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Modelling bus bunching and holding control with vehicle overtaking and distributed passenger boarding behaviour

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Abstract:

Headway fluctuation and bus bunching are commonly observed in transit operations, while holding control is a proven strategy to reduce bus bunching and improve service reliability. A transit operator would benefit from an accurate forecast of bus propagation in order to effectively control the system. To this end, we propose an 'ad-hoc' bus propagation model taking into account vehicle overtaking and distributed passenger boarding (DPB) behaviour. The latter represents the dynamic passenger queue swapping among buses when bunching at bus stops occurs and where bus capacity constraints are explicitly considered. The enhanced bus propagation model is used to build the simulation environment where different holding control strategies are tested. A quasi first-depart-first-hold (FDFH) rule is applied to the design of headway- and schedule-based holding control allowing for overtaking, with the objective to minimise the deviation from the targeted headway. The effects of control strategies are tested in an idealized bus route under different operational setting and in a real bus route in Guangzhou. We show that when the combined overtaking and queue-swapping behaviour are considered, the control strategies can achieve better headway regularity, less waiting time and less on-board travel time than their respective versions without overtaking and DPB. The benefit is even greater when travel time variability is higher and headway is smaller, suggesting that the control strategies are preferably deployed in high-frequency service.

Keywords: Public transport; Holding control; Bus bunching; Overtaking; Distributed passenger boarding

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1. Introduction

The effectiveness of public transport system can be measured by its reliability. In uncontrolled bus systems, bus bunching is prevalent especially in the peak hours. Bus bunching occurs when two or more buses along the same route arrive at a designated stop simultaneously. This is undesirable for both passengers and transit operator since it leads to unexpectedly longer waiting times and degraded service reliability of public transport system (Hollander and Liu, 2008).

A series of factors contribute to bus bunching, such as stochastic running times and demand, vehicle capacity, driving manoeuvres, and passenger boarding behaviour. Among the driving manoeuvres, bus overtaking is one that is commonly observed in real life. Such phenomenon can take place between stops or at bus stops. The former is mainly due to stochastic travel times, whereas the latter often corresponds to scenarios whereby a late arrival bus departs earlier due to fewer queuing passengers. The performance of bus scheduling is closely related to both temporal and spatial distributions of passengers and available fleet (Sorratini et al., 2008; Liu and Sinha, 2007). Intuitively, there are two processes going on during bus service at stops. One is passengers' boarding and alighting process, and the other is the bus arrival process which forms bus bunching at the stopping area (Bian, et al, 2015). Accordingly, passengers would make their decisions as to which bus to board in response to bus arrival status at stops. When bus bunching occurs, passengers waiting at the stop may not always board the first arriving bus, instead they may autonomously swap queues over bus platoon to reduce their waiting time, assuming that the other bus also serves the same destinations. These microscopic behaviour are likely to have an impact on the performance of bus bunching and holding control.

To reduce bus bunching, a variety of corrective actions have been proposed in the literature. Within the family of dynamic control strategies, holding is the most commonly used. Holding control works by delaying buses at stops to regularize bus headway and reduce the overall passenger waiting time, possibly at the expense of extending on-board waiting time and total riding time. A well-designed holding strategy can improve the efficiency of a transit system by increasing its effective capacity and vehicle utilization. However, if poorly designed, the overall bus frequency would be reduced and the efficiency of a transit system worsened. One of the greatest problems facing transit agencies is maintaining service reliability while achieving high efficiency. It is clearly beneficial to mitigate the negative effects of holding control. Since overtaking provides some flexibility for bus motion, more efficiency could be expected by allowing buses to overtake each other. At the same time, the passenger queue swapping behaviour can also balance the queue lengths and thus the load over buses. Most of the existing literature on bus propagation and holding control strategies presents simplified models without consideration of overtaking or passenger queue swapping behaviour. To increase the operational efficiency and behavioural realism, we set out in this paper to investigate bus propagation and holding control with these realistic characters.

Our primary objective in this paper is to identify possible measures that could help operators and decision makers to realize the full potential of holding control schemes, more specifically by including overtaking and passenger queue swapping behaviour in the design of the control strategies. We achieve this firstly by developing a new bus motion model which accounts explicitly for the stochastic attributes and overcrowding effect caused by vehicle capacity. Secondly, the new bus motion model is further extended to embed holding control rules. We develop the holding control strategies for both the schedule- and headway-based approaches. The new holding control strategies are tested through case studies both for a hypothetic and a simulated real-life bus route. Our findings show that the inclusion of overtaking and passenger queue swapping behaviour can greatly increase the efficiency and accuracy of holding control strategies. We thus suggest that the performance of control policies can be improved in an ad-hoc manner, which provides managerial insights for bus operational control.

The remainder of this paper is organized as follows. In Section 2, we discuss the relevant literature. In Section 3, simulation frameworks for bus propagation are developed. In section 4, new holding control models are developed based on the new bus propagation model. In Section 5, a number of indicators are proposed. In Section 6, we verify the effectiveness of the proposed methods through an idealized bus line and a real bus line in Guangzhou, China. Finally, Section 7 draws conclusions of the study and discusses the practical implications on bus operational control.

2. Literature Review

There is an extensive literature on bus control strategies for improving service reliability. The analysis of bus bunching was pioneered by Newell and Potts (1964) for a single bus line. They described how a small initial delay from a designated bus stop propagates along the bus route, and the conditions for service recovery. Fonzone et al. (2015) studied the impact of passengers' timetable behaviour on bus bunching. They showed that the bus bunching phenomenon is in part due to such passengers' timetable behaviour. Schmöcker et al. (2016) investigated the influence of common line stops on bus bunching, and they found that the presence of common lines have positive effects when overtaking is possible. Their analysis, however, ignores bus capacity constraints. Since the Newell and Potts' model, a variety of solutions has been proposed to improve bus service reliability. A sampling of control strategies includes: holding control (e.g., Wu et al, 2016; Hernandez et al, 2015; Dessouky et al, 1999; Dagazno, 2009; Hickman, 2001), boarding limits (Delgado, et al, 2012), bus speed control (Daganzo and Pilachowski, 2011) and stop skipping (Sun and Hickman, 2005). Among them, bus holding control strategy is the most commonly adopted method. The design of a holding strategy is to determine whether a vehicle should be held and for how long at a given control point. The objective of holding control is to keep the sequence of vehicle headway regularity, or minimize the total passenger cost along the route.

The holding controlling approaches can be classified into three groups, namely, schedule-based control, headway-based control and optimization-based control. The first two approaches are triggered by bus arrival time deviations and headway variations, respectively, while the third approach optimizes holding times by formulating holding control as a mathematical programming problem where the objective function is cost or time based. They are implemented through building slacks in the schedule at designated time points, in which the slacks are predetermined and static for schedule-based control while in headway and optimization based holding strategies the slacks are determined in real-time. Under schedule-based control policy, drivers are instructed to hold until scheduled departure time in case of early arrival, while late arriving buses leave the stop immediately after completing serving passengers (Wirasinghe and Liu, 1995). Osuna and Newell (1972) studied the holding problem at a single service point for an idealized cycle route, aiming at minimizing the total waiting time of passengers over a long time. Hickman (2001) derived an analytical model to determine the optimal holding time at a control stop along a bus route considering the stochastic running time and the interaction between passengers and buses. Zhao et al. (2006) investigated the optimal slack time under schedule-based control, with the objective of minimizing passengers' expected waiting time. Recently, Wu et al (2016) introduced a schedule-based holding control with time window into the bus schedule coordination problem.

Headway-based holding control approach is mainly triggered by headway deviation. Daganzo (2009) explored a headway-based control scheme, in which the dynamic holding times are determined by taking advantage of the real-time forward headway information. The results showed that by using headway-based control approach, faster speed can be achieved in comparison to the schedule-based approach, and thus reducing the travel time for on-board passengers. Later, Xuan et al (2011) proposed a set of control strategies by integrating both the forward headway and backward headway information. It was found that the headway regularity can be further improved compared to the previous headway-based methods. The work is extended by Argote-Cabanero et al. (2016) to be scalable for multi-line systems, taking into account the interaction among different bus lines.

Alternatively, optimization-based models determine holding decisions through mathematical programming formulation, with the objective to minimize the passenger waiting time or cost, either for the waiting passenger at-stops only or in combination with passengers in-vehicle. Eberlein et al. (2001) proposed a model of dynamic holding to minimize the at-stop waiting time, assuming the availability of real-time information. The holding problem was formulated as a deterministic quadratic program. Delgado et al. (2012) jointly optimized holding times and the number of boarding passengers. They adopted a boarding limit to restrict the number of passengers boarding a vehicle even though there is still capacity remaining on the bus, with the goal of minimizing waiting times both in-vehicle and at-stops. Berrebi et al. (2015) proposed a stochastic bus holding model to dispatch buses on a loop-shaped route using real-time information. The problem was formulated as a stochastic decision process. Recently, Sánchez-Martínez et al. (2016) formulated an effective dynamic holding control model that can reflect dynamic running times and demand. The model was evaluated in a stochastic simulation environment under a variety of cases having different dynamics.

However, if the transit system relies solely on these strategies, the overall operational efficiency could be reduced. For example, a drawback of holding control is that it may result in lengthened bus dwell times and overall travel time. Although stop-skipping scheme could increase the commercial speed, it also increases the waiting time of those passengers at the stops which are skipped. The operational efficiency of a bus system involves both bus and passenger motions. If more flexibility can be provided by either process, the operational efficiency could be enhanced. One possible solution is to allow overtaking among buses. When the slow-moving bus hinders the following buses from passing, it often causes queueing behind it and forms the moving bottleneck. Since moving bottleneck is one of the largest contributors to traffic congestion, allowing for overtaking avoids the moving bottleneck effect along the bus route. Bus overtaking happens between bus stops (due to variations in travel conditions), as well at bus stops (due to variations in passenger demand). Therefore, strategic overtaking may be desirable to manage disruption. On the other hand, although overtaking may result in leap-frogging bunch arrivals at stops, the bunch can be released again from overtaking operation, for example, a latter bus can leave the stop earlier than the front bus when fewer passengers are queueing for it, thus saving the loss time holding at stops. Another solution is to distribute passenger queueing in response to bus arrivals and crowding levels, either in a guided or autonomous way. For example, passengers are asked to board a latter bus if the current bus is congested, or they switch queues towards less congested buses once bus bunching occurs. Instead of using boarding limits to redistribute passengers among vehicles as Delgado et al (2012) suggested, in this research we consider a behaviour-driven distribution of passengers among bunched vehicles. This is approximated by using a proportional assignment approach based on the available residual capacity. We call such solutions as 'ad hoc control strategy' in that they take advantage of the internal driving and distributed passenger boarding (DPB) behaviour instead of relying on passive control actions imposed on vehicles.

Although there are many literature with various methods of operational control, to simplify the models, most of the existing studies present an oversimplified bus model, notably without overtaking. As stated by Sánchez-Martínez et al (2016), "the relaxation of overtaking constraints in holding control would require structural changes to models". Another significant simplification in previous studies is the assumption that passengers always board the first bus that arrived at the bus stop unless reaching vehicle capacity or boarding limit. However, in reality, passengers may swap queues to a latter incoming bus that might depart earlier as less passengers are queuing for it. Such autonomous behaviour should be reinforced when bus capacity is considered. A recent work for common line design problem considering these two features is Schmöcker et al (2016). They present cases wherein passengers form queues of equal length for only two buses, assuming that buses have infinite capacity to accommodate all passengers awaiting at stops. The existence of capacity constraint has further increased the challenge of solving the problem. The key challenge in considering bus capacity while allowing for overtaking in holding control lies in two aspects. First, as it turns out (see Remark 2 and 3), there is an interaction between overtaking and capacity constraint in the calculation of bus load along the route, which also causes difficulty in accounting for the boarding, alighting and "leftover" passenger demand. Second, compared to holding control without overtaking, the sequence of holding actions should be re-organized according to bus arrival times and random departure times, thus a solution is required to obtain minimal total dwell (holding) time.

The main contribution of this paper is an enhanced bus propagation model which explicitly considers bus overtaking and the dynamic queuing and boarding behaviour of passengers. Bus overtaking can take place both en-route and at stops, while the queuing passengers can redistribute themselves among all dwelling buses which considering spare capacity on buses. As a secondary contribution, this paper proposes a quasi-first-depart-first-hold (FDFH) principle to allow practical implementation of holding control strategies in such a realistic bus propagation environment. We provide mathematical formulation of the bus propagation model and two holding control strategies under FDFH principles. The performance of the bus propagation model with the enhanced features on bus overtaking and dynamic passenger behaviour, and under the holding control, is demonstrated in a simulation environment. As far as we are aware, this is the first time that overtaking and passenger queue swapping behaviour have been considered in the bus bunching and holding control models with capacity constraint.

