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K-space Filter Deconvolution and Flip Angle Self-Calibration in 2D Radial Hyperpolarised $^3$He Lung MRI

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Short Title: K-space Filter Deconvolution in $^3$He MRI
Abstract

In hyperpolarised $^3$He lung MRI with constant flip angles the transverse magnetisation decays with each RF excitation, imposing a k-space filter on the acquired data. For radial data acquired in an angularly-sequential order this filter causes streaking, angular shading and loss of spatial resolution in the images. Radial trajectories sample the centre of k-space with every projection, thus self-tracking the signal decay. The average flip angle per slice was calculated from this signal decay, and the values were found to correspond well with conventional flip angle maps, providing a means of flip angle self-calibration. The inverse of the signal decay function was used to retrospectively deconvolve the RF depolarisation k-space filter effects, and the method was demonstrated in 2D radial imaging in phantoms and human lungs. A golden angle radial acquisition was shown to effectively suppress artefacts caused by the RF depletion k-space filter.

Keywords: hyperpolarised $^3$He, lung MRI, radial acquisition, k-space filter
List of abbreviations used (excluding standard abbreviations):

\(^{3}\text{He}\) helium-3

\(^{129}\text{Xe}\) xenon-129

\text{SNR}\) signal to noise ratio

\text{RF}\) radio frequency

\text{pO}_2\) oxygen partial pressure

\text{N}_2\) nitrogen

\text{FOV}\) field of view

\text{BW}\) bandwidth

\text{ROI}\) region of interest

\(^{13}\text{C}\) carbon-13
INTRODUCTION

Hyperpolarised gas ($^{3}$He or $^{129}$Xe) MRI is a valuable tool for the visualisation of lung and airways ventilation in obstructive lung diseases. Images are most commonly acquired using 2D spoiled gradient echo sequences with low flip angle RF pulses at a constant flip angle, and a breath-hold of 15 seconds is a typical acquisition time for imaging of static lung ventilation in humans (1).

Radial sampling offers potential benefits for hyperpolarised gas imaging (2-6). It is robust to angular undersampling (7) which allows acceleration of image acquisition. Frequent resampling of the k-space centre by radial acquisition enables the tracking of signal over time, an attribute which has been used for dynamic imaging of $^{3}$He inhalation and exhalation (2,3,5,6). The frequent resampling of the k-space centre also makes radial acquisition robust to motion, which is useful because the lungs experience cardiac pulsatility and diaphragmatic motion even at breath-hold. Finally, the short echo times possible with radial sampling can help mitigate losses in SNR through the effects of diffusion attenuation of the gas signal by the dephasing action of the imaging gradients (4).

In hyperpolarised gas MRI the longitudinal magnetisation is non-renewable. The transverse signal ($S_n$) in a spoiled sequence with constant flip angle decays with each RF excitation according to the relation:

$$ S_n = S_0 (\cos \theta)^{n-1} \exp\left[-t/T_1(pO_2)\sin \theta\right] $$  \[1\]

where $S_0$ is the initial longitudinal magnetisation, $n$ is the excitation number and $\theta$ is the constant flip angle used. The gas $T_1$ depends on the regional oxygen partial pressure, $pO_2$, and is approximately 20s in fully oxygenated lung but changes during the breath-hold (8). The effect of $T_1$ in equation [1] is negligible on the timescale of a 2D imaging experiment at short TR (9) and thus equation [1] simplifies to:
\[ S_n = S_0 (\cos \theta)^{n-1} \sin \theta \]  

This signal decay imposes a filter on the k-space data in the phase encoding or angular encoding incrementation dimension. There are additional effects which contribute towards the decay of transverse signal over the course of the data acquisition, including diffusion attenuation due to the imaging gradients and \( T_2^* \) decay. However, in experiments in human lungs these are less significant than the signal decay caused by RF depletion (10), and so are not considered in this treatment. In Cartesian sampling, the signal decay causes loss of effective spatial resolution for centric phase encoding and loss of signal to noise ratio (SNR) for sequential phase encoding (10). A variable flip angle scheme, where the flip angle is increased throughout the acquisition in order to keep the amount of transverse magnetisation constant, has been introduced to counter this effect (11). However, it is difficult to implement in a robust fashion due to \( B_1 \) inhomogeneity and slice profile effects in 2D (12), and is not commonly used at present. Retrospective adaptive k-space filtering has been proposed to compensate for signal decay in a constant flip angle Cartesian acquisition, and was shown to slightly improve image quality (13).

