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Overcoming the Downs-Thomson Paradox by transit subsidy policies

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Abstract

Consider a competitive highway/transit transportation system in which travelers either drive on the bottleneck-constrained highway or take scheduled trains from home to the workplace in the morning peak hours. This paper explores the impact of bottleneck capacity expansion on transit operating schemes (fleet size and fare) and travelers' departure time and mode choices. Due to the potential occurrence of the Downs-Thomson (D-T) Paradox after highway capacity expansion, the paper investigates whether the D-T Paradox can be circumvented by implementing transit subsidy policies. The effects of different transit subsidy schemes are explored: subsidizing the transit company (cost subsidy) or the passengers (passenger subsidy) with the financial support from either government funding or road pricing revenue. For each combination of subsidy method and financial sourcing, the condition for overcoming the D-T Paradox is established.

Keywords: Transit operating schemes, Downs-Thomson Paradox, Departure time and mode choices, Transit subsidy, Road pricing

1. Introduction

Downs (1962) remarked on the possibility that adding road capacity cannot relieve traffic congestion at all. Retiming of trips, route shifting, and mode shifting from public transportation modes have been observed after the road expansion and are attributed for its failure. In an independent study, Thomson (1977) noted that driving and transit are close substitutes in many cities. The increase of road capacity, by attracting enough transit riders to drive, could lead to a decline in transit service or rises in fares due to the scale economies of transit service. The users of both modes may end up worse off. This phenomenon is then analytically formulated by Mogridge et al. (1987) and is named the "The Downs-Thomson Paradox (D-T Paradox)" thereafter. Furthermore, many other paradoxes have been studied in the transportation literature, including Braess paradox (Braess, 1968, 2005; Lin and Lo, 2009; Zhao et al., 2014; Di et al., 2014; Zverovich and Avineri, 2015; Hwang and Cho, 2015; Wang et al., 2016), emission paradox (Nagurney, 2000), capacity paradox (Yang and Bell, 1998),

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stochastic assignment paradox (Sheffi and Daganzo, 1978; Yao and Chen, 2014), transit assignment paradox (Szeto and Jiang, 2014) and so on. This paper focuses on examining the D-T Paradox introduced by Downs (1962) and Thomson (1977).

A substantial literature has been developed both analytically and empirically to investigate when and why the D-T Paradox occurs; see Arnott and Small (1994) and Zhang et al. (2014, 2016) for synthesis reviews. In particular, much attention is paid to the mode shifting between transit and driving and how transit service changes when the road is expanded. For example, Mogridge (1997) suggested that the transit service should be improved with the highway capacity expansion to raise commuters' travel utility in the two-mode system. Similarly, Arnott and Yan (2000) showed that the travel cost might increase after the expansion of highway capacity if the transit operator does not consider the impacts of its decision on highway travelers. It is further suggested by Basso and Jara-Díaz (2012) that the occurrence of the D-T Paradox highly depends on the status quo of the transit side. In the same spirit, Zhang et al. (2014, 2016) analyzed the effects of road expansion when the transit service is operated in different administrative regimes. Zhang et al. (2014, 2016) showed that the paradox is likely to occur when the transit operator is self-sustaining and the two modes are perfect substitutes. Based on these studies, this paper takes one step further to re-examine the D-T Paradox in a more general setting where trip retiming, crowding effect, and policy remedies are in effect.

First, previous studies on the D-T Paradox only consider travelers' mode shifting while the effect of trip retiming is neglected. However, the benefits from urban highway expansion would generally be underappreciated if shortening of the rush hour is not noticed. To explore this, the direct benefit of shortening rush hour and indirect adverse impacts from induced travel demand need to be compared. This paper formulates the bi-modal equilibrium following Kraus (2003, 2012) in which travelers either drive on the bottleneck-constrained highway or take scheduled trains, and each traveler makes decisions of his/her departure time choice and mode choice simultaneously to minimize the total travel cost.

Second, the discomfort in mass transit during the peak hour is a noticeable factor that influences individual's departure time and mode choices which is under-investigated by the studies on the D-T paradox. Crowdedness both at train stations and in the carriages could bring passengers unpleasant travel experiences, and a bunch of literature has been developed to investigate its impact on people's travel, e.g., Horowitz and Sheth (1977), Huang (2000, 2002), Tian et al. (2007, 2009), Wang et al. (2014), Tian and Huang (2015), Lu et al. (2015). Particularly, previous studies (e.g., de Palma et al., 2015; Zhang et al., 2015) have shown that the crowdedness in the transit mode will lead to a tendency that transit users depart early or late to avoid the over-crowdedness or switch to driving. This paper investigates the D-T Paradox by taking into account the effect of body congestion in carriages on travelers' departure time and mode choices following the framework of Wu and Huang (2014).

Moreover, while existing literature mostly focuses on the occurrence of the paradox, this study attempts to find the way out, i.e., how to overcome the D-T paradox. The major objective of this study is to bridge this research gap through specifically studying the prevention of the D-T Paradox with various transit subsidy policies while travelers' departure time choice and crowding effects in transit services are explicitly considered. If the transit operation is subsidized by the government funding or the revenue of road toll levied from car usage, the adjustment of the transit operator in response to the expansion of highway capacity (raise fare or reduce frequency or both) would change, then the D-T Paradox may be avoided.

In many countries and regions, different types of transit subsidy policy have been proposed, i.e., allowing commuters to deduct monetary expenses of taking public transport from the income tax liability, subsidizing transit fares, reducing indirect tax rates for transit company and so on. To date, there is no standard definition of transit subsidies in the existing research works, all of the above examples can be seen as transit subsidies (Tscharaktschiew and Hirte, 2012). OECD (2005) states that "*a subsidy in general is a result of a government action that confers an advantage on consumers or producers in order to supplement their income or lower their costs*". Frankena (1981) proposed three kinds of transit subsidy policy: (i) one-time subsidy - amount of money which is exogenously determined and independent on the operating performance of transit company, (ii) cost subsidy - amount of money which is dependent on the operating cost of transit company, and (iii) passenger subsidy - amount of money which is exogenously determined and distributed to each transit commuter. In this paper, we focus on the latter two transit subsidy schemes. That is, we consider two classes of transit subsidy policy: cost subsidies which finance the transit company to keep within budget and passenger subsidies which subsidize the transit commuters to reduce travel expenses. Specifically, we examine the impact of cost or passenger subsidies, which source from either the government funding or the road pricing revenue, on the transit operating strategies and travelers' departure time and mode choice behavior. For each combination of transit subsidy and financial sourcing, the conditions for overcoming the D-T Paradox are established. The framework is shown in Fig. 1. Responsive transit service 1, 2, and 3 respectively refers to the scenario where 1) the transit mode is not subsidized, 2) the transit mode is under cost subsidy, and 3) the transit mode is under passenger subsidy. This study provides a better understanding on how to avoid the occurrence of D-T Paradox with the help of transit subsidy policies, and is a significant extension of the existing two-mode problem and D-T Paradox studies.

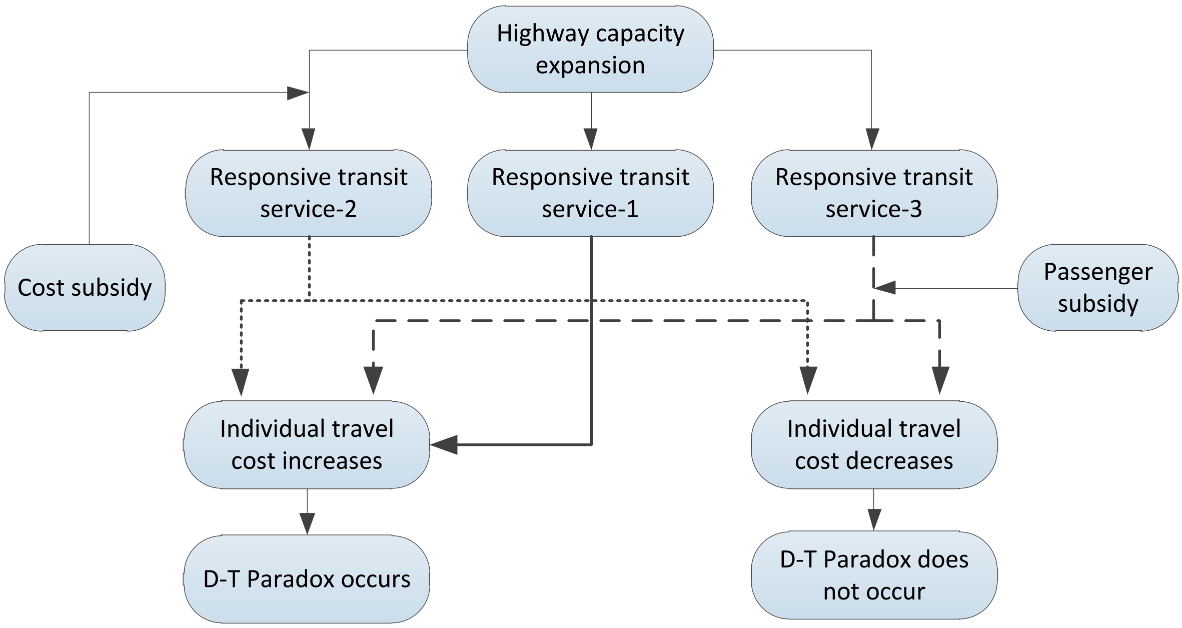

Fig. 1. The impact of transit subsidy policies on the D-T Paradox.

This article is structured as follows. The bi-modal equilibrium and the occurrence of the

D-T Paradox are proposed in Section 2. Section 3 provides alternative transit subsidy policies to overcome the D-T Paradox. Section 4 presents numerical results for understanding theoretical results. Conclusions are given in Section 5.

2. Bi-modal equilibrium and Downs-Thomson Paradox

2.1. Basic model

Consider the case when there are two modes of travel between the residential area and the working place. Mode 1 represents the highway with a single bottleneck, which has a deterministic capacity. Mode 2 denotes the mass transit (i.e., railway). Commuters know the generalized travel cost for each mode well and choose the travel mode and departure time from home in the morning rush hours. The essential notations used throughout the paper are listed below.

V	the number of commuters
V_1	the number of auto commuters
V_2	the number of transit commuters
α	unit cost of travel time
β	schedule penalty for a unit time of early arrival
γ	schedule penalty for a unit time of late arrival
$\delta \equiv \frac{\beta\gamma}{\beta+\gamma}$	
s	bottleneck capacity
s_0	original bottleneck capacity
t	departure time from home
t^*	work starting time
t_s	the time at which the queue on the highway begins
t_e	the time at which the queue on the highway ends
$\bar{t}$	departure time at which an auto users arrives on time t^*
$r_1(t)$	departure rate of auto commuters
$T(t)$	the commuting time of an auto user who departs at time t
$k(t)$	the fine/time-varying toll
$C_1(t)$	generalized cost of an auto user who departs at time t

T_1	free-flow travel time on highway
τ_1	monetary cost of an auto commuter
T_2	on-train moving time
τ_2	transit fare
$r_2(t)$	departure rate of commuters on the railway
t_s'	the time at which train commuters start to departure
t_e'	the time at which train commuters end to departure
m	the number of scheduled trains
λ	time interval between successive trains during time window $[t_s, t_e]$
φ	unit cost of discomfort generated by the body congestion in carriages
$C_2(t)$	generalized cost of a train commuter who departs at time t
$\tilde{t}$	departure time at which train commuters can arrive punctually t^*
c	variable cost of one transit passenger
R	variable cost of one dispatching train
F	the fixed cost of transit, which consists of facility costs and constant maintenance cost
Z	profit of transit operator
N	total revenue from road toll

The basic assumptions used in this paper are made in the following.