3. A bus propagation model allowing bus overtaking and DPB

This paper seeks to develop an enhanced holding control models, more specifically by modelling overtaking and DPB, and investigates how the resulting policies behave under various operational settings. To this end, we firstly present an uncontrolled bus propagation model with bus overtaking and DPB. We then develop bus holding strategies for both schedule- and headway-based control from the enhanced bus propagation model. The models are implemented in a Monte Carlo simulation framework. To demonstrate the benefit of the enhanced holding control schemes, they are compared to those without overtaking and DPB. The following sections present the model assumptions and formulations, and their implementation in a simulation framework.

3.1 Model framework, assumptions and notations

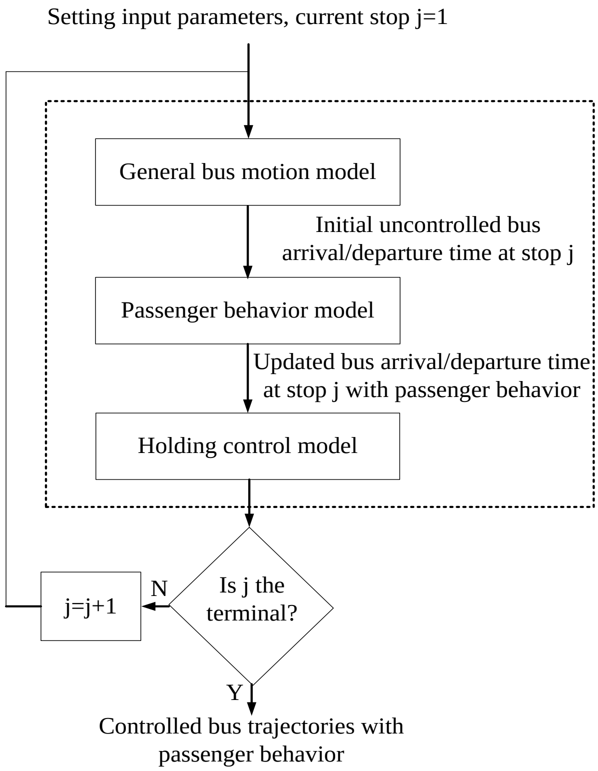


Fig.1. Simulation framework

When overtaking is allowed, the bus order may change from stop to stop. Since the boarding demand depends on bus headway, it requires sorting the bus order in increasing order of arrival times at each stop. As a consequence, the bus trajectories should be calculated iteratively stop-by-stop, including the bus arrival and departure time at each stop. Fig.1 illustrates how bus trajectories process across the different model components.

The overall simulation framework consists of three components: a general bus motion model, a passenger queuing behaviour model and a holding control model. The general bus motion model generates the initial bus trajectories, from which dwell times and leftover demand are also yielded. Afterwards, the passenger queuing behaviour model updates the dwells time and boarding demand according to a redistribution law of queueing passengers, resulting in corresponding updated information such as departure times and leftover demand. The holding control model imposes holding actions on bus motions. The passenger queueing behaviour model and all bus propagation models are deterministic in that they output deterministic values instead of a probability density function in each simulation run.

To facilitate the model development, the following assumptions are made:

(A1) Passenger arrivals at bus stops follow uniform distributions. This is a reasonable assumption for high frequency service, as validated and commonly used by many researchers (e.g. Salek and Machemehl, 1999; Sánchez-Martínez et al., 2016)

(A2) In this paper, we consider the attributing factor to bus bunching being the variability in bus link travel time, as opposed to an initial delay considered in some other literature (e.g. Newell and Potts, 1964; Schmöcker et al, 2016).

(A3) when there is more than one bus at a bus stop available for boarding, the waiting-to-board passengers would split according to the available bus capacity.

In addition, we consider a stable period in which there is no significant variation in passenger demand or travel speed. The irregularity in bus travel times in our study is assumed to have come from stochastic traffic phenomena. The changes in passenger demand or travel speed is another source of perturbation, and the very phenomenon exists in the transitional period (e.g., from peak to off-peak hours or vice versa) during which demand and running time varies significantly. When considering such dynamic effect, the passenger demand and running time can be formulated as time-dependent variables as suggested in Sánchez-Martínez et al. (2016). This dynamic effect is however beyond the scope of this study.

The following notations in Table 1 are adopted in this paper.

Table 1 List of primary symbols, definitions and units

Symbol	Definition	Unit
Indices		
i, k	Subscripts used to denote buses (bus k is the bus immediately ahead of bus i)	—
j	Subscript index of bus stop	—
Model parameters and model inputs		
C	The vehicle capacity	pax/veh
λ_j	The passenger arrival rate at stop j	pax/min
ρ_j	The alighting proportion at stop j	%
b	The average boarding rate	pax/min
α	Alighting time per passenger	min

δ	Minimum safety interval	min
β	Minimum headway factor	—
g_{max}	Maximum holding time	min
H	The departure headway of a bus line	min
Auxiliary variables		
$h_{i,j}$	The inter-departure headway between bus i and the preceding bus at stop j	min
$s_{i,j}$	The schedule departure time of bus i at stop j for schedule-based control	min
$a_{i,j}$	The arrival time of bus i at stop j	min
$d_{i,j}$	The departure time of bus i at stop j	min
$g_{i,j}$	The holding time of bus i at stop j	min
$t_{i,j}$	The travel time between stop j and $j+1$ for bus i	min
$B_{i,j}$	The total number of waiting passengers for bus i at stop j during inter-departure headway	pax
$B'_{i,j}$	The number of arriving passengers for bus i at stop j during inter-departure headway	pax
$\bar{B}_{i,j}$	The actual number of boarding passengers for bus i at stop j during inter-departure headway	pax
$\bar{B}'_{i,j}$	The actual number of arriving passengers who are able to board during inter-departure headway	pax
$B^a_{k,j}$	The number of passengers who have boarded bus k at stop j during time $[a_{k,j}, a_{i,j}]$	pax
$\bar{B}^u_{i,j}$	The actual number of boarding passenger for bus i at stop j after passenger redistribution	pax
$B^r_{i,j}$	The swapping number of boarding passengers from bus k to bus i at stop j	pax
$A_{i,j}$	The alighting demand for bus i at stop j	pax
$\bar{D}_{i,j}$	The actual dwell time for bus i at stop j	min
$D_{i,j}$	The boarding time for bus i at stop j with consideration of capacity	min
$D'_{i,j}$	The boarding time for bus i at stop j accommodating total waiting passengers	min
$l_{i,j}$	The number of leftover passengers of bus i when it departs from stop j	pax
$L_{i,j}$	The number of on-board passengers of bus i between stop j and $j+1$	pax
$Lr_{i,j}$	The available space for bus i at stop j after passenger alighting	pax
$r_{i,j}$	the proportion of swapping boarding demand of bus i at stop j	%

3.2 A bus motion model with capacity constraint and allowing for overtaking

Generally, a bus motion model consists of calculations of three components: departure times from bus stops, dwell times at stops and link travels times. We consider link travel time as a random variable and this variation causes bus bunching and triggers bus overtaking effect. The bus dwell time at a bus stop is affected not only by the demand for boarding, but also by the available capacity of the bus at that stop. There may be passengers who fail to board if the bus is full and who have to wait for the next bus. Without holding control, the buses depart as soon as all passengers are boarded (or when the capacity is full).

The arrival time of bus i at stop j is the departure time from stop $j-1$ plus the random link travel time between stop $j-1$ and j:

$$a_{i,j} = d_{i,j-1} + t_{i,j-1} \tag{1}$$

The bus departure time is determined by its dwell time and a holding time:

$$d_{i,j} = a_{i,j} + \overline{D}_{i,j} + g_{i,j} \tag{2}$$

where the holding time $g_{i,j}$ is obtained from Section 6 when holding control strategies are used.

Let the buses be numbered by their dispatching sequence from the terminal, i.e., $i = \{1,2,\cdots,M\}$. Due to the variability in travel times, the order of buses when arriving at a stop may have changed. Therefore, the ranking order of the subject bus that is adjacent to bus i is not necessarily bus $i-1$. The formula of inter-departure headway is:

$$h_{i,j} = d_{i,j} - d_{k,j} \tag{3}$$

where bus k is the one immediately ahead of the subject bus i at bus stop j, and it can be derived as:

$$k = \underset{k \in K}{arg\,max}\{d_{k,j} | d_{k,j} < d_{i,j}\} \tag{4}$$

where K is the set of buses in a bus line.

Assuming uniform passenger arrivals (A1), the total number of passengers waiting to board bus i and stop j will include those who arrived over the inter-departure headway plus those fail to board the previous departure bus k:

$$B_{i,j} = \lambda_j h_{i,j} + l_{k,j} \tag{5}$$

The alighting demand is assumed to be proportional to the number of passengers onboard:

$$A_{i,j} = L_{i,j-1}\rho_j \tag{6}$$

where ρ_j is a stop-specific alighting proportion, and $L_{i,j-1}$ is the bus load before it arrived at stop j.

The actual number of boarding passengers cannot exceed the remaining capacity, thus we have

$$\overline{B}_{i,j} = min\{B_{i,j}, C - L_{i,j-1} + A_{i,j}\} = min\{B_{i,j}, C - L_{i,j-1}(1-\rho_j)\} \tag{7}$$

We assume here that the waiting passengers are loaded in a random manner, and the left-over passengers are not given priority to board over later arrivals, which resembles situations such as where passengers mingle on waiting platforms.

Therefore, when the number of passengers who want to board exceed the remaining capacity, the actual number of arriving passengers who are able to board is

$$\overline{B}'_{i,j} = \frac{\lambda_j h_{i,j}}{\lambda_j h_{i,j} + l_{k,j}} \overline{B}_{i,j} \tag{8}$$

As a result, the number of passengers prevented from boarding is the difference between waiting passengers and that of boarding ones.

$$l_{i,j} = B_{i,j} - \overline{B}_{i,j} \tag{9}$$

While the number of on-board passengers of bus i when it departs from stop j becomes:

$$L_{i,j} = L_{i,j-1} + \bar{B}_{i,j} - A_{i,j} \tag{10}$$

If all waiting passengers at the stop could be accommodated by the bus, the boarding time can be expressed as follows, assuming each passenger takes an average time to complete the boarding process.

$$D'_{i,j} = \frac{1}{b}\left[\lambda_j\left(d_{i,j} - d_{k,j}\right) + l_{k.j}\right] = \frac{1}{b}\left[\lambda_j\left(a_{i,j} + D'_{i,j} - d_{k,j}\right) + l_{k.j}\right] \tag{11}$$

With Eq. (11), the boarding time for all passengers $B_{i,j}$ can be further simplified to

$$D'_{i,j} = \frac{\lambda_j}{b-\lambda_j}\left(a_{i,j} - d_{k,j}\right) + \frac{l_{k,j}}{b-\lambda_j} \tag{12}$$

With the available capacity constraint, the actual boarding time is

$$D_{i,j} = min\left\{\frac{\lambda_j}{b-\lambda_j}\left(a_{i,j} - d_{k,j}\right) + \frac{l_{k,j}}{b-\lambda_j}, \frac{1}{b}\left[C - L_{i,j-1}(1-\rho_j)\right]\right\} \tag{13}$$

Since in general the boarding and alighting simultaneously take place during the stop service, the total dwell time at a stop is the greater of boarding and alighting times

$$\bar{D}_{i,j} = max\left\{D_{i,j}, \alpha A_{i,j}\right\} \tag{14}$$

Remark 1: Note that the formulations of dwell time (Eqs. (11) to (13)) entail two underlying assumptions: (a) Passengers keep boarding the preceding bus until it reaches its capacity or boarding limit. In other words, the issue that passengers may spontaneously swap to the later arriving bus has not been considered. (b) In order to make the boarding time $D_{i,j} \geq 0$ hold, the rear bus should always arrive after the front bus leaves the stop, i.e., $a_{i,j} \geq d_{k,j}$. Evidently, this assumption is too strong since in practice there might be buses arriving before the previous bus departs from the stop, i.e., $a_{i,j} < d_{k,j}$, in which circumstance the passengers queueing for bus k may swap to board bus i. Typically, the following bus might even leave before the preceding bus when less passengers are queueing for it, exactly overtaking at the stop occurs. Therefore, to make the model more realistic, Eq.(11) and the resulting Eq.(13) should be modified in the provision of overtaking at stops and DPB, see detailed derivation in Section 3.3.

