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A Joint Control-Communication Design for Reliable Vehicle Platooning in Hybrid Traffic

Bingyi Liu, Dongyao Jia, Kejie Lu, Dong Ngoduy, Jianping Wang, and Libing Wu

Abstract—Recent studies have shown that traffic safety and efficiency can be substantially improved by vehicle platooning, in which vehicles periodically broadcast their kinetic status to neighbors, known as beacon message dissemination. As a networked control system, vehicle platoon has attracted significant attention from both the control and networking areas. However, few studies consider the practical traffic scenario with both platoons and individual vehicles, and the proposed beaconing schemes lack the deep understanding of relationship between the beaconing performance and the requirements of the control mechanism. To address these challenging issues, we propose a joint control-communication design to achieve reliable vehicle platooning in a more realistic traffic scenario, wherein the traffic consists of both platoons and individual vehicles, and both periodic beacon messages and event-based safety messages shall be delivered together. Specifically, we first develop a comprehensive control-theoretical analysis to understand how the vehicular communication can affect features of platoon driving; based on the understanding, we then propose and analyze an adaptive platoon-based message dissemination scheme; finally, we conduct extensive numerical experiments to validate the effectiveness of the protocol and to confirm the accuracy of the our theoretical analysis.

Index Terms—platoon, stability, consensus control, protocol design, beacon.

I. INTRODUCTION

With the advances of control and vehicular communication technologies, the vehicles with some common interests can cooperatively drive on the road, e.g., platoon-based driving pattern, which may significantly improve the traffic safety and efficiency [1]–[4]. To maintain a safe and efficient platoon, vehicles in the platoon have to obtain information from neighboring vehicles via inter-vehicle communication (IVC), and then adopt a suitable control law to achieve certain objective, e.g., maintaining a constant inter-vehicle spacing within the same platoon [5], [6].

Clearly, for such a networked control system, it is necessary to design not only the advanced control mechanism for vehicles in the same platoon, but also the efficient IVC protocol to deliver control messages. In the past few years, these two areas have been hot topics. Some typical platoon control strategies include adaptive cruise control (ACC) based on the preceding vehicle information [7], sliding-mode control with the leader-follower information [5], consensus-based control with neighboring information [8], etc. On the other hand, for IVC protocol design, periodic beacon messages and event-based safety messages dissemination has been extensively studied based on the dominating IEEE 802.11p standard [9]–[12], in which the channel access time is divided into synchronized intervals (SI) with the control channel interval (CCHI) and service channel interval (SCHI). Moreover, several IVC communication patterns have been proposed specifically to support vehicle platooning application, most of which try to adjust beaconing frequency or transmit power to achieve a higher beacon reception ratio and maintain the channel load at a desired value [13]–[16].

Although the aforementioned studies are fundamentally important, they are not sufficient because most existing studies consider the two areas separately. For instance, for the platoon control, seldom work fully considered the realistic IVC implementation. On the other hand, most existing communication protocols were designed without in-depth understanding on whether the beaconing performance can meet the requirements of platoon control.

Moreover, few studies consider the realistic hybrid traffic flow that consists of both platoons and individual vehicles (i.e., vehicles not in any platoon) running on the same road, as shown in Fig. 1, and few of them consider the common communication scenario with co-existing beacon dissemination messages and event-based safety messages. In fact, overloaded safety messages on channel will seriously deteriorate the performance of beacons dissemination, and vice versa. Therefore, it is imperative to jointly design the control mechanism and communication protocol so as to not only guarantee the safety and stability of vehicles’ cooperative driving, but also deliver the emergency messages reliably with low delay.

In this study, we systematically investigate how to support reliable vehicle platooning in the hybrid traffic scenario by a joint control-communication design. Specifically, we pro-
pose the coupled design which combines the consensus-based control theory and adaptive communication protocols. To the best of our knowledge, this is a first attempt that vehicle platooning is integrally designed from both communication and control perspective, which deeply explores the interplay between them.

Our main contributions in this paper are as follows:

- We consider a more realistic application scenario with mixed traffic flow on road, which leads to the co-existence of both periodic beacon dissemination messages and event-based safety messages.
- We propose a joint control-communication design for reliable vehicle platooning. Specifically, a consensus-based platoon control scheme is adopted to theoretically explore the relationship between message dissemination and platooning performance. Based on the control-theoretical analysis, we propose an adaptive message dissemination strategy, in which we provide different beaconing strategies for both intra-platoon and inter-platoon, and regulate the event-based safety message dissemination for individual vehicles, respectively.
- We develop an analytical model and conduct extensive simulation experiments to evaluate the proposed control algorithms and communication protocols. Both analytical and numerical results confirm the efficiency of our joint design on the vehicle platooning system.