A1. The relation $\gamma > \alpha > \beta$ holds according to Small (1982).

A2. During peak hour the time interval of dispatching trains λ would be very short, and the waiting costs of all users at the train station are then very small and would be neglected (Wu and Huang, 2014).

A3. During the train intervals users are observed continually arriving at the station (de Palma et al., 2015).

A4. Both mode 1 and mode 2 would be all used, which is reasonable in reality (Huang, 2000, 2002; Zhang et al., 2014; Wu and Huang, 2014).

A5. The body congestion cost is dependent on passenger volumes in carriages and the train moving time (Huang, 2000; Tian and Huang, 2015).

For the auto mode, due to the physical restriction of the highway capacity, traffic congestion occurs before the bottleneck facility. Some auto commuters arrive at the workplace earlier or later than the work starting time t^* to experience less queuing time before the bottleneck. Assume that commuters' generalized travel cost is comprised of in-vehicle travelling time, the schedule delay and monetary cost. The generalized travel cost for each auto commuter is then derived as:

$$C_1(t)=\begin{cases}\alpha T(t)+\beta\left(t^*-t-T(t)\right)+\tau_1, & t\in\left[t_s,\bar{t}\right]\\ \alpha T(t)+\gamma\left(t+T(t)-t^*\right)+\tau_1, & t\in\left[\bar{t},t_e\right]\end{cases} \tag{1}$$

Suppose that auto commuters minimize their generalized travel cost by choosing the departure time from home. In equilibrium, each auto commuter is unable to improve the travel utility through unilaterally changing the departure time. Namely, $C_1(t)$ should be the same for all time $t\in\left[t_s,t_e\right]$. It is easy to derive the departure time at which the queue begins (t_s), queue ends (t_e) and an individual arrives at the destination punctually ($\bar{t}$), respectively, as well as the departure rate from home ($r_1(t)$) and the generalized travel cost of each auto commuter:

$$t_s=t^*-\frac{\delta V_1}{\beta s}-T_1,\quad t_e=t^*+\frac{\delta V_1}{\gamma s}-T_1,\quad \bar{t}=t^*-\frac{\delta V_1}{\alpha s}-T_1 \tag{2}$$

$$r_1(t)=\begin{cases}\dfrac{\alpha s}{\alpha-\beta}, & t\in\left[t_s,\bar{t}\right]\\ \dfrac{\alpha s}{\alpha+\gamma}, & t\in\left[\bar{t},t_e\right]\end{cases} \tag{3}$$

$$C_1=\alpha T_1+\tau_1+\frac{\delta V_1}{s} \tag{4}$$

Interested readers can refer to Vickrey (1969), Arnott et al. (1990a) and Tabuchi (1993) for detailed calculation of these results.

For the transit mode, commuters' discomfort arises as the crowdedness increases at train stations and in carriages. Thus, travelers may depart earlier or later from home to avoid the heavy body congestion. The generalized cost per train commuter consists of in-vehicle travelling time, schedule delay, monetary travel cost and the crowding cost, which can be derived as follows:

$$C_2(t)=\begin{cases}\alpha T_2+\tau_2+\beta\left(\tilde{t}-t\right)+\varphi\lambda T_2 r_2(t), & t\in\left[t'_s,\tilde{t}\right]\\ \alpha T_2+\tau_2+\gamma\left(t-\tilde{t}\right)+\varphi\lambda T_2 r_2(t), & t\in\left[\tilde{t},t'_e\right]\end{cases} \tag{5}$$

where $\tilde{t}=t^*-T_2$.

At equilibrium, each train commuter cannot select a departure time which could further improve his or her travel utility. That is, $C_2(t)$ should be the same for all time during $\left[t'_s,t'_e\right]$. According to Wu and Huang (2014), we can obtain the train commuters' departure time window ($[t_s', t_e']$), the departure time at which passengers can arrive punctually ($\tilde{t}$), the departure rate from home ($r_2(t)$) and the generalized travel cost per train commuter:

$$t_s' = \tilde{t} - \frac{\sqrt{2\delta\varphi\lambda T_2 V_2}}{\beta}, \quad t_e' = \tilde{t} + \frac{\sqrt{2\delta\varphi\lambda T_2 V_2}}{\gamma}, \quad \tilde{t} = t^* - T_2 \tag{6}$$

$$r_2(t) = \begin{cases} \dfrac{\beta(t - t_s')}{\varphi\lambda T_2}, & t \in [t_s', \tilde{t}] \\ \dfrac{\gamma(t_e' - t)}{\varphi\lambda T_2}, & t \in [\tilde{t}, t_e'] \end{cases} \tag{7}$$

$$C_2 = \alpha T_2 + \tau_2 + \sqrt{2\delta\varphi\lambda T_2 V_2} \tag{8}$$

Given the relationship of the number of scheduled trains (m) and the time interval between successive trains (λ) during $[t_s', t_e']$, i.e., $m\lambda = t_e' - t_s'$ (Wu and Huang, 2014), combining Eq. (6) and $m\lambda = t_e' - t_s'$ leads to:

$$m = \sqrt{\frac{2\varphi T_2 V_2}{\lambda\delta}}, \quad \lambda = \frac{2\varphi T_2 V_2}{m^2\delta} \tag{9}$$

Integrating Eqs. (8) and (9), we can re-obtain the generalized travel cost per train commuter:

$$C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m} \tag{10}$$

Using Eq. (10), we also re-obtain:

$$t_s' = \tilde{t} - \frac{2\varphi T_2 V_2}{m\beta}, \quad t_e' = \tilde{t} + \frac{2\varphi T_2 V_2}{m\gamma} \tag{11}$$

$$r_2(t) = \begin{cases} \dfrac{m^2\delta\beta(t - t_s')}{2\varphi^2 T_2^2 V_2}, & t \in [t_s', \tilde{t}] \\ \dfrac{m^2\delta\gamma(t_e' - t)}{2\varphi^2 T_2^2 V_2}, & t \in [\tilde{t}, t_e'] \end{cases} \tag{12}$$

The bi-modal equilibrium can be achieved when no commuter is able to improve the travel utility by means of unilaterally switching to the other travel mode. Therefore, the equilibrium travel cost for each mode are the same, i.e., $C_1 = C_2$.

2.2. Downs-Thomson Paradox

Zhang et al. (2014) generally defined the occurrence of the D-T Paradox in the following:

Definition 1. The D-T Paradox is defined to occur at a given equilibrium point of $s = s_0$ if

$$\left.\frac{dC_{1/2}}{ds}\right|_{s=s_0} > 0 \tag{13}$$

when there is a small increase in the road capacity.

If Eq. (13) holds at any $s \in [s_0, s_1]$[1], the equilibrium travel cost monotonically increases with the road capacity. Then, the D-T Paradox continuously occurs in $s \in [s_0, s_1]$.

Fig. 2 presents the fundamental analysis of occurrence of the D-T Paradox using a diagram similar to Mogridge (1997). The difference is that in Mogridge's (1997) analysis, the transit travel cost includes the waiting time cost. Thus increasing service frequency would reduce the average waiting cost and thus lead to lower travel cost and large patronage. In this study, the waiting for transit is excluded while the crowding effect is added. Therefore, denser service on one hand reduces the density of passengers in each train and on the other increases the total patronage. According to Eq. (10), the change of transit cost is determined by V_2/m.

In Fig. 2, the distance between the horizontal axes is the total number of commuters and the auto travel cost is measured by the axis on the left-hand side and the transit travel cost is measured by the right one. When capacity increases from s_0 to s_1, the auto cost curve pivots clockwise from $C_1(s_0)$ to $C_1(s_1)$. The slop of transit cost depends on how transit operator adjusts its service. If the transit service remains the same (i.e., m is unchanged), the transit cost increases with the number of transit commuters. In this case, curve $C_2(s)$ is downward-sloping (represented by the solid curve $C_2(s)$), and the resulting equilibrium cost $C_E(s_1)$ after capacity expansion is less than the original $C_E(s_1)$. In another case where the transit operator reduces the service frequency as patronage increases, (i.e., the change direction of m is opposite to V_2), the transit cost C_2 increases even faster with V_2, and the equilibrium cost is lower under s_1 than s_0.

However, if the transit service frequency and patronage move in the same direction as in Mohring's (1972) classic model, the slope of transit cost curve depends on V_2/m. When the change in m is sharper than V_2, the transit cost curve would be upward-sloping (represented by the dashed curve $C_2'(s)$). In other words, as long as the transit service exhibits scale economies (the transit travel cost decreases with the patronage), and the new

[1] s_0 and s_1 are the highway capacities before and after expansion $(s_0 < s_1)$.

equilibrium cost $C'_E(s_1)$ is greater than the original $C'_E(s_0)$. Without considering departure time choices, Zhang et al. (2014) have proved that when the transit operator is a self-financing welfare-maximizer, the transit cost curve is upward-sloping and the D-T paradox occurs whenever the highway is expanded. In contrast, when the transit operator is a profit-maximizer, the curve is downward-sloping and the D-T paradox never occurs. This paper further examines how the transit service is adjusted when the departure equilibrium is included and how to overcome the D-T paradox.

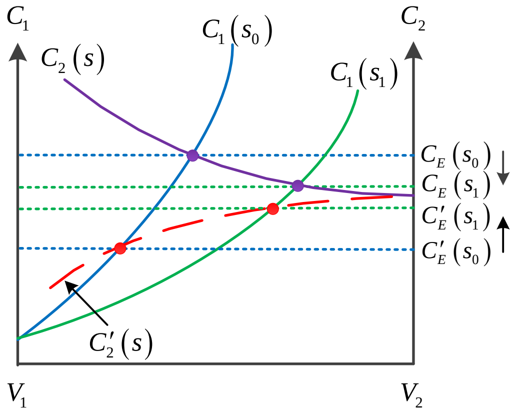

Fig. 2. The fundamental of the D-T Paradox.

2.3. The occurrence of Downs-Thomson Paradox with no transit subsidy

This section explores the effect of bottleneck capacity expansion on the departure and mode choice equilibrium, how transit operator adjusts the service and fare, and the occurrence of the D-T Paradox. As the benchmark, no transit subsidy is included. We can prove that the D-T paradox will not occur if the transit operator is a profit-maximizer. The proof can be referred to in Appendix A. As the major objective is to investigate how the D-T paradox can be avoided, we will not extend the analysis for this scenario; instead, the focus will be given to the scenario where the transit operator is a self-financing welfare-maximizer in which the D-T paradox is likely to occur. The objective of the transit operator is to minimize individual travel cost (ITC) while maintaining breakeven. The transit operator determines the transit fare and the number of scheduled trains (departure interval) to achieve its goal. For any given bottleneck capacity s, the transit operation strategy is determined as follows:

ITC-1:

$$\min_{\tau_2, m} C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m} \tag{14}$$

s.t.

(4) and (10)

$$(\tau_2 - c)V_2 - mR - F = 0 \tag{15}$$

$$C_1 = C_2 \tag{16}$$

$$V_1 + V_2 = V \tag{17}$$

The objective (14) is to minimize the ITC. Eq. (15) is the zero-profit constraint, the transit fare and the number of scheduled trains are set to ensure breakeven. Eqs. (4), (10), (16) and (17) are the bi-modal equilibrium conditions. The above problem has at least one optimal solution due to the continuity of the objective function and the compactness of the constraints. The uniqueness of the optimal solution is proved in Appendix B. Then we can get the following proposition:

Proposition 1. If the transit operator determines the transit fare and the number of trains under different values of bottleneck capacity to minimize ITC with a self-financing constraint, then when the bottleneck capacity increases,

(i) the optimal transit fare monotonically increases;
(ii) the optimal number of trains monotonically decreases;
(iii) the number of transit passengers monotonically decreases;
(iv) the ITC monotonically increases, indicating that the D-T Paradox happens.