3.3 Capacity-constrained bus motion with overtaking and DPB

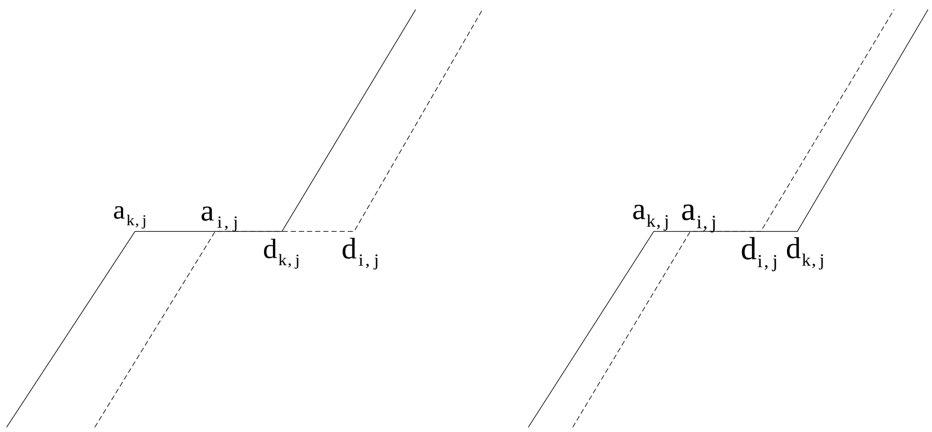
We consider the scenario where a preceding bus is still dwelling when the next bus arrives at the same stop. In such circumstance, some of the waiting passengers may choose to board the latter bus. To illustrate the concept of such dynamic passenger boarding behaviour, we consider a scenario whereby bus i arrives when bus k is still serving stop j. Consequently, passengers arriving at the stop during the departure time between bus k and the one departing just prior to it, more specifically, the originally calculated boarding demand of bus k at stop j, $B_{k,j}$ (Eq. (5)) should be split over buses k and i. There are two possible outcomes: (A) only one bus arrives during the dwelling of the preceding bus; and more generally (B) more than one bus arrives during the dwelling of the preceding bus.

When the queuing passengers are about to board a vehicle, whether or not the boarding is successful depends on the available space in the vehicle, and they may react to crowding by adapting their boarding choice. We assume that each available space in buses is equally likely to be favoured by the queueing passengers, and the boarding probability of a bus is proportional to its available capacity. More specifically, the probability of boarding bus $k(\text{or } i)$ can be calculated as the ratio of the remaining available space of bus $k(\text{or } i)$ to the total available space. Note that such boarding choice behavior could also be formulated as a discrete choice model. For example, there is existing literature that has addressed how travellers' behavioural response to crowding or discomfort, typically by departing earlier or later to avoid crowding (Tian et al., 2007; Pel et al., 2014; Xu et al., 2017) and waiting for a less crowded vehicle/service (Palma et al., 2015). However, applying such choice models would require well calibrated parameters from a field survey, as such, this work has been left for future studies. Another feature of DPB is that the late arrival bus is usually positioned some distance away from the queueing passengers, therefore extra access time would be required to account for boarding the latter bus. Such extra access time can be alleviated with the provision of real-time information, so that passengers can position themselves on the platform upon vehicle approaching.

In the following, we discuss how passengers are assigned for the two cases.

Case A. Only bus i arrives when bus k is dwelling at stop j, i.e., $a_{k,j} < a_{i,j} < d_{k,j}$

By distinguishing whether the latter bus overtakes at the stop, two patterns are emerged as shown in Fig.2. One is that the latter bus departs after the predecessor, i.e., $d_{k,j} < d_{i,j}$ (Fig. 2a), while the other is that the latter bus depart earlier as less passengers are queueing for it such that overtaking occurs, i.e., $d_{k,j} \geq d_{i,j}$ (Fig. 2b).



(a) (b)

Fig.2. Bus arriving cases for passenger re-assignment: (a) later arrival departs late; and (b) later arrival departs early

At the instant bus i arrives at stop j, the available space on the bus is the difference between the capacity and the number of on-board passengers from the previous stop, plus the number of alighting passengers. The underlying assumption here is that boarding dominates alighting during the simultaneous boarding and alighting process, and the alighting time is negligible compared with the boarding time. Therefore, we obtain:

$$Lr_{i,j} = C - L_{i,j-1} + A_{i,j} \tag{15}$$

The available space for bus k should consider the number of passengers who have boarded before bus i arrives. With this we further obtain the available space of bus k as in Eq. (16)

$$Lr_{k,j} = C - L_{k,j-1} + A_{k,j} - B_{k,j}^{a} \tag{16}$$

where $B_{k,j}^{a}$ denotes the number of passengers who have boarded bus k during loading time $[a_{k,j}, a_{i,j}]$, and is calculated as

$$B_{k,j}^{a} = \frac{a_{i,j} - a_{k,j}}{d_{k,j} - a_{k,j}} \bar{B}_{k,j} \tag{17}$$

The number of passengers needed to be re-assigned is the difference between the originally total boarding demand and the load, i.e.,

$$B_{k,j} - B_{k,j}^{a} \tag{18}$$

With the available space for bus i and k, the proportion of swapping boarding demand of bus i can be obtained by

$$r_{i,j} = \frac{Lr_{i,j}}{Lr_{k,j} + Lr_{i,j}} \tag{19}$$

The swapping number of boarding passengers from bus k to bus i can be calculated as the number of re-assigned passengers that is discounted by a factor $r_{i,j}$, while not exceeding the available space

$$B_{i,j}^{r} = min\{(B_{k,j} - B_{k,j}^{a})r_{i,j}, Lr_{i,j}\} \tag{20}$$

Correspondingly, the number of passengers that still board bus k is the difference between the number of total redistributed passengers and that of switching to bus i, while not exceeding the available space, i.e.,

$$B_{k,j}^{r} = min\{B_{k,j} - B_{k,j}^{a} - B_{i,j}^{r},\ Lr_{k,j}\} \tag{21}$$

Now we update the number of boarding passengers of bus k and bus i after passenger redistribution. The updated number of boarding passengers of bus k is the sum of the number of already on-board passengers and that of not swapping passengers, and that is

$$\bar{B}_{k,j}^{u} = B_{k,j}^{a} + B_{k,j}^{r} \tag{22}$$

As a result, the updated boarding time for bus k is then reduced to $\bar{B}_{k,j}^{u}/b$, with which we can obtain the updated departure time $\bar{d}_{k,j}$ for bus k.

The boarding demand of bus i consists of two groups: one is the swapping demand from the bus k (let's call it Group X); and the other, if exists, is the newcomers and the possible leftover passengers after bus k leaves this stop (Group Y). The time required to load the swapping demand is $B_{i,j}^{r}/b$. Since bus i starts to serve Group X when it arrives at the stop, the end time for serving this group is obtained by adding the loading time to the arrival time, that is, $a_{i,j} + B_{i,j}^{r}/b$. At this moment, if the front bus k has left the stop, bus i should continue to serve Group Y, which corresponds to the scenario for Fig.2(a). On the other hand, if the front bus k is still serving at the stop, bus i could overtake the front bus and leave the stop, which corresponds to the scenario for Fig.2(b).

To sum up, whether the latter bus should serve Group Y depends on the relationship between the departure time of the preceding bus and end time for serving Group X. More specifically, when the end time for serving Group X is larger than the updated departure time of bus k, i.e. $a_{i,j}+B_{i,j}^{r}/b>\bar{d}_{k,j}$, bus i should serve Group Y. Accordingly, the calculation of the boarding time without capacity constraint can also be obtained by modifying Eq.(11), in which the boarding demand should also account for the swapping demand. The formulation takes the following form:

$$D_{i,j}'=\frac{1}{b}\left[\lambda_j\left(d_{i,j}-\bar{d}_{k,j}\right)+l_{k,j}^{u}+B_{i,j}^{r}\right]=\frac{1}{b}\left[\lambda_j\left(a_{i,j}+D_{i,j}'-\bar{d}_{k,j}\right)+l_{k,j}^{u}+B_{i,j}^{r}\right] \tag{23}$$

where $l_{k,j}^{u}$ represents the updated leftover demand after passenger redistribution and is obtained by substituting $\bar{B}_{k,j}^{u}$ (Eq.(22)) into Eq.(9).

Clearing $D_{i,j}'$ in the right hand side of Eq.(23) we get an expression of $D_{i,j}'$

$$D_{i,j}'=\frac{\lambda_j}{b-\lambda_j}\left(a_{i,j}-\bar{d}_{k,j}\right)+\frac{l_{k,j}^{u}+B_{i,j}^{r}}{b-\lambda_j} \tag{24}$$

On the other hand, if the latter bus i has completed loading (the swapping passengers) while the preceding bus k has not, i.e., $a_{i,j}+B_{i,j}^{r}/b\le\bar{d}_{k,j}$, the latter bus will depart immediately and overtake the preceding bus at stop, and the unconstrained dwell time is $B_{i,j}^{r}/b$. As a result, the actual boarding time with capacity constraint of bus i is summarized as the following piecewise function:

$$D_{i,j}=\begin{cases} min\left\{\frac{\lambda_j}{b-\lambda_j}\left(a_{i,j}-\bar{d}_{k,j}\right)+\frac{l_{k,j}^{u}+B_{i,j}^{r}}{b-\lambda_j},\frac{1}{b}\left[C-L_{i,j-1}(1-\rho_j)\right]\right\}, & for\ \ a_{i,j}+\frac{B_{i,j}^{r}}{b}>\bar{d}_{k,j} \\ min\left\{\frac{B_{i,j}^{r}}{b},\frac{1}{b}\left[C-L_{i,j-1}(1-\rho_j)\right]\right\}, & otherwise \end{cases} \tag{25}$$

Case B. More than one buses arrive when bus k is dwelling at stop j, where $a_{k,j}<a_{\vec{\imath},j}<d_{k,j}$

Now we extend Case A to a more general case where more than one buses arrive during the dwelling of the first bus. Herein, $\vec{\imath}$ represents the series of bus order. When there are more than one bus arrives during the dwelling of the first bus, the time interval is generally very short in a real situation. Therefore, it is reasonable to ignore the interval and treat these approaching buses as bus platoon for simplicity. The queueing passengers then choose to board one of the buses in proportion to available space.

Similar to the case of one arriving bus, the number of aboard passengers in bus k is

$$B_{k,j}^{a}=\frac{a_{i_1,j}-a_{k,j}}{d_{k,j}-a_{k,j}}\ \bar{B}_{k,j} \tag{26}$$

where $i_1\in min\{\vec{\imath}\}$ denotes the first arriving bus in bus platoon $\vec{\imath}$.

The total number of passengers need to be distributed over bus platoon can be given by Eq.(18).

The proportion of swapping boarding demand of bus i can be obtained by

$$r_{i,j}=\frac{Lr_{i,j}}{Lr_{k,j}+\sum_i Lr_{i,j}},\ \forall i\in\vec{\imath} \tag{27}$$

Given the total number of redistributed passengers, the swapping number of boarding passengers from bus k to bus i can be further obtained by Eq.(20).

The number of passengers that still board bus k is the difference between the number of total redistributed passengers and that of switching to bus platoon $\vec{\iota}$, while not exceeding the available space

$$B_{k,j}^{r} = min\{B_{i,j} - B_{k,j}^{a} - \sum_i B_{i,j}^{r}, C - L_{k,j-1} + A_{k,j} - B_{k,j}^{a}\} \quad (28)$$

In line with Case A, the updated dwell time of bus k and bus i can be further obtained in a similar way. With this the updated departure time and passenger flows can also be obtained.

3.4 Bus trajectories constraints when overtaking is not permitted

To investigate the potential benefit of overtaking manoeuvre, we also perform the bus motions without overtaking for the purpose of comparison. To do this, constraints should be embedded in the simulation model to ensure that the bus order would not change overtime. In other words, the buses depart in the same order as they arrive at the bus stop, and the arrival order is the same as that from the terminal. The solution is that, if the preceding bus is caught by the following bus during the simulation, i.e.,$a_{i,j} - a_{i-1,j} < 0$, similar to Nagatani (2003), let the preceding bus restarts after a delay time δ, which we call it minimum safety interval, i.e.,

$$a_{i,j} = a_{i-1,j} + \delta \quad (29)$$

Similarly, when the unconstraint departure time of the preceding bus is bigger than the following bus, i.e.,$d_{i,j} - d_{i-1,j} < 0$, we also set the preceding bus restarts after a minimum safety interval.

$$d_{i,j} = d_{i-1,j} + \delta \quad (30)$$

3.5 Solution algorithms for bus trajectories evolution

With the above formulations, Algorithm 1 outlines the general simulation framework in which alighting process, capacity constraint and leftover passengers are incorporated. The algorithm is made up of three components, calculating respectively the departures of buses, link travel times and dwell times. The bus departures and dwell time components described in the algorithm should capture the capacity effect, which has been formulated in Section 3.2. To discourage bunching at the beginning of the simulation as much as possible, headways are set deterministically for the first dispatching bus, thereafter headways may become variable due to the variability in bus link travel time. Moreover, buses are assumed to depart from the terminal on time, whereas it can be easily relaxed by considering uneven initial headway which is not the focus of this paper.