The rest of this paper is organized as follows. In Section II, we first discuss related work about platooning control and IVC schemes. In Section III, we present the system model, and then adopt a control-theoretical approach to analyze the relationship between platoon stability and control message dissemination in Section IV. Based on the understanding, we propose a comprehensive dissemination scheme for both periodical beaconing messages and event-based safety messages in Section V, and we theoretically analyze the performance of the proposed scheme in Section VI. Finally, in Section VII, we validate our design and analysis through extensive simulation experiments, before concluding the paper in Section VIII.

II. RELATED WORK

This section discusses the related work in terms of platoon control and message dissemination respectively.

A. Platoon control algorithms

In the last decades, a large number of platoon control schemes have been proposed, which can be classified according to different communication information, communication topology, as well as the control laws. For instance, [17] presents a cooperative adaptive cruise control (CACC) design with the predecessor-follower information, [5] develops a sliding-mode control with the leader-follower information, [18] adopts the gas-kinetic theory to model the mixed traffic of manual and ACC vehicles, [19] studies the influence of information flow topology on the internal stability and scalability of homogeneous vehicular platoons moving in a rigid formation, and more studies can be found in the recent surveys [1], [20] and references there-in. Among different control strategies, the consensus-based approach has recently been applied into the platoon control [6], [8], [21], because it can efficiently facilitate the convergence of collective behavior among multiple agents [22], and that can well adapt to the characteristics of the time-varying communication topology of IVC. In this study, we will investigate consensus-based control and will particularly focus on the stability of platoon under realistic imperfect IVC, which has not been well addressed in the past.

B. Message dissemination

To improve the performance of message dissemination for vehicular networking, many communication schemes have been proposed which can be either contention-free or contention-based. The main idea for typical contention-free solutions is that vehicles are grouped into a cluster in which the cluster head is responsible for allocating time division multiple access (TDMA) slots to other cluster members [11], [23], [24]. As for typical contention-based solutions, the networking parameters, such as the beacon frequency, beacon dwelling time, transmit power and contention window size, are adjusted adaptively in accordance with the changing traffic conditions to achieve better system performance [9], [10], [14], [25]–[30]. For example, the Adaptive Traffic Beacon (ATB) protocol in [31] adjusts the beacon rate based on two key metrics: message utility and channel quality. For more details, please see [1] and references there-in.

Some beaconing strategies were designed specifically for platooning. In [15], the authors developed an algorithm, named Dynamic beaconing (DynB), with which each vehicle decreases/increases its beacon rate if the channel load is higher/lower than the desired one. To cope with the heavy communication load among vehicles in the same platoon and possible data collisions between adjacent platoons in [5], one CCHI is divided into several time-slots that are allocated to the vehicles based on their relative positions in the platoon. [16] proposes Jerk, a dynamic information dissemination protocol for platooning that exploits vehicle dynamics to send beacons only when needed. The protocol shows that the beaconing frequency can be less than 10Hz when the control qualities do not change. In this way, the channel load can be reduced and thus the protocol may improve the delivery of safety messages.

Although the aforementioned protocols are important to support vehicle platooning, few of which have been designed based on the theoretical analysis of the relationship between the beaconing and the platoon control performance. Moreover, the realistic traffic scenario, such as the multi-platoon driving and the coexistence of beacons and safety messages, have not been fully considered in the literature. In this study, we design and analyze an adaptive beacon/safety message dissemination scheme that not only can meet the requirements for platoon control but also takes into account realistic hybrid traffic conditions, which has not been addressed in the literature.

