas, they do not share the same highway or public transit, thus there is no direct flow interaction among them. However, as they travel to the same downtown with parking limitation, they have to compete with each other for parking spaces. For commuters living in the same origin, they not only compete with each other for parking, but also interact with each other at the shared highway and public transit.

In the many-to-one network depicted in Figure 1, for the i-th O-D pair, i.e., from O_i to D, let t_i denote the free flow travel time, s_i the capacity of the highway bottleneck i, and N_i, $N_{a,i}$, $N_{t,i}$, $P_{t,i} = c_{t,i}\left(N_{t,i}\right)$ the total travel demands, the numbers of auto commuters and transit commuters, and transit cost function, where $i \in I = \{1, 2, ..., \mathrm{n}\}$, and I is the set of O-D pairs. Total number of parking spaces available at the city center is M .

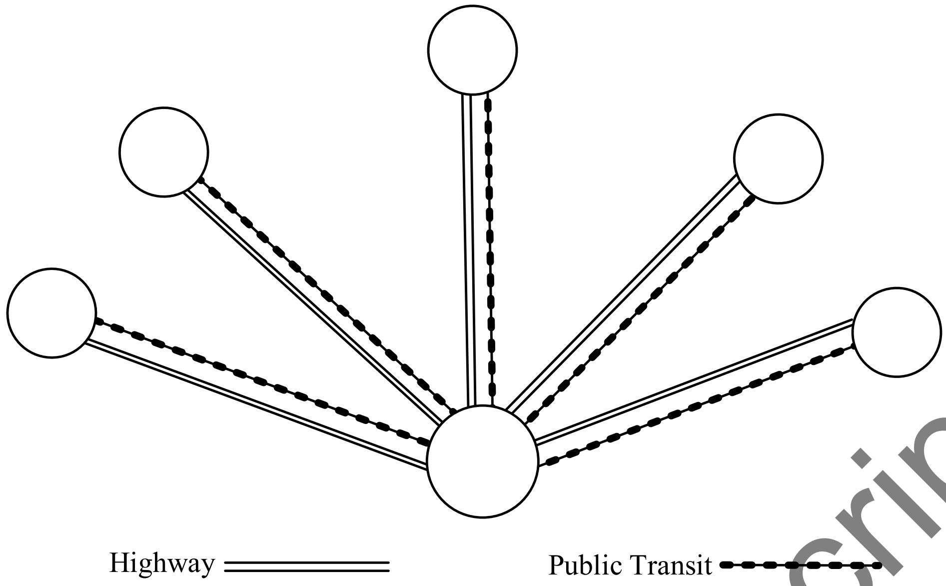


Figure 1. The many-to-one network with highway and public transit

3.1. Commuting equilibrium

Firstly, suppose M is large enough thus there is no parking space constraint. For given origin O_i, under no parking space constraint, the equilibrium travel costs of auto commuters and transit commuters should be equal, i.e., $P_{a,i}\left(\bar{N}_{a,i}\right)=P_{t,i}\left(\bar{N}_{t,i}\right)$, where $\bar{N}_{a,i}+\bar{N}_{t,i}=N_i$, and $\bar{N}_{a,i}$ and $\bar{N}_{t,i}$ are the numbers of auto and transit commuters from origin O_i respectively. Without a parking space constraint, the commuting equilibria for different O-D pairs are independent of each other. Therefore, the combined equilibrium in the bi-modal many-to-one network will reduce to a simple summation of equilibrium without parking space constraint for the n O-D pairs. In this study, we will focus on the case with binding parking space constraints. A binding parking space constraint in the many-to-one network implies that

$$M<\bar{N}_a=\sum_i \bar{N}_{a,i}, \tag{10}$$

where $\bar{N}_{a,i}$ is the potential auto demand or parking demand for traveler from the i-th origin O_i. In this case, travelers from at least one origin will have to compete for the parking spaces. The commuting equilibrium with parking space constraints in the bi-modal many-to-one network can be determined by a similar approach as that in Zhang et al. (2011), with the help of Eq.(9) to determine the parking space ending time. Later we will present the procedure to compute the equilibrium in the many-to-one network with parking space constraints and parking reservations, which incorporates that in Zhang et al. (2011) as a special case.

Now we turn to the commuting equilibrium when parking reservations are introduced. Denote the allocation of reserved parking spaces (or parking reservations) among commuters from all origins by $\mathbf{M^r} = \{M_i^r\}$, where $0 \le M_i^r \le \bar{N}_{a,i}$ for any $i \in I$. Then, M_i^r is the parking reservations assigned to commuters from origin O_i. Note that the commuting equilibrium without introducing reservations is a special case of our model with $M_i^r = 0$ (later in numerical analysis, we use UE to refer to this situation). For O-D pair i, denote the auto demand function with respect to the public parking space ending time, as defined in Eq.(9) by $\pi^i(t)$. Let t_{end} be the public parking space (those not for reservation) ending time. Since the total number of public parking spaces is $M - \sum_i M_i^r$, we have $\sum_i \pi^i(t_{end}) = M - \sum_i M_i^r$. The O-D pairs can be classified into three groups as follows:

1) $t_{u,e}^i(0^+) \ge t_{end}$ and $M_i^u = 0$, $i \in I_1$;
2) $t_{u,e}^i(M_i^u) = t_{end}$ and $M_i^u > 0$, $i \in I_2$;
3) $t_{u,e}^i(M_i^u) < t_{end}$ and $M_i^u \ge 0$, $i \in I_3$;

where $I_1 \cup I_2 \cup I_3 = I$. For commuters from O-D $i \in I_1$, competing for parking spaces is too costly when compared with taking transit thus no one will choose to drive without a reserved space. For commuters from O-D $i \in I_2$, some of the travelers choose to compete for parking spaces (u-commuter) and the last u-commuters of these O-D arrive at the destination just at the ending time of public parking space. For commuters from O-D $i \in I_3$, given the allocation $\mathbf{M^r} = \{M_i^r\}$, the parking space constraint is not binding for them, and those without reservations can always obtain a public parking space before the ending time. Note that I_k, where $k = 1,2,3$, might be empty while at least one of them should be non-empty.

Given the allocation $\mathbf{M^r} = \{M_i^r\}$, we propose the bi-section based procedure (denoted as Procedure I, which will also be used later in Section 4) to calculate the dynamic user equilibrium with parking space constraints and mixed supply of parking spaces (two classes: those for reservation and those open for competition). In Procedure I, Stage 1 will compute the public parking space ending time through a bi-section based procedure, and Stage 2 will determine the exact equilibrium traffic pattern for each O-D pair i depending on M_i^r and the resulting $\frac{\gamma}{\delta} \cdot (M_i - m_i)$. Note that, for O-D $i \in I_3$, the auto commuting traffic pattern at equilibrium is just the same as that if there is no parking constraint, which is a limiting case of Category II. The travel costs in this case can still be calculated by using the formulations for Category II.

Procedure I: Computing the User Equilibrium Solution

Stage 0: Initialization

Input: N_i, $N_{a,i}$, $N_{t,i}$, $P_{t,i} = c_{t,i}\left(N_{t,i}\right)$, M; specific reservation allocation M_i^r.

Compute the $\bar{N}_{a,i}$ for each O-D pair.

Stage 1: Compute the public parking space ending time

Step 1-0: Initialization

Define $\tau_e = \min_i \left\{t_{u,e}^i\left(0\right)\right\}$ and $\tau_l = \max_i \left\{t_{u,e}^i\left(\bar{N}_{a,i} - M_i^r\right)\right\}$.

Set a sufficiently small positive number ε as the convergence criterion.

Let $k = 1$.

Step 1-1:

Let $\tau^{(k)} = \frac{\tau_e + \tau_l}{2}$.

If $\left|\left(\tau_l - \tau_e\right)/\tau_l\right| \le \varepsilon$, $t_{end} = \tau^{(k)}$; otherwise, go to Step 1-2.

Step 1-2:

If $\sum_i \pi^i\left(t_{end}\right) > \left(M - \sum_i M_i^r\right)$, $\tau_l = \tau^{(k)}$; otherwise, $\tau_e = \tau^{(k)}$.

Let $k = k+1$, go to Step 1-1.

Stage 2: Determine the equilibrium for each origin-destination pair

Step 2-0: Initialization

Calculate the number of u-commuters for each O-D pair by $M_i^u = \pi^i\left(t_{end}\right)$.

Calculate $M_i = M_i^r + M_i^u$, and $N_{t,i} = N_i - M_i$.

Step 2-1:

Calculate m_i with $N_{t,i} = N_i - M_i$ from Step 2-0.

Step 2-2:

For every $i \in I$, compare M_i^r with $\frac{\gamma}{\delta}\cdot\left(M_i - m_i\right)$:

If $M_i^r \ge \frac{\gamma}{\delta}\cdot\left(M_i - m_i\right)$, the equilibrium for O-D pair i is in Category I;

Otherwise, the equilibrium for O-D pair i is in Category II.

Note: $\varepsilon = 10^{-5}$ is applied in this paper.

To ease later analysis of the commuting equilibrium in the bi-modal many-to-one network, we now summarize several facts about the latest arrival time of the u-commuters defined in Eq.(8) in the following Lemma 1.

Lemma 1. The latest arrival time of the u-commuters defined in Eq.(8) satisfying

$$\frac{dt_{u,e}\left(M^u\right)}{dM^u} > 0;\ \frac{\partial t_{u,e}\left(M^u\right)}{\partial N} < 0;\ \frac{\partial t_{u,e}\left(M^u\right)}{\partial M^r} > 0;\ \frac{\partial t_{u,e}\left(M^u\right)}{\partial t_f} > 0;\ \frac{\partial t_{u,e}\left(M^u\right)}{\partial s} \le 0. \tag{11}$$

Proof. With Eq.(5) and Eq.(8), it follows

$$t_{u,e}\left(M^u\right) = t^* - \frac{1}{\beta}\cdot\left(c_t\left(N - M^r - M^u\right) - \alpha\cdot t_f\right) + \frac{M^u}{s}. \tag{12}$$

Let $c_t' = \frac{dc_t\left(N-M^r-M^u\right)}{d\left(N-M^r-M^u\right)}$, then $c_t' > 0$ as $c_t(\cdot)$ is increasing. With Eq.(12), we have

$$\begin{aligned}
&\frac{dt_{u,e}\left(M^u\right)}{dM^u} = \frac{1}{\beta}\cdot c_t' + \frac{1}{s} > 0 \\
&\frac{\partial t_{u,e}\left(M^u\right)}{\partial N} = -\frac{1}{\beta}\cdot c_t' < 0 \\
&\frac{\partial t_{u,e}\left(M^u\right)}{\partial M^r} = \frac{1}{\beta}\cdot c_t' > 0 \\
&\frac{\partial t_{u,e}\left(M^u\right)}{\partial t_f} = \frac{1}{\beta}\cdot \alpha > 0 \\
&\frac{\partial t_{u,e}\left(M^u\right)}{\partial s} = -\frac{M^u}{s^2} \le 0
\end{aligned} \quad . \tag{13}$$

This completes the proof. #

Lemma 1 can be applied to the functions of latest arrival time of the u-commuters for all O-D pairs, i.e., $t_{u,e}^i(\cdot)$ for $i \in I$.