Note that no overtaking is a special case of overtaking where constraints Eqs.(29) and (30) are added into the iterative calculation of bus propagation. Since the bus order may change overtime for the case of overtaking, it requires sorting buses according to their arrival times at the stop to achieve the effective headway between successive buses. Therefore, an additional branching algorithm (Algorithm 2) is presented in the overtaking case for calculation of dwell times and the corresponding passenger flows. The reader is referred to the Appendix 1 for the algorithm used.

4. A model for bus holding control allowing bus overtaking and DPB

Holding control only regularizes the inter-departure headway at stops, but bus overtaking between stops and the bunch arrivals at stops could still take place due to the stochastic nature of traffic flow, or the insertion and removal of vehicles throughout the day (Sánchez-Martínez et al, 2016). In a high-frequency transit system, it is common to encounter disturbances en-route such as delay from signal intersections, which is reflected by travel time variability, under which circumstance the following bus could catch up with the preceding one before or upon reaching the next stop although the inter-departure headway at stops has been regularized. Analogously, when the bunch arrives at a stop and less passengers are queueing for the following bus, the following bus can leave before the preceding bus when overtaking at stops is allowed.

The enhanced bus propagation model presented in Section 3 presents challenges to the traditional implementation of bus holding controls. Traditional holding controls work by comparing individual buses actual headway or departure time (for headway- or schedule-based holding controls) with those planned. When bus overtaking happens, that direct comparison between the actual and the planned headway and departure times is lost since the sequence of holding actions changes. It then requires additional rules to govern which bus(es) to hold and for how long. We propose a practical quasi first-depart-first-hold (FDFH) rule to enable bus holding strategies to be implemented in a system which allows bus overtaking and dynamic passenger boarding. In this section, we present the formulation for two types of holding control with FDFH rules: the schedule-based and the headway-based holding control.

4.1 Schedule-based holding control

Under the schedule-based (SH) holding control, buses either depart on schedule or immediately after serving passengers if they arrive late at the time point. Therefore, the departure times without overtaking from a time point takes the following piecewise function:

$$d_{i,j} = \begin{cases} s_{i,j}\,, & a_{i,j} < s_{i,j} - \bar{D}_{i,j} \\ a_{i,j} + \bar{D}_{i,j}, & a_{i,j} \geq s_{i,j} - \bar{D}_{i,j} \end{cases} \tag{31}$$

In Eq. (31), $s_{i,j} - \bar{D}_{i,j}$ is the critical arrival time after which the bus has to depart later than the scheduled departure time $s_{i,j}$.

The scheduled departure time at a designated stop $s_{i,j}$ is obtained by adding the scheduled link riding time and slack time to scheduled departure time from the previous stop $s_{i,j-1}$. The reliability of bus operation under schedule-based holding is related to the allocated slack time. Naturally, a longer slack time will lead to better schedule adherence, but at the expense of reduced commercial speed and increased the operating cost. Thus in practice transit agency need to balance the trade-off between operating cost (via slack time) and performance (measured in terms of headway variation).

The departure rules with overtaking is not straightforward, however, because the bus order may change stop-by-stop. Therefore, the actual bus arrival order at a stop may not comply with the predetermined schedule departure sequence. To allow overtaking at stops and reduce the total holding times, here we propose that the departures of buses should follow a FDFH principle. The concept of FDFH is analogous to the classical first-in-first-out (FIFO) principle, by converting the “influx” and “outflux” events of FIFO to “depart” and “hold” events for bus holding sequence. More specifically, a FDFH control states that: for a set of buses serving passengers at a bus stop, the first bus predicted to complete serving the passengers is to be held first.

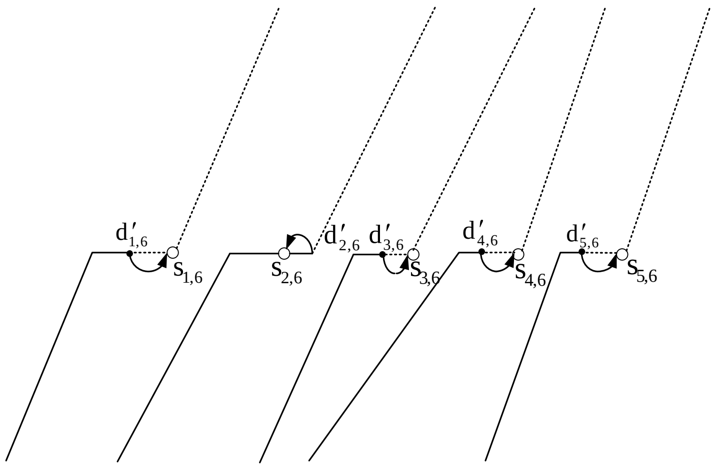


Fig.3. Illustration of the quasi FDFH principle for schedule-based holding control

Fig.3 illustrates the quasi FDFH principle. We first sort bus departures $d_{i,j}$ in order of increasing departure times at stop j, such that $d'_{n-1,j} \leq d'_{n,j}$, where n represent the bus in the n[th] position and $d'_{n,j}$ is the corresponding departure time. Then the random departure time of n[th] bus $d'_{n,j}$ and the scheduled departure time $s_{n,j}$ are connected by a fictitious link which accommodates the holding stage. The connecting direction depends on the relationship between the random departure time and the scheduled departure time. The backward connection (e.g., $d'_{2,6}$ to $s_{2,6}$) represents that the departure time of n[th] bus is bigger than the n[th] scheduled departure time, thus no holding is required for n[th] bus and it should dispatch immediately after serving passengers, while the forward connection (e.g., $d'_{4,6}$ to $s_{4,6}$) represents n[th] bus is hold until $s_{n,j}$, and the holding time is $s_{n,j} - d'_{n,j}$. As a result, by modifying Eq. (31), the departure time from a time point with the overtaking effect takes the following piecewise function:

$$d'_{n,j} = \begin{cases} s_{n,j}\,, & a'_{n,j} < s_{n,j} - \overline{D}'_{n,j} \\ a'_{n,j} + \overline{D}'_{n,j}, & a'_{n,j} \geq s_{n,j} - \overline{D}'_{n,j} \end{cases} \tag{32}$$

In Eq. (32), $s_{n,j} - \overline{D}'_{n,j}$ stands for the critical arrival time after which the bus has to depart later than the scheduled departure time $s_{n,j}$.

4.2 Headway-based holding control

Headway-based holding control approach is another category of bus holding control strategies, which is triggered by headway deviation. In this paper, we use a heuristic headway-based (HH) holding control adapted from Sánchez-Martínez et al. (2016), where a bus is held if the headway to the preceding bus is less than the plan headway or is dispatched immediately in other case. However, as reported by previous research (Oort et al., 2010; Delgado et al., 2012), such rule may lead to significantly high holding times and travel time especially for short headway scenario. To reduce the excess travel time, we introduce a maximum holding time to prevent anyone from experiencing extremely long travel times. In addition, we define a minimal headway requirement βH, $0 < \beta \leq 1$, for the holding criterion. β is a threshold ratio parameter which defines the minimum allowable headway relative to the plan headway. Previous studies show that the optimal threshold parameter ranges between 0.6 and 0.8 (e.g., Fu and Yang, 2002; Cats et al., 2011). For the case without overtaking, the unconstrainted departure time of the preceding bus is never less than the next bus by incorporating the constraint Eq. (30). Therefore, the expression of holding time is

$$g_{i,j} = min\{max[0, \beta H - (d_{i,j} - \bar{d}_{i-1,j})], g_{max}\} \tag{33}$$

where $d_{i,j}$ stands for the departure time of the controlled bus when no holding is used, and $\bar{d}_{i-1,j}$ stands for the previous departure time from the control point, and that is $\bar{d}_{i-1,j} = d_{i-1,j} + g_{i,j}$. g_{max} is the maximum holding time used to prevent the unnecessary long holding time and the resulting domino effect.

For the case with overtaking, however, the dispatching law of buses is more complex. Since the objective is to keep headway regularity while reducing the total dwell time, the quasi FDFH principle can be utilized similar to that of schedule-based approach. Since there is no reference on scheduled departure time under headway-based control, the concept of FDFH for such control strategy is simply that the holding is applied to the bus in chronological sequence of predicted departure time (arrival time plus dwell time). Therefore, we sort bus departures $d_{i,j}$ in increasing order, such that $d'_{n-1,j} \le d'_{n,j}$. The fundamental idea is holding n^{th} bus to ensure that the headway between n^{th} and $(n-1)^{\text{th}}$ bus is not less than the designed headway. However, the preceding departure time $d'_{n-1,j}$ may exceed the next departure time $d'_{n,j}$ after adding the holding time, and that is $\bar{d}'_{n-1,j} > d'_{n,j}$. In this case, we propose that the holding time for the n^{th} bus is the interval between $\bar{d}'_{n-1,j}$ and $d'_{n,j}$, plus the predetermined design headway. The idea behind this is to deploy holding to address very particular situation, when we need not to compensate the deviation, but also keep the same minimum headway.

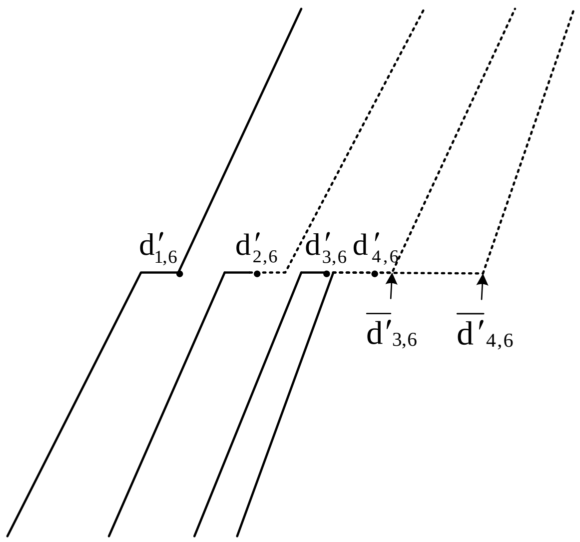


Fig.4. Illustration of the quasi FDFH principle for headway-based holding control

To illustrate this concept, as shown in Fig.4, let the departure time of 3th and 4th buses without holding be $d'_{3,6}$ and $d'_{4,6}$, respectively. Let us assume that the departure time of 3th bus from the control point $\bar{d}'_{3,6}$ exceeds $d'_{4,6}$. As a result, the 4th bus first holds to time $\bar{d}'_{3,6}$ for compensating the deviation, it proceeds to hold for a minimal design headway. Taken together, the holding time for the n^{th} bus at stop *j* is formulated as follows:

Ⅰ. If $0 \leq d'_{n,j} - \bar{d}'_{n-1,j} \leq \beta H$

$$g'_{n,j} = min\{max[0, \beta H - (d'_{n,j} - \bar{d}'_{n-1,j})], g_{max}\} \quad (34)$$

Ⅱ. If $d'_{n,j} - \bar{d}'_{n-1,j} < 0$

$$g'_{n,j} = min\{(\bar{d}'_{n-1,j} - d'_{n,j}) + \beta H, g_{max}\} \quad (35)$$

where $\bar{d}_{n-1,j}$ represents the departure time of the $(n-1)^{th}$ bus from the control point, and that is $\bar{d}'_{n-1,j} = d'_{n-1,j} + g'_{n-1,j}$.

5. Performance measures

In this paper, we attribute the initial cause of bus bunching as the stochastic travel time. The stochastic travel time is drawn from a lognormal distribution, following the assumptions made in previous studies (Hickman, 2001; Delgado et al., 2012; Sánchez-Martínez et al., 2016), and is generated using a Monte Carlo simulation. Multiple simulation runs are made, and from which we generate distributions of performance measures.

In order to quantify the effect of overtaking and passenger distributed boarding behaviour in the holding controls, we select the following three performance measures to cover the perspectives of both the operators and the passengers.

5.1 Average travel time

The average travel time for buses is an important operational performance measure for the operators. For each simulation run, an average travel time is derived as the arithmetic mean over all buses simulated. Then an expected average travel time $E(\overline{Tr})$, derived from multiple simulation runs is used as the performance measure.

5.2 Headway variability

Uneven headway gives rise to spatially unbalanced load and thus bus bunching. Related to this, we utilize the standard deviation of headways as an indicator of headway regularity. Irregular headway leads to increases in passenger waiting times. An expected standard deviation of headway $E(\sigma_h)$, as sampled through all inter-bus headways at all bus stops and over the multiple simulation runs, is used as the second performance measure.