With radial imaging the trade-off between image SNR and spatial resolution should be less critically dependent upon k-space acquisition order, because both low and high spatial frequencies are acquired with each projection. The signal decay filter in the angular (RF encoding) dimension does however, cause streaking, angular shading and loss of spatial resolution in the image. Sampling the centre of k-space (\( k=0 \)) with every projection allows the signal decay to be self-tracked throughout the acquisition. In this work the effect of the signal decay k-space filter on image quality for 2D radial hyperpolarised \( ^3 \)He lung imaging at breath-hold was investigated for both sequential and golden angle (14) radial sampling strategies. The use of the signal decay function to determine the average flip angle applied to a slice was
investigated. A method which uses the inverse of the sampled k=0 decay function to retrospectively compensate the data for the signal loss incurred by RF depolarisation is proposed and is demonstrated in phantom and breathold human lung imaging experiments with $^3$He.

METHODS

**Effect of RF depletion on k-space filter**

The signal decay caused by RF depletion (eq. 2) imposes a k-space filter on the acquired data. The effect of this filter in image space was investigated for sequential radial acquisition (angle incremented between projections ($\Delta \phi = 180^\circ$/number of projections) and for golden angle radial acquisition ($\Delta \phi = 111.246^\circ$) (14) where the RF depletion filter function is distributed more evenly throughout 2D k-space, but the angular sampling density is slightly non-uniform.

*Simulations - Effect of Flip Angle on Signal for Radial Sampling*

The transverse signal in a spoiled sequence with constant flip angle ($\theta$) is given by equation [2]. The signal amplitude of the k=0 point of k-space is a measure of the signal level in the resulting image. For Cartesian centric encoding the k=0 amplitude is proportional to $S_0 \sin \theta$, and for Cartesian sequential encoding the k=0 amplitude is proportional to $S_0 (\cos \theta)^{(N/2)-1} \sin \theta$ where $N$ is the total number of phase encodes (10). In radial imaging all projections contribute to the final gridded k=0 amplitude, so the signal level in the image is proportional to the mean k=0 value of all the projections:

$$\text{signal} \propto \sum_{n=1}^{N} S_n / N \quad [3]$$
Using equations [2] and [3], k=0 signal amplitudes were calculated for radial acquisitions with 201, 128, 96 and 64 projections over a range of constant flip angle values from 1° to 15°, and for Cartesian sequential and centric acquisitions with 201, 128, 96 and 64 phase encodes.

**Simulations - sampling patterns and PSFs**

The RF depolarisation k-space filter (eq. 2) and its point-spread function (PSF) were simulated for radial acquisition using Matlab (MathWorks, Natick, MA, USA). Three scenarios were simulated; no k-space filter (constant $M_{xy}$), the k-space filter for a sequential acquisition and the k-space filter for a golden angle acquisition. A full echo radial trajectory with 128 read encode samples in the k, direction and 201 angular projections was populated with values of one. The number of projections ($N_\theta$) were chosen to satisfy the Nyquist criterion in the angular direction for a read-direction matrix ($N_r$) of 128 ($N_\theta = \pi N_r / 2$). The simulated radial data were regridded to Cartesian k-space and Fourier transformed to visualise the PSF. For the simulations of the k-space filter, a constant flip angle ($\theta$) of 6.4°, excitation numbers ($n$) from 1 to 201 and an initial signal value ($S_1$) of one were input into eq. 2 to model the RF depletion. Sequential and golden angle trajectories were filled with the calculated RF decay values, regridded and Fourier transformed. The chosen flip angle of 6.4° depleted the signal to 29% of its initial value by the last projection.

**Imaging**

Hyperpolarised $^3$He radial spoiled gradient echo (SPGR) images were acquired from volunteers and phantoms using a 3T whole body system (Philips Intera, Best, Netherlands, maximum gradient strength = 40mTm$^{-1}$, maximum slew rate = 200mTm$^{-1}$s$^{-1}$) and data were processed in Matlab. $^3$He was polarised to ~25% with a Helispin polariser (GE Healthcare, Amersham, UK) under a site-specific UK regulatory licence and imaging was performed with ethics committee approval. All radial data were reconstructed using regridding.
**Phantom Data**

A 1L Tedlar bag phantom (Jensen Inert Products, Coral Springs, FL, USA) containing 100ml hyperpolarised $^3$He and 900ml $N_2$ was imaged with both a sequential radial acquisition and a golden angle radial acquisition. A prototype Helmholtz coil (Pulseteq, UK) of 20cm diameter loops was used for linear transmit-receive. The same axial 2D slice was imaged with a two minute pause between the acquisitions to allow hyperpolarised $^3$He with no RF depolarisation to mix by diffusion with the partially depolarised imaging slice. Other imaging parameters were; FOV = 38cm$^2$, matrix = 128 x 201 projections with the first projection aligned right-left, full echo, slice thickness = 15mm, TE = 2.3ms, TR = 5.4ms, BW = 56kHz and $\theta = 5.3^\circ$.