Proposition 1. At equilibrium, i) for $i \in I_1$, $M_i^u = 0$; ii) for $i \in I_2$, $N_i \uparrow$, $M_i^r \downarrow$, $t_i \downarrow$, $s_i \uparrow$ indicate $M_i^u \uparrow$; iii) for $i \in I_3$, $M_i^u = \bar{N}_{a,i} - M_i^r$.

In Proposition 1, the results in (i) and (iii) are straightforward. Proposition 1(ii) states that for O-D pair i with $t_{end} = t_{u,e}^i\left(M_i^u\right)$, a larger N_i, a smaller M_i^r, a smaller t_i, or a larger s_i (while other parameters are identical) indicates a larger equilibrium M_i^u. This can be derived from Lemma 1. Suppose that for two O-D pairs $i, j \in I_2$, all parameters are identical except $N_i > N_j$. Given $t_{end} = t_{u,e}^i\left(M_i^u\right) = t_{u,e}^j\left(M_j^u\right)$, from Lemma 1 and Eq.(13), we have $M_i^u > M_j^u$. Similar analysis can be applied for M_i^r, t_i and s_i.

3.2. System performance with parking reservations

Given the allocation of reservations among commuters of different O-D pairs, $\mathbf{M^r}$, the total travel cost of all commuters at equilibrium can be written as

$$TC\left(\mathbf{M^r}\right) = \sum_i TC_i\left(\mathbf{M^r}\right) = \sum_i \left[P_{a,i}^r \cdot M_i^r + P_{a,i}^u \cdot M_i^u + P_{t,i}\cdot\left(N_i - M_i^r - M_i^u\right)\right], \tag{14}$$

where for all $i \in I$, M_i^u, $P_{a,i}^r$ and $P_{a,i}^u = P_{t,i} = c_{t,i}\left(N_i - M_i^r - M_i^u\right)$ are from the resulting equilibrium computed according to Section 3.1. Note that $P_{a,i}^r$ is in the form given in Eq.(6).

In Eq.(14), $TC_i\left(\mathbf{M^r}\right)$ is the total travel cost of commuters from origin O_i. The socially optimal allocation $\mathbf{M^{r*}}$ can be determined by solve the following problem:

$$\min_{\mathbf{M^r}} : TC\left(\mathbf{M^r}\right) \tag{15}$$

s.t.

$$\sum_i M_i^r \le M \; ; \tag{16}$$

$$M_i^r \le \bar{N}_i^a \text{ for all } i \; ; \tag{17}$$

$$M_i^r \ge 0 \text{ for all } i \, . \tag{18}$$

Now we consider the case when all parking space are reserved to commuters, i.e., $\sum_i M_i^r = M$, and $M_i^u = 0$ for all i. Denote the allocation that satisfies $\sum_i M_i^r = M$ as $\bar{\mathbf{M}}^{\mathbf{r}}$. In this case, minimizing total travel cost is equivalent to solving the problem in Eq.(15) while replacing the constraint in Eq.(16) by $\sum_i M_i^r = M$, and remaining the other constraints in Eqs.(17) and (18). Under its optimal solution $\bar{\mathbf{M}}^{\mathbf{r}*}$, total travel cost should be no less than that by solving Eq.(15) under constraints in Eqs.(16)-(18), i.e., $TC\left(\bar{\mathbf{M}}^{\mathbf{r}*}\right) \ge TC\left(\mathbf{M^{r*}}\right)$. This is straightforward because $\bar{\mathbf{M}}^{\mathbf{r}*}$ is the optimal solution within a subset of the feasible region defined by Eqs.(16), (17) and (18). Therefore, it might be socially preferable to retain some parking spaces open for competition.

Proposition 2. For any $i \in I$ and $j \in I$, at equilibrium we have

$$-1 \le \frac{dM_j^u}{dM_i^r} \le 0 \, . \tag{19}$$

Proof. See Appendix.

Proposition 2 states that once the number of reservations assigned to travelers from a specific origin O_i increases, the numbers of u-commuters from each origin will decrease or at least do not increase. Moreover, for O-D pair i, as shown in the proof of Proposition 2, the total number of auto commuters, i.e., $M_i = M_i^r + M_i^u$ (summation of those with and without reservation), will probably increase. Note that it is possible that M_i does not increase, e.g., when there is only one O-D pair, or other O-D pairs all belong to group 1 or 3 (boundary equilibrium) such that M_i remains constant.

In most cases, M_i will increase as M_i^r increases. If the commuting traffic equilibrium for O-D pair i belongs to Category I, from Eq.(6), we know that the travel cost of r-commuters (those with reservation) will be $P_{a,i}^{r} = \alpha \cdot t_i + \delta \cdot M_i^r / s_i$, which increases with M_i^r. If the commuting traffic equilibrium belongs to Category II, $P_{a,i}^{r} = \alpha \cdot t_i + \gamma \cdot (M_i - m_i)/s_i$ will increase with M_i^r as long as M_i increases as well. This is because, if an increase in M_i^r leads to an increase in M_i, then $m_i = m_i(M_i)$ decreases. However, for the case of a one-to-one network, the total number of drivers $M = M^r + M^u$ remains constant even if we increase M^r. Therefore, travel cost of r-commuters, $P_a^r = \alpha \cdot t_f + \gamma \cdot (M - m)/s$, will be a constant as discussed in Section 2.2.

Proposition 3. For any $i \in I$ and $j \neq i \in I$, where $|I| \geq 2$, , we have

$$\frac{dTC_j}{dM_i^r} \geq 0 \text{, if } j \text{ belongs to Category I;} \tag{20}$$

$$\frac{dTC_j}{dM_i^r} \geq 0 \text{, if } j \text{ belongs to Category II and } M_j^r \to 0; \tag{21}$$

where TC_j is defined by Eq.(14).

Proof. Based on Eq.(14),

$$\frac{dTC_j}{dM_i^r} = \frac{dP_{a,j}^r}{dM_i^r} \cdot M_j^r + \frac{dP_{t,j}}{dM_j^r} \cdot \left(N_j - M_j^r\right). \tag{22}$$

For j that belongs to Category I, with Eq.(6), we have $dP_{a,j}^r / dM_i^r = 0$. Therefore,

$$\frac{dTC_j}{dM_i^r} = \frac{dc_{t,j}}{dN_{t,j}} \cdot \left(-\frac{dM_j^u}{dM_i^r}\right) \cdot \left(N_j - M_j^r\right), \tag{23}$$

where $\frac{dc_{t,j}}{dN_{t,j}} = \frac{dP_{t,j}\left(N_j - M_j^r - M_j^u\right)}{d\left(N_j - M_j^r - M_j^u\right)} > 0$. Since $-1 \leq dM_j^u / dM_i^r \leq 0$, we have $dTC_j / dM_i^r \geq 0$. For j that belongs to Category II, with Eq.(6) and Eq.(22), we have

$$\frac{dTC_j}{dM_i^r} = \left[\left(\frac{\gamma}{s_j} + \frac{\gamma}{\beta}\frac{dc_{t,j}}{dN_{t,j}}\right) \cdot M_j^r - \frac{dc_{t,j}}{dN_{t,j}} \cdot \left(N_j - M_j^r\right)\right] \cdot \frac{dM_j^u}{dM_i^r}. \tag{24}$$

If $M_j^r \to 0$, $\left(\frac{\gamma}{s_j} + \frac{\gamma}{\beta}\frac{dc_{t,j}}{dN_{t,j}}\right) \cdot M_j^r$ will also approach zero, and be less than $\frac{dc_{t,j}}{dN_{t,j}} \cdot \left(N_j - M_j^r\right)$. Again, as $-1 \leq dM_j^u / dM_i^r \leq 0$, we have $dTC_j / dM_i^r \geq 0$. #

Proposition 3 is a direct result of Proposition 2. It indicates that if one increases the number of reservations assigned to commuters of one specific O-D pair, the commuters of other O-D pairs belonging to Category I would be worse off or at least not better off. Furthermore, if O-D pair j belongs to Category II, $dTC_j / dM_i^r \geq 0$ might not hold when M_j^r is relatively large. This is explained as follows. By increasing the number of reservations assigned to travelers from origin O_i, the number of commuters from origin O_j who choose auto mode, i.e., $M_j^r + M_j^u$, can decrease, and the highway can be less congested with less traffic. Thus, $dP_{a,j}^r / dM_i^r < 0$. It is also worth mentioning that assigning more reservations to commuters from one specific origin, while might increase total travel cost of commuters from other origins (as stated in Proposition 3), will not necessarily reduce the total travel cost of commuters from this origin. This will be discussed in the following.