5.3 Average waiting time

Passenger waiting time relates directly to bus headways. For passengers arriving at stop j with a uniform rate λ_j, the average passenger waiting time at stop j waiting for bus i can be estimated as $h_{i,j}/2$ (Chen, et al, 2015; Liu, et al, 2013; Ceder and Marguier, 1985). The total waiting time for passengers has two components: one for the newly arriving passengers before the arrival of bus i, denoted by $\bar{B}'_{i,j}$; and the other for those passengers who miss bus i and have to wait for the next bus, their additional waiting time should be $h_{i,j}$. As a result, the total waiting time is expressed as follows:

$$w = \frac{1}{2}\sum_i \sum_j \bar{B}'_{i,j} h_{i,j} + \sum_i \sum_j l_{i-1,j} h_{i,j} \tag{36}$$

Thus the averaged waiting time for passengers is obtained via dividing the total waiting time by the total number of boarding passengers

$$\bar{w} = \frac{w}{\sum_i \sum_j \bar{B}_{i,j}} = \frac{\sum_i \sum_j (\bar{B}'_{i,j} + 2l_{i-1,j}) h_{i,j}}{2\sum_i \sum_j \bar{B}_{i,j}} \tag{37}$$

Since each individual headways are aggregated in the formulation, the average waiting time reflects both the mean and standard deviation of headway. An expected average waiting time $E(\bar{w})$, is drawn from multiple simulation runs and is used as our third performance measure.

6. Model experiments and application

To validate the enhanced bus holding control strategies with overtaking and DPB in this paper, a small numerical test and an empirical test based on Guangzhou are conducted in this section. The main purpose of the test on the small example presented in Section 6.1 is to highlight the relative effect of overtaking and DPB on each control policy (i.e., no holding, SH and HH) under given operational settings, while the test on a bus route in Guangzhou presented in Section 6.2 is to analyse the trend along the route for relevant measures.

6.1 Numerical test

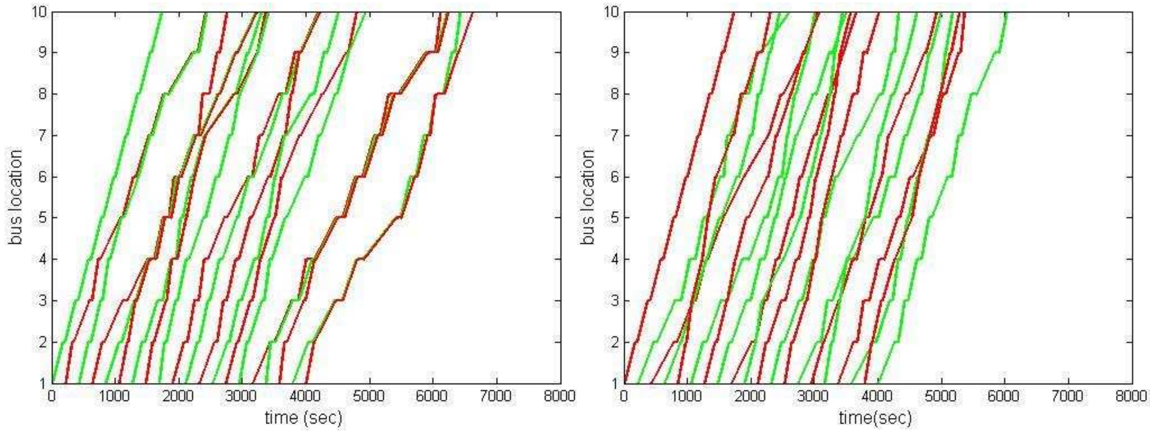
Consider a simple bus route with 10 stops, of which the passenger arrival rates and alighting proportions are listed in Table 2. The minimum safety interval and the boarding rate are set at $\delta = 0.3$ min and $b = 15 pax/min$, respectively. The link riding times t are drawn from a log-normal distribution with the natural logarithmic mean and standard deviation of 1.0 min and 0.5 min respectively, i.e., $ln(t) = N(1.0, 0.5^2)$. Buses are set to depart from the terminal on time, and the departure headway and vehicle capacity are set as H = 3.5 min and C = 50 pax/veh respectively for the base case. For SH scheme, we set the scheduled link riding time and slack time as 2.7 min and 0.3 min, respectively.

The detailed output from a typical simulation run includes vehicle trajectories, bus arrival and departure times at each stop, vehicle load, and number of leftover passengers. To make the system evolve to be chaotic enough for bus bunching to appear, the fleet size is set sufficiently large (here we set it as 20) for each simulation. From a number of simulation runs we can attain the expected value of indicators as presented in Section 5, and the number of Monte Carlo simulations is set as 1000.

Table 2 Input parameters for the simple bus line

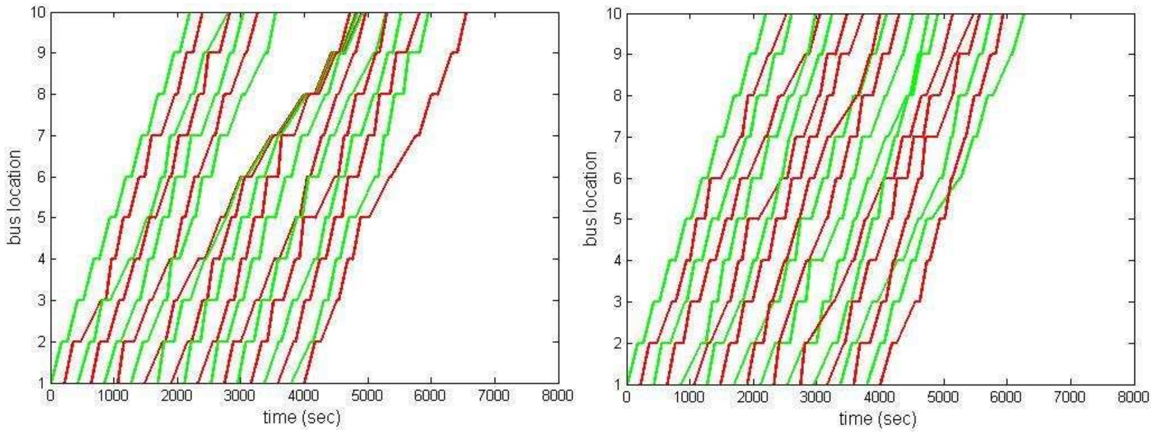
Stop	1	2	3	4	5	6	7	8	9	10
λ_j (pax/min)	2	2.5	3	3	2	2	1.5	2.4	1.75	0
ρ_j (%)	0	10	20	30	20	30	30	50	60	100

Now we simulate the effect of holding controls on bus bunching and overall system performance. Fig. 5 presents the simulated bus trajectories for no holding (NH) control for a typical simulation run, while Figs. 6 and 7 show the results under schedule-(SH) and headway-based (HH) holding controls respectively. Here the control parameters for HH is set as $\beta = 1$, $g_{max} = \infty$ (HH1), which is equivalent to the conventional rule where a bus is held to ensure that the headways are never less that a target headway and holding times are not binding. In each case, with and without bus overtaking are distinguished. To illustrate the effect of overtaking clearly, the consecutive buses dispatched from the terminal are drawn in different colors. When overtaking is not permitted, the adjacent trajectories are always in different colors, vice versa.



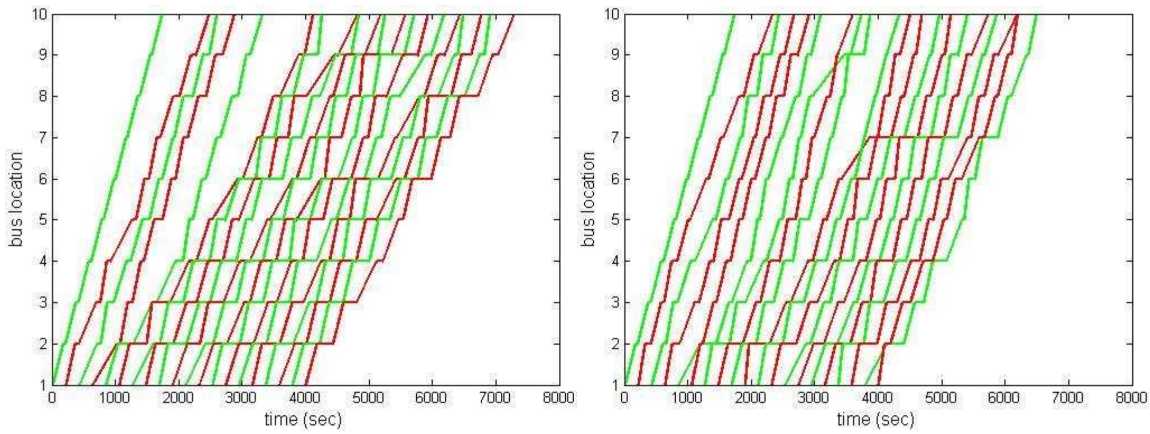
(a) (b)

Fig.5. Bus trajectories for cases of no holding control (NH): (a) no overtaking; (b) overtaking



(a) (b)

Fig.6. Bus trajectories for cases of schedule-based holding control (SH): (a) no overtaking; (b) overtaking



(a) (b)

Fig.7. Bus trajectories for cases of headway-based holding control (HH): (a) no overtaking; (b) overtaking

Compared to Fig. 5, it is clear from Figs. 6 and 7 that holding controls are effective in reducing bus bunching. However, any holding delay to one bus may cause knock-on delays to following buses, resulting in extended travel times, which is evident for the SH and HH schemes. For each control policy (i.e., NH, SH and HH), the trajectories with overtaking appear to be less dispersed compared to those without overtaking. This suggests that overtaking is effective in reducing travel time and bus bunching. The reason is that for the case of no overtaking, the motion of the following bus is constrained by the leader when the follower is about to catch up, such that an initial delay imposed to the leader can also knock on to the following bus. Therefore, the disturbances are more likely to propagate to the subsequent downstream trips when overtaking is not allowed, as such impose adverse impacts to other vehicles and lead to bus bunching problems.

Table 3 Performance measures for the three types of control policies

Scenario		$E(\sigma)$ (min)	$E(\overline{w})$ (min)	$E(\overline{Tr})$ (min)	Bunching (%)
NH	Overtaking	2.81/0.39[a]	2.46/0.19	**32.4**/1.14	50.2
	No overtaking	4.52/0.78	2.65/0.30	39.8/3.01	68.1
SH	Overtaking	2.84/0.37	2.41/0.17	**34.1**/1.16	48.3
	No overtaking	4.32/0.75	2.56/0.26	40.0/2.89	64.0
HH1	Overtaking	1.64/0.32	2.08/0.10	**44.8**/2.42	14.1
	No overtaking	2.39/0.57	2.44/0.18	51.0/4.19	16.3
HH2	Overtaking	2.00/0.32	2.03/0.14	**37.3**/1.63	17.1
	No overtaking	2.91/0.58	2.43/0.19	44.6/3.65	17.8
HH3	Overtaking	2.21/0.34	2.09/0.17	**35.8**/1.23	28.0
	No overtaking	3.48/0.75	2.59/0.24	41.8/2.73	36.9

a. The standard deviation of indicators

Table 3 presents several measures of performance for the test scenarios. The share (or occurrence probability) of bus bunching is another reflection of the headway variability. According to TCRP's Transit Capacity and Quality of Service Manual, the share of bunched buses can be defined as the percentage of headways that are shorter or longer than of half of the planned headway (TCRP, 2003). Given that the conventional HH1 produces some long holdings propagating to the following buses, two additional scenarios are tested for HH scheme: (HH2) $\beta = 0.7$, $g_{max} = \infty$; (HH3) $\beta = 0.7$, $g_{max} = 1.5min$. Instead of applying the control actions suggested by HH1, the two scenarios apply only a fraction of them. The results confirm that compared to the no holding case, the holding control improves the headway regularity and reduces the passenger waiting time at the expense of longer travel time. Importantly, the resulting performance allowing for bus overtaking is consistently better than that of without overtaking. The share of bunching decreased significantly when HH or overtaking strategies are applied.

In addition, Table 3 shows that by imposing the minimal headway ratio α, HH2 performs better than HH1 in the waiting and travel times due to the reduced mean headway. Compared to HH2, although HH3 slightly increases the waiting time and service irregularity, its implementation results in considerably shorter travel time, and its reliability indicator is much better than under the NH and SH strategies. Therefore, by introducing a minimal headway ratio and a maximum holding time, the HH scheme avoids overacting to the stochastic elements, improves service reliability and reducing passenger waiting time.