Proof. The proof is given in Appendix C.

Proposition 1 indicates that when the bottleneck capacity increases, the transit side would lose passengers and cannot keep self-sustaining. Then, the transit operator needs to raise the fare and reduce the number of trains to maintain breakeven (Proposition 1(i)(ii)). Such service adjustment would lead to even fewer passengers (Proposition 1(iii)) and even more congested highway. At last, the generalized travel cost of both modes would increase after the capacity expansion, and thus the D-T Paradox happens (Proposition 1(iv)). Fig. 3 shows the chain reaction. This means that the highway capacity expansion could make all the commuters worse-off when the transit operator is a self-financing welfare-maximizer. By incorporating travelers' departure time choice, Proposition 1 proves that the Proposition 4 in Zhang et al. (2014) is valid in the more general setting.

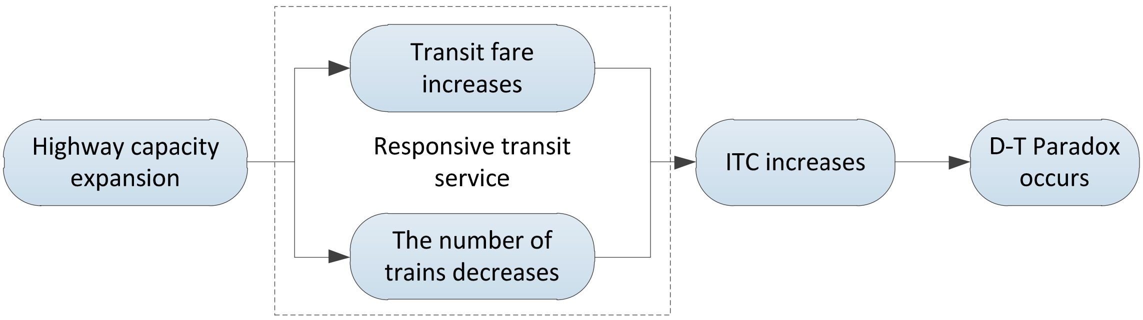

Fig. 3. The chain reaction after the highway capacity expansion.

3. The prevention of Downs-Thomson Paradox with transit subsidy policies

In this section, we investigate the effect of transit subsidy policies on transit authority's operating schemes and the prevention of D-T Paradox under highway capacity expansion. Two classes of transit subsidy policy are considered: (i) cost subsidies which aim to finance the transit company to cover costs and (ii) passenger subsidies which are provided directly to passengers by reducing the fares. A transit subsidy policy would result in higher transit service level, lower travel cost, and more ridership, which may effectively avoid the occurrence of the undesirable D-T Paradox under highway capacity expansion.

3.1. Cost subsidy

3.1.1. Government's financial subsidy

Transit subsidies in most countries have grown faster than almost any other type of government expenditure during the last few decades (Basso et al., 2011; Basso and Jara-Díaz, 2012). Undoubtedly, the public transit subsidies can support transit authority to maintain or expand services. Under the subsidies from government, the transit authority's operating schemes may change, which consequently affects travelers' mode choice behaviors. In the end, the D-T Paradox may be avoided. With the constant financial assistance provided by the government (denoted by B), the transit operator determines the transit fare and the number of trains to maintain breakeven:

$$(\tau_2 - c)V_2 - mR - F + B = 0 \tag{18}$$

For any given bottleneck capacity, if the subsidized transit operator optimizes the fare and number of trains to minimize ITC while maintaining breakeven, we have the following minimization problem:

ITC-2:

$$\min_{\tau_2, m} C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m}.$$

subject to Eqs. (4), (10), (16-18).

Proposition 2. If the ITC-minimizing and subsidized transit operator adjusts the fare and number of trains to ensure the operation breakeven in response to the varying bottleneck capacity, then when the bottleneck capacity increases,

(i) the optimal number of trains monotonically decreases;
(ii) the optimal transit fare monotonically decreases if $B > F$ and increases if $B < F$;
(iii) the number of transit commuters monotonically decreases;
(iv) the ITC monotonically decreases if $B > F$ (implying the D-T Paradox does not occur, if the government subsidy is greater than the fixed cost for providing the trains service like the costs for construction and rolling stock) and increases if $B < F$ (indicating the D-T Paradox occurrence when the subsidy is smaller than the fixed cost of train service).

Proof. The proof is given in Appendix D.

Proposition 2(i) states that as the bottleneck capacity increases, the transit operator would reduce the number of trains to keep no-deficit even with the cost subsidy, which is mainly due to the decreasing transit passengers (Proposition 2(iii)). When the cost subsidy (B) is smaller than the fixed cost of transit company (F), the optimal transit fare after subsidization increases with the bottleneck capacity, then the ITC goes up and the D-T Paradox happens. However, as long as B is larger than F, the optimal transit fare decreases with the bottleneck capacity, then the ITC declines and the D-T Paradox is eliminated (Proposition 2(ii)(iv)). This is because the transit operator has to balance the trade-off between covering cost and minimizing ITC when the highway capacity is expanded. The transit operator has to raise the fare to maintain no-deficit when its fixed cost is partly subsidized ($B < F$). But once its fixed cost is entirely subsidized (i.e., $B > F$), the transit operator can reduce the fare to minimize ITC. Furthermore, the optimal number of scheduled trains decreases in proportion to transit ridership, thus ITC is mainly dependent on the transit fare.

Proposition 2(i)(ii) also shows that the change direction of transit fare (number of trains) can (cannot) be influenced by the concerned transit subsidy policy. Proposition 2(iii) establishes the intuitive result that when the transit operator is minimizing ITC, it cannot offset the loss of ridership by adjusting either the number of scheduled trains or transit fare, or both. Proposition 2(iv) indicates that the transit subsidy policy can effectively eliminate the D-T Paradox as long as the cost subsidy is large enough to cover the fixed cost for providing train service which includes the costs for construction and rolling stock. However, one obvious weakness of this direct cost subsidy policy is that the government has to bear high level of financial expenditure other than the investment on the highway capacity expansion.

3.1.2. Subsidy financed by road toll revenue

In the previous subsection, one can find that the government's direct financial subsidy to the transit company could effectively avoid the D-T paradox. However, the government needs to spend more financial expenditure for subsidy in addition to the highway capacity expansion investment, which would bring huge financial burden to the government. In this section, we assume that the government would not use direct government expenditure for the transit subsidy. Alternatively, the government can levy road toll to the highway users to reduce the bottleneck congestion, and at the same time use the revenue collected from road tolls to subsidize the transit company (Huang, 2000). In this study, we would examine that, under what conditions, the D-T paradox can be avoided, if the road toll revenue is used for transit subsidy. We assume that all the toll revenue would be used to subsidize the transit authority. It should be noted that road toll is imposed to reduce bottleneck congestion (or eliminate the queue) and the toll revenue is endogenously determined by the travel equilibrium achieved in the two-mode system.

It is known that the queuing time on the highway is a significant loss, so that at the social optimum there is no queue and hence total queuing time cost is zero. The social optimum can be achieved by employing the following fine/time-varying toll:

$$k(t)=\begin{cases}\dfrac{\delta V_1}{s}-\beta\left(t^*-T_1-t\right), & t_s\le t<\overline{t}\\ \dfrac{\delta V_1}{s}, & t=\overline{t}\\ \dfrac{\delta V_1}{s}-\gamma\left(t+T_1-t^*\right), & \overline{t}\le t<t_e\end{cases}$$

Under the fine toll, the departure rate of auto commuters is the same with the bottleneck capacity within $[t_s, t_e]$, then the queuing will be eliminated. Correspondingly, the total revenue from fine toll is $N=\dfrac{\delta V_1^2}{2s}$. However, the fine toll does not change the private travel cost (an individual's queuing time in the no-toll equilibrium is replaced by the fine toll in the social optimum), thus it does not affect the modal choice[2].

With the transit subsidy generated by the fine toll on the highway[3], the transit operator determines the transit fare and the number of scheduled trains to maintain no-deficit:

$$(\tau_2 - c)V_2 - mR - F + \frac{\delta(V - V_2)^2}{2s} = 0 \tag{19}$$

For any given bottleneck capacity, if the subsidized transit operator provides the optimal fare and number of trains to minimize ITC, we have the following minimization problem:

ITC-3:

$$\min_{\tau_2, m} C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m}.$$

subject to Eqs. (4), (10), (16), (17) and (19).

Proposition 3. If the ITC-minimizing and subsidized transit operator adjusts the fare and number of trains in response to the continuously varying bottleneck capacity, then when the bottleneck capacity goes up,

(i) the optimal number of trains monotonically decreases;

(ii) the optimal transit fare monotonically increases if $N < \bar{N}$ and decreases if $N > \bar{N}$, where $\bar{N}$ is a critical value and $\bar{N} = \frac{2sF^2}{\delta V^2}$;

(iii) the number of transit users monotonically decreases;

(iv) the ITC monotonically increases if $N < \bar{N}$ (implying the D-T Paradox happens) and decreases if $N > \bar{N}$ (implying the D-T Paradox does not happen).

Proof. The proof is given in Appendix E.

Proposition 3(i) indicates that if the bottleneck capacity is expanded, the transit operator would cut the number of trains to maintain zero-profit even under the cost subsidy, which is mainly due to the reduced transit passengers (Proposition 3(iii)). If the transit subsidy is relative small ($N < \bar{N}$), the optimal transit fare after subsidization increases with the bottleneck capacity, then the ITC goes up and the D-T Paradox occurs. In contrast, if the transit subsidy is relative large ($N > \bar{N}$), the optimal transit fare decreases with the bottleneck capacity, then the ITC declines and the D-T Paradox is eliminated (Proposition 3(ii)(iv)). Since s, F, δ, and V are all exogenous variables, the value of $\bar{N}$ is also exogenously determined. It obviously increases with the fixed cost of transit operation, F.

[2] Interested readers can refer to Vickrey (1969), Arnott et al. (1990a) and Tabuchi (1993) for detailed interpretation of the fine toll.

[3] In the future study, we will consider the uniform/flat toll or course toll for auto users in the bi-modal system (Xiao et al., 2011, 2012).

The reason is that when road capacity is expanded, some travelers will switch from transit to driving. The transit operator needs to balance the trade-off between being fully self-financed and minimizing ITC by adjusting the number of scheduled trains and transit fare. On one hand, because ridership drops, to continue satisfying the self-financing constraint, the transit operator has to reduce the number of scheduled trains. On the other hand, when the fine toll revenue is less than a critical value ($N < \bar{N}$), the transit authority has to raise the fare to maintain budget. However, as long as the fine toll revenue is sufficient ($N > \bar{N}$), the transit operator can reduce the fare to minimize the ITC. As the optimal number of scheduled trains decreases in proportion to transit ridership, the change direction of the ITC is consistent with the transit fare. Moreover, comparing the changes of transit scheduling and pricing schemes in response to the loss of patronage, one can observe that the transit subsidy policy has obvious effects on the change direction of transit fare; nevertheless, the number of scheduled trains always decreases.