III. THE SYSTEM MODEL

This section describes our system model with main assumptions and specifications in terms of inter-vehicle communication and cooperative driving control, respectively.
facilitate further discussions, we first summarize the symbols and notations in Table I.

| $V_I$ | individual vehicle |
| $V_L$ | leader of the platoon |
| $V_M$ | member of the platoon |
| $x_i$ | position of vehicle |
| $v_i$ | velocity of vehicle $i$ in the platoon |
| $\alpha$ | acceleration of $V_L$, $\alpha$ is the maximum |
| $\tau$ | beacon dissemination delay |
| $N_m$ | number of member vehicles |
| $N$ | platoon size, i.e., $N = N_m + 1$ |
| $N_i$ | number of individual vehicles |
| $d$ | intra-platoon spacing |
| $\lambda_1, \gamma_1, \gamma_2$ | positive control parameters in consensus algorithms |
| $t$ | length of road |
| $R$ | transmission range |
| $\lambda_d$ | number of vehicles per meters |
| $\gamma^f$ | duration of a slot for beaconing |
| $\gamma$ | duration of backoff slot |
| $E$ | communication channel quality |
| $k_m$ | number of slots for beaconing of $V_M$ |
| $P_I$ | beacon transmission ratio for $V_L$ |
| $P_m$ | beacon transmission ratio for $V_M$ |
| $P_{ir}$ | safety message transmission ratio for $V_L$ |
| $P_{inr}$ | safety message reception ratio for $V_L$ |
| $P_{inr}$ | safety message reception ratio for $V_I$ |
| $P_{uns}$ | probability that a $V_I$ transmits in a randomly slot under unsaturated situation |
| $P_{uns}$ | probability that a $V_I$ transmits in a randomly slot under unsaturated situation with our scheme |
| $T_{CCHI}$ | duration of a CCH |
| $T_l$ | duration allocated for beaconing of $V_L$ |
| $T_f$ | duration for TDMA-based period in CCHI |
| $T_c$ | duration for CSMA-based period in CCHI |

### A. Inter-vehicle Communication

In this paper, we consider the traffic flow on a road which consists of platoons driving in a dedicated lane and individual vehicles in other lanes, as shown in Fig. 1. Thus the message dissemination under such hybrid traffic flow includes the platooning beacons from the platoon and safety messages from individual vehicles, respectively.

To support platoon-based driving, each vehicle is equipped with on-board sensors and GPS (Global Positioning System) to measure its absolute position, speed and acceleration as well as time stamp, and adopts the WAVE suite (the de facto vehicular networking standards) as the IVC protocol. In addition, the application layer is assumed to be aware of channel CCH/SCH and the platooning beacons are disseminated at each available CCHI time.

For vehicular communications, we consider that the communication topology among platoon members can be represented as a directed graph (digraph) $G = (V, E, A)$, where $V = 1, 2, \ldots, n$ is the set of vehicles, $E \subseteq V \times V$ is the set of edges, and $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is an adjacency matrix with nonnegative elements which represents the communication link between vehicle $i$ and $j$. In this paper, we assume $a_{ij} = 1$ in the presence of a communication link from node $j$ to node $i$, otherwise $a_{ij} = 0$. In addition, we assume no self-loops in the directed graph, i.e., $a_{ii} = 0$ for all $i = 1, \ldots, n$. The degree matrices $D = \text{diag}(d_1, \ldots, d_n)$ are diagonal matrices, whose diagonal elements are given by $d_i = \sum_{j=1}^{n} a_{ij}$. The Laplacian matrix of the weighted digraph is defined as $L = D - A$. To study the leader-following problem, we also define a diagonal matrix $B = \beta \cdot \text{diag}(b_1, \ldots, b_n) \in \mathbb{R}^{n \times n}$ to be a leader adjacency matrix associated with the system consisting of $n$ vehicles and one leader (labeled with 0), where $\beta$ is the control weight, $b_i = 1$ in presence of a communication link from leader 0 to node $i$, otherwise $b_i = 0$. In case of switching topology (i.e., the communication topology among vehicles changes due to the packet loss), all adjacency matrices are labeled with the subscript $\sigma$, i.e., $A_{\sigma}$. All possible topology set is defined as $\Lambda = \{ G_0, G_1, \ldots, G_K \}$, where $K$ denotes the total number of all possible communication graphs.

### B. Platoon-based Cooperative Driving

In this paper, the consensus-based control algorithms are applied on the vehicle platooning, which has been verified as an effective method to convergence collective behavior of the multi-agents under the time-varying communication topology [22]. To this end, we need to formulate the vehicle platooning into a consensus control problem at first.

Since we only focus on the intra-platoon driving performance in this paper, we consider the hybrid traffic system consisting of $N_i$ individual vehicles denoted by $V_I$s and one platoon with one leader $V_L$ and $N_m$ members $V_M$s. Some specifications and assumptions regarding the control system are made as follows.

1) Each vehicle has the same fixed transmission range $R$ and all vehicles in the same platoon can connect to each other, i.e., $R > N_m d$.