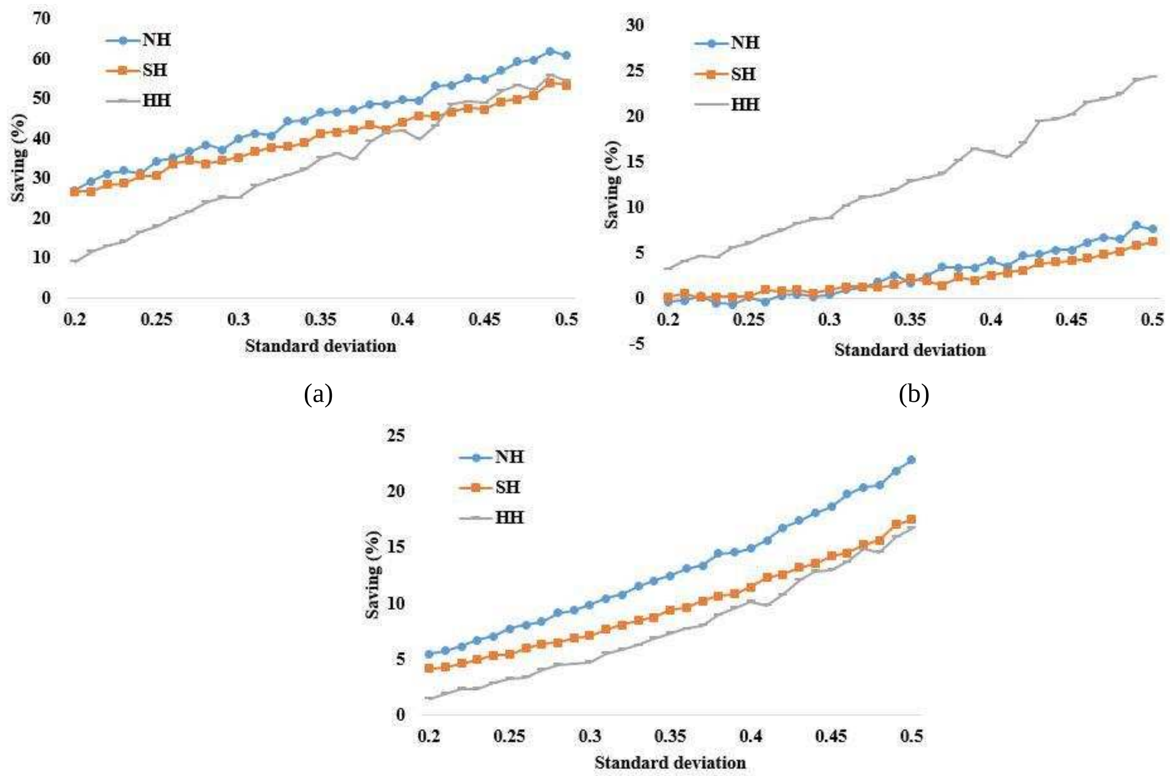
Hereafter, the HH used in the sensitivity analysis refers to the case with parameter settings $\beta = 0.7$, $g_{max} = 1.5min$. Optimizing the control parameters for different objectives has been left for future work.

To further verify the effectiveness of the proposed model, we conduct sensitivity analysis under various operational settings to examine the improvements by overtaking and DPB. To highlight the effect of overtaking and DPB, the results of our proposed model (i.e., with overtaking and DPB) are compared with the reference case either without overtaking or DPB. Since the travel time variability and boarding operations are central sources triggering bunching, in what follows we first investigate the sensitivity to link travel time stochasticity, followed by the sensitivity to headways and vehicle capacity, in which overtaking is not allowed for the reference case. Finally we analyse the effect of DPB on performance measures. To achieve this, the feature of DPB is not adopted in the reference case. All control policies are compared under the same operational setting, except where they are the subject of the sensitivity test. This is done in the interest of presenting the incremental improvement, though it might be possible to improve performance further by optimizing headway. The performance improvement is calculated as $(reference - proposed) \times 100/proposed$.

6.1.1 Sensitivity to travel time variability

In this section, we analyse the performance improvement from overtaking under various levels of travel time variability. The travel time variability is reflected by the standard deviation of lognormally distributed link travel time. The results are presented in Fig.8 and they show that the resulting performance allowing for bus overtaking is consistently better than that of without overtaking. For three types of policies, the savings for all indicators generally increase with the increasing travel time variability. This is because, as the travel time variability gets higher, buses are easier to bunch together where overtaking can take more effect. Since overtaking operation contributes to reducing unnecessary dwell time at stops, less travel time can be expected with various travel time variability (Fig.8(c)). In comparison, the headway regularity and travel time improvement for NH are consistently higher than those with holding control, which is due to the fact that overtaking occurs more frequently under NH policy and thus the improvement is higher.

Better still, the benefit of passenger waiting time gained by the overtaking is greater with HH schemes (Fig.8(b)). The reason is that the average waiting time is related to both the mean and standard deviation of headway. While the headway regularity improvement is not the largest for HH strategies, the waiting time improvement is significantly due to running more frequent service and the resulting reduced mean headway stemming from overtaking operation. This reveals promising application potentials for bus operational control.



(a) (b)

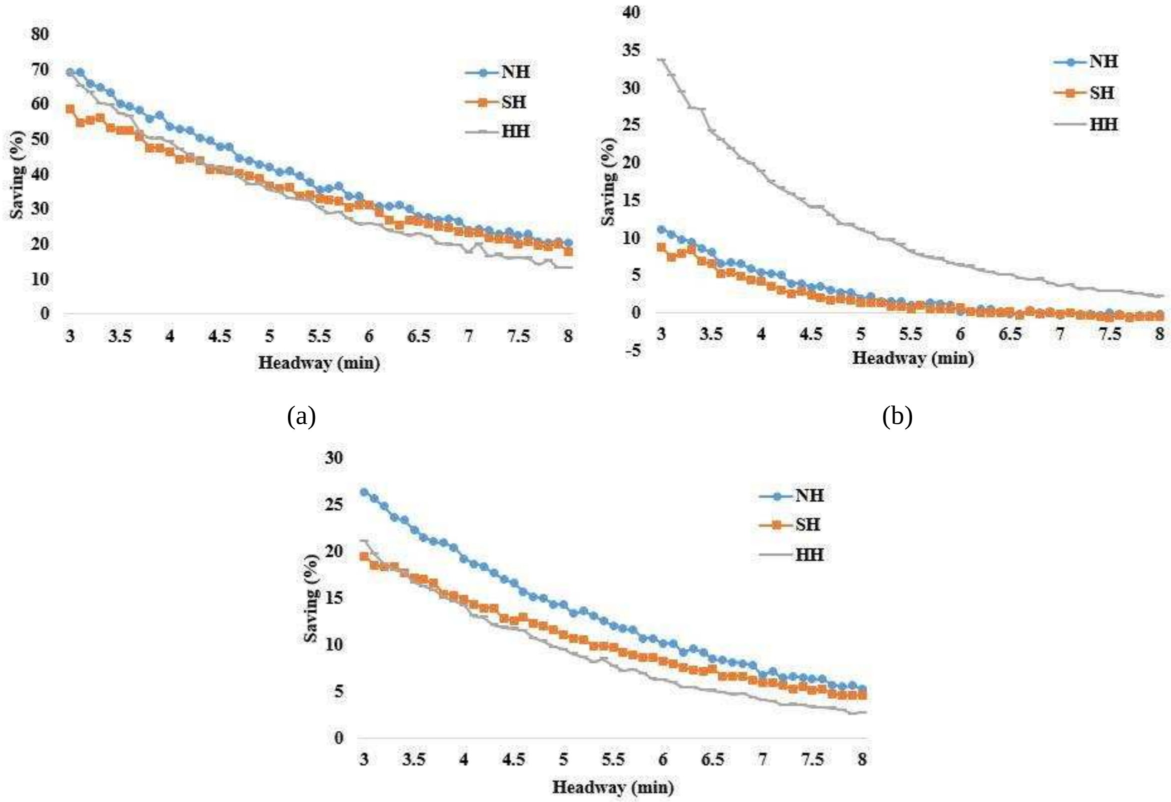
(c)

Fig.8. Percentage reduction in: (a)E(σ), (b)E($\overline{w}$), and (c)E($\overline{Tr}$) under different standard deviations of travel time.

6.1.2 Sensitivity to headways

Bus service headway is determined by the demand for the service, and would normally not change if the demand doesn't. Since incorporating variation in demand into the headway variation would add to the complications in the analysis, in this test we keep the demand constant and vary only the headways to examine the impact of control strategies on the system performance under different headways.

Fig.9 presents the effect of the size of headway on the performance measures for different policies. To isolate from the effect of capacity binding or excess demand, the capacity is never reached within the headway of interests. For NH and SH policies, the savings for all indicators decrease with the longer headway, and they are negligible when reaching a critical headway, which is as expected since longer headway leads to less chance of bunching/overtaking. This gives us a practical insight that, to achieve the benefit of overtaking, a maximum design headway (e.g., about 8 min for HH) is required. In other words, overtaking is more effective under a relatively small headway, which provides a possible way for operational management for high-frequency transit service. Among them, the critical headway for HH is relatively smaller than other control policies, which is owing to the strong self-adaptive effect of HH method.



(a) (b) (c)

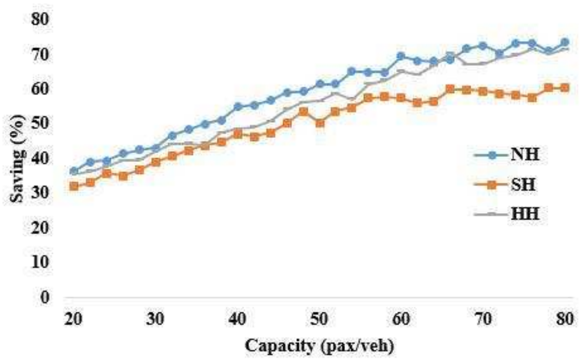
Fig.9. Percentage reductions in: (a)E(σ), (b)E($\overline{w}$), and (c)E($\overline{Tr}$) under different headways

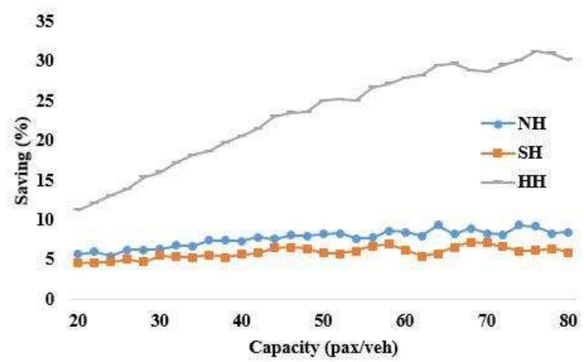
Similar to the effect of travel time variability, the benefit of passenger waiting time is also greater with headway-based holding controls (as shown in Fig.9(b)). The reason is that the shorter the scheduled headway between buses, the greater is the expected knock-on delay and hence the greater the expected mean headway. Thus accounting for overtaking allows running significantly more frequent service, which in turn resulting in significantly less passenger waiting time.

6.1.3 Sensitivity to capacity

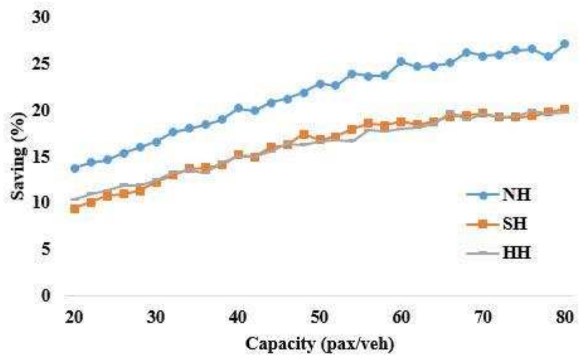
Similar to service headway, bus capacity is also linked closely with passenger demand. In this test, however, we consider variations in bus capacity without changing the passenger demand so as to examine the effect of capacity on holding control performances.

Fig.10 presents the effect of the size of vehicle capacity on the performance measures for different policies. We observe that the savings for all indicators generally increase with the capacity size. There are two possible reasons for this. First, when the vehicle size gets larger, it may be able to provide more swapping flexibility for the queueing passengers when buses are bunched. Second, when the capacity gets larger, any bus suffering a delay will be able to serve more waiting passengers in the subsequent stops of the route. As a result, the commercial speed of this bus drops due to increasing dwell times at stops. Such side effect would be amplified when overtaking is not allowed since the front bus will act as a movable bottleneck. Since overtaking operation contributes to eliminating the bottleneck effect and improve service reliability as discussed previously, greater benefit from overtaking operation can be expected when the vehicle capacity becomes larger. However, overtaking takes less effect when holding control is applied, we therefore observe the improvement in headway regularity and travel time for NH policy is relatively larger. In addition, the benefit of waiting time is greater with headway-based holding controls when the vehicle capacity gets larger, which suggests that the enhanced HH is more beneficial when the capacity constraint is less active.