In all, if fine toll scheme is implemented to reduce bottleneck congestion, the collected toll revenue can be used as the transit subsidy to eliminate D-T Paradox, which would save the direct government financial expenditure. However, the effectiveness of avoiding D-T paradox is conditional on the value of total revenue from road pricing, i.e., if the toll revenue is greater than the critical value of $\bar{N} = \dfrac{2sF^2}{\delta V^2}$, the D-T Paradox can be eliminated, and otherwise, D-T Paradox still occurs. It is easy to know that the critical value $\bar{N}$ decrease with increasing bottleneck capacity s. Fig. 4 and 5 show the change in N and $\bar{N}$ with respect to s and the condition of overcoming the D-T Paradox[4].

Remark. If the transit subsidy sourced from the road toll revenue is not enough to overcome the D-T Paradox, the government may have to make up the deficiency by using financial expenditure. Then, the transit subsidy is financed by the combination of government expenditure and road pricing revenue, which can be easily modeled in the same way as previously stated.

[4] As we do not know how N changes with s, we assume $\dfrac{dN}{ds} < 0$ in Fig. 4 and $\dfrac{dN}{ds} < 0$ in Fig. 5.

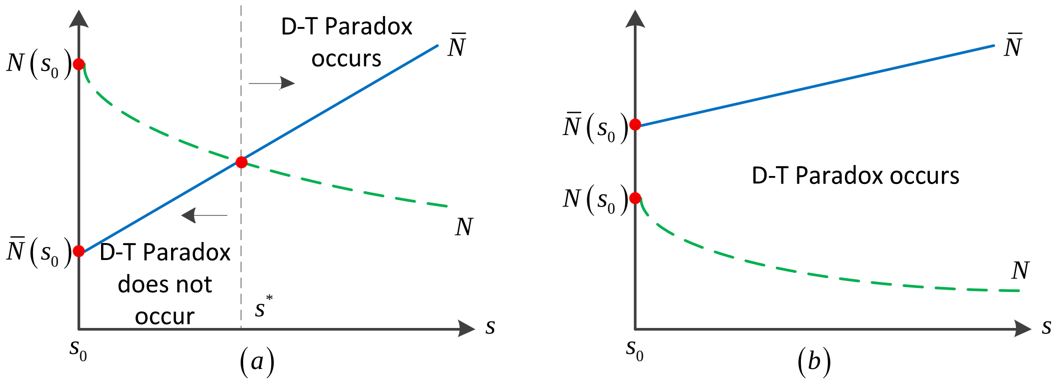

Fig. 4. The change in the value of N and $\bar{N}$ with bottleneck capacity s ($\frac{dN}{ds} < 0$).

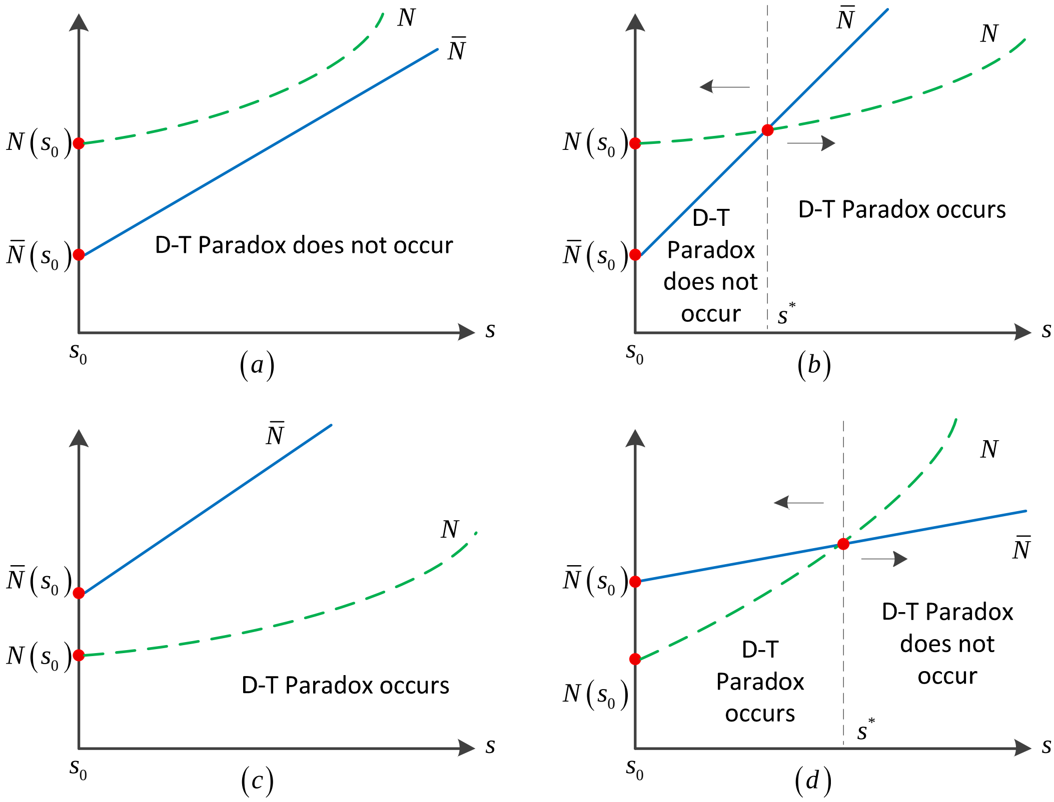

Fig. 5. The change in the value of N and $\bar{N}$ with bottleneck capacity s ($\frac{dN}{ds} > 0$).

Propositions 2 and 3 imply that cost subsidy policy can be applied to effectively avoid the D-T Paradox and improve the two-mode system performance under certain conditions. Fig. 6 schematically shows the mechanism of cost subsidy policies on the prevention of D-T Paradox.

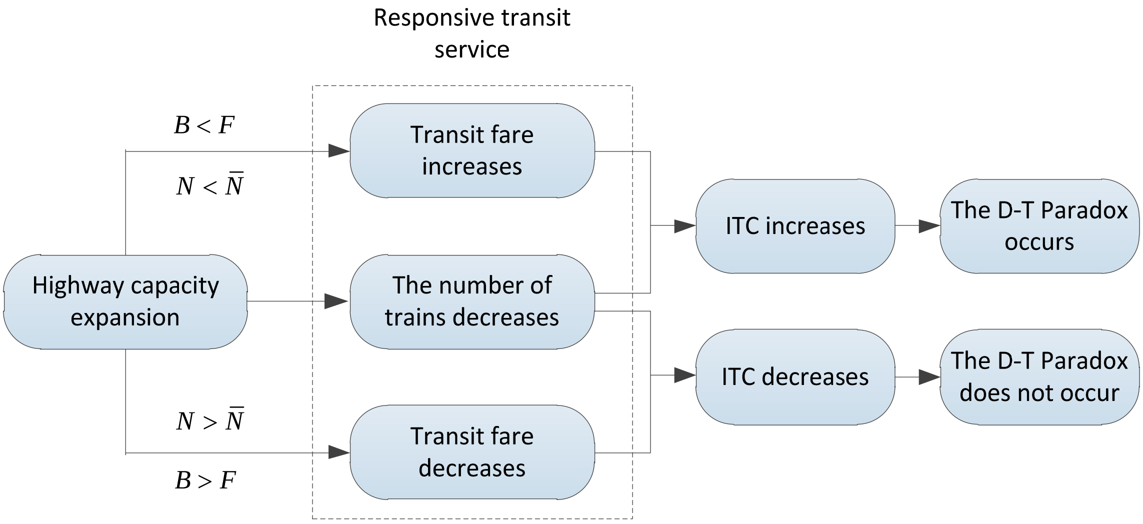

Fig. 6. The impact of cost subsidy policies on the prevention of D-T Paradox.

3.2. Passenger subsidy

According to the OECD (2005), a cost subsidy is able to increase the transit authority's income or reduce its cost, while a passenger subsidy (subsidizing public transport fares) can lower down transit users' travel cost. The latter (i.e., the passenger subsidy) could be implemented to avoid the occurrence of the D-T Paradox under highway capacity expansion as well.

3.2.1. Government's financial subsidy

The government's financial expenditures can be used to subsidize the transit passengers. If the total amount of the subsidy is B, then every transit user receives B/V_2. The generalized travel cost per train commuter becomes:

$$C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m} - \frac{B}{V_2} \tag{20}$$

For any given bottleneck capacity, if the transit operator optimizes the fare and number of trains to minimize ITC, we have the following minimization problem:

ITC-4:

$$\min_{\tau_2, m} C_2$$

subject to Eqs. (4), (15-17) and (20)[5].

Proposition 4. If the ITC-minimizing transit operator adjusts the fare and number of trains in response to the varying bottleneck capacity after government's financial subsidies towards passengers, then when the bottleneck capacity increases,

(i) the optimal number of trains monotonically decreases;

[5] It should be noted that Eq. (15) is the zero-profit constraint. When the passenger subsidy policy is applied, there is no direct subsidy provided to the transit operator. Therefore, the transit operator should maintain the break-even between the farebox revenue and the operation cost.

(ii) the optimal transit fare monotonically increases;
(iii) the number of transit passengers monotonically decreases;
(iv) the ITC monotonically increases if $B < F$ (implying the D-T Paradox happens) and decreases if $B > F$ (implying the D-T Paradox does not occur).

Proof. The method for the proof is exactly the same to that for Proposition 2, and therefore the detailed proof is not elaborated here.

Proposition 4(i)(ii) imply that if the subsidy is provided directly to the passengers, it would not change the decisions of the transit operator, i.e., the operator reduces the number of trains and raises the fare as in the no subsidy case (see Proposition 1). One possible explanation is that, even though passengers enjoy the best service (ITC-minimizing transit policies and subsidies) provided by the transit authority and the government, some transit passengers would give up public transport and use highway after the tradeoff between the decline of transit fare due to subsidy and the decrease of queueing time due to bottleneck capacity expansion (Proposition 4(iii)). As a result, the loss of transit patronage breaks the balance between income and expenditure, and the transit company would have to lower down the service (raise the transit fare and reduce the number of scheduled trains) to maintain breakeven, which induces a higher automobile volume and a higher perceived travel costs of both modes.

Proposition 4(iv) indicates that the passenger subsidy policy can successfully avoid the occurrence of the D-T Paradox if government's financial expenditures exceeds the transit company's fixed cost ($B > F$). The reason is that even the transit authority raises the transit fare and reduces the number of scheduled trains, an enough passenger subsidy ($B > F$) can lead to a decrease in generalized travel costs and makes every user better off. The higher subsidization, the lower commuters' costs in bi-modal equilibrium, and the more likely the D-T Paradox can be avoided.

3.2.2. Subsidy financed by road toll revenue

The fine toll revenue can also be used to subsidize transit passengers. As the total amount of the subsidy is $N = \frac{\delta V_1^2}{2s}$, then every transit user receives $\frac{\delta V_1^2}{2sV_2}$. The generalized travel cost per train commuter becomes:

$$C_2 = \alpha T_2 + \tau_2 + \frac{2\varphi T_2 V_2}{m} - \frac{\delta V_1^2}{2sV_2} \tag{21}$$

For any given bottleneck capacity, the optimal transit fare and number of scheduled trains solves the following minimization problem:

ITC-5:

$$\min_{\tau_2, m} C_2$$

subject to Eqs. (4), (15-17) and (21).

Proposition 5. If the ITC-minimizing transit operator adjusts the fare and number of trains in response to the continuously varying bottleneck capacity after the redistribution of the toll revenue towards passengers, then with respect to the bottleneck capacity,

(i) the optimal number of trains monotonically decreases;
(ii) the optimal transit fare monotonically increases;
(iii) the number of transit users monotonically decreases;
(iv) the ITC monotonically increases if $N < \bar{N}$ (implying the D-T Paradox happens) and decreases if $N > \bar{N}$ (implying the D-T Paradox does not happen).