**Volunteer Data - Helmholtz Transmit-Receive Coil**

300ml of hyperpolarised $^3$He mixed with 700ml of $N_2$ was inhaled by a healthy volunteer. Axial radial images of the lungs were acquired at breath-hold using a prototype Helmholtz coil for transmit-receive (FOV = 32cm$^2$, matrix = 128 x 201 projections with the first projection aligned right-left, full echo, slice thickness = 15mm, 8 slices, TE = 2.4ms, TR = 5.6ms, BW = 47kHz, mean $\theta = 5.8^\circ$ and a sinc function apodised with a Gaussian kernel (sinc-Gaussian) RF pulse envelope).

**Volunteer Data - Birdcage Transmit-Receive Coil**

In order to study the effect of the flip angle spatial distribution on the RF depletion k-space filter, further volunteer experiments were carried out using an elliptical quadrature birdcage coil (Rapid Biomedical, Germany) for transmit-receive, which had a more homogeneous $B_1$ field than the Helmholtz coil. Axial radial images of the lungs were acquired at breath-hold (300ml $^3$He / 700ml $N_2$ per breath-hold, FOV = 32cm$^2$, matrix = 128 x 201 projections, full echo, slice thickness = 15mm, 8 slices, TE = 2.1ms, TR = 6.4ms, BW = 56kHz and a sinc-Gaussian RF pulse envelope). In one experiment projections were sequentially acquired with the first
projection aligned right-left; during one breath-hold a mean flip angle of $4.2^\circ$ was used and during a second breath-hold a mean flip angle of $6.3^\circ$ was used. In another experiment sequential radial data and a golden angle radial data were acquired in separate breath-holds with a mean flip angle of $6.4^\circ$ and the first projection aligned anterior-posterior.

**Flip angle calculation**

In order to best use the available magnetisation in hyperpolarised gas MRI, the flip angle must be calibrated for each subject because the delivered flip angle will change according to coil loading at 97MHz. In a standard flip angle calibration typically a small amount of hyperpolarised gas is inhaled by the volunteer and a series of spoiled pulse acquire acquisitions are made to sample the RF depletion of the signal, from which the flip angle is determined according to equation [2]. Here the use of a radial acquisition for flip angle self-calibration was investigated.

In order to compare the radial flip angle calculation to a 'standard' flip angle calibration, radial and Cartesian data were acquired at consecutive breathholds of 100ml of hyperpolarised $^3$He and 900ml of $N_2$. Data were acquired using; (i) a sequential radial SPGR sequence (Helmholtz coil, FOV = 32cm$^2$, read matrix = 64, 64 projections, slice thickness = 15mm, 8 slices, TE = 2.3ms, TR = 5.5ms, BW = 31kHz), and (ii) a Cartesian sequence with the phase encoding turned off (all other parameters the same as the radial sequence). For both radial and Cartesian data sets, the average flip angle of each slice was calculated from eq. 2. This was calculated twice, using all projection data and also using data from only the first 20 projections. Simulations (not shown) using the sinc-Gaussian RF envelope used in the sequence showed that the deviation from the ideal flip angle after 20 RF pulses due to the effects of the flip angle distribution across the slice profile described in (10) is negligible.
The flip angle was also calculated in this fashion for the Helmholtz coil volunteer radial data and the birdcage coil volunteer data (average flip angle of 6.4°) described in the 'effect of RF depletion on k-space filter' section. Calculations were performed slice by slice using all projection data, using data from only the first 64 projections, and using data from only the first 20 projections. To produce flip angle maps for comparison, Cartesian images (matrix = 64 x 64, 3 dynamic frames, θ matched, 300ml ³He mixed with 700ml N₂, Helmholtz coil images: TE = 1.8ms, TR = 4.2ms, BW = 32kHz, and birdcage coil images: TE = 2.1ms, TR = 6.4ms, BW = 28kHz) of the same axial slices were acquired at breath-hold of hyperpolarised ³He following the radial data acquisitions. Flip angle maps were produced by fitting the signal decay in successive images to equation 2 pixel by pixel. The images were masked to exclude background noise using a threshold (15 times the standard deviation of the noise). Again, the effects of the slice profile on underestimating the RF decay (10) were neglected in this process but simulations indicate an error of less than 10%. The mean and standard deviation of the resulting B₁ maps were calculated for each slice. In addition, the golden angle volunteer radial data were processed to produce self-referenced flip angle maps using a method similar to that proposed by Frederick et al (15). Images were reconstructed from three groups of 64 projections to produce three images separated by a mean of 64 RF pulses and then fitted to produce flip angle maps.