(a) (b)



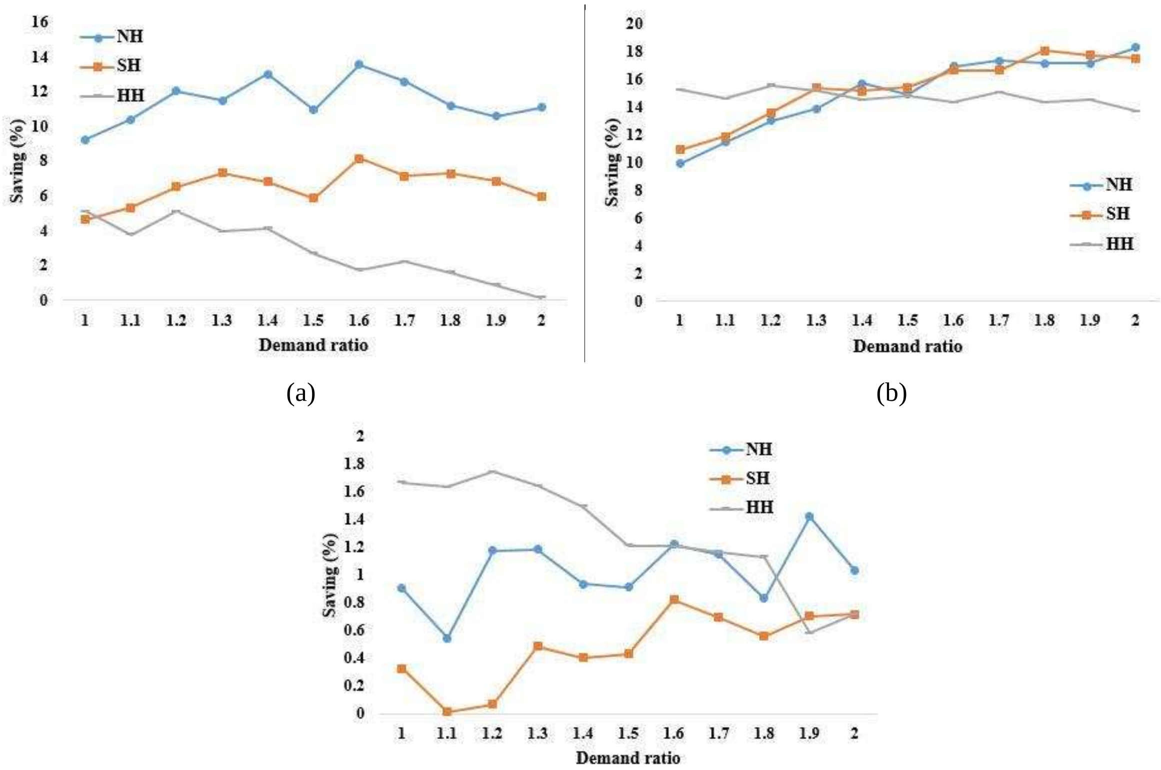
(c)

Fig.10. Percentage reductions in: (a)$\mathrm{E}(\sigma)$, (b)$\mathrm{E}(\overline{w})$, and (c)$\mathrm{E}(\overline{Tr})$ under different capacity levels

6.1.4 Impact of DPB

Distributed passenger boarding behaviour, where the waiting passengers choose which bus to board when more than one buses are present at the stop, is a new feature in our model and in the design of holding control strategies. In this section, we analyse the effect of DPB on bus performance, and compare that without the boarding choices.

To highlight the effect of DPB, our proposed model (i.e., with overtaking and DPB) is compared with the reference case with overtaking but without DPB for a range of passenger demand ratios. The results of the percentage changes in the system performance measures are shown in Fig.11. We find that the inclusion of DPB is beneficial. In terms of headway regularity, the improvement is most significant with the NH policy. Although the improvement of average travel time is not quite outstanding, it yields a considerable reduction of headway variations and expected waiting time as high as 14% and 18%, respectively. This suggests that such dynamic passenger behaviour of swapping queues can lead to service improvement under various demand levels. This is because, the ‘flexible’ passengers could take effect in helping redistribute the queues to balance the loads among vehicles and mitigate the bus crowding, which promotes a more efficient utilization of vehicles. In addition, we observe that HH is generally a smaller improvement in headway regularity and expected waiting time. This is due to the fact that DPB is in effect only when the bunch arrivals occur, whereas the bunch arrivals are less likely to occur under HH scheme compared to NH and HH schemes.



(a) (b)

(c)

Fig.11. Savings due to DPB for different demand ratios: (a)$\mathrm{E}(\sigma)$, (b)$\mathrm{E}(\overline{w})$, and (c)$\mathrm{E}(\overline{Tr})$

6.2 Empirical test

The main purpose of this section is to analyse the trend along the route for relevant measures. In this experiment, the proposed control methods featuring overtaking and DPB are validated based on the data of Bus Route 256, a busy route in Guangzhou City serving 23 stops, as shown in Fig.12. The busy route circles the city of about 16 km, and the passenger flow is about 25,000 on average in one day. All buses serving this line are of the same vehicle size and have a capacity of 100 pax/veh. The boarding passenger flow data are provided by the local bus company. We use data during the morning peak hour (9:00-10:00am) in one of directions of this route (from Guangzhou Railway Station to Zhudao Garden Station). The link travel time data are obtained from on-board GPS tracking devices, from which the mean and standard deviation of travel time between stops are calculated and listed in Table 4.

Fig.12. A map of Bus Route Number 256

Following Liu et al. (2013) and Chen et al. (2015), we assume that passengers boarding at a designated stop will evenly alight at the remaining stops, thereafter the average alighting rate at each stop can be obtained from the boarding rate. The passenger flow distribution at stops and derived alighting distributions are shown in Fig.13. The departure headway of the bus route in the period of interests is 7 min. Therefore, the departure headway falls within the domain of high-frequency service and we can reasonably assume that passenger arrivals follow uniform distribution. To examine the evolution of bus movement, we have the buses depart the terminal on time at the beginning of the time period.

Table 4. Statistics of link travel times (unit: min)

Bus	1	2	3	4	5	6	7	8	9	10	11
Mean	0.79	2.54	2.02	3.16	0.87	1.10	1.30	1.70	1.78	1.04	2.93
STD	0.21	0.83	1.60	1.27	0.90	0.20	0.18	1.72	0.33	0.31	0.93
Bus	12	13	14	15	16	17	18	19	20	21	22
Mean	2.54	0.94	2.99	3.10	2.68	1.10	1.94	2.87	0.41	1.19	0.31
STD	0.77	0.51	1.11	1.77	0.94	0.52	1.93	1.51	0.11	0.21	0.08

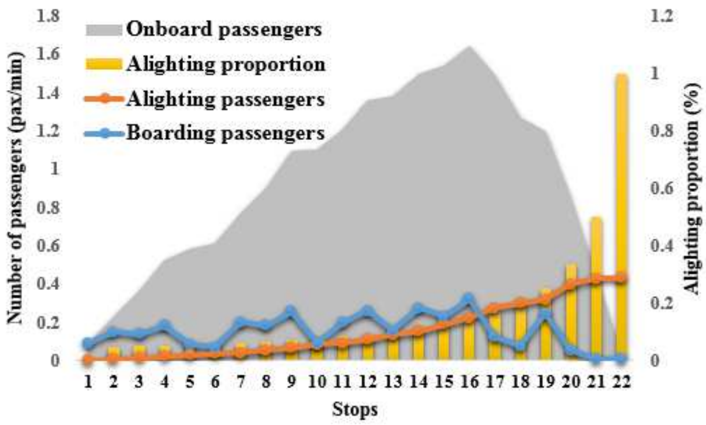


Fig.13. Passenger flow on 256 Route during 9:00-10:00am

We select only stop 8 and 17 as the control points for tests, which shares 4.5% and 7.4% of total through passengers. A comparison is conducted among three control policies, i.e., NH, SH and HH, with and without overtaking. We present the evolution of bus motions along the route to identify the mechanism of overtaking and DPB. The performance measures include the standard deviation of headways at stops along the bus route, average passenger waiting time, and the cumulative running time. Again the link travel times are drawn from lognormal distribution, and the simulation is run 1000 times.

We can see in Fig.14 that, as expected, service unreliability propagates along the bus route. Headway variability and average waiting time decrease considerably immediately after a control point stop, restraining the continuous increase in service irregularity. This indicates that the SH and HH methods can considerably reduce the headway variability along intermediate stops, compared to the NH policy. In this regard, the HH strategies reduce headway variability substantially compared with SH holding. However, HH strategies results in shorter passenger waiting times at the expense of slightly longer running times compared with SH holding.

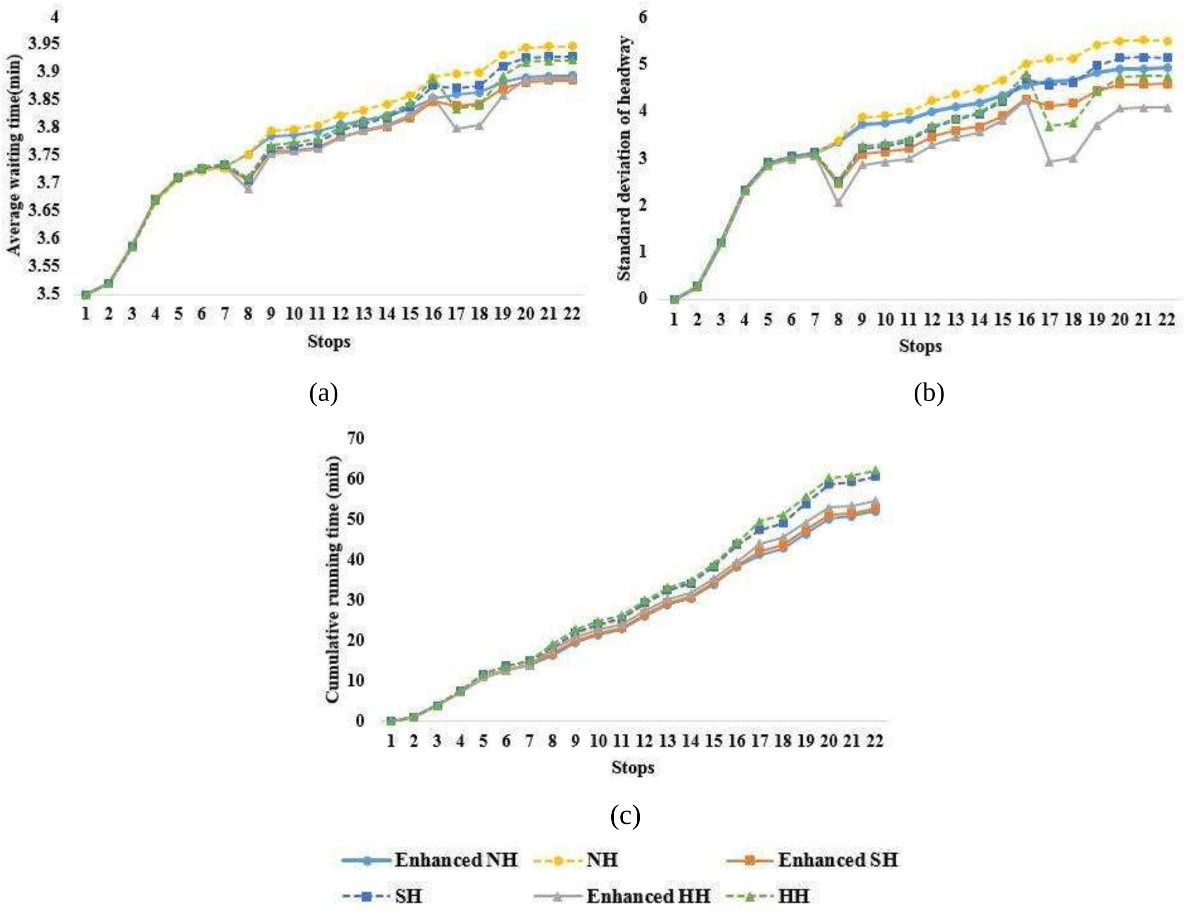


Fig.14. Evolution of bus movement along the route: (a) headway variability; (b) average waiting time; (c) cumulative running time

Confirming the observation in Section 6.1, the resulting performance allowing for bus overtaking is consistently better than that of without overtaking, especially for the case of NH policy. From the view of evolution, performance measures of the enhanced models at stops are initially close to the base models before reaching certain location, thereafter branching and the gaps keep enlarging along the remaining segment.

The results suggest that the control methods can improve service reliability and reduce in-vehicle travel time in the presence of overtaking and DPB, and these features could improve performance by a greater degree when the route is longer, particularly for the NH policy. In other words, ignoring overtaking and DPB would underestimate the efficacy of bus propagation and holding control models.

7. Concluding Remarks

A significant body of research has focused on various forms of bus holding strategies to reduce bus bunching, largely resting on simplified assumptions of no overtaking or distributed passenger boarding behaviour. This paper proposed a bus propagation model and holding control model by explicitly taking these real-life features into account. To allow for overtaking while minimizing the dwell time, we proposed a quasi first-depart-first-hold (FDFH) principle to be combined with schedule- and headway-based holding strategies, which states that: for a set of buses serving passengers at a bus stop, the first bus predicted to complete serving the passengers is to be held first. Another key contribution of the new model is the formulation of a dynamic passenger boarding (DPB) process that considers passengers' ability to board any of the bunched buses serving a stop at the same time.