Proof. The method for the proof is exactly the same to that for Proposition 3, and therefore the detailed proof is not elaborated here.

Proposition 5(i)(ii) indicate that, in presence of the bottleneck capacity expansion, the transit operator has to raise the fare and reduce the number of trains. As can be seen, the passenger subsidy has no effect on transit authority's pricing and scheming decision compared to the case without subsidy (see Proposition 1). It is reasonable since such passenger subsidy cannot change the result of passenger loss after highway capacity expansion (Proposition 5(iii)) and eventually affect transit operator's decision.

Proposition 5(iv) also implies that the D-T Paradox could be avoided by the passenger subsidy policy under the specific road toll revenue condition ($N > \bar{N}$). This can be easily explained. Sufficient passenger subsidy can lead to a decrease in the perceived travel cost of both modes under the bi-modal equilibrium and makes everyone better off. The higher passenger subsidies, the lower ITC, and the more likely the D-T Paradox can be prevented. Fig. 7 shows the mechanism of passenger subsidy policies on the prevention of D-T Paradox.

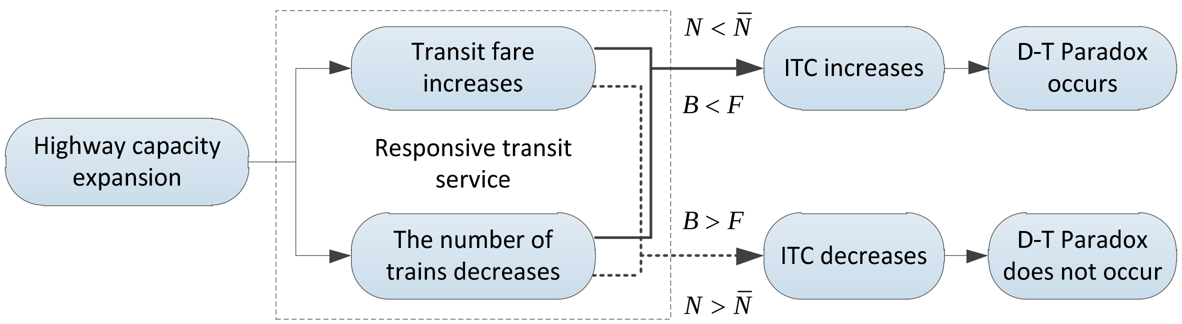

Fig. 7. The impact of passenger subsidy policies on the prevention of D-T Paradox.

4. Numerical examples

This section provides an example to derive numerical solution of the proposed models. Consider the two-mode network described in Section 2 with the following parameters: $V = 10,000$ (commuters), $(\gamma, \alpha, \beta) = (3.0, 1.2, 0.6)$ (HK\$/minute), λ=2.5 (minute), $t^* = 9:00$ a.m., $T_1 = 30$ (minute), $T_2 = 45$ (minute), $\tau_1 = 30.0$ (HK\$), $\varphi = 0.0001$ (HK\$/discomfort), $c = 10$ (HK\$/commuter), $R = 100$ (HK\$/veh), and $F = 2000$ (HK\$).

4.1. No transit subsidy

Fig. 8 presents the numerical results under Scenario ITC-1 when the bottleneck capacity increases from 50 to 300. It is found that the non-subsidized transit operator would raise the transit fare and reduce the number of scheduled trains in response to the bottleneck capacity expansion as shown in Fig. 8(a)(b). As a result, the ITC goes up with the increasing bottleneck capacity as shown in Fig. 8(c), which shows that the D-T Paradox continuously happens.

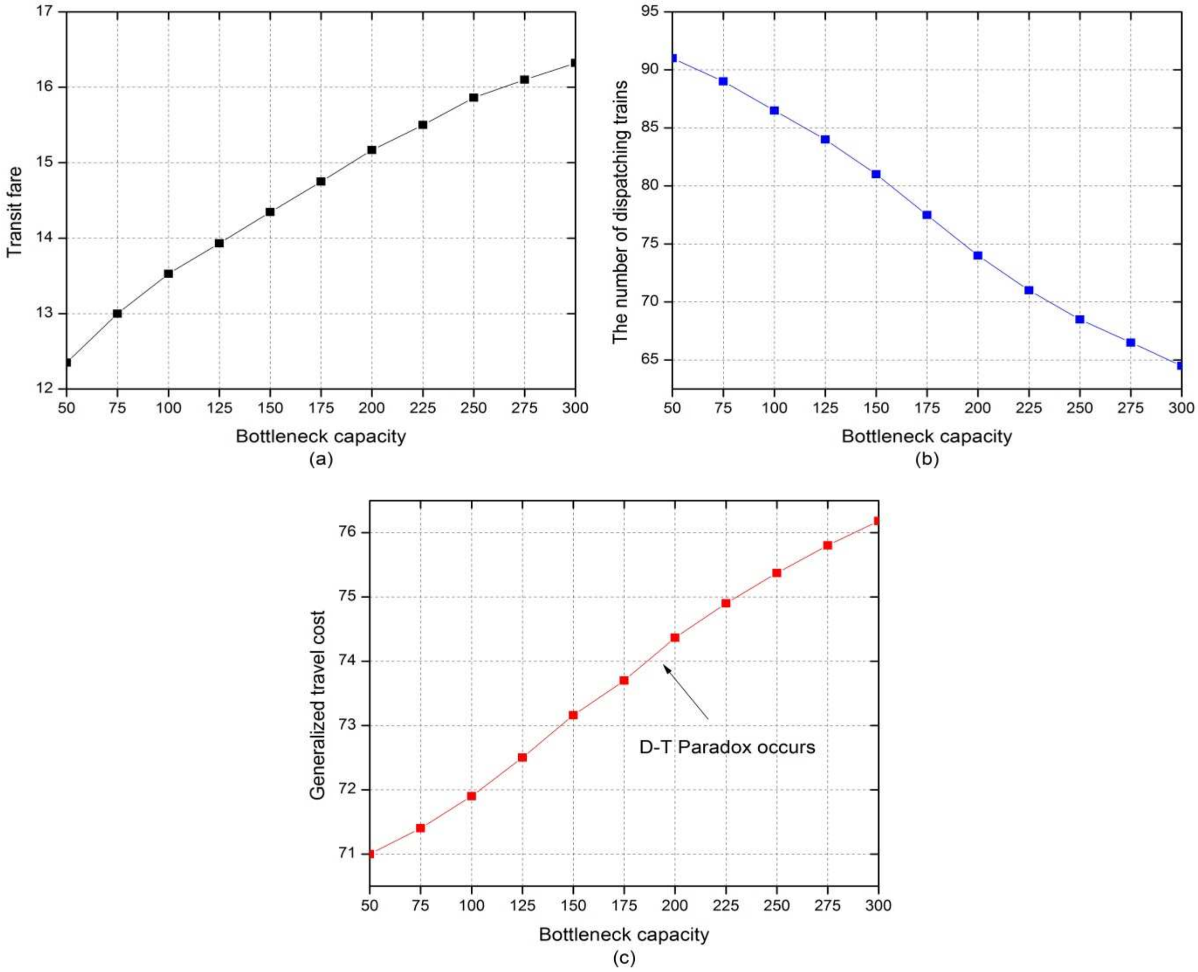

Fig. 8. Transit fare, the number of scheduled trains and ITC without transit subsidy.

Fig. 9 shows the departure patterns of commuters on highway and railway and modal share under different bottleneck capacities. As revealed in Fig. 9(a), the departure time for auto commuters at which the queue begins, ends and a commuter reaches the destination punctually does not change after the bottleneck capacity expansion, which is mainly due to the linear relationship between the number of auto commuters and the bottleneck capacity. Moreover, the auto departure rate during $[t_0, \bar{t}]$ and $[\bar{t}, t_1]$ is monotonically increasing with the bottleneck capacity, which could be obviously observed from Eq. (3).

As can be seen in Fig. 9(b), the time window of transit commuters' departure $[t_s', t_e']$ does not vary with the bottleneck capacity, which can be explained by the linear relationship between passenger volumes and the number of scheduled trains. Furthermore, the growth rate of the departure rate of transit commuters during $[t_s', \tilde{t}]$ and the drop rate of the one during $[\tilde{t}, t_e']$ are monotonically decreasing with the bottleneck capacity, which can be understood by the relationship of the number of passengers and the number of scheduled trains.

Fig. 9(c) shows that the modal share for auto increases and transit decreases when bottleneck capacity goes up, which has been interpreted before.

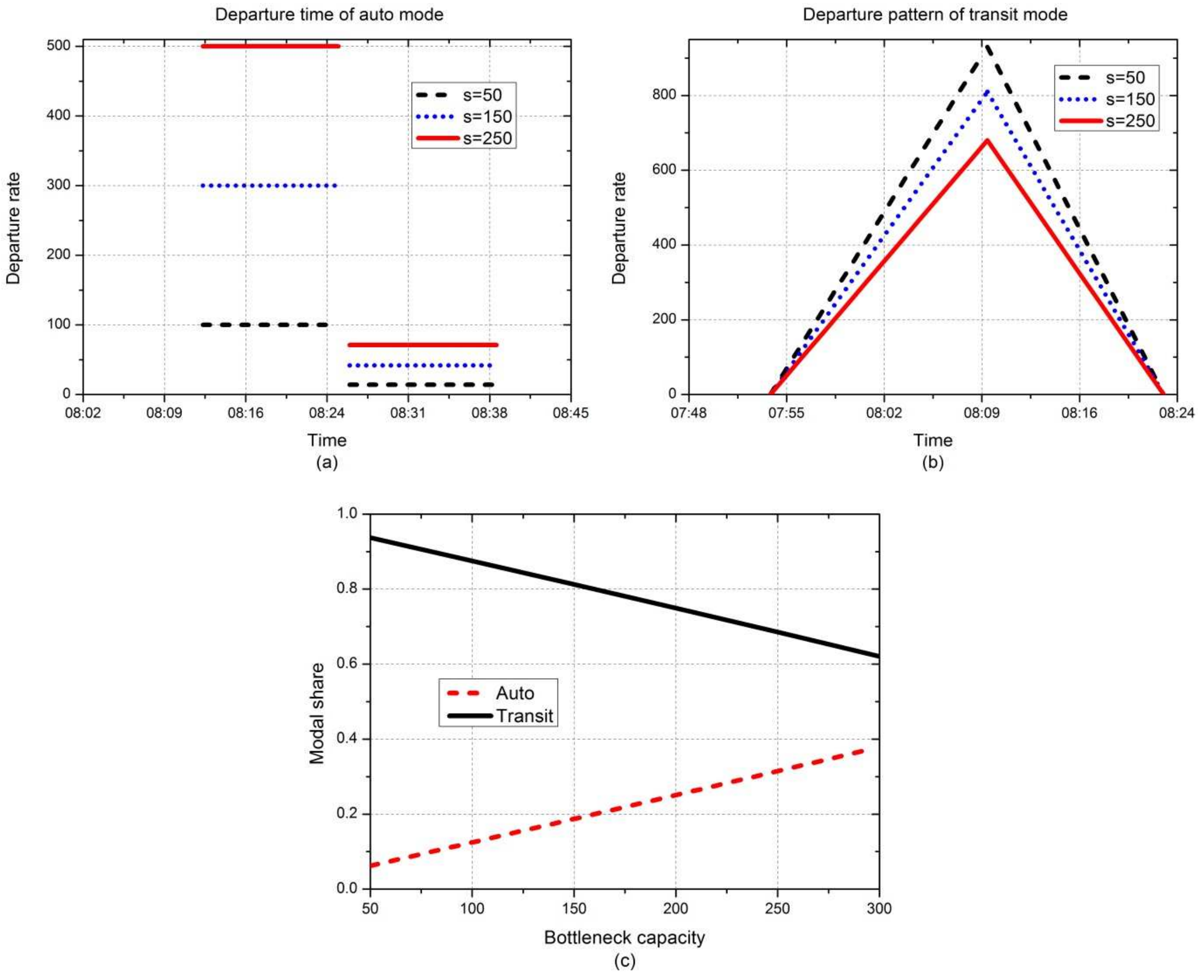

Fig. 9. Departure rates on highway and railway and modal shares without transit subsidy.