2) The beacon frequency is set to $1/\tau$ (which is typically 10Hz), and the consensus control is implemented at each end of CCHI.

3) The position and velocity function of vehicle is time-continuous, and the leader’s acceleration is assumed with an upper bound $\bar{\alpha}$: $|\dot{v}_0| = |\ddot{v}(t)| \leq \bar{\alpha}$.

It shall be noted that, due to the presence of system uncertainties and physical limitations, including actuator lags and sensing delays, precisely modelling vehicle dynamics is very cumbersome. To simplify the system analysis, we model the continuous-time dynamics of vehicle $i$ in a platoon as a second-order equation, which has been widely adopted in the literature:

$$\dot{x}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = u_i(t)$$

where $x_i \in \mathbb{R}$ and $v_i \geq 0$ are the position and velocity of vehicle $i$ in the platoon, $u_i \in \mathbb{R}$ is the control input which can be adjusted based on the neighboring information.

\[\text{This assumption is reasonable under the current vehicular communication capability. However, it does not mean the platoon is fully connected at any time due to the packet loss.}\]
The platoon control objective is to let each member follow the leader asymptotically and maintain the identical inter-vehicle spacing $d$, i.e. to achieve consensus of platoon, which can be expressed by:

$$x_i(t) \to x_0(t) - i \cdot d, \quad v_i(t) \to v_0(t)$$

where $x_0(t), v_0(t)$ are the position and speed of the leader, respectively.

IV. Platoon Control

In this section, we investigate the impact of the vehicular communication performance, such as packet loss and delay, on the consensus-based platoon control system. Moreover, the factors of relative position of vehicles in the platoon and the leader’s dynamics are also taken into account. The analysis results will be utilized as the guideline of the message dissemination design in Section V.

A. Consensus-based Platoon Control

We design the consensus control algorithms for the members to follow the leader’s state. To deal with the packet loss of leader’s information, we adopt the last available state of leader to estimate the current state of vehicles, which means the leader’s states (velocity and position) are always globally reachable to all followers. Thus the consensus algorithms are proposed as follows.

$$u_i(t) = \sum_{j=1}^{N} a_{ij} \{ \gamma_1[x_j(t) - x_i(t)] + v_0(t - \tau_0)\tau_j - (i - j) \cdot d \}$$  \hspace{1cm} (4a)

$$+ \gamma_2[v_j(t - \tau_j) - v_i(t)]$$ \hspace{1cm} (4b)

$$+ \beta_1[x_0(t - \tau_0) + v_0(t - \tau_0)\tau_0 - x_i(t) - i \cdot d]$$  \hspace{1cm} (4c)

$$+ \gamma_2[v_0(t - \tau_0) - v_i(t)]$$ \hspace{1cm} (4d)

where $a_{ij}$ is the $(i,j)$th entry of the adjacency matrix and $\beta_1$ and $\gamma_2$ are the positive control parameters. $\tau_j$ is the time-varying communication delays when information is transmitted from vehicle $j$ to other members within the same platoon. The desired acceleration is determined by the state difference (position and velocity) between itself and neighbours:

- (4a) represents the estimated position error between the gap of member $i$ and $j$ at the time $t$ with respect to the desired gap of $(i - j) \cdot d$. Due to time-delay $\tau_j$ of $x_j$, the item of $v_0(t - \tau_0)\tau_j$ is added as the desired gap supplement between member $i$ and $j$, assuming member $j$ follows the speed of leader 0.
- (4b) denotes the velocity error between member $i$ and $j$.
- (4c) denotes the estimated position error between the gap of member $i$ and leader 0 at the time $t$ with respect to the desired gap of $i \cdot d$.
- (4d) denotes the velocity error between member $i$ and leader 0.

Defining the position and speed errors with respect to the leader as $\bar{x}_i \triangleq x_i + i \cdot d - x_0$ and $\bar{v}_i \triangleq v_i - v_0$, substituting Eq. (4) into Eq. (1)-Eq. (2), we can obtain the closed-loop dynamics of members:

$$\dot{\hat{x}}_i(t) = \hat{v}_i(t)$$

$$\dot{\hat{v}}_i(t) = \sum_{j=1}^{N} a_{ij} \{ \gamma_1[\bar{x}_j(t) - \bar{x}_i(t)]$$

$$+ \gamma_2[\bar{v}_j(t - \tau_j) - \bar{v}_i(t)]\} - \beta_1[\bar{x}_i(t) + \gamma_2\bar{v}_i(t)]$$

$$+ \sum_{j=1}^{N} a_{ij} \{ \gamma_1[v_0(t - \tau_0)\tau_j - x_0(t - \tau_0)]$$

$$+ \gamma_2[v_0(t - \tau_0) - v_0(t)]\}$$

$$+ \beta_1[v_0(t - \tau_0)\tau_0 - x_0(t - \tau_0)]$$

$$+ \gamma_2[v_0(t - \tau_0) - v_0(t)]\} - \alpha$$

(6)