**Compensation of k₀-space filter**

![Figure 1](image)

For the sequential radial data described above, a 3rd order polynomial was fitted to the decay of k=0 signal as a function of projection (fig 1a) for each slice. A fit to the data was used, rather than using the data values directly, in order to reduce the contribution of noise to the
compensation process. The inverse of the polynomial normalised by its minimum value (fig 1b) was used as a compensation deconvolution function. The data from each projection was multiplied by the compensation function value for that projection ($k_\theta$ compensation). The compensated k-space data were then reconstructed using regridding. The $k_\theta$ compensation method was also applied to the golden angle radial data.

The proposed method of $k_\theta$ compensation could potentially result in the contribution of later projections which have a lower signal to noise ratio (SNR) being amplified. To evaluate the effect of the compensation method on image quality, a region of interest (ROI) assessment of the original and $k_\theta$ compensated images was carried out. The noise of radial images is spatially correlated (16) and the streak artefacts are spatially variable, meaning that a ROI measurement as a metric of SNR (defined as the mean signal of an object ROI divided by the standard deviation of a background ROI) has inherent flaws. Here the intention was to provide a rough measure of image quality taking into account the presence of both streaking artefacts and noise.

Figure 2

ROIs (examples shown in fig 2) were prescribed on the original image that encompassed the whole object (ROI 1, inside solid line), everything outside of the object (ROI 2, outside solid line), a background region within the signal smear artefact (ROI 3, dashed line) and a background region away from visually-obvious artefacts (ROI 4, dotted line). The mean and standard deviation (SD) of pixel magnitudes in each ROI were calculated from the pre and post correction images. Three ratios of the ROI statistics were calculated; ratio A = mean(ROI 1)/SD(ROI 2), ratio B = mean(ROI 1)/SD(ROI 4), and ratio C = mean(ROI 3)/mean(ROI 1). Ratio A is an overall measure of image quality; signal is inside the object (where it dominates over noise and artefacts), and noise and artefacts are present throughout the image. Ratio B is an
approximation of image SNR, which aims to reduce the artefact contribution to the measurement. Ratio C gives a sense of the artefact level in the image.

RESULTS

Effect of RF depletion on k-space filter

Simulations - Effect of Flip Angle on Signal for Radial Sampling

Figure 3

The simulation results of the flip angle influence on total k=0 signal for radial sampling are shown in figure 3. The flip angle value which gives the maximum signal for radial acquisition increases as the number of projections decreases (left plot). The optimum flip angle is 6.4° for 201 projections, 8.0° for 128 projections, 9.2° for 96 projections and 11.3° for 64 projections. The optimum flip angle for radial acquisition is higher than that for sequential Cartesian acquisition using the same number of projections and phase encode lines (right plot, e.g. 6.4° for 201 radial projections compared to 5.7° for 201 sequentially acquired Cartesian phase encode lines). The signal peak is broader for radial acquisition than for sequential Cartesian acquisition suggesting less sensitivity to flip angle setting, a useful factor when considering coils with limited B₁ homogeneity.