We test the performance of different combinations of the behaviour models and control policies under various operational settings in a Monte Carlo simulation environment. Experiments underline the importance of the introduced effect of overtaking and DPB by revealing the performance improvement that have not been described in the literature before. We find that allowing for overtaking among buses improves service regularity, and the benefit is greater when the travel time variability is higher and the designed headway is smaller. This suggests that the new strategy is most beneficial for the high frequency transit service. Another interesting finding is that the inclusion of distributed passenger boarding behaviour can alleviate the negative impact of bus bunching and holding control on the performance measures, suggesting that using information on the crowding levels in buses to influence passenger boarding choices could help reduce bus bunching and improve bus services. Finally, we verify the effectiveness of the proposed methods through a case study in a real bus line in Guangzhou City, and the results suggest that inclusions of overtaking and distributed passenger boarding behaviour in the design of holding controls enhances the performances of the control strategies. Since in reality vehicles already overtake and passengers already distribute themselves, so these performance improvements would not be seen. The real implication is that previous models don't capture these two aspects of real operations, so they are producing control instructions based on less accurate information/forecasts.

This study opens up new research directions. For example, while this paper only investigates the spontaneous passenger redistribution when buses are bunched, future research can investigate the effects of an advanced public transport information system on passengers' decision-making and the resulting performance of bus holding control strategies. In addition, the proposed model framework could be extended to help design whole system (e.g., optimization-based) control strategies and to incorporate more complex driving manoeuvres (e.g., considering vehicles cycle and schedule recovery) in the bus propagation and control models.

Acknowledgement

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Appendix 1. Vehicle trajectories evolution algorithms

Let M denotes the fleet size of the modelled bus line, and N the number of bus stops on the corridor served by the bus line.

Algorithm 1. Trajectories evolution algorithm with and without overtaking

Initialization: Set input parameters and the counter of simulations

Procedure:

Step 1: Generate the departure times for all trips from the terminal

for bus i=1: M **do**

Compute the departure time for the bus line, satisfying $d_{i,1} := d_{1,1} + (i-1)H$

end

Step 2: Generate the stochastic bus link travel time

for bus i=1: M **do**

for stop j=2: N **do**

Compute the bus link travel time $t_{i,j-1}$ from a lognormal distribution.

end

end

Step 3: Generate the full trajectories of the first bus

for stop j=2: N **do**

Compute the arrival time of bus 1 at stop j, satisfying $a_{1,j} := d_{1,j-1} + t_{1,j-1}$

Compute the departure time of bus 1 from stop j, satisfying $d_{1,j} := a_{1,j} + \lambda_j H/b$

Compute the number of on-board passengers, satisfying $L_{1,j} := L_{1,j-1}(1-\rho_j) + \lambda_j H$

Let the leftover demand, $l_{1,j} := 0$

end

Step 4: Generate the trajectories for the remaining trips of the bus line

for stop j=2: N **do**

for bus i=2: M **do**

Compute the arrival time of bus i at stop j using Eq.(1) (plus the constraint Eq.(29) only for the case of no overtaking)

end

For the case of no overtaking:

for bus i=2: M **do**

Check whether $a_{i,j} > d_{i-1,j}$, if yes, compute the boarding time using Eq. (13) and passenger flows using Eqs.(5)-(10); if not, compute the boarding time using Eq. (25) and update the passenger flows by Section 3.3. Then compute the departure time of bus i at stop j using Eq.(2).

end

For the case of overtaking:

Switch to Algorithm 2.

end

Algorithm 2. Obtain the passenger flows and times at stop j for the overtaking case

Step 1: Sort vehicles in order of bus arrival times at stop j and compute the corresponding passenger flows
Step 1.1 Sort $a_{i,j}$ in increasing order and store in set $AU(j)$, i.e., $AU(j) = [\hat{a}_{1,j}, \hat{a}_{2,j}, \cdots, \hat{a}_{n,j}, \cdots, \hat{a}_{M,j}]$, where $\hat{a}_{n,j}$ denotes the arrival time of n^{th} bus in set $AU(j)$, such that $\hat{a}_{n-1,j} \leq \hat{a}_{n,j}$.
Step 1.2 Obtain $\xi(i,j)$ that represents the ranking order of bus i in $AU(j)$, which is used to mapping a set of bus information to those ordered ones.

for bus n=2: M **do**

Update the load of n^{th} bus at stop $j-1$ with $L_{p(n,j)),j-1}$, where $p(n,j)$ denotes the bus dispatching order (from the terminal) corresponding to n^{th} bus at stop j.

Compute the boarding time of n^{th} bus at stop j using Eq.(13) for the case when $\hat{a}_{n,j} > \hat{d}_{n-1,j}$, while using Eq.(25) for the case when $\hat{a}_{n,j} \leq \hat{d}_{n-1,j}$.

Compute the passenger flows and dwell time of n^{th} bus using Eqs.(5)-(10) and Eq. (14), from which the departure time $\hat{d}_{n,j}$ of n^{th} bus is obtained.

If holding control is in place, update the departure time according to Section 4.1 and 4.2 for schedule- or headway-based control model.

The results are stored into a matrix P, where n^{th} row corresponds to a scalar passenger flows and (arrival, dwell, and departure) times of n^{th} bus.

end

Step 2: Obtain the information of the original bus order from those of the ranking order

for bus i=2: M **do**

Obtain the passenger flows and (arrival, dwell, and departure) times of bus i at stop j by mapping to the $\xi(i,j)^{\text{th}}$ row in matrix P

end

Remark 2: To obtain the alighting and on-board demand of bus i at stop j, the number of on-board passengers of bus i at the last stop $j-1$ is required (see Eqs. (9) and (10)). However, when overtaking is allowed, the ranking order for a given bus at the subject stop may not be equal to the ranking order for this bus at the last visited stop, which is different to the case of no overtaking. In other words, the n^{th} bus at stop j is not necessarily the n^{th} bus at stop $j-1$, underlying the interaction between overtaking and capacity.

Remark 3: Since the algorithm processes along the sequence of bus stops, the information of passenger flows at the previous stop ($j-1$) has been available before calculating those at the subject stop (j). To address the interaction between overtaking and capacity, it requires identifying the original dispatching bus order from the last visited stop. Therefore, we introduce $p(n,j)$ that represents the bus dispatch order from the terminal corresponding to the n^{th} bus at stop j. As a result, the alighting demand of the n^{th} bus at stop j is updated by $L_{p(n,j),j-1}\rho_j$.

Remark 4: For the cases with overtaking but without DPB, there would be no swapping passengers among vehicles. Therefore, when $a_{i,j} > d_{k,j}$, the dwell time of bus i can be simply set as the alighting time since no passengers would swap from bus k.

Reference

Argote-Cabanero, J., Daganzo, C.F., Lynn, J.W., 2015. Dynamic control of complex transit systems. Transportation Research Part B, 81, 146-160.

Berrebi, S., Watkins, K., Laval, J., 2015. A real-time bus dispatching policy to minimize passenger wait on a high frequency route. Transportation Research Part B, 81, 377-389.

Bian, B., Zhu, N., Ling, S., Ma, S., 2015. Bus service time estimation model for a curbside bus stop. Transportation Research Part C, 57, 103-121.

Cats, O., Larijani, A.N., Koutsopoulos, H.N., Burghout, W., 2011. Impacts of holding control strategies on transit performance, Transportation Research Record, 2216, 51-58.

Ceder, A., Marguier, P.H.J., 1985. Passenger waiting time at transit stops. Traffic engineering and control, 26(6), 327-329.

Chen, J., Liu, Z., Zhu, S., Wang, W., 2015. Design of limited-stop bus service with capacity constraint and stochastic travel time, Transportation Research Part E, 83, 1-15.

Daganzo, C. F., 2009. A headway-based approach to eliminate bus bunching: Systematic analysis and comparisons. Transportation Research Part B, 43(10), 913-921.

Daganzo, C.F., Pilachowski, J., 2011. Reducing bunching with bus-to-bus cooperation. Transportation Research Part B, 45, 267–277.

Delgado, F., Munoz, J.C., Giesen, R., 2012. How much can holding and limiting boarding improve transit performance? Transportation Research Part B, 46(9), 1202-1217.

Dessouky, M., Hall, R., Nowroozi, A., Mourikas, K. 1999. Bus dispatching at timed transfer transit stations using bus tracking technology. Transportation Research Part C, 7(4), 187-208.

Eberlein, X.J., Wilson, N.H.M., Bernstein, D., 2001. The holding problem with real-time information available. Transportation Science, 35(1), 1-18.

Fu, L., Yang, X., 2002. Design and implementation of bus-holding control strategies with real-time information. Transportation Research Record, 1791, 6-12.

Cats, O., Larijani, A., Koutsopoulos, H.N., Burghout, W., 2011. Impacts of holding control strategies on transit performance. Transportation Research Record, 2216, 51-58.

Fonzone, A., Schmöcker, J.D., Liu, R., 2015. A model of bus bunching under reliability-based passenger arrival pattern. Transportation Research Part C, 59, 164-182.

Hernandez, D., Munoz, J.C., Giesen, R., Delgado, F., 2015. Analysis of real-time control strategies in a corridor with multiple bus services. Transportation Research Part B, 78, 83-105.

Hickman, M.D., 2001. An analytic stochastic model for the transit vehicle holding problem. Transportation Science, 35(3), 215-237.

Hollander, Y. and Liu, R (2008) Estimation of the distribution of travel times by repeated simulation. Transportation Research, 16C, 212-231.

Liu, R. and Sinha, S. (2007) Modelling urban bus service and passenger reliability. Paper presented at the International Symposium on Transportation Network Reliability, The Hague, July, 2007.

Liu, Z., Yan, Y., Qu, X., Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. Transportation Research Part C, 35, 46-56.

Nagatani, T., 2003. Dynamical hehavior of N shuttle buses not passing each other: chaotic and periodic motions. Physica A, 327, 570-582.

Newell, G.F. and Potts, R.B., 1964. Maintaining a bus schedule. Proceedings 2nd Australian Road Research Board, 2, 388-393.

Osuna, E.E. Newell, G.F., 1972. Control strategies for an idealized bus system. Transportation Science, 6(1), 52-71.

Palma, A., Kilani, M., Proost, S., 2015. Discomfort in mass transit and its implication for scheduling and pricing. Transportation Research Part B, 71, 1-18.

Pel, A.J., Bel, N.H., Pieters, M., 2014. Including passengers' response to crowding in the Dutch national train passenger assignment model. Transportation Research Part A, 66, 111-126.

Oort, N., Wilson, N.H.M., Nes, R., 2010. Reliability improvement in short headway transit services. Transportation Research Record, 2143, 67-76.

Salek, M.D., Machemehl, R.B., 1999. Characterizing bus transit passenger waiting times. SWUTC Research Peport 99, Austin, Texas.

Sánchez-Martínez, G.E., Koutsopoulos, H.N., Wilson, N.H.M., 2016. Real-time holding control for high-frequency transit with dynamics. Transportation Research Part B, 83, 1-19.

Schmöcker, J.D., Sun, W., Fonzone, A., Liu, R., 2016. Bus bunching along a corridor served by two lines. Transportation Research Part B, 93, 300-317.

Sorratini, J., Liu, R. and Sinha, S (2008) Assessing bus transport reliability using micro-simulation. Transport Planning & Technology, 31(3), 303-324.

Sun, A., Hickman, M., 2005. The real-time stop-skipping problem. Journal of Intelligent Transportation System, 9(2), 91-109.

TCRP Report 100: Transit Capacity and Quality of Service Manual, 2nd ed., 2003. Transportation Research Board of the National Academies, Washington, D.C.

Tian, Q., Huang, H., Yang, H., 2007. Equilibrium properties of the morning peak-period commuting in a many-to-one mass transit system. Transportation Research Part B, 41, 616-631.

Wirasinghe, S.C., Liu, G., 1995. Determination of the number and locations of time points in transit schedule design-Case of a single run. Annuals of Operations Research, 60, 161-191.

Wu, W., Liu, R. Jin, W., 2016. Designing robust schedule coordination scheme for transit networks with safety control margins. Transportation Research Part B, 93, 495-519.

Xu, S., Liu, R. Liu, T-L., and Huang, H-J, 2017. Pareto-improving policies for an idealized two-node city serviced by two congestible models. In Proceedings of the 22nd International Symposium on Transportation and Traffic

Theory.

Xuan, Y.G., Arogte, J., Daganzo, C.F., 2012. Dynamic bus holding strategies for schedule reliability: optimal linear control and performance analysis. Transportation Research Part B, 45(10), 1831-1845.

Zhao, J., Dessouky, M., Bukkapatnam, S., 2006. Optimal slack time for schedule-based transit operations. Transportation Science, 40(4), 529-539.