Accordingly, the system can be decoupled into two parts: the neighboring consensus system and the leader’s state error system. For the time-continuous velocity of leader, $x_0(t) - x_0(t - \tau_j) = \int_{t - \tau_j}^{t} v_0(\sigma) d\sigma = v_0(t - \tilde{\tau}_j)\tau_j$, where $\tilde{\tau}_j \in [0, \tau_j]$.

Letting $\bar{x} \triangleq [\bar{x}_1, ..., \bar{x}_n]^T$, $\bar{v} \triangleq [\bar{v}_1, ..., \bar{v}_n]^T$, $\hat{x} \triangleq [\hat{x}^T, \hat{v}^T]^T$, Eq. (5)-Eq. (6) can be transformed into:

$$\dot{\hat{x}}(t) = A_{0,\sigma}\hat{x}(t) + \sum_{j=1}^{N} A_{j,\sigma}\hat{x}(t - \tau_j) + \Delta$$

where

$$A_{0,\sigma} = \begin{bmatrix} \frac{I_{N \times N}}{\gamma_1 D_{\sigma} + \beta I} & -\frac{I_{N \times N}}{\gamma_2 D_{\sigma} + \beta I} \end{bmatrix},$$

$$A_{j,\sigma} = \begin{bmatrix} 0_{N \times N} & 0_{N \times N} \\ \gamma_1 A_{j,\sigma} & \gamma_2 A_{j,\sigma} \end{bmatrix},$$

$$A_{j,\sigma} = \begin{bmatrix} 0 & \cdots & a_{1j} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & a_{2j} & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & \cdots & a_{Nj} & \cdots & 0 \end{bmatrix},$$

$$\Delta = \begin{bmatrix} 0_{N \times 1} \\ \delta_{N \times 1}(t) \end{bmatrix} = \begin{bmatrix} 0_{N \times 1} \\ \gamma_1 A_{\sigma} T_{N \times 1} \end{bmatrix} \cdot (v_0(t - \tau_0) - v_0(t - \tilde{\tau}_j))$$

$$+ \begin{bmatrix} 0_{N \times 1} \\ \gamma_2 A_{\sigma} \end{bmatrix} \cdot (v_0(t - \tau_0) - v_0(t))$$

$$+ \begin{bmatrix} 0_{N \times 1} \\ \beta_1 \gamma_1 1_{N \times 1} \end{bmatrix} \cdot \tau_0 (v_0(t - \tau_0) - v_0(t - \tilde{\tau}_0))$$

$$+ \begin{bmatrix} 0_{N \times 1} \\ \beta_2 \gamma_2 1_{N \times 1} \end{bmatrix} \cdot (v_0(t - \tau_0) - v_0(t))$$

$$+ \begin{bmatrix} 0_{N \times 1} \\ \alpha 1_{N \times 1} \end{bmatrix} \cdot T_{N \times 1} = [\tau_1, ..., \tau_N]^T,$$

and

$$\delta_{N \times 1}(t) = [\delta_1(t), ..., \delta_1(t)]^T.$$

Due to periodical implementation of the consensus algorithms (see assumption 2 of system model), if the leader’s beacon reception ratio is $P_{br}$, for the given confidence level
$P_0 < 1$, the maximum number of intervals $\pi$ can be estimated by:
\[
\sum_{r=1}^{\pi} (1 - P_{tr})^{r-1} P_{tr} \geq P_0, \quad \text{for } r = 1, 2, \ldots
\] (8)

Accordingly, we can further estimate the bounded value of $\delta_i(t)$:
\[
|\delta_i(t)| \leq |\bar{\delta}| \equiv (NP_{tr} + \beta)(\gamma_1 \tau / 2 + \gamma_2)(\pi - 1)\tau + 1)\alpha
\] (9)