Simulations - sampling patterns and PSFs

Figure 4

Simulation results (fig 4) show the k-space filter and PSF with no RF depletion (top row), with RF depletion and sequential angular acquisition (middle row), and with RF depletion and golden angle acquisition (bottom row). The RF depletion in a sequential radial acquisition
leads to an asymmetric broadening of the PSF with a long vertical component and a diagonally-oriented oval smoothing kernel. These produce vertical streak artefacts, angular shading and loss of spatial resolution in the image. Golden angle radial acquisition distributes the polarisation decay more evenly over k-space resulting in a more benign k-space filter. The centre of the golden angle PSF is similar to that of the no RF depletion PSF; there is little broadening of the PSF main peak. However, golden angle sampling leads to weak side lobes appearing in the PSF, which would correspond to angular undersampling radial streak artefacts in the image. This is because the Nyquist criterion is not satisfied angularly for all of k-space with the golden angle acquisition simulated as a result of the non-uniform $\Delta k_\phi$. The golden angle sampling pattern has three different angular gaps (when the number of projections is not equal to a Fibonacci number), the largest of which determines the number of projections required to satisfy the Nyquist criterion for all of k-space. For this reason golden angle sampling requires more projections to satisfy the Nyquist criterion than uniform radial sampling (14).

**Imaging**

**Phantom Data**

Figure 5

Figure 5a shows a $^3$He phantom image acquired with the sequential radial acquisition and figure 5d shows the same slice acquired with the golden angle radial acquisition. The SNR of the golden angle image is less because it was acquired after the sequential image from the same hyperpolarised $^3$He sample. Vertical streak artefacts are prominent in the sequential image but are not present in the golden angle image.

**Volunteer Data**
Figure 6 shows $^3$He images of healthy volunteers acquired with the Helmholtz coil (a) and birdcage coil (d-m). The volunteer images sequentially acquired with higher flip angles show strong streak artefacts (e.g. figure 6a (mean slice $\theta = 5.8^\circ$), 6g (mean slice $\theta = 6.4^\circ$) and 6j (mean slice $\theta = 6.8^\circ$)). The images sequentially acquired with a lower flip angle show faint streak artefacts (e.g. figure 6d, posterior edge of the left lung (mean slice $\theta = 4.1^\circ$)) because the RF decay k-space filter is milder. The k=0 signal was depleted to 64% of its original value by the last projection when the mean slice flip angle was 4.1° compared to 26% of the original value when the mean slice flip angle was 6.4°. In figures 6a, 6d and 6g the streaks are anterior-posterior because the first k-space projection was aligned right-left, and in figure 6j the streaks are right-left because the first k-space projection was aligned anterior-posterior.

The RF depolarisation k-space filter artefacts are not present in the golden angle volunteer images (e.g. figure 6m). In addition there are no angular undersampling radial streak artefacts visible in the golden angle images.

**Flip angle calculation**

Similar flip angle values were returned for Cartesian and radial flip angle calibrations that used the same number of projections. Using all 64 projections, the average flip angle (averaged over all slices) calculated from the Cartesian acquisition was 9.4°. For the radial acquisition it was 9.6°, with a maximum difference of 1.7° and a minimum difference of 0.1° between the Cartesian and radial values on a slice by slice basis. Using the first 20 projections only, the average flip angle (averaged over all slices) calculated from the Cartesian acquisition was 12.4°. For the radial acquisition it was 13.1°, with a maximum difference of 1.9° and a minimum difference of 0.1° between the Cartesian and radial values on a slice by slice basis. This implies that there are small rotational effects of radial sampling *in vivo* (e.g. angle-
dependent echo shifting) which may be due to background gradients caused by magnetic susceptibility differences. For both 64 and 20 projections the maximum difference between Cartesian and radial values was found in the slice nearest the diaphragm. Mis-registration due to slightly different breathhold positions may have contributed to the differences between flip angle values in these lower slices. In similar phantom experiments (not fully described for brevity) the same flip angle was returned for Cartesian and radial data in both cases.

The largest effect on the returned flip angle is caused by the number of projections used in the calculation. Using all 64 projections returns a lower average flip angle (Cartesian/radial = 9.4°/9.6°) than using only the first 20 projections in the calculation (Cartesian/radial = 12.4°/13.1°). This is primarily due to slice profile evolution; signal from the slice edges, which experience lower flip angles, weights the later projections leading to a shallower signal decay function for slice-selective acquisitions (10). In addition, diffusion of 'fresh' gas with no RF history from outside the excited slice adds to the in-slice signal throughout the acquisition (17), although this effect will be small for the gas diffusion coefficients found in vivo at this sequence TR. These RF history effects increase over time, so data from the first few projections give a better estimate of the true flip angle applied.