4.2. Cost subsidy

4.2.1. Government's financial subsidy

Fig. 10 displays the results under Scenario ITC-2 when the bottleneck capacity increases as an illustration of Proposition 2. As is shown in Fig. 10, no matter what is the size of subsidies the government provides to the transit operator, the number of trains is always dropping when the bottleneck capacity goes up. In contrast, when the transit subsidy (B) is less (greater) than the fixed cost of transit[6] (F), the transit fare increases (decreases) along with bottleneck capacity. Then, the ITC would indeed increase (decrease) after the bottleneck capacity expansion, which means that the D-T Paradox does (not) happen.

[6] Let $B = 1500$ ($B < F$) or $B = 2500$ ($B > F$).

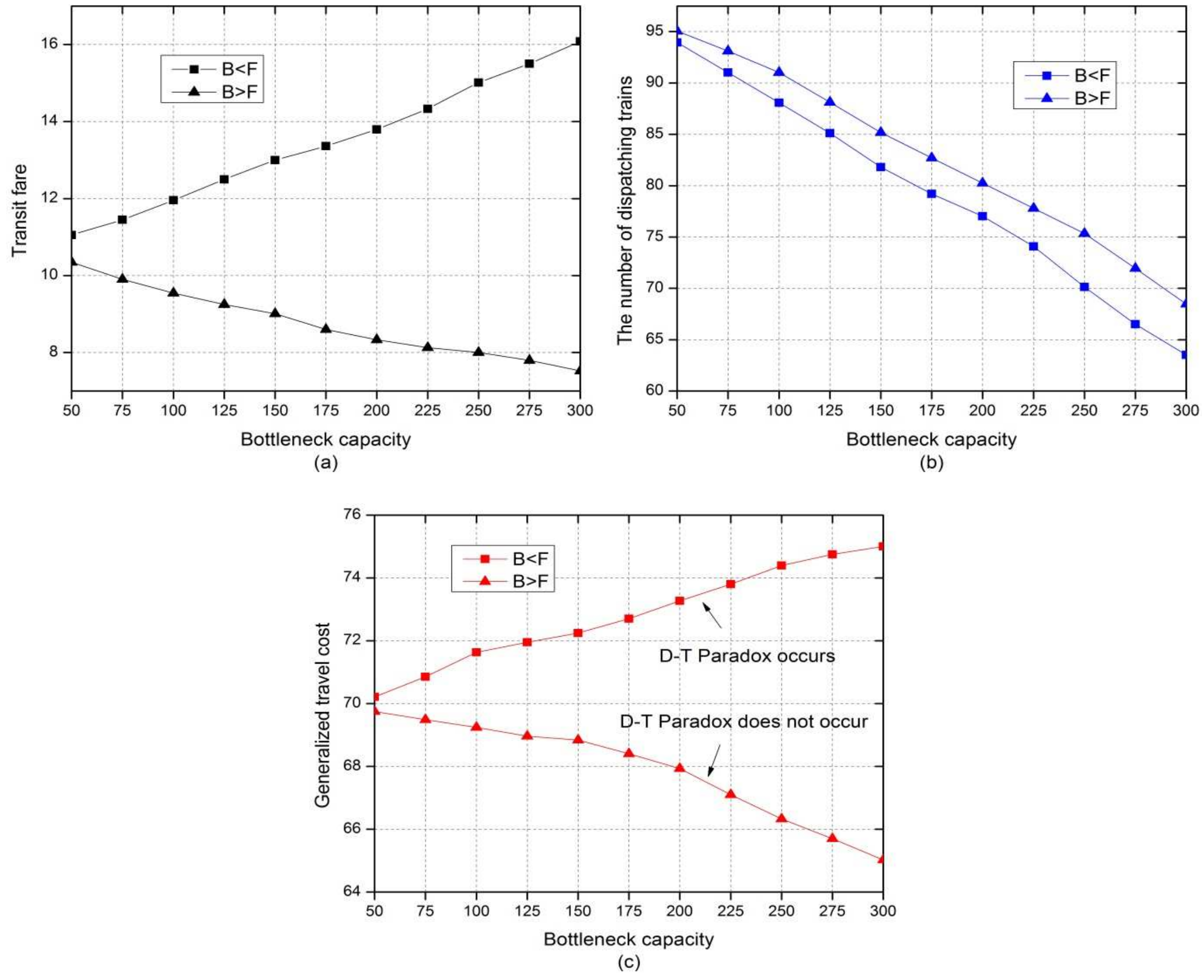

Fig. 10. Transit fare, the number of trains and ITC with cost subsidy from government.

4.2.2. Subsidy financed by road toll revenue

Fig. 11 provides the numerical results under Scenario ITC-3 when the bottleneck capacity increases. It is clear from Fig. 11(a), (b) that along with the increase of the bottleneck capacity, the transit fare firstly goes up and then descends while the number of scheduled trains always decreases. Therefore, the generalized travel cost of travelers firstly increases and then decreases with the bottleneck capacity as presented in Fig. 11(c), which states the D-T Paradox can be prevented under certain road toll revenue condition (i.e., $N > \bar{N}$). It should be noted that the train timetabling or scheduling (e.g., Li et al., 2013, Wu et al., 2015) has not been explicitly considered in this paper, which could be further addressed in the future study.

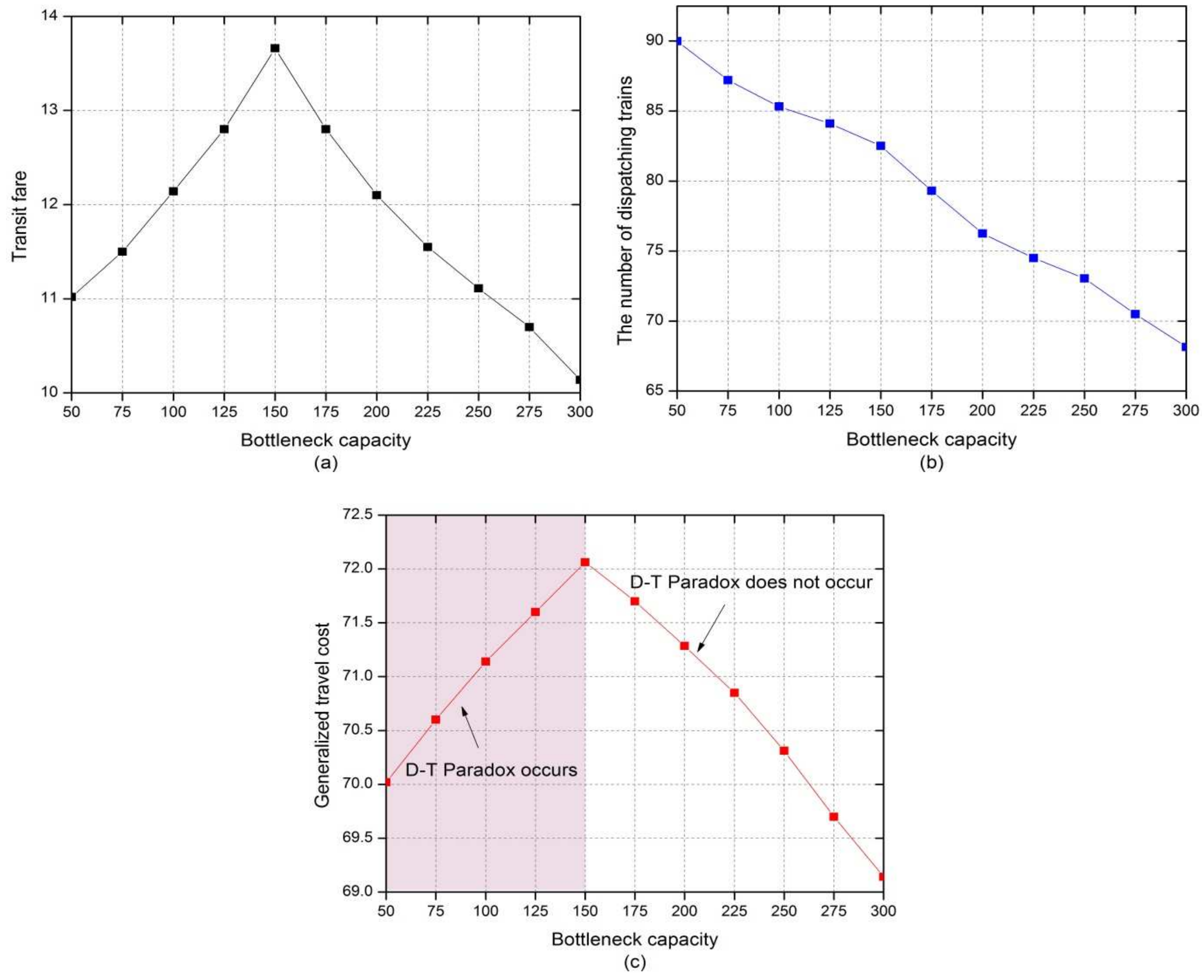

Fig. 11. Transit fare, the number of trains and ITC with cost subsidy from road toll revenue.

4.3. Passenger subsidy

4.3.1. Government's financial subsidy

Fig. 12 presents the numerical result under ITC-4 with increasing bottleneck capacity. Regardless of the size of government' financial subsidy, the optimal number of scheduled trains is dropping while the optimal transit pricing is increasing when the bottleneck capacity goes up. If the passenger subsidy (B) is less (greater) than the fixed cost of transit (F), the ITC would indeed increase (decrease) after the bottleneck capacity expansion, which means that the D-T Paradox does (not) occur.

