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# Extended Multipoint Approximation Method

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**Abstract.** Stemming from polynomial metamodels, multipoint approximation method (MAM) and moving least square method (MLSM) focus on the development of metamodels for the objective and constraint functions in solving a mid-range optimization problem with a trust region. Although both of these methods could solve problems successfully, there is still some room for improvement on the computational effort and search capability. To address this problem, the extended multipoint approximation method is proposed to seek the optimal solution in this paper. The developed method assimilating the advantage of Taylor's expansion used in MLSM demonstrates its superiority over other methods in terms of the computational efficiency and accuracy by some well-established benchmark problems.

## Introduction

For better meeting the challenges in temporary tough competition in engineering products, metamodel-based optimization method has become increasing popular cause it could relieve the computation burden during analysis and minimize the total cost during experiments than normal detailed simulation methods (e.g. finite element method)

The Multipoint Approximation Method (MAM) [1] is one of the best-known metamodel-based optimization methods, which replaces the original optimization problem by a succession of simpler mathematical programming. This method is based on an assembly of multiple surrogates into a single surrogate using linear regression. The coefficients of the model assembly are not weights of the individual models but tuning parameters determined by the least squares method [2].

Although the MAM shows good performance on solving continuous problems, its full potential is not yet utilized. In this paper, an extended MAM has been studied and validated to be more efficient in searching ability and accuracy. In current research, another new surrogate called Tylor's expansion surrogate is added together with other five surrogates into an assembly to build the metamodel. This approach is intrigued by the Moving Least Squares Method (MLSM), which is a metamodel building technique that has been suggested for the use in the meshless form of the Finite Element method [3]. And it has been proposed for the applications to design optimization[4–6].

However, for the aim of optimization, the weighting coefficients here is no longer a function of Euclidian distance but a function related to the constraint and function values.

In current research, several benchmark problems have been tested by both the MAM and the Extended MAM with comparison to other algorithms. Analytical analysis validates that the extended MAM has better optimization ability.

## Multipoint Approximation Method

In the present work, the metamodel is built by two subsequent steps. In the first step, single surrogate $\varphi_l$ is identified using the weighted least squares method as follows:

$$\sum_{p=1}^{P} w_p^j \left[F_j(\boldsymbol{x_p}) - \varphi_l(\boldsymbol{x_p}, \boldsymbol{a_j})\right]^2 \rightarrow min \quad (1)$$

Where the coefficients $w_p^j$ refer to the weights that reflect the inequality of data obtained in different sampling points $P$ [7] and $\boldsymbol{a_j}$ is the tuning parameters with respect to the specific surrogate.

In the second step, based on the known parameters $\boldsymbol{a_j}$ and keeping the same design of experiments fixed, different approximate models could be assembled into one metamodel in the same manner:

$$\sum_{p=1}^{P} w_p^j \left[F_j(\boldsymbol{x_p}) - \widetilde{F}_j(\boldsymbol{x_p}, \boldsymbol{b_l})\right]^2 \rightarrow min \quad (2)$$

In other words, the assembly metamodel could be expressed as:

$$\tilde{F}(x) = \sum_{l=1}^{NF} b_l \cdot \varphi_l(x) \quad (3)$$

That leads to solving the linear system of $NF$ equations with $NF$ unknowns $b_l$ where $NF$ is the number of regressors in the model bank $\{\varphi_l(x)\}$. Here, the coefficients $b_l$ are regression coefficients that should not be considered as weight factors, e.g. could be positive or negative.

## Extended MAM

Intrigued by the Moving Least Square Method (MLSM), which builds the highly-dependent metamodel in the local space around the specific point, an extended MAM could be developed to explore the full potentials of the polynomial regression metamodels.

Other than the surrogates built by Liu and Toropov [2], one new surrogate called the Taylor's Expansion surrogate is introduced in the extended MAM by the following equation:

$$\varphi(x) = \varphi_0 + \sum_{i=1}^{N} \left(\frac{\partial \varphi_0}{\partial x_i} \cdot \Delta_i\right) \quad (4)$$

Where

$$\Delta_i = x_i - {x_i}^0 \quad (5)$$

${x_i}^0$ is the starting point e.g. the sub-optimal point during the optimization loop and $\varphi_0$ is the primary function value at the starting point.

Unlike the metamodel built by the MLSM, the weight $w_p^j$ is no longer the functions related to the Euclidian distance from a sampling point to the specific point where the surrogate model is evaluated. Actually, the points that belong to the boundary of the feasible region should be treated with great weights. In the current work, this can be achieved by the formula $\omega = e^{-4|F_j(\boldsymbol{x_p})-1|}$.