### B. Stability Analysis

By the Leibniz-Newton formula, we have $\chi(t) = \chi(t) - \int_{-\tau}^{0} \bar{\chi}(t + s)ds = \bar{\chi}(t) - \sum_{i=0}^{N} A_{i,\sigma} \int_{-\tau}^{0} \bar{\chi}(t + s - \tau_i)ds - \int_{-\tau}^{0} \Delta(t + s)ds$, where $\tau_0 \equiv 0$. Substituting this equation into Eq. (7), we can obtain
\[
\dot{\chi}(t) = F_{\sigma} \chi(t) - \sum_{j=1}^{N} \sum_{i=0}^{N} A_{j,\sigma} A_{i,\sigma} \int_{-\tau_j}^{0} \bar{\chi}(t + s - \tau_i)ds
\]
\[
- \sum_{j=1}^{N} A_{j,\sigma} \int_{-\tau_j}^{0} \Delta(t + s)ds + \Delta
\] (10)

where
\[
F_{\sigma} = \sum_{i=0}^{N} A_{i,\sigma} = \begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -\gamma_1 H_{\sigma} & -\gamma_2 H_{\sigma} \end{bmatrix}, \quad H_{\sigma} = L_{\sigma} + \beta I
\]

In the proposed beacon dissemination scheme, a successful beacon dissemination from the leader indicates there always exists a spanning tree with the root of the leader in all switching topologies $\Lambda$ of platoon. In this case, matrix $H_{\sigma}$ is positive stable according to [32].

**Lemma 1:** $F_{\sigma}$ is Hurwitz stable if
\[
\frac{\gamma_2}{\sqrt{\gamma_1}} > \max_{1 \leq \sigma < K} \left\{ \max_{\theta_i \in \sigma(H_{\sigma})} \frac{|Im(\theta_i)|}{\sqrt{|Re(\theta_i)|} \cdot |\theta_i|} \right\}
\]

where $\sigma(H_{\sigma})$ is the set of all eigenvalues of $H_{\sigma}$.

The proof follows a similar line to that of Lemma 4 in [8] and is omitted here. Thus we have the following theorem.

**Theorem 1:** Consider Eq. (7), if the control parameters $\gamma_1$ and $\gamma_2$ satisfy
\[
\frac{\gamma_2}{\sqrt{\gamma_1}} > \max_{1 \leq \sigma < K} \left\{ \max_{\theta_i \in \sigma(H_{\sigma})} \frac{|Im(\theta_i)|}{\sqrt{|Re(\theta_i)|} \cdot |\theta_i|} \right\}
\] (11)

then there exists a sufficient small constant $\tau_0 > 0$, such that when $0 \leq \tau_j \leq \tau_0$ \(i=1,...,N\), the state error between the members and the leader is uniformly ultimately bounded by:
\[
\lim_{t \to \infty} ||\bar{\chi}|| \leq C_0
\] (12)

for some constant $C_0$ depending on $\alpha$, beacon delivery ratio $P_{tr}$, platoon size $N$. Moreover, if $\alpha = 0$, then $\lim_{t \to \infty} \bar{\chi} = 0$.

**Remark 1:** It shall be noted that, in this paper, we only consider the local stability from the perspective of consensus control, i.e., the state errors between the vehicle and its neighbors are bounded by some factors, e.g., acceleration and time delay. Such stability is known as the bounded stability. The string stability, which reflects the attenuation of disturbance along the vehicles, will be investigated in our future work.

The detailed proof of Theorem 1 is given in Appendix. Theorem 1 shows that states (position and velocity) error $\bar{\chi}$ can be converged within a certain bound in case of beaconing packet loss. Moreover, the value of bound is determined by system parameters, such as platoon size, beacon delivery ratio, and acceleration perturbation magnitude, as shown in Eq. (9).

Specifically, the leader’s state information plays a critical role on platoon stability. In general, in case of time-varying velocity of the leader (i.e. $\alpha \neq 0$), given other parameters, the state error can be mitigated by improving beaconing frequency and beacon delivery ratio. In addition, in case of constant velocity of leader, the states error will converge to 0. The theoretical results can be utilized as the guideline for the beaconing protocols design in the following section.

### V. Beacon and Safety Message Dissemination

In this section, we provide a complete Adaptive Beacons and event-based Safety messages Dissemination scheme (ABSD), wherein a more common scenario with multiple platoons and a number of individual vehicles on a road is considered.

A distinctive feature of ABSD is combining the beaconing scheme with the stability requirement for platoon-based driving. The main idea of ABSD is: 1