We examine the specific allocation $\bar{\mathbf{M}}^{\mathbf{r}}$, under which we have $\sum_i M_i^r = M$. In this case, according to our classification of equilibria, the traffic equilibrium of each O-D is a limiting case of Category I where $M_i^u = 0$. Also note that if we reduce M_i^r by a certain amount which is small enough, the traffic equilibrium of each O-D will still belong to Category I. We focus on the situation when $\mathbf{M}^{\mathbf{r}}$ is close to $\bar{\mathbf{M}}^{\mathbf{r}}$ thus the traffic equilibria of all O-D pairs belong to Category I. Given the above consideration, for a given M, the total travel cost is

$$TC\left(\mathbf{M}^{\mathbf{r}}\right) = \sum_i TC_i^1 = \sum_{i=1}^{n}\left[\left(\alpha \cdot t_i + \delta \cdot \frac{M_i^r}{s_i}\right) \cdot M_i^r + c_{t,i}\left(N_i - M_i^r - M_i^u\right) \cdot \left(N_i - M_i^r\right)\right], \quad (25)$$

where $c_{t,i}(\cdot)$ is the transit cost function for commuters from origin i, and TC_i^1 is the special form of TC_i defined in Eq.(14). Note that each M_i^u in Eq.(25) is the equilibrium number of u-commuters for O-D pair i which fully depends on $\mathbf{M}^{\mathbf{r}}$. The first-order derivative of Eq.(25) with respect to M_i^r is

$$\frac{dTC\left(\mathbf{M}^{\mathbf{r}}\right)}{dM_i^r} = \quad (26)$$

$$\alpha \cdot t_i + 2\delta \cdot \frac{M_i^r}{s_i} \quad (27)$$

$$-c_{t,i}\left(N_i - M_i^r - M_i^u\right) - \frac{dc_{t,i}\left(N_i - M_i^r - M_i^u\right)}{d\left(N_i - M_i^r - M_i^u\right)} \cdot \left(1 + \frac{dM_i^u}{dM_i^r}\right) \cdot \left(N_i - M_i^r\right) \quad (28)$$

$$-\sum_{j \neq i} \frac{dc_{t,j}\left(N_j - M_j^r - M_j^u\right)}{d\left(N_j - M_j^r - M_j^u\right)} \cdot \frac{dM_j^u}{dM_i^r} \cdot \left(N_j - M_j^r\right), \quad (29)$$

where $i = 1, 2, ..., n$. By assigning additional reservation to travelers of O-D i, Eq.(27) is the marginal increase of total travel cost of all r-commuters of O-D i, and Eq.(28) is the marginal decrease of total travel cost of all other commuters from O-D i, and Eq.(29) is the marginal increase of total travel cost of all other commuters from O-D $j \neq i$. Note that Eq.(29) is non-negative according to Proposition 3. While Eq.(27) is only valid when the commuting equilibrium belongs to Category I, Eqs.(28) and (29) are valid for both Categories I and II.

Proposition 4. i) If

$$\max\left\{\left.\frac{dTC\left(\mathbf{M^r}\right)}{dM_i^r}\right|_{\mathbf{M^r}=\bar{\mathbf{M}}^{\mathbf{r}*}}\right\}>0, \tag{30}$$

where $dTC\left(\mathbf{M^r}\right)/dM_i^r$ is defined in Eq.(26), it is socially preferable to retain some parking spaces open for competition. ii) Furthermore, for any $i\in I$, if

$$\left.\frac{dTC\left(\mathbf{M^r}\right)}{dM_i^r}\right|_{\mathbf{M^r}=\bar{\mathbf{M}}^{\mathbf{r}*}}>0, \tag{31}$$

a small decrease of $M_i^r=\bar{M}_i^{r*}$ will lead to a decrease in total cost.

Proposition 4 is straightforward. If the first-order derivative of total cost at $\bar{\mathbf{M}}^{\mathbf{r}*}$ with respect to a specific M_i^r is positive, total travel cost can be reduced through decreasing M_i^r by a small amount. In the following Proposition 5, we provide a stronger condition for further intuition. Before doing so, we define

$$\Delta MC_i\left(M_i^r\right)=\left(\alpha\cdot t_i+2\delta\cdot\frac{M_i^r}{s_i}\right)-\left(c_{t,i}\left(N_i-M_i^r\right)+\frac{dc_{t,i}\left(N_i-M_i^r\right)}{d\left(N_i-M_i^r\right)}\cdot\left(N_i-M_i^r\right)\right), \tag{32}$$

for any $i\in I$. $\Delta MC_i\left(M_i^r\right)$ is the difference between the marginal cost of the auto side and the marginal cost of the transit side when all parking spaces are reserved to commuters (then the number of transit users $N_{t,i}=N_i-M_i^r$).

Proposition 5. i) As

$$\max\left\{\left.\Delta MC_i\left(M_i^r\right)\right|_{\mathbf{M^r}=\bar{\mathbf{M}}^{\mathbf{r}*}}\right\}>0, \tag{33}$$

it is socially preferable to retain some parking spaces open for competition. ii) Furthermore, for any $i\in I$, if

$$\left.\Delta MC_i\left(M_i^r\right)\right|_{\mathbf{M^r}=\bar{\mathbf{M}}^{\mathbf{r}*}}>0, \tag{34}$$

a small decrease of $M_i^r=\bar{M}_i^{r*}$ will lead to a decrease in total cost.

Proof. According to Proposition 4, regarding the first-order derivative of Eq.(25) with respect to M_i^r, as defined in Eq.(26), we have

$$\begin{gathered}\frac{dTC\left(\mathbf{M^r}\right)}{dM_i^r}\geq\\ \alpha\cdot t_i+2\delta\cdot\frac{M_i^r}{s_i}-c_{t,i}\left(N_i-M_i^r-M_i^u\right)-\frac{dc_{t,i}\left(N_i-M_i^r-M_i^u\right)}{d\left(N_i-M_i^r-M_i^u\right)}\cdot\left(N_i-M_i^r\right)\end{gathered}. \tag{35}$$

At $\mathbf{M}^{\mathbf{r}} = \bar{\mathbf{M}}^{\mathbf{r}*}$, $M_i^u = 0$ for any $i \in I$, it follows

$$\left.\frac{dTC\left(\mathbf{M}^{\mathbf{r}}\right)}{dM_i^r}\right|_{\mathbf{M}^{\mathbf{r}}=\bar{\mathbf{M}}^{\mathbf{r}*}} \geq \Delta MC_i\left(M_i^r\right)\Big|_{\mathbf{M}^{\mathbf{r}}=\bar{\mathbf{M}}^{\mathbf{r}*}}. \tag{36}$$

Therefore, Eq.(33) in Proposition 5 implies that Eq.(30) in Proposition 4 holds as well. It is socially preferable to retain some parking spaces unreserved. Similarly, we can show that Eq.(34) indicates that a small decrease of $M_i^r = \bar{M}_i^{r*}$ will lead to a decrease in total cost. #

Proposition 5 indicates that when all parking spaces are reserved to travelers, if the marginal cost in the auto side is larger than that in the public transit side, it is beneficial to the system to retain some public parking spaces open for competition in the morning peak. Furthermore, if we consider the transit cost is nearly constant, i.e., $dc_{t,i}\left(N_i - M_i^r\right) / d\left(N_i - M_i^r\right) \to 0$, it follows that Eq.(32) approaches $\Delta MC_i\left(M_i^r\right) = \alpha \cdot t_i + 2\delta M_i^r / s_i - c_{t,i}\left(N_i - M_i^r\right)$. When M_i^r is relatively large, i.e., closer to $\bar{N}_{a,i}$, $\Delta MC_i\left(M_i^r\right)$ is more likely to be positive, indicating it is more likely to be beneficial by retaining some parking spaces open for competition. However, when parking is very limited, i.e., M is relatively small, and M_i^r will be small as well, then $\Delta MC_i\left(M_i^r\right)$ is likely to be negative, indicating that it is unlikely to reduce travel cost by maintaining some public parking spaces. This is verified in numerical studies.

4. Efficiency loss due to trading of reservations among commuters

4.1. One-to-one network case

Now we consider the case where commuters can trade their parking reservations (without consideration of transaction cost). It is similar with tradable parking permit considered in Zhang et al. (2011), Liu et al., (2014b). Firstly, we look at the case of a one-to-one network. Under a parking space constraint, a reserved parking space would yield a travel cost saving, i.e., $P_a^r < P_t$, where P_t is the transit travel cost and P_a^r is the travel cost of r-commuters, which is discussed in Section 2.2. Thus, travelers would be willing to pay a price no larger than $P_t - P_a^r$ to obtain a parking reservation. Indeed, given the total number of reservations M^r, the equilibrium price of a parking reservation should be exactly equal to the cost saving from traveling with a reservation, which is

$$p^p = P_t - P_a^r. \tag{37}$$

For the one-to-one network case, given parking capacity M, the transit cost P_t is constantly equal to $c_t\left(N - M\right)$, while P_a^r given by Eq.(6) is increasing over M^r for $M^r \geq \frac{\gamma}{\delta}\left(M - m\right)$, i.e., Category I, and constant for $M^r < \frac{\gamma}{\delta}(M-m)$, i.e., Category II. Therefore, the price of the reservation given in Eq.(37) will be decreasing for Category I, while it will be constant for Category II, which are shown in Figure 2. Indeed, M^r can be regarded as the total supply of parking spaces for reservation, with the increase of supply, the price of reservation goes down or at least does not go up. Also note that, when $M \le m$, where m is defined by Eq.(5), as the difference of $M-m$ will be always non-positive, $M^r \ge \frac{\gamma}{\delta}(M-m)$ holds. It follows that the equilibrium in Category II will never arise. In this case, the price of reservation is depicted by the right panel in Figure 2.

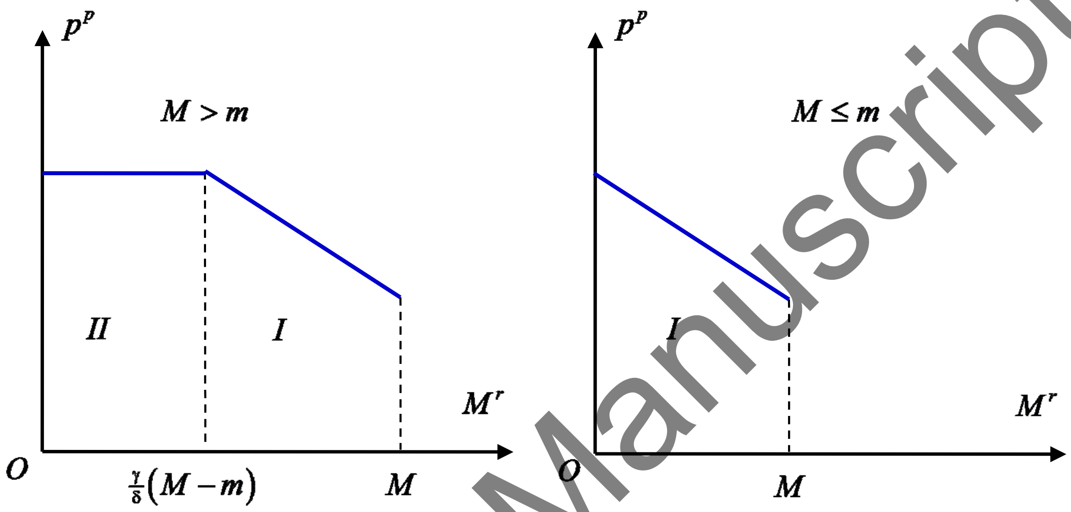


Figure 2. The price of parking reservation under the commuting equilibrium in the one-to-one network

Note that the left panel of Figure 2 for price of reservation in Category II is only valid in the one-to-one network case. This is because, as discussed in Section 3.2 after Proposition 2, in the case of a many-to-one network, an increase M_i^r for one specific O-D i might yield an increase in $P_{a,i}^r$ even if the equilibrium belongs to Category II (due to the network-wide interaction among travelers through parking competition). Then, the price (value) of a reservation for travelers of O-D i will not be a constant. This is also briefly discussed in the following section.