## Examples

### Design of Tension/Compression Spring

This problem was first described by Belegundu [8] and Arora[9]. The design objective is to minimize the weight of a tension/compression spring subject to constraints on shear stress, surge frequency and minimum deflections as shown in Figure 1. The design variables include the wire diameter $d$; the mean coil diameter $D$, and the number of active coils $N$. (see detailed formulation in [8])

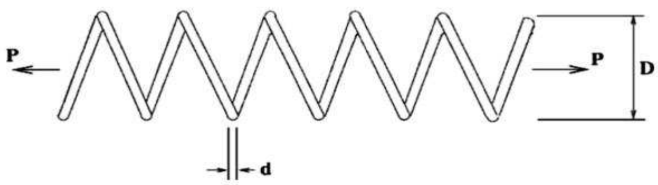


Figure 1. Schematic of the tension/compression spring with indication of design variables

Table 1 and Table 2 compare the best results obtained by present algorithms and those of the other researchers. As is shown in table 1, the MAM and extended MAM found the best design overall (0.0126653). In fact, the lighter design found by Kaveh actually violates the first two optimization constraints which could be seen in Table 2.

By choosing different starting points, it can be seen as showed in Table 3.9 that the evaluations called by the extended MAM are 10% less than MAM. And the average optimal weight also demonstrates the extended MAM has better search performance (0.01274 VS 0.01283). In conclusion, from this typical case, the sixth regressor added in the extended MAM enhances the robustness and the accuracy of metamodel.

Table 1. Comparison of present optimized designs with literature for the spring

| Methods | $d$ | $D$ | $N$ | weight |
|---|---|---|---|---|
| Belegundu [8] | 0.050000 | 0.315900 | 14.250000 | 0.0128334 |
| Arora [9] | 0.053396 | 0.399180 | 9.185400 | 0.0127303 |
| Coello [10] | 0.051480 | 0.351661 | 11.632201 | 0.0127048 |
| Coello & Montes [11] | 0.051989 | 0.363965 | 10.890522 | 0.0126810 |
| Montes & Coello [12] | 0.051643 | 0.355360 | 11.397926 | 0.012698 |
| Kaveh & Talatahari [13] | 0.051744 | 0.358532 | 11.165704 | 0.0126384 |
| MAM | 0.051604352 | 0.35468326 | 11.409247 | 0.0126653 |
| Extended MAM | 0.051656017 | 0.35592318 | 11.3357128 | 0.0126653 |

Table 2. Comparison of present constraint values with literature for the spring

| Methods | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
|---|---|---|---|---|
| Belegundu [8] | -0.000014 | -0.003782 | -3.938302 | -0.756067 |
| Arora [9] | 0.000019 | -0.000018 | -4.123832 | -0.698283 |
| Coello [10] | -0.002080 | -0.000110 | -4.026318 | -4.026318 |
| Coello & Montes [11] | -0.000013 | -0.000021 | -4.061338 | -0.722698 |
| Montes & Coello [12] | -0.001732 | -0.0000567 | -4.039301 | -0.728664 |
| Kaveh & Talatahari [13] | 8.78603e-6 | 0.0011043 | -4.063371 | -0.726483 |
| MAM | -1.0843e-7 | -6.10541e-8 | -4.0497478 | -7.291416 |
| Extended MAM | -6.3091e-7 | -3.2158e-7 | -4.052208 | -7.282805 |

Table 3. Statistical results from present algorithms for the spring

| Methods | | | MAM | | Extended MAM | |
|---|---|---|---|---|---|---|
| Number of sample points | | | 8 | | 8 | |
| Starting point $(d, D, N)$ | | | Output Value | No. of iteration | Output Value | No. of iteration |
| 0.05 | 0.4 | 9 | 0.01269 | 17 | 0.01267 | 19 |
| 0.08 | 1.0 | 10 | 0.01311 | 58 | 0.01277 | 28 |
| 0.06 | 0.5 | 11 | 0.01267 | 11 | 0.01266 | 17 |
| 0.07 | 0.8 | 8 | 0.01303 | 15 | 0.01296 | 25 |
| 0.05 | 0.5 | 11 | 0.01269 | 10 | 0.01266 | 14 |
| Average | | | 0.01283 | 22 | 0.01274 | 20 |

## Design of a Pressure Vessel

The second case is the design optimization of the cylindrical pressure vessel capped at both ends by hemispherical heads (Figure 2). The main topic of this problem is to minimize the total manufacturing cost of the vessel including the combination of welding, material and forming costs. The design variables consist of the shell thickness $T_s$, the spherical head thickness $T_h$, the radius of cylindrical shell $R$, and the shell length $L$. The detailed problem formulation could be seen in [14].