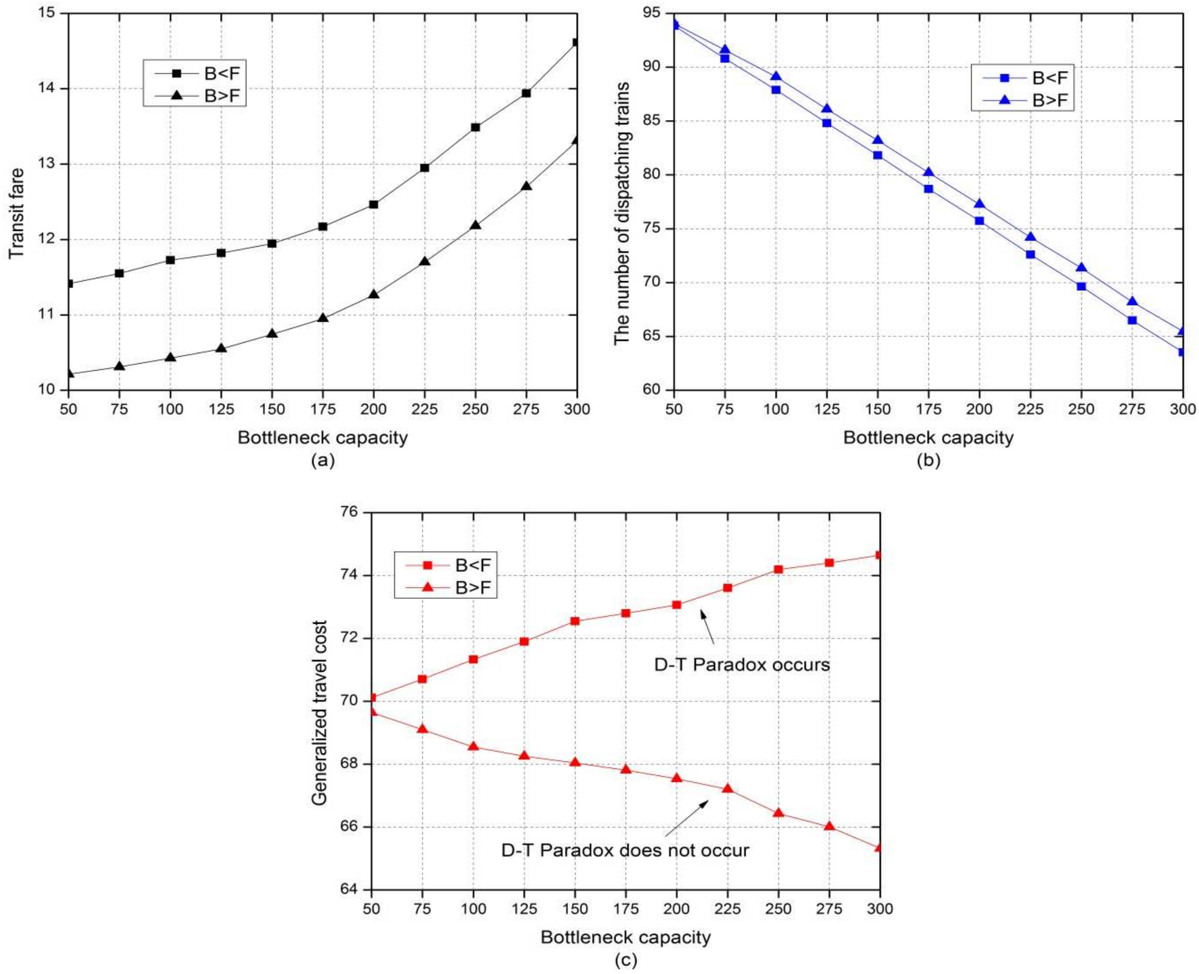

Fig. 12. Transit fare, the number of trains and ITC with passenger subsidy from government.

4.3.2. Subsidy financed by road toll revenue

Fig. 13 depicts the results under Scenario ITC-5 when the bottleneck capacity increases. It can be seen from Fig. 13(a)(b) that along with the increase of the bottleneck capacity, the transit fare firstly goes up and the number of scheduled trains decreases. The individual commuter's generalized travel cost firstly increases and then decreases with the bottleneck capacity as depicted in Fig. 13(c), which implies the D-T Paradox can be prevented under the specific road pricing revenue condition (i.e., $N > \bar{N}$).

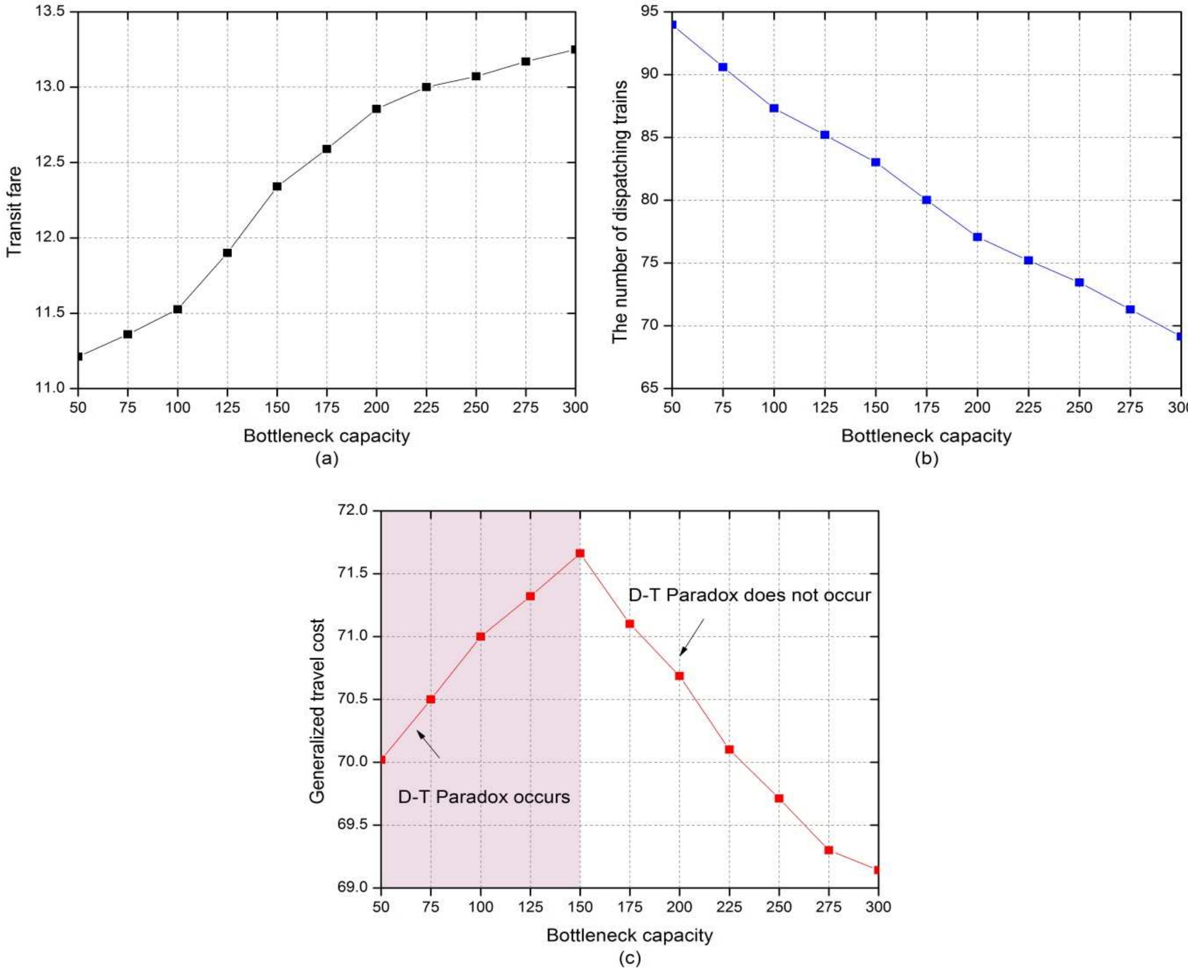

Fig. 13. Transit fare, the number of trains and ITC with passenger subsidy from road toll revenue.

4.4. Comparison of ITC under different transit subsidy policies

Fig. 14 presents the ITC under different transit subsidy (cost and passenger subsidy) policies. As shown in Fig. 14, when the bottleneck capacity is expanded, the ITC without transit subsidy is always the largest and the one with government's abundant subsidy ($B > F$) is always the lowest; the ITC with transit subsidy from road toll revenue is often lower than the one from government's insufficient financial expenditure ($B < F$). This means that the single transport policy, i.e., highway capacity expansion, may have undesirable effects. However, the combined transport policy of road capacity investment and appropriate transit subsidies can improve the two-mode system performance. From Fig. 14, we can also see that there are similarities between the effects of cost subsidy and the passenger subsidy in overcoming the D-T Paradox. On one hand, with the financial support from the government funding, both the cost and passenger subsidy can (cannot) avoid the occurrence of the D-T Paradox if the subsidy is larger (lower) than the fixed cost of transit operator. On the other hand, with the financial support from the road toll revenue, both the cost and passenger subsidy can eliminate the D-T Paradox under the same condition. A possible explanation is that we do not consider travelers' heterogeneity in the current framework. If commuters' heterogeneity is incorporated into the analysis of the D-T Paradox, the difference of the two subsidy schemes will be expected, which could be further addressed in the future work.

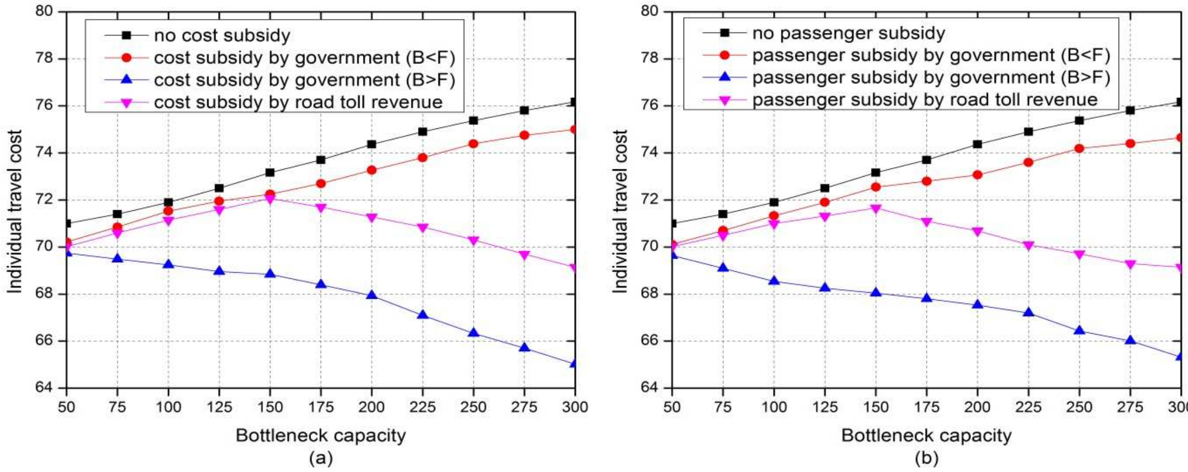

Fig. 14. The ITC under different transit subsidy policies.

5. Conclusions

This study investigates the occurrence conditions and preventive measures of the D-T Paradox considering travelers' departure time and mode choices. When the transit operator minimizes ITC while maintaining break-even, it is found that the highway capacity expansion has a significant effect on transit operating (pricing and dispatching) schemes as well as travelers' departure time and mode choices, which gives rise to the phenomenon of the D-T Paradox.

In order to overcome the D-T Paradox, transit subsidies (including cost and passenger subsidies) sourced from either government funding or road toll revenue are designed in this research. The impact of cost or passenger subsidy on transit operating schemes and ITC are explicitly explored. It is shown that the transit subsidy financed by government funding (or road toll revenue) can effectively avoid the occurrence of the D-T Paradox if the subsidy is larger than the fixed cost of transit operator (or a critical value).

There are a number of directions in which the study could be extended. Further analysis should consider: (1) change of departure pattern under different transit subsidy policies, (2) non-linear body congestion (Huang, 2002; de Palma et al., 2015), (3) heterogeneous commuters (Xiao et al., 2011; Xiao and Zhang, 2014; Du and Wang, 2014; Wang and Du, 2016), (4) oligopoly transit operators (Liu et al., 2011), (5) three travel modes: auto-only mode, transit-only mode, and park-and-ride mode (Liu et al., 2009; Lu et al., 2015; Wang and Du, 2013), (6) congestion interactions between autos and buses (Huang et al., 2007; Basso et al., 2011), (7) parking fees at the destination (Qian et al., 2011, 2012; Qian and Rajagopal, 2014).