Figure 7

Figure 7b shows that the average slice flip angles calculated from the Helmholtz coil volunteer radial data (red, all 201 projections) correspond well with those calculated from Cartesian flip angle maps (blue, error bars show inter-pixel standard deviation), e.g. radial = 6.2°, $B_1$ map = 6.2 ± 2.6° for slice 6 (smallest difference) and radial = 5.3°, $B_1$ map = 6.2 ± 2.6° for slice 8 (largest difference). Fig 7a shows the Cartesian flip angle map of slice 6 for comparison. The spatial inhomogeneity of the $B_1$ field of the Helmholtz coil in the axial plane is evident. The average slice flip angles calculated from the first 64 projections of the radial data (green) and
from the first 20 projections of the radial data (yellow) are also plotted in figure 7b. The flip angle values increase as the number of projections used in the calculation decreases. The reason for this is demonstrated in figure 7c, where the log k=0 values of all the radial projections are plotted (black crosses, slice 6), along with fits to all the projections (red line), the first 64 projections only (green line) and the first 20 projections only (yellow line). The 3 cyan triangles approximate the positions on the RF decay curve at which the Cartesian k=0 values were sampled to create the $B_1$ maps. The Cartesian flip angle values correspond well to the radial flip angle values calculated using all projections because both datasets were affected to a similar extent by slice profile evolution effects.

Comparisons of flip angles calculated from radial data and Cartesian $B_1$ mapping for the birdcage coil volunteer data are shown in figure 7 (d, e, f). The golden angle radial values and data are shown in plots (e) and (f), sequential radial data gave similar results (not shown for clarity). The flip angle values calculated from the radial data (201, 64 and 20 projections) fall within plus or minus the standard deviation of the mean Cartesian $B_1$ map values, with the flip angle values calculated from 64 radial projections being closest to the Cartesian values. The $B_1$ field of the birdcage coil (d) is more homogeneous than that of the Helmholtz coil (a) which results in a lesser deviation from a linear fit in the log plot (f) than (c). In an inhomogeneous $B_1$ field the k=0 values of the first projections are dominated by signal from spatial regions which experience larger flip angles (at the anterior-posterior edges here). As RF excitations are applied, signal in high flip angle regions decays faster, so signal from regions which experience smaller flip angles contribute more to later k=0 values. The influence of $B_1$ inhomogeneity acts in addition to slice profile evolution effects, causing a bigger variation between flip angles calculated with different numbers of projections for the Helmholtz coil (b) than for the birdcage coil (e).
Using the radial k=0 image data as proposed returns the average flip angle delivered over the excited slice. The flip angle calculation discussed above is not a substitute for $B_1$ mapping methods (e.g. (18)), but does give the slice by slice variation of the average flip angle without the need for additional data acquisition.

Figure 7(g) is the self-referenced flip angle map produced from the golden angle radial data. The spatial flip angle distribution is similar to that of the Cartesian $B_1$ map (7d) but the values are generally lower in magnitude (slice 3; Cartesian $B_1$ map, mean ± std = 7.2 ± 1.1; golden angle radial $B_1$ map, mean ± std = 6.3 ± 1.4). The inaccuracy of the golden angle flip angle map is likely due to the angular undersampling artefacts and noise in the source images.

**Compensation of $k_\Phi$-space filter**

Figure 5 shows (a) the original phantom image and (b) the $k_\Phi$ compensated image. Figure 5(c) shows the difference between the original and $k_\Phi$ compensated images (b-a) normalised by the maximum image values.

The top row of figure 6 demonstrates the compensation method for volunteer data acquired with the Helmholtz coil. The prominent vertical streak artefacts are removed in the compensated images (b). The normalised difference image (c) highlights changes in angular shading and the appearance of some features between the original and compensated images (for example in the right anterior lung, white arrows).

The birdcage coil volunteer data are shown in the other four rows of figure 6. For sequentially acquired data the compensation method (central column) removes the streaking artefacts from the original image (left column) and restores signal in some areas e.g. near the blood vessel in the central right lung of j,k,l (white arrows). Applying the $k_\Phi$ compensation method to the golden angle data (m,n,o) gave no visible improvement in image quality.
The ROI image quality ratios are displayed in table 1. The proposed retrospective compensation method gave an improvement in overall image quality for all sequentially acquired data (ratio A is increased by the compensation method). Ratio B, which aims to approximate image SNR without the artefact contribution, was higher for the original images than the $k_\theta$ compensated images for the birdcage coil data and the Helmholtz coil phantom data, indicating that the compensation method does moderately reduce image SNR. However, ratio B was essentially unchanged between the original image and the $k_\theta$ compensated image for the Helmholtz coil volunteer data. This may be due to the modest signal gain around the vessels within the lungs seen in the $k_\theta$ compensated volunteer image (fig 6b,c) offsetting the noise amplification in later projections introduced by the compensation method. The artefact level (ratio C) was reduced by the compensation method for all the sequential data acquisitions, but not for the golden angle data acquisition where it is inherently low.