4.2. Many-to-one network case

In the many-to-one network, the parking reservations might be traded among commuters from different origins. In this case, as the total number of reservations M^r distributed to the population is identical, the resulting allocation of reservations among O-D pairs, $\mathbf{M^r} = \{M_i^r\}$ will be identical, which is a result of joint equilibrium of modal-split and traffic pattern in the network and trading of reservations in the market, i.e., equilibrium of both travel and trading.

Trading of reservations will occur between commuters with different values of reservation, as long as commuters with lower values still have positive number of reservations.

For ease of presentation, the set of all O-D pairs I is divided into two subsets I_a and I_b in the way that at equilibrium, if $M_i^r > 0$, $i \in I_a$ and if $M_i^r = 0$, $i \in I_b$. Note here when we consider trading of reservations, M_i^r is used to denote the equilibrium number of reservations for commuters of O-D i after trading, which is also the equilibrium number of r-commuters for this O-D. Further study may consider issues related to initial distribution of reservations to travelers as considered in Liu et al. (2014b). For commuters of O-D $i \in I_a$, the equilibrium parking reservation price is given by

$$p^p = p_i^p = P_{t,i} - P_{a,i}^r . \tag{38}$$

The price should be identical for any O-D $i \in I_a$. Otherwise, trading of parking reservations will occur, i.e., those with higher p_i^p will buy reservations from those with lower p_i^p, and both sides of the trading can benefit from it. For commuters of O-D pair $i \in I_b$, the value of a reservation is

$$p_i^p = P_i^T - P_{r,i}^A \le p^p , \tag{39}$$

where p^p is determined by Eq.(38). Note that the "equal sign" of the inequality in Eq.(39) holds in the boundary case, where $M_i^r = 0$ and p_i^p is just equal to the price determined in Eq.(38). As mentioned, for $i \in I_b$, $M_i^r = 0$. This means that, travelers from these origins will benefit from selling out their reservations (if some reservations are assigned to them at the initial reservation distribution) even they have to compete for parking or take transit; and if they do not have a reservation, they will not buy from the market as the reservations are too expensive for them, i.e., the price of a reservation is greater than the cost saving associated with the reservation.

As discussed in Section 4.1, in a one-to-one network, the price of reservation is constant (independent of M^r) if the equilibrium is in Category II because P_a^r is constant. However in the many-to-one network, as discussed in Section 3.2, $P_{a,i}^r$ might increase with M_i^r even if the equilibrium for O-D i belongs to Category II. It follows that the value of reservation for commuters from this origin might decrease with M_i^r (in the case of Category II).

The optimal allocation of reservation $\mathbf{M}^{\mathbf{r}*}$ minimizes the total travel cost given in Eq.(14), and we have

$$\left.\frac{\partial TC\left(\mathbf{M}^{\mathbf{r}}\right)}{M_i^r}\right|_{\mathbf{M}^{\mathbf{r}}=\mathbf{M}^{\mathbf{r}*}} = \left.\frac{\partial TC\left(\mathbf{M}^{\mathbf{r}}\right)}{M_j^r}\right|_{\mathbf{M}^{\mathbf{r}}=\mathbf{M}^{\mathbf{r}*}} , \tag{40}$$

for any $i, j \in I$, where $TC(\mathbf{M^r})$ is given in Eq.(14). However, the equilibrium allocation of parking reservations after trading will satisfy the following conditions,

$$p_i^p = P_{t,i} - P_{a,i}^r \le p^p, \tag{41}$$

$$M_i^r \cdot \left(p_i^p - p^p\right) = 0. \tag{42}$$

The trading equilibrium determined by Eqs.(41) and (42) usually deviates from Condition Eq.(40). This means that trading of reservation among travelers will probably lead to non-optimal allocation of reservations among travelers from a system perspective. This is later verified in numerical studies.

At the joint equilibrium of travel and trading, Eqs.(41) and Eq.(42) should hold in addition to the traffic equilibrium conditions. By taking the advantage of the Procedure I in Section 3.1, we then propose the heuristic (Procedure II) to compute the joint equilibrium. In Procedure II, Step 0 computes the traffic equilibrium given the current $\mathbf{M^r}$, and Step 1 adjusts $\mathbf{M^r}$ in the direction to satisfy conditions for trading equilibrium.

Procedure II: Computing the User Equilibrium with Trading

Initialization:

Input: N_i, $N_{a,i}$, $N_{t,i}$, $P_{t,i} = c_{t,i}\left(N_{t,i}\right)$, M, M^r;

Compute the $\bar{N}_{a,i}$ for each O-D pair.

Set $M_i^r = \frac{\bar{N}_{a,i}}{\sum_i \bar{N}_{a,i}} \cdot M^r$ as an initial solution.

Adjust the M_i^r:

Step 0:

Use **Procedure I** to compute the traffic equilibrium.

Set $k = 1$.

Step 1:

For every $i \in I$, calculate $p_i^p = P_{t,i} - P_{a,i}^r$; calculate $p^p = \frac{\sum_i p_i^p \cdot M_i^r}{M^r}$

Loop 1 (for every $i \in I$):

If $p_i^p \ge p^p$, $M_i^{r(k+1)} = \min\left\{\bar{N}_{a,i}, M_i^{r(k)} + \bar{N}_{a,i} \cdot \frac{1}{k} \cdot \left(\frac{p_i^p}{p^p} - 1\right)\right\}$, and let $i \in I'$;

Otherwise, $M_i^{r(k+1)} = M_i^{r(k)}$ and let $i \in I''$. (End of **Loop 1**)

Loop 2 (for every $i \in I$):

If $p_i^p \ge p^p$, $M_i^{r(k+1)} = M_i^{r(k)}$;

Otherwise, $M_i^{r(k+1)} = \frac{M_i^{r(k)}}{\sum_{j \in I''} M_j^{r(k)}} \cdot \left(M^r - \sum_{j \in I'} M_j^{r(k)}\right)$. (End of **Loop 2**)

Step 2:

If $\left|\frac{\sum_i p_i^p \cdot M_i^{r(k+1)} - \sum_i p_i^p \cdot M_i^{r(k)}}{\sum_i p_i^p \cdot M_i^{r(k)}}\right| < \varepsilon'$, stop, and let $M_i^r = M_i^{r(k+1)}$;

Otherwise, let $M_i^r = M_i^{r(k+1)}$, and $k = k+1$, then go to Step 1-0.

Note: $\varepsilon' = 10^{-7}$ is applied in this paper.

5. Numerical study

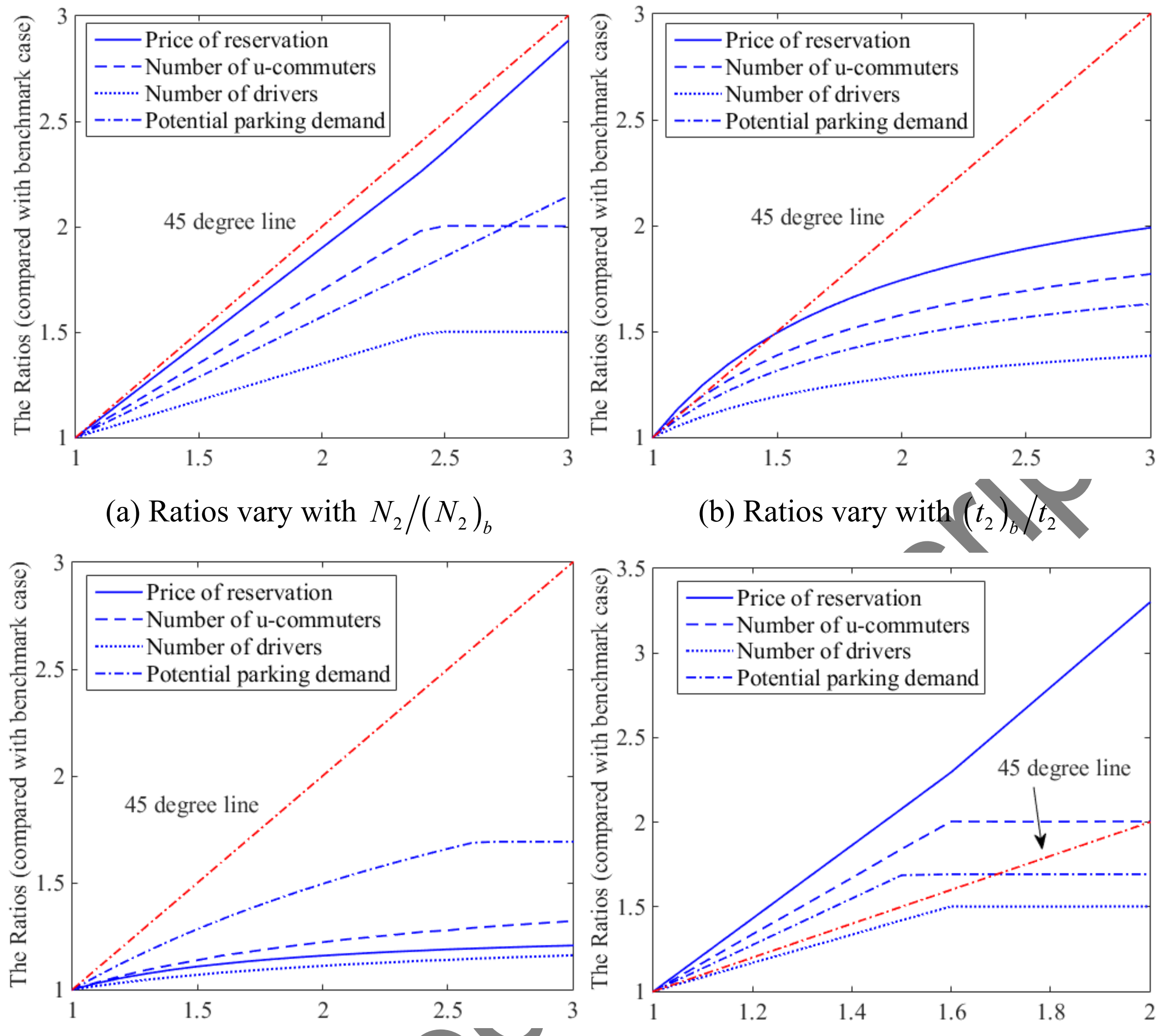
In this section, we will present some numerical analysis to illustrate the essential ideas in the paper. In our analysis, following Liu et al. (2015b), the value of time and schedule delay penalties are: $\alpha = 9.91\text{ (EUR\$/hour)}$, $\beta = 4.66\text{ (EUR\$/hour)}$, $\gamma = 14.48\text{ (EUR\$/hour)}$.