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Appendix A. Proof of nonoccurrence of D-T Paradox

We firstly consider a transit operator who maximizes its profit. For any given bottleneck capacity, the optimal transit fare and number of trains solves the following optimization problem with the bi-modal equilibrium constraint:

$$\max_{\tau_2, m} Z = \left(\tau_2 - c\right) V_2 - mR - F \tag{22}$$

subject to Eqs. (4), (10), (16) and (17).
The existence and uniqueness of the optimal solution of the above problem can be guaranteed by the continuity of the objective function and the compactness of the feasible region.

The necessary conditions for an interior optimal $\left(\tau_2^*, m^*\right)$ of the above problem are:

$$V_2\left(\delta m + 2\varphi s T_2\right) - \left(\tau_2 - c\right) sm = 0 \tag{23}$$

$$2\left(\tau_2 - c\right)\varphi s T_2 V_2 - mR\left(\delta m + 2\varphi s T_2\right) = 0 \tag{24}$$

To obtain the relationship between the optimal $\left(\tau_2^*, m^*\right)$ and s, we take total derivative of both sides of Eqs. (23) and (24) with respect to s:

$$\frac{d\tau_2}{ds} = -\frac{\delta V}{2s^2} < 0, \quad \frac{dm}{ds} = \frac{m\left(2V_2 - V\right)}{2sV_2}.$$

Thus, $\frac{dV_2}{ds} = \frac{2V_2 - V}{2s}$, according to Definition 1, the non-existence of the D-T Paradox can be shown by:

$$\frac{dC_2}{ds} = -\left(m\delta + 2s\varphi T_2\right)\delta V < 0.$$

Appendix B. Proof for the uniqueness of the optimal solution of Problem (14-17)

The uniqueness of the optimal solution of Problem (14-17) can be proved by contradiction. Assume there are two different optimal solutions $\left(\tau_{21}^*, m_1^*\right)$ and $\left(\tau_{22}^*, m_2^*\right)$, such that $\tau_{21}^* \neq \tau_{22}^*$ or $m_1^* \neq m_2^*$, then we have two distinct equilibrium solutions $\left(V_{11}^*, V_{21}^*\right)$ and $\left(V_{12}^*, V_{22}^*\right)$. Obviously, the objective values of the two optimal solutions should be identical, i.e., $\alpha T_2 + \tau_{21}^* + \frac{2\varphi T_2 V_{21}^*}{m_1^*} = \alpha T_2 + \tau_{22}^* + \frac{2\varphi T_2 V_{22}^*}{m_2^*}$. According to the constraints (4), (10) and (16), we can conclude that $V_{11}^* = V_{12}^*$ or $V_{21}^* = V_{22}^*$ must hold. Then, it can be evidently obtained from Eqs. (4), (9), (10) and (16) that $\tau_{21}^* = \tau_{22}^*$ and $m_1^* = m_2^*$ must be simultaneously true. This contradicts with the assumption that the two optimal solutions are different.

Appendix C. Proof of Proposition 1

From the zero-profit condition (15), the transit fare can be expressed as:

$$\tau_2 = \frac{cV_2 + mR + F}{V_2} \tag{25}$$

Substituting Eq. (25) into the equilibrium conditions (4), (10), (16) and (17), we have:

$$G\left(V_2, m, s\right) = \alpha T_1 + \tau_1 + \frac{\delta\left(V - V_2\right)}{s} - \alpha T_2 - \frac{cV_2 + mR + F}{V_2} - \frac{2\varphi T_2 V_2}{m} = 0 \tag{26}$$

The partial derivatives of V_2 with respect to m and s are given by:

$$\frac{\partial V_2}{\partial m} = \frac{RV_2 m^2 s - 2\varphi T_2 V_2^3 s}{-\delta V_2^2 m^2 - m^3 sR + m^2 sF - 2\varphi T_2 V_2^2 ms},$$

$$\frac{\partial V_2}{\partial s} = \frac{\delta\left(V - V_2\right)V_2^2 m}{-\delta s V_2^2 m - m^2 s^2 R + ms^2 F - 2\varphi T_2 V_2^2 s^2}.$$

Evidently, $\partial V_2 / \partial s < 0$ is true by the monotonicity of function $C_1\left(V_2, s\right)$ [7], then it means that:

$$ms\left(F + mR\right) - V_2^2\left(m\delta + 2s\varphi T_2\right) < 0.$$

According to Eq. (25), rewrite ITC-1 as:

ITC-1':

$$\min_m C_2 = \alpha T_2 + \frac{cV_2 + mR + F}{V_2} + \frac{2\varphi T_2 V_2}{m} \tag{27}$$

subject to Eq. (26).

The optimal m^* can be derived by:

$$Rm^2 - 2\varphi T_2 V_2^2 = 0 \tag{28}$$

To obtain the relationship between the optimal $\left(\tau_2^*, m^*\right)$ and s, we take total derivative of both sides of Eq. (28) with respect to s:

$$\frac{dm}{ds} = \frac{2\psi\delta T_2 V_2^3\left(V - V_2\right)}{Rsm\left(sF - \delta V_2^2\right)} < 0,$$

$$\frac{d\tau_2}{ds} = -\frac{\delta F\left(V - V_2\right)}{s\left(sF - \delta V_2^2\right)} > 0.$$

Thus,

$$\frac{dC_2}{ds} = -\frac{\delta F V_2^2\left(V - V_2\right)}{s\left(sF - \delta V_2^2\right)} > 0.$$

It is implied that the D-T Paradox would happen.

[7] $C_1\left(V_2, s\right) = \alpha T_1 + k_1 + \delta V_1 / s = \alpha T_1 + k_1 + \delta\left(V - V_2\right)/s$.

Appendix D. Proof of Proposition 2

According to the zero-profit condition (15), the transit fare can be obtained as:

$$\tau_2 = \frac{cV_2 + mR + F - B}{V_2} \tag{29}$$

Substituting Eq. (29) into the equilibrium conditions (4), (10), (16) and (17), we have:

$$G(V_2, m, s) = \alpha T_1 + \tau_1 + \frac{\delta(V - V_2)}{s} - \alpha T_2 - \frac{cV_2 + mR + F - B}{V_2} - \frac{2\varphi T_2 V_2}{m} = 0 \tag{30}$$

The partial derivatives of V_2 with respect to m and s are expressed by:

$$\frac{\partial V_2}{\partial m} = \frac{RV_2 m^2 s - 2\varphi T_2 V_2^3 s}{-\delta V_2^2 m^2 - m^3 sR + m^2 s(F - B) - 2\varphi T_2 V_2^2 ms},$$

$$\frac{\partial V_2}{\partial s} = \frac{\delta(V - V_2)V_2^2 m}{-\delta s V_2^2 m - m^2 s^2 R + ms^2(F - B) - 2\varphi T_2 V_2^2 s^2}.$$

Apparently, $\partial V_2/\partial s < 0$ is true by the monotonicity of function $C_1(V_2, s)$, then it implies that:

$$ms(F - B + mR) - V_2^2(m\delta + 2s\varphi T_2) < 0.$$

From Eq. (29), rewrite ITC-2 as:

ITC-2':

$$\min_m C_2 = \alpha T_2 + \frac{cV_2 + mR + F - B}{V_2} + \frac{2\varphi T_2 V_2}{m} \tag{31}$$

subject to Eq. (30).

The optimal m^* can be obtained by:

$$Rm^2 - 2\varphi T_2 V_2^2 = 0 \tag{32}$$

To derive the relationship between the optimal (τ_2^*, m^*) and s, we take total derivative of both sides of Eq. (32) with respect to s:

$$\frac{dm}{ds} = \frac{2\varphi T_2 \delta(V - V_2)V_2^3}{Rsm\left[s(F - B) - \delta V_2^2\right]} < 0,$$

$$\frac{d\tau_2}{ds} = \frac{\delta(V - V_2)(B - F)}{s\left[s(F - B) - \delta V_2^2\right]}.$$

Then,

$$\frac{dC_2}{ds}=\frac{\delta(V-V_2)(B-F)}{s\left[s(F-B)-\delta V_2^2\right]}.$$

Therefore, if $B<F$, then $\frac{d\tau_2}{ds}>0$ and $\frac{dC_2}{ds}>0$; and if $B>F$, then $\frac{d\tau_2}{ds}<0$ and $\frac{dC_2}{ds}<0$.

Appendix E. Proof of Proposition 3

Using the zero-profit condition (15), the transit fare can be determined by:

$$\tau_2=\frac{cV_2+mR+F-\frac{\delta(V-V_2)^2}{2s}}{V_2} \tag{33}$$

Substituting Eq. (33) into the equilibrium conditions (4), (10), (16) and (17), we have:

$$G(V_2,m,s)$$
$$=\alpha T_1+\tau_1+\frac{\delta(V-V_2)}{s}-\alpha T_2-\frac{cV_2+mR+F-\frac{\delta(V-V_2)^2}{2s}}{V_2}-\frac{2\varphi T_2V_2}{m}=0 \tag{34}$$

The partial derivatives of V_2 with respect to m and s are expressed as:

$$\frac{\partial V_2}{\partial m}=\frac{2RV_2m^2s-4\varphi T_2V_2^3s}{\left(2smR+2sF-\delta V^2-\delta V_2^2\right)m^2-4\varphi T_2V_2^2ms},$$

$$\frac{\partial V_2}{\partial s}=\frac{\delta(V-V_2)(V+V_2)V_2m}{\left(2smR+2sF-\delta V^2-\delta V_2^2\right)sm-4\varphi T_2V_2^2s^2}.$$

Obviously, $\partial V_2/\partial s<0$ is true by the monotonicity of function $C_1(V_2,s)$, then it indicates that:

$$\left(2smR+2sF-\delta V^2-\delta V_2^2\right)m-4\varphi T_2V_2^2s<0.$$

According to Eq. (33), rewrite ITC-3 as:

ITC-3’:

$$\min_m C_2=\alpha T_2+\frac{cV_2+mR+F-\frac{\delta(V-V_2)^2}{2s}}{V_2}+\frac{2\varphi T_2V_2}{m} \tag{35}$$

subject to Eq. (34).

The optimal m^* can be given by:

$$Rm^2-2\varphi T_2V_2^2=0 \tag{36}$$

To obtain the relationship between the optimal $\left(\tau_2^*, m^*\right)$ and s, we take total derivative of both sides of Eq. (36) with respect to s:

$$\frac{dm}{ds} = \frac{2\varphi T_2 V_2^2 \delta\left(V - V_2\right)\left(V + V_2\right)}{Rsm\left(2sF - \delta V^2 - \delta V_2^2\right)} < 0,$$

$$\frac{d\tau_2}{ds} = \frac{\delta\left(V - V_2\right)\left(-\delta V V_2 - 2sF + \delta V^2\right)}{sV_2\left(2sF - \delta V^2 - \delta V_2^2\right)}.$$

Therefore,

$$\frac{dC_2}{ds} = \frac{\delta\left(V - V_2\right)\left(-\delta V V_2 - 2sF + \delta V^2\right)}{sV_2\left(2sF - \delta V^2 - \delta V_2^2\right)}.$$

Consequently, if $0 < V_2 < V - \frac{2sF}{\delta V}$, that is $\frac{\delta V_1^2}{2s} > \frac{2sF^2}{\delta V^2}$, then $\frac{d\tau_2}{ds} < 0$ and $\frac{dC_2}{ds} < 0$; and if $V_2 > V - \frac{2sF}{\delta V}$, that is $\frac{\delta V_1^2}{2s} < \frac{2sF^2}{\delta V^2}$, then $\frac{d\tau_2}{ds} > 0$ and $\frac{dC_2}{ds} > 0$.