DISCUSSION AND CONCLUSIONS

The effect of the RF depletion k-space filter in radial imaging depends on the projection acquisition order. For sequential radial acquisition the k-space filter causes streaking, angular shading and loss of spatial resolution in the image. Golden angle radial acquisition, which distributes the effects of the k-space filter more evenly over k-space, gives reduced image artefacts from RF depletion. In theory golden angle acquisition introduces angular undersampling artefacts compared to the same number of uniformly acquired projections, but in volunteer and phantom images (using 201 projections with a 128 $k_r$ matrix) these effects were not observed.

Artefacts due to the RF depletion k-space filter depend intrinsically on the delivered flip angle. Decreasing the flip angle can greatly reduce the RF decay artefacts but simulations show that
signal is less than optimum for flip angle values where this is the case. The problem is more
difficult to avoid for a coil with an inhomogeneous $B_1$ field due to the large range of flip angle
values present over the imaging volume, such that even for a low average slice flip angle there
are likely to be some regions which experience high enough flip angles to cause RF decay
artefacts.

Using a radial acquisition for RF calibration returns a similar flip angle to the conventional
method of using a series of spoiled pulse acquire acquisitions, with the added utility of
providing a $^3$He image which can be used as a scout for planning of further scans. The
provision of a $^3$He scout image from the RF calibration scan is particularly valuable when using
$^3$He coils without proton traps (e.g. references (19) and (20)) where it is not possible to
acquire proton images for planning purposes. The $B_1$ self-calibration property of radial $^3$He
imaging can be used to calculate the average slice flip angle without an additional $B_1$ mapping
scan, which may prove useful to compensate for $B_1$ inhomogeneity in the slice direction.

For golden angle radial acquisition self-referenced flip angle maps can be generated with no
additional data acquisition. The flip angle maps produced are prone to errors introduced by
angular undersampling artefacts and noise in the source images but give a reasonable
approximation of the flip angle distribution.

The oversampling of the centre of k-space inherent to radial acquisition provides a self-
measure of the signal decay. This means that the effect on the decay of differences between
the intended and delivered flip angle due to $B_1$ transmit inhomogeneity (which is not trivial for
close coupled body transmit coils at high $B_0$) are taken into account by the compensation
method. The proposed retrospective compensation method gives a substantial improvement
in image quality for sequential acquisition, however it does amplify noise in the later
projections resulting in a decrease in apparent image SNR. A $k_0,k_r$ compensation method,
where a variable Gaussian kernel is applied in the $k_r$ direction to suppress noise amplification at high $k_r$ in the later projections, can be applied to increase apparent SNR at the expense of reduced spatial resolution due to the smoothing effect of the Gaussian filter in $k_r$. Overall, golden angle sampling appears to be the best option for the suppression of RF decay artefacts because it allows the use of the optimum flip angle without any noise amplification due to the retrospective compensation technique.

This work focused on radial imaging at breath-hold of hyperpolarised $^3$He. Although the $k$-space filter effect is also present in dynamic imaging of inhaled gas, it is convolved with the unquantified inflow or outflow of hyperpolarised gas which makes the transverse signal behaviour more complex (21). Whilst demonstrated here for $^3$He with a 2D radial sequence, the proposed techniques are directly relevant to the imaging of other hyperpolarised nuclei such as $^{129}$Xe and $^{13}$C with any non-Cartesian pulse sequence that repeatedly samples central $k$-space, and could equally well be adapted to 3D radial acquisition.

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EPSRC grant #EP/D070252/1, GE Healthcare for polariser support, Matthew Clemence for help with $k$-space trajectory mapping, Rachel Chan at the University of Toronto for regridding code, and Sebastian Kozerke at King’s College London for loan of the birdcage $^3$He coil.
REFERENCES


4. Wild JM, Cooper KJ. Radial Projection MRI of Hyperpolarised 3He Gas in the Lungs. 2007; ISMRM Non-Cartesian Workshop, Sedona, AZ, USA.


**Tables**

**Table 1 - Image compensation ROI ratios**

Ratios of image ROIs pre and post correction. The ratios are stated in the 'compensation of k-space filter' section.