5.1. Two-to-one case

Firstly, we look at the two-to-one case, and we take the symmetric case as a benchmark, where $t_i = 25\text{(min)}$, $N_i = 2500$, $s_i = 30(veh/\text{min})$, $c_{t,i}\left(N_{t,i}\right) = c_{0,i} + c_{1,i} \times N_{t,i}$, $c_{0,i} = 6\text{ (EUR\$)}$ and $c_{1,i} = 0.001\text{ (EUR\$/person)}$ for $i = 1,2$. By "symmetric", we mean the characteristics and parameters for both O-D pairs are identical. The potential auto demand for each O-D pair is then $\bar{N}_{a,i} = 1477$. We consider $M = 2000$, and $M_i^r = 500$ for both $i = 1,2$. At the bi-modal equilibrium, $M_i^u = 500$, the price of reservation is $p^p = p_i^p = 2.39\text{ (EUR\$)}$ for both $i = 1,2$.

When all the parameters for O-D 1 are fixed, we now look at how changing those for O-D 2, i.e., N_2, t_2, s_2 and $c_{0,2}$, would affect the commuting equilibrium. Figure 3 shows how these four ratios, i.e., $p_2^p/\left(p_2^p\right)_b$, $M_2^u/\left(M_2^u\right)_b$, $M_2/\left(M_2\right)_b$ and $\bar{N}_{a,2}/\left(\bar{N}_{a,2}\right)_b$, vary with the ratios $N_2/\left(N_2\right)_b$ (in Figure 3(a)), $\left(t_2\right)_b/t_2$ (in Figure 3(b)), $s_2/\left(s_2\right)_b$ (in Figure 3(c)) and $c_{0,2}/\left(c_{0,2}\right)_b$ (in Figure 3(d)). Note that the subscript 'b' corresponds to the benchmark case (symmetric case).

Figure 3(a) shows that the four ratios increase if we increase the travel demand N_2. This indicates that if there is more travel demand from origin 2, the cost saving (value or price) of a reservation is higher, more commuters will choose to drive without reservation (however, it is upper bounded by 1000, and $M_2^u/\left(M_2^u\right)_b$ is upper bounded by $1000/500 = 2$ due to the parking limitation), total number of drivers (both those with and without reservation) increases (however, it is upper bounded as $M_2^r = 500$ and M_2^u is upper bounded by 1000), and the potential parking demand increases.



(a) Ratios vary with $N_2/(N_2)_b$ (b) Ratios vary with $(t_2)_b/t_2$

(c) Ratios vary with $s_2/(s_2)_b$ (d) Ratios vary with $c_{0,2}/(c_{0,2})_b$

Figure 3. Comparison of ratios of $p_2^p/(p_2^p)_b$, $M_2^u/(M_2^u)_b$, $M_2/(M_2)_b$ and $\bar{N}_{a,2}/(\bar{N}_{a,2})_b$

Figures 3(b), 3(c) and 3(d) show that the four ratios increase if we decrease the free flow time t_2, i.e., increase $(t_2)_b/t_2$, and if we increase the roadway capacity s_2, and if we increase fixed transit cost $c_{0,2}$. By decreasing t_2, increasing s_2, and increasing $c_{0,2}$, it means for commuters from origin 2, driving becomes more attractive (either driving becomes less costly or taking transit becomes more costly). As a result, the cost saving (value or price) of a reservation is higher, more commuters will choose to drive without reservation, total number of drivers (with or without reservation) increases, and the potential parking demand increases. Figure 3(b) shows that the increasing rate of these four ratios are decreasing as $(t_2)_b/t_2$ increases. Indeed, the four ratios are all upper bounded by the values when $t_2 \to 0$. When we increase s_2, the ratios $p_2^p/(p_2^p)_b$, $M_2^u/(M_2^u)_b$, $M_2/(M_2)_b$ are upper bounded by the values when $s_2 \to \infty$. Figure 3(c) shows that when $s_2/(s_2)_b \geq 2.5$, these three ratios almost remain constants.

Furthermore, the ratio of potential demand $\bar{N}_{a,2} / (\bar{N}_{a,2})_b$ is upper bounded by $1.69 = 2500/1477$ as $\bar{N}_{a,2} \le N_2 = 2500$. In Figure 3(d), it is shown that $p_2^p / (p_2^p)_b$ always increases as $c_{0,2} / (c_{0,2})_b$ increases. This is because, when transit becomes more costly, the saving from a reservation would be larger. In Figure 3(d), similar to those in Figure 3(a), $M_2^u / (M_2^u)_b$ is upper bounded by 2, and $M_2 / (M_2)_b$ is upper bounded by 1.5; and similar to that in Figure 3(c), $\bar{N}_{a,2} / (\bar{N}_{a,2})_b$ is upper bounded by 1.69.

When all parameters for O-D pair 1 are fixed, Figure 4(a) presents how increasing M_2^r would influence p_1^p, p_2^p, p_{ave}^p, where $p_{ave}^p = (M_1^r \cdot p_1^p + M_2^r \cdot p_2^p) / M^r$. Figure 4(b) shows how this would affect M_1^u and M_2^u, as well as M_1 and M_2. Figure 4(c) presents how travel costs vary with M_2^r. Figure 4(d) shows when commuters can trade their reservations, how the resulting equilibrium travel cost and the price of reservation will be different from those when trading is not allowed. Note that in the cases without trading of reservations, the average price $p_{ave}^p = (M_1^r \cdot p_1^p + M_2^r \cdot p_2^p) / M^r$ can be regarded as the average cost saving from a reservation (for all commuters). In the cases with trading, it is equal to the price of a reservation at the trading market, and also the cost saving by driving with a reservation.

As shown in Figure 4(a), as we increase M_2^r, the price (or value) of a reservation for commuters from origin 2, i.e., p_2^p, decreases. However, p_1^p decreases with M_2^r. This is because, at equilibrium, M_1^u decreases with M_2^r, as shown in Figure 4(b). It follows that, for commuters of O-D 1, traveling without reservations (either compete for parking or take transit) is more costly. Then the value of a reservation is larger. It is straightforward that p_{ave}^p is between p_1^p and p_2^p.

Figure 4(b) shows that not only M_1^u, but also M_2^u decreases with M_2^r, which is consistent with Proposition 2 in Section 3. Furthermore, in Figure 4(b), $M_2 = M_2^r + M_2^u$ increases with M_2^r. Note that, as shown in Figure 4(b), when $M_1^r = M_2^r = 500$ (the benchmark case), we have $M_1^u = M_2^u = 500$, and $M_1 = M_2 = 1000$. As shown in Figure 4(a) and Figure 4(c), the price of reservation, and the total travel cost of commuters are identical for each O-D pair.

Figure 4(c) shows the travel cost of commuters of O-D 1 increases with M_2^r, which is consistent with Proposition 3 in Section 3. As mentioned in Section 3, assigning more reservations to commuters from a specific origin does not necessarily reduce total travel cost of them. This is depicted in Figure 4(c) as travel cost of commuters of O-D 2 (the blue dash-dotted line) increases with M_2^r after it reaches its minimum at $M_2^r = 965$. In Figure 4(c), the red dashed line is the half of the total travel cost of all commuters, which is the average of the blue dotted line and blue dash-dotted line.

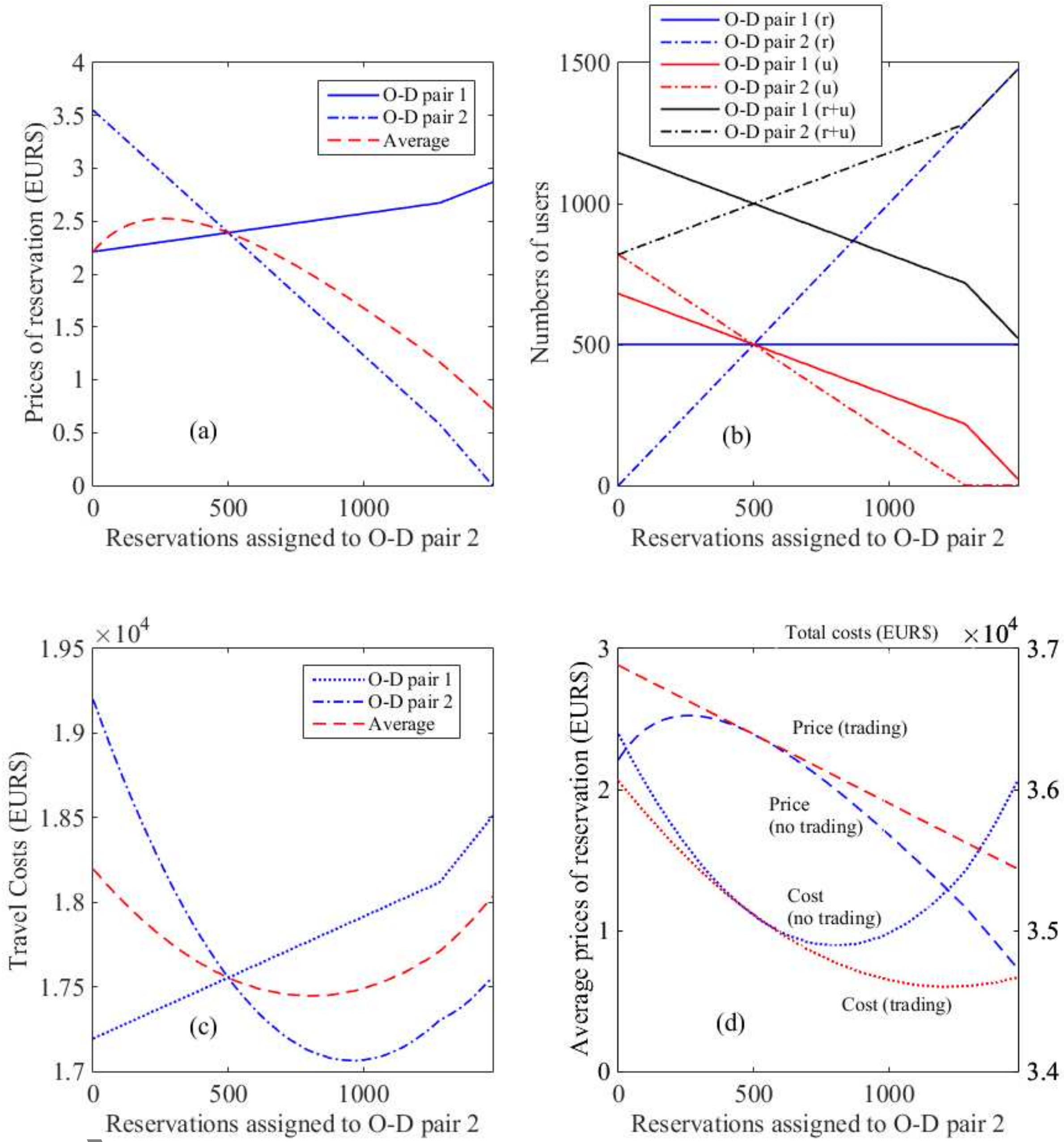


Figure 4. Equilibrium under varying M_2^r in the two-to-one case

Figure 4(d) further shows that if we allow travelers to trade their reservations, how the equilibrium travel cost and the average price of reservation vary with M_2^r. Note here, since all characteristics for the two O-D pairs are identical except the reservations, given the total number of reservations, the optimum (in terms of minimizing total travel cost) coincides with the trading equilibrium. Therefore, in Figure 4(d), the red dotted line (for trading equilibrium) is under the blue dotted line (for non-trading equilibrium). Furthermore, the price of reservation at the trading equilibrium is higher than the average price at the non-trading equilibrium (the red dashed line is above the blue dashed line), indicating averagely the cost saving through a reservation is higher in the trading equilibrium case.

Table 1. Parameters for three cases: symmetric, asymmetric 1 and asymmetric 2

	Symmetric case		**Asymmetric case 1**		**Asymmetric case 2**	
$c_{2,i}\left(N_{2,i}\right)$	$6.0+10^{-3}\times N_{2,i}$		$6.5+10^{-3}\times N_{2,i}$		$7.5+10^{-3}\times N_{2,i}$	
N_2	2500		2750		3000	
t_2 (min)	25		22		18	
s_2 (veh/min)	30		25		22	
$\bar{N}_{a,1},\bar{N}_{a,2}$	1477,1477		1477,1676		1477,2051	
Figure 5	(a)	(b)	(c)	(d)	(e)	(f)
M	1500	2500	1500	2500	1500	2500
M_1^r, M_2^r	0,0	0,0	0,0	0,0	0,0	0,0
$TC(UE)$	38750	36250	42722	40106	48474	45753
M_1^r, M_2^r	750,750	800,800	679,821	813,897	555,945	841,1099
$TC(SO)$	35522	33763	38776	36829	43178	40864
M_1^r, M_2^r	750,750	800,800	600,900	746,964	357,1143	682,1258
$TC(UET)$	35522	33763	38814	36855	43437	41031
M_1^r, M_2^r	750,750	1250,1250	605,895	1129,1371	388,1112	929,1571
$TC(SOB)$	34420	31508	37510	34071	41564	37301
$\frac{TC(UE)-TC(SO)}{TC(UE)-TC(SOB)}$	75%	52%	76%	54%	77%	58%

Note: All costs TC are in EUR$

Table 1 summarizes the parameters of O-D pair 2 for the examples in Figure 5 while those of O-D pair 1 are identical to the benchmark case. In Table 1, $TC(UE)$ is the total travel cost when reservation is not introduced, which corresponds to the point (0,0) in Figure 5; and $TC(SO)$ is the total travel cost when the optimal allocation is introduced given the parking capacity, which corresponds to the star-marked point in Figure 5; and $TC(UET)$ is the total travel cost when total number of reservations is identical to that under $TC(SO)$, but we allow travelers to trade, which corresponds to the circle-marked point in Figure 5; and $TC(SOB)$ is the minimum total travel cost can be achieved (congestion at highway is eliminated, the joint result of allocation of parking spaces among different O-D pairs and modal-split for each O-D pair are optimal).

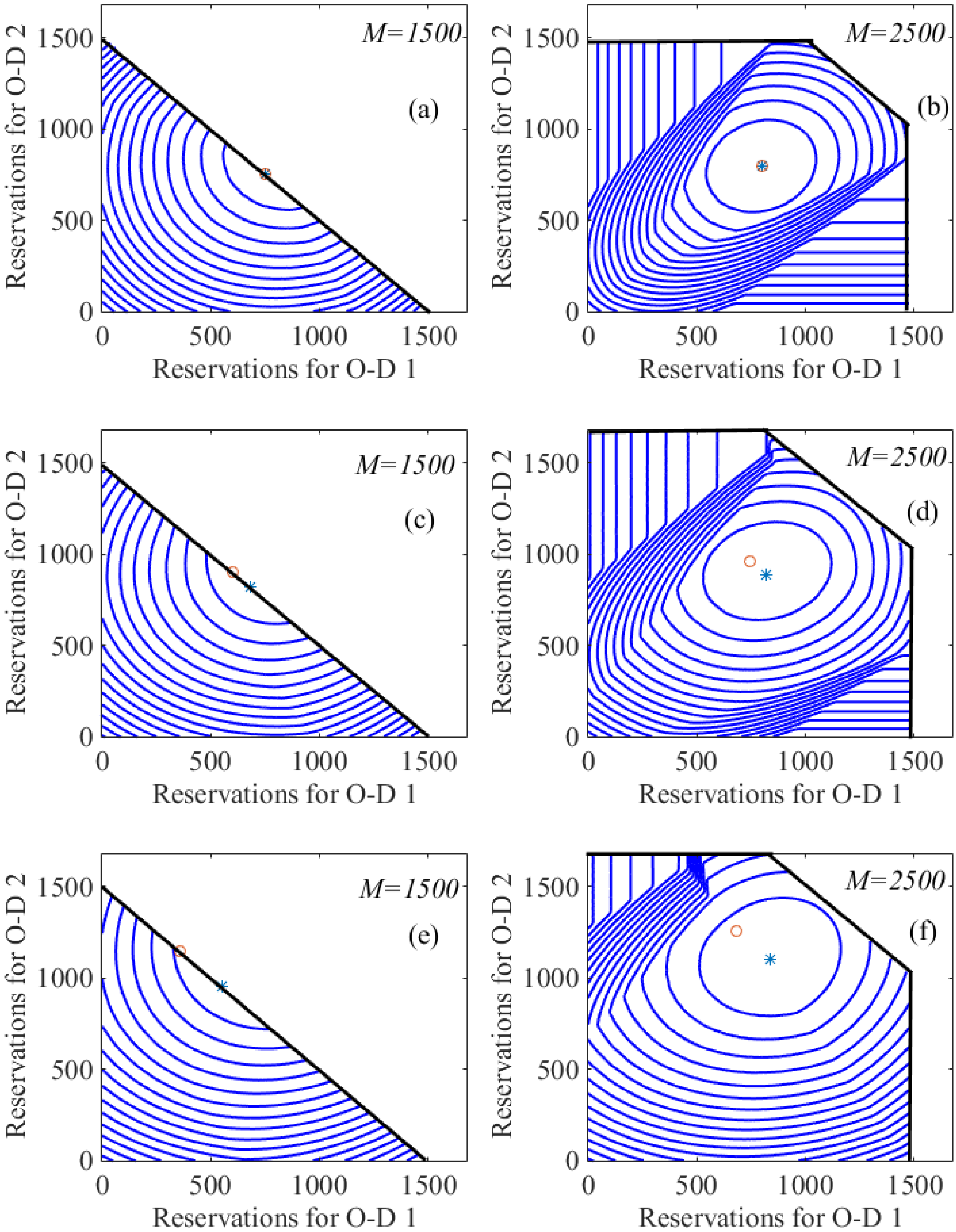


Figure 5. Total cost contours for six cases

For Figures 5(a), 5(c) and 5(e), parking capacity is relatively small, i.e., $M = 1500$, which is $0.51\bar{N}_a$ for symmetric case, $0.48\bar{N}_a$ for asymmetric case 1 and $0.43\bar{N}_a$ for asymmetric case

2. $TC(SO)$ is achieved at the boundary of the feasible domain of $\left(M_1^r, M_2^r\right)$, denoted by the star-marked point, indicating it is socially preferred to reserve all parking spaces to commuters to avoid costly competition for parking. However, in Figures 5(b), 5(d) and 5(f), parking capacity is relatively large although it is less than the potential demand, i.e., $M = 2500$, which is $0.85\bar{N}_a$ for symmetric case, $0.80\bar{N}_a$ for asymmetric case 1 and $0.71\bar{N}_a$ for asymmetric case 2. $TC(SO)$ is achieved at the interior of the feasible domain of $\left(M_1^r, M_2^r\right)$, denoted by the star-marked point, indicating it is socially preferred to retain some parking spaces open for competition to separate arrivals of r-commuter and u-commuters, and thus temporally relieve traffic congestion.

As we see from Table 1, from symmetric case $\rightarrow$ asymmetric case 1 $\rightarrow$ asymmetric case 2, the characteristics for the two O-D pairs become more asymmetric. It follows that the total cost contours in the feasible domain of $\left(M_1^r, M_2^r\right)$ become more asymmetric, as shown in Figure 5. We also note that, the combination of M_1^r and M_2^r at the trading equilibrium (the circle-marked point) moves further away from the optimal allocation (the star-marked point) as the two O-D pairs becomes more asymmetric. However, in the current example, even if the two O-D pairs are relatively asymmetric (Asymmetric case 2), the efficiency loss due to trading of reservations is relatively small for both $M = 1500$ and $M = 2500$, as one can see in Table 1 (43437 and 41031 for UET while 43178 and 40864 for SO). However, if the two O-D pairs become more asymmetric, the efficiency will be even larger. Later in the five-to-one network example, we will show the efficiency loss due to trading can be fairly large.

In Table 1, $\frac{TC(UE)-TC(SO)}{TC(UE)-TC(SOB)}$ is the ratio of travel cost reduction by the reservation system to the maximum travel cost reduction (the maximum potential), given the current travel demand, parking capacity, highway capacity and transit cost. We can see the reservation system is quite efficient, as in Table 1, $\frac{TC(UE)-TC(SO)}{TC(UE)-TC(SOB)}$ is all above 52%, and can be up to 77%. Furthermore, as the two O-D pairs become more asymmetric, the ratio $\frac{TC(UE)-TC(SO)}{TC(UE)-TC(SOB)}$ increases, i.e., $75\% \rightarrow 76\% \rightarrow 77\%$ for the case with $M = 1500$, and $52\% \rightarrow 54\% \rightarrow 58\%$ for $M = 2500$. Besides, $\frac{TC(UE)-TC(SO)}{TC(UE)-TC(SOB)}$ is larger for cases with smaller parking capacity. This is because, when parking capacity is smaller, the costly schedule delays due to competition for parking, which are the major inefficiencies, can be sharply reduced through reservation.

5.2. Five-to-one example

Now we present a numerical example of a five-to-one network for further insights. Table 2 summarizes characteristics for each O-D pair in such a network.

Table 2. Summary of values of parameters and variables

O-D specific Parameters	O-D pair				
	1	2	3	4	5
Travel demand N_i	3000	2000	3000	2000	2500
Free flow travel time t_i (min)	24	30	26	35	20
Highway capacity s_i (veh/min)	25	18	25	25	22
Fixed transit cost $c_{0,i}$ (EUR$)	5.5	6.0	5.8	6.0	6.5
Variable transit cost $c_{1,i}$ (10^{-3} EUR$/person)	1	1	1	0.5	1
Potential auto demand $\bar{N}_{a,i}$	1354	714	1345	428	1552

Table 3 further presents the results of the five-to-one network in nine situations. In Table 3, OUE corresponds to the case without parking spaces constraints, and UE corresponds to the case without reservation introduced (but with a parking space constraint), and SO is the case with optimal allocation of reservations among O-D pairs. When total number of reservations is equal to that for SO, UEP is the case that the numbers of reservations assigned to each O-D pair are proportional to their potential demands, and UET is when trading of reservations is allowed. Note that UE(1), SO(1), UEP(1) and UET(1) are for the case with $M = 4880$, while UE(2), SO(2), UEP(2) and UET(2) are for the case with $M = 2000$.

In Table 3, for each situation, we present the numbers and prices of reservations for each O-D pair and as well as the total travel cost. TC_{SOB} corresponds to the minimum travel cost given the travel demand, parking capacity, highway capacity and transit cost (generally cannot be achieved by reservation system only). Taking TC_{SOB} as the benchmark for efficiency analysis, we define the ratio $\theta = \left(TC(UE) - TC\right) / \left(TC(UE) - TC_{SOB}\right)$ to measure the relative efficiency of the specific situation. Note that $TC(UE) - TC$ is the travel cost reduction compared with the UE, and $TC(UE) - TC_{SOB}$ is the maximum travel cost reduction.

As shown in the third row of Table 3, given the same parking capacity M and M^r, for a specific O-D pair i, p_i^p decreases with M_i^r (for the three situations of SO, UEP and UET). Also, in three cases, i.e., SO(1), UEP(1) and UET(1), for O-D 4, $M_4^u = 0$ when a positive number of public parking spaces are open for competition, indicating competing for parking is too costly for commuters from O-D 4. In this case, O-D 4 belongs to group 1 where $t_{u,e}^i\left(0^+\right) > t_{end}$ as discussed in Section 3.1. When we look at O-D pair 5 in the case of UEP(1), we see that the total number of drivers is $M_5 = 916 + 636 = 1552$, which equals the potential demand in Table 2. In this case, O-D pair 5 then belongs to group 3. The current allocation of reservation assigns quite a lot reservations to commuters of this O-D pair, and those without reservation can always obtain a public parking space before the ending time as discussed in Section 3.1. Similarly, in the situation of UE(1), O-D pair 4 belongs to group 3. Given the parking allocation, for commuters from O-D pairs belong to group 3, the parking space constraint is not binding for them. Therefore, the travel cost of those with and without reservation is identical, which is shown in the fourth row of Table 3, and the value of a reservation is zero, which is shown in the third row.

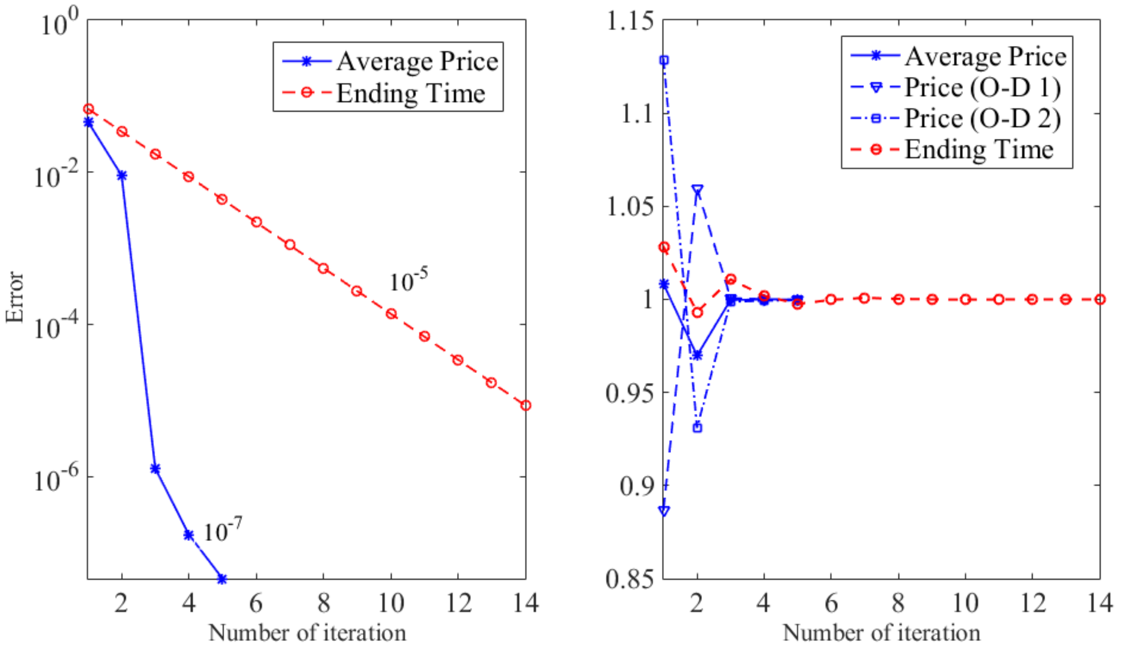
Table 3. Summary of different cases in the bi-modal five-to-one network

	OUE	UE(1)	SO(1)	UEP(1)	UET(1)	UE(2)	SO(2)	UEP(2)	UET(2)
M	5393	4880	4880	4880	4880	2000	2000	2000	2000
M^r	0	0	3183	3183	3183	0	2000	2000	2000
(p^p)	(0)	(1.49)	(1.36)	(0.84)	(0.77)	(3.40)	(2.79)	(2.77)	(2.69)
M_i^r, M_i^u	0,1354	0,1220	773,454	799,460	847,451	0,552	578,0	502,0	552,0
(p_i^p)	(0)	(1.85)	(1.49)	(1.30)	(0.77)	(3.98)	(2.56)	(2.85)	(2.69)
	0,714	0,662	574,108	421,146	502,133	0,146	216,0	265,0	86,0
	(0)	(0.92)	(0.49)	(1.10)	(0.77)	(2.90)	(2.13)	(1.92)	(2.69)
	0,1345	0,1212	772,447	794,454	845,444	0,545	583,0	499,0	543,0
	(0)	(1.83)	(1.47)	(1.33)	(0.77)	(3.96)	(2.55)	(2.83)	(2.69)
	0,428	**0,428**	**372,0**	**252,0**	**157,0**	0,0	25,0	159,0	0,0
	(0)	**(0.00)**	(0.16)	(0.50)	(0.77)	(1.22)	(1.15)	(0.77)	(1.22)
	0,1552	0,1362	692,686	**916,636**	833,668	0,757	599,0	576,0	820,0
	(0)	(2.87)	(2.47)	(**0.00**)	(0.77)	(4.94)	(3.50)	(3.58)	(2.69)
$P_{t,i}, P_{a,i}^u$	7.15	7.28	7.27	7.24	7.20	7.95	7.92	8.00	7.95
$(P_{a,i}^r)$	(7.15)	(5.43)	(5.78)	(5.94)	(6.43)	(3.96)	(5.32)	(5.14)	(5.26)
	7.29	7.34	7.32	7.43	7.37	7.85	7.78	7.74	7.91
	(7.29)	(6.42)	(6.83)	(6.33)	(6.59)	(4.96)	(5.66)	(5.82)	(5.24)
	7.46	7.56	7.58	7.55	7.51	8.25	8.22	8.30	8.26
	(7.46)	(5.76)	(6.11)	(6.22)	(6.74)	(4.29)	(5.66)	(5.47)	(5.57)
	6.79	**6.79**	6.81	6.87	6.92	7.00	6.99	6.92	7.00
	(6.79)	**(6.79)**	(6.65)	(6.37)	(6.15)	(5.78)	(5.84)	(6.15)	(5.78)
	7.45	7.64	7.62	**7.45**	7.50	8.24	8.40	8.42	8.18
	(7.45)	(4.77)	(5.15)	**(7.45)**	(6.73)	(3.30)	(4.90)	(4.84)	(5.49)
TC_i	21438	21841	20664	20682	20954	23843	22263	22561	22362
	14572	14676	14355	14400	14343	15707	15111	14963	15597
	22365	22764	21606	21598	21882	24765	23164	23490	23314
	13572	13572	13569	13621	13722	14000	13946	13720	14000
	18622	19096	**17346**	18620	**18106**	20607	18907	18997	18247
TC	90570	91949	87540	88922	89008	98922	93392	93730	93520
TC_{SOB}	81953	82701	82701	82701	82701	91866	91866	91866	91866
θ	-	0	**48%**	33%	**32%**	0	78%	74%	77%
Category	2,2,2,2,2	2,2,2,2,2	2,1,1,1,2	2,1,2,1,2	2,1,2,1,2	1,1,1,1,1	1,1,1,1,1	1,1,1,1,1	1,1,1,1,1

Again, we see the reservation scheme is efficient in reducing travel cost, as θ is above 32% for the six situations where reservation is introduced, and can be as large as 78%. However, when comparing SO(1) and UET(1), we see a efficiency loss of 16% due to trading of reservations. This means that we probably should prohibit commuters from trading their reservations. Furthermore, we see the major contributor of this loss is the increase of travel cost of commuters of O-D pair 5, as can be seen by looking at the increase of TC_5 (from 17346 to 18106). This increase is due to a more congested highway as travelers from this origin will buy reservations from other commuters, i.e., the increase in M_5^r ($833 > 692$) leads to an increase in total number of drivers ($833 + 668 > 692 + 686$).

5.3. Convergence of Procedure I and Procedure II

Figure 6 shows convergence of the proposed Procedure I and Procedure II in the two-to-one example. Figure 6(a) depicts the errors defined in Procedure I and Procedure II for computing traffic equilibrium (without trading), i.e., $\left|(\tau_l - \tau_e)/\tau_l\right|$, and for computing joint equilibrium of travel and trading, i.e., $\left|\frac{\sum_i p_i^p \cdot M_i^{r(k+1)} - \sum_i p_i^p \cdot M_i^{r(k)}}{\sum_i p_i^p \cdot M_i^{r(k)}}\right|$, against the number of iterations; while Figure 6(b) depicts how the prices of reservation and public parking space ending time evolve over iteration.



(a) Errors against iteration (b) Prices of reservation and ending time

Figure 6. Convergence of the proposed Procedures I and II

Note that when computing the joint equilibrium of travel and trading, in each iteration, we have to run Procedure I to compute the traffic equilibrium. Thus, there would be one error curve for ending time (red dashed line in Figure 6(a)) and curves of ending time (red dashed line in Figure 6(b)). In Figure 6, we choose only one as an illustrative example. The values in Figure 6(b) are normalized through being divided by the final solution, so that all the curves approaches 1 after certain numbers of iterations.

6. Concluding remarks

In this paper we examine how the binding parking capacity constraints re-shape the rush-hour traffic patterns in a bi-modal many-to-one network. Several properties of the commuting equilibrium with parking space constraints and parking reservation are discussed. Procedures for computing the dynamic user equilibrium with a parking space constraint (either trading of reservations is allowed or not) have been proposed.

Particularly, we show that parking reservation system can help reduce deadweight loss due to parking competition and roadway congestion. Furthermore, it is shown that assigning more reservations to travelers from a specific origin does not necessarily reduce total travel cost of them, while doing so can lead to an increase in travel cost of travelers from other origins. Our analysis also indicates that when parking supply is less than the potential demand but relatively large, it is socially preferred to retain some parking spaces open for competition. This is because, by retaining public parking spaces, we indeed separate departures and arrivals of those with and without reservations. Thus, traffic congestion is temporally relieved. However, when the total parking supply is relatively small, all parking spaces should be reserved to travelers to avoid costly competition for parking.

We also show that trading of reservations among travelers can yield an efficiency loss. As shown in the numerical example, this loss is less when the characteristics of all the O-D pairs are more similar. However, as shown in the five-to-one network example, the loss of efficiency can be up to 16%, thus trading should be prohibited. However, when the loss due to trading is relatively small, we may allow people to trade as prohibiting trading can be costly in practice.

While the current study adopts a network with multiple independent traffic corridors, further study may consider a linear corridor with multiple origins and one destination as in Tian et al. (2007). In this case, if we only consider single bottleneck congestion at the end of the corridor, the model would be very similar to the one-to-one traffic corridor case discussed in Section 2 and Yang et al. (2013) (the difference is that, commuters from different origins along the corridor will have different free-flow travel times). However, if we incorporate dynamic traffic congestion along the corridor as Arnott and de Palma (2011), the problem would be much more challenging, which is one of our priorities for future research. We may also analyze the impacts of limited parking in a general multi-modal network. We expect similar trends as those in this study even for a general network with more complex traffic dynamics. This is because, the qualitative impacts of parking limitation and parking reservation will not change, i.e., parking limitation forces people to travel earlier or shift to public transport; parking reservation offers flexibility in mode and departure time choices.

The current paper considers all the travelers as regular morning commuters. Future study may take into account two classes of travelers: regular and occasional commuters. In this case, parking operators might be more willing to provide parking reservation services to the regular customers (e.g., by charging less). We will explore how to set the differentiated reservation prices, and efficiently allocate the reservations among these two classes of travelers. Besides considering prices of parking, we may also link parking duration with scheduling of daily work activities, morning-evening commutes such as that considered in Zhang et al. (2005).

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Appendix. Proof of Proposition 2

Proof. We prove Proposition 2 by looking at the case that $|I_h| \geq 2$ where $h = 1,2,3$, while the other case where $|I_h| < 2$ is much simpler and Proposition 2 can be verified. Suppose there is a marginal increase $\Delta M_i^r > 0$ in M_i^r, which can be sufficiently small. Note that the case with $\Delta M_i^r < 0$ is similar.

Part (i): $i \in I_1$

Suppose $i \in I_1$, as $\Delta M_i^r > 0$ is small enough, then for $j \in I_1 \cup I_3$, $\Delta M_j^u = 0$. Now we look at those $j \in I_2$. Firstly, there must exist at least one O-D pair denoted by $k \in I_2$ such that M_k^u decreases, i.e., $\Delta M_k^u < 0$, otherwise we have the contradiction that $\sum_i \left(M_i^r + M_i^u\right) > M$.

We now show that if there exists an $k \in I_2$, thus $\Delta M_k^u < 0$, then for any $j \in I_2$, $\Delta M_j^u < 0$ by contradiction. Suppose for some j, we have $\Delta M_j^u \geq 0$. Since $\Delta M_k^u < 0$ and M_k^r remains constant, $M_k^r + M_k^u$ decreases. It follows that $c_{t,k}\left(N_k - M_k^r - M_k^u\right)$ increases. Further with Eq.(12) and $\Delta M_k^u < 0$, the ending time of public parking spaces t_{end} becomes earlier. For those j with $\Delta M_j^u \geq 0$, we have that $M_j^r + M_j^u$ increases or at least does not decrease, and $c_{t,j}\left(N_j - M_j^r - M_j^u\right)$ decreases or at least does not increase. With Eq.(12) and $\Delta M_j^u \geq 0$, the ending time of public parking spaces t_{end} becomes later or at least not earlier, which is a contradiction.

According to the above, we easily see that given $\Delta M_i^r > 0$, for any $j \in I$, we have $\Delta M_j^u \le 0$. As $\Delta M_i^r > 0$ can be sufficiently small, we have $dM_j^u / dM_i^r \le 0$. Furthermore, by noting the fact that $\sum_{j=1}^{n} dM_j^u / dM_i^r = -1$, we have $dM_j^u / dM_i^r \ge -1$.

Part (ii): $i \in I_2$

Suppose $i \in I_2$, as $\Delta M_i^r > 0$ is small enough, then for $j \in I_1 \cup I_3$, it can be verified that $\Delta M_j^u = 0$. Similarly, there must exist at least one O-D pair denoted by $k \in I_2$ such that M_k^u decreases, i.e., $\Delta M_k^u < 0$, otherwise we have the contradiction that $\sum_i \left(M_i^r + M_i^u \right) > M$. Given $\Delta M_k^u < 0$, we consider two possible cases: $k = i$ or $k \ne i$. Similar to the proof for Part(i), we can show that if there exists an $l \in I_2 \ne i$, thus $\Delta M_l^u < 0$, then for any $j \in I_2 \ne i$, $\Delta M_j^u < 0$ by contradiction. Now we discuss the two possible cases: $k = i$ or $k \ne i$.

If $k \ne i$, since $\Delta M_k^u < 0$, based on the analysis above, for any $j \in I_2 \ne i$, $\Delta M_j^u < 0$, and it follows that the ending time of public parking spaces becomes earlier. Given this result, it suffices to show that $\Delta M_i^u < 0$. Suppose $\Delta M_i^u \ge 0$, $M_i^r + M_i^u$ increases, and $c_{t,i}\left(N_i - M_i^r - M_i^u \right)$ will decrease. With Eq.(12) and $\Delta M_i^u \ge 0$, the ending time of public parking spaces becomes later, which is a contradiction, and $\Delta M_i^u < 0$ holds. This means for any j, whether equal to i or not, we have $\Delta M_j^u < 0$, and $dM_j^u / dM_i^r \le 0$.

If $k = i$, and $\Delta M_k^u < 0$. Given the above analysis, it suffices to show that for any $j \in I_2 \ne i$, $\Delta M_j^u \ge 0$ will lead to a contradiction. Suppose for some $j \ne i$, we have $\Delta M_j^u \ge 0$. Then, similarly, the ending time of public parking spaces becomes later or at least not earlier. Note that this ending time is also that for travelers from origin $k = i$, as $\Delta M_k^u < 0$, with Eq.(12), one can verify that $c_{t,k}\left(N_k - M_k^r - M_k^u \right)$ should decrease. It then follows that $M_k^r + M_k^u$ will increase, and note that this also means $\left| \Delta M_k^r \right| > \left| \Delta M_k^u \right|$. As for any $j \ne i$, we have $\Delta M_j^u \ge 0$ and M_j^r remains constant, thus $M_j^r + M_j^u$ will increase or at least do not decrease. We thus have $\sum_{j=1}^{n} \left(M_j^r + M_j^u \right) > M$, which contradicts the fact that $\sum_{j=1}^{n} \left(M_j^r + M_j^u \right) = M$. We then conclude that for any j, whether equal to $k = i$ or not, $\Delta M_j^u < 0$, and $dM_j^u / dM_i^r \le 0$.

In summary, given that $i \in I_2$ and $\Delta M_i^r > 0$, for any $j \in I$, we have $\Delta M_j^u \le 0$. As $\Delta M_i^r > 0$ can be sufficiently small, we have $dM_j^u \big/ dM_i^r \le 0$. Since $\sum_{j=1}^{n} dM_j^u \big/ dM_i^r = -1$, we then have $dM_j^u \big/ dM_i^r \ge -1$.

Part (iii): $i \in I_3$

Suppose $i \in I_3$, as the parking space constraint is not binding for commuters from origin i, it follows $M_i^u = \bar{N}_{a,i} - M_i^r$. As $\Delta M_i^r > 0$, we have $\Delta M_i^u = -\Delta M_i^r < 0$. For all other $j \ne i$, we have $\Delta M_j^u = 0$. It is straightforward to see $-1 \le dM_j^u \big/ dM_i^r \le 0$. This completes the proof. #

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