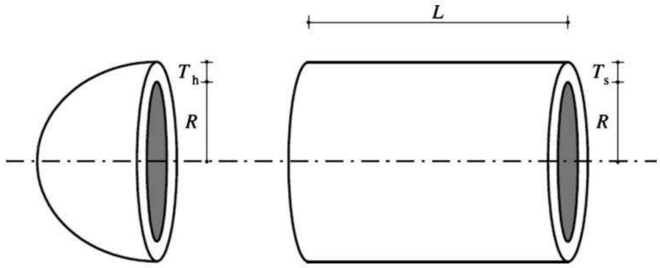


Figure 2. Schematic of the spherical head and cylindrical wall of the pressure vessel with indication of design variables

Table 4. Comparison of present optimized designs with literature for the pressure vessel

| Methods | $T_s$ | $T_h$ | $R$ | $L$ | cost |
|---|---|---|---|---|---|
| Kannan & Kramer [14] | 1.125000 | 0.625000 | 58.29100 | 43.6900 | 7198.0428 |
| Deb [15] | 0.937500 | 0.500000 | 48.32900 | 112.6790 | 6410.3811 |
| Coello [10] | 0.812500 | 0.437500 | 40.32390 | 200.0000 | 6288.7445 |
| Coello & Montes [11] | 0.812500 | 0.437500 | 42.09739 | 176.6540 | 6059.9463 |
| Montes & Coello [12] | 0.812500 | 0.437500 | 42.098087 | 176.64051 | 6059.7456 |
| Kaveh & Mahdavi [16] | 0.779946 | 0.385560 | 40.409065 | 198.76232 | 5889.911 |
| MAM | 0.7781687 | 0.3846492 | 40.319619 | 200.000 | 5885.268 |
| Extended MAM | 0.7781687 | 0.3846492 | 40.319619 | 200.000 | 5885.268 |

It can be seen from Table 4 that the present algorithms found the best design that is 0.8% less than the best-known design quoted in literature (5885.268 vs. 5889.911). Table 5 supports that the optimized designs are feasible cause all constraints are not violated. Statistical results given in Table 6 indicate that the extended MAM reduces the number of response analysis by 8%.

Table 5. Comparison of present constraint values with literature for the pressure vessel

| Methods | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
|---|---|---|---|---|
| Kannan & Kramer [14] | 1.6e-5 | -0.0689 | -21.220 | -196.31 |
| Deb [15] | -4.7e-3 | -0.0390 | -3652.877 | -127.321 |
| Coello [10] | -3.4e-2 | -0.0528 | -27.106 | -40.000 |
| Coello & Montes [11] | -2.0e-5 | -0.0359 | -27.886 | -63.346 |
| Montes & Coello [12] | -7.0e-6 | -0.0371 | 2.94791 | -63.360 |
| Kaveh & Mahdavi [16] | -5.1e-5 | -0.0013 | -19.195 | -41.237 |
| MAM | -5.3e-8 | -0.0012 | -0.01962 | -40.000 |
| Extended MAM | -5.3e-8 | -0.0012 | -0.01962 | -40.000 |

Table 6. Statistical results from present algorithms for the pressure vessel

| Methods | | | | MAM | | Extended MAM | |
|---|---|---|---|---|---|---|---|
| Number of points | | | | 9 | | 9 | |
| Starting point ($T_s, T_h, R, L$) | | | | Output Value | No. of iteration | Output Value | No. of iteration |
| 1.0 | 1.0 | 100 | 150 | 5885.268 | 10 | 5885.268 | 10 |
| 0.8 | 0.5 | 50 | 150 | 5885.268 | 10 | 5885.268 | 9 |
| 0.5 | 0.5 | 100 | 100 | 5885.268 | 21 | 5885.268 | 17 |
| 1.5 | 1.5 | 50 | 50 | 5885.268 | 9 | 5885.268 | 10 |
| Average | | | | 5885.268 | 13 | 5885.268 | 12 |

## Summary

The extended multipoint approximation method is proposed to solve complex optimization problems with the high efficiency and accuracy. It significantly reduces the calls for evaluations on objective and constraint functions, which is really important when solving complex optimization problems. The robustness of the extended multipoint approximation method is validated by some benchmark examples. The potential of this developed method demonstrates that it can be easily implemented in MAM to solve mixed-variable problems and outperforms the other methods in terms of the computational accuracy.

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