<table>
<thead>
<tr>
<th></th>
<th>ratio A</th>
<th>ratio B</th>
<th>ratio C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>phantom data - Helmholtz coil</strong></td>
<td></td>
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<td></td>
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<tr>
<td>original sequential image, mean $\theta = 5.3^\circ$</td>
<td>25.2</td>
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<td>21.5</td>
<td>0.15</td>
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<tr>
<td>compensated golden angle image, mean $\theta = 6.8^\circ$</td>
<td>11.3</td>
<td>13.2</td>
<td>0.16</td>
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</table>
FIGURE CAPTIONS

Figure 1

Determination of the k-space filter compensation function for the phantom data shown in figure 4. (a) k=0 signal decay (points) and the fitted polynomial decay function (line) with respect to projection number, and (b) the \( k_\phi \) compensation function (the normalised inverse of the polynomial in (a)). (c) shows the \( k_\phi, k_r \) compensation kernel with \( k=0 \) \( k_\phi \) compensation determined by the function in (b) and a Gaussian filter in the \( k_r \) direction. The compensation functions are shown regridded onto a Cartesian k-space grid \( (k_x, k_y) \) in (d) \( k_\phi \) compensation only and (e) \( k_\phi, k_r \) compensation.

Figure 2

The ROIs described in the 'methods; compensation of k-space filter’ section are shown drawn on the original sequential images for (a) phantom data and (b) volunteer data. The solid line outlines ROI 1 (everything inside) and ROI 2 (everything outside), the dashed line outlines ROI 3, and the dotted line outlines ROI 4.

Figure 3

Simulation results of average signal as a function of flip angle for (left plot) radial acquisition with 201, 128, 96 and 64 projections and (right plot) radial acquisition with 201 projections, Cartesian sequential acquisition with 201 phase encoding lines and Cartesian centric acquisition.
Figure 4

Simulation results of the k-space filter (left column) and PSF (second column) for, (top row) no RF depletion, (middle row) RF depletion and sequential radial acquisition, and (bottom row) RF depletion and golden angle radial acquisition. The third column shows the centre of the PSF, and the right column shows the log PSF, for each case. The full PSFs (middle column) are displayed with intensity values saturated at 25% of the maximum signal value in order to see the side lobe patterns.

Figure 5

The effect of sampling pattern and the proposed compensation method on phantom images. (a) is the original image acquired with sequential radial sampling, (b) is the \( k_x \) compensated image and (c) is the normalised difference image (b-a). (d) shows the same slice acquired with golden angle sampling (original image). The SNR of (d) is lower than that of (a) because it was acquired after (a) from the same dose of \( ^3\)He.

Figure 6

In-vivo \(^3\)He images; top row acquired using the Helmholtz coil, all other images acquired using the birdcage coil. (left column) the original image, (central column) the \( k_x \) compensated image and (right column) the normalised difference image (central minus left). Sequential radial images with the first projection aligned right-left are shown acquired with a mean slice flip angle of 5.8\(^\circ\) (top row), a mean slice flip angle of 4.1\(^\circ\) (second row) and a mean slice flip angle of 6.4\(^\circ\) (third row). Images acquired with a mean slice flip angle of 6.8\(^\circ\) and the first projection
aligned anterior-posterior are shown for a sequential radial acquisition (forth row) and a golden angle radial acquisition (bottom row). White arrows indicate changes in the appearance of features between original and compensated images.

Figure 7

Comparison of flip angles calculated from radial data and Cartesian B₁ mapping. (a) Average slice flip angle of Helmholtz coil volunteer data calculated; using a Cartesian flip angle mapping method (blue, standard deviation shown by error bars), from the k=0 signal of all 201 radial projections (red), from the k=0 signal of only the first 64 radial projections (green), and from the k=0 signal of only the first 20 radial projections (yellow). (b) Cartesian flip angle map for slice 6. (c) Log plot of the radial k=0 data (black crosses) with fits to all projections (red), the first 64 projections only (green), and the first 20 projections only (yellow). The 3 cyan triangles approximate the positions on the RF decay curve at which the Cartesian k=0 values were sampled to create the Cartesian B₁ maps. The insert in the top right corner of (c) is a magnification of the data for the first projections. (d), (e) and (f) are the corresponding figures for the birdcage coil golden angle data. (g) is the self-reference flip angle map produced from the golden angle radial data (c.f. (c) the Cartesian data flip angle map of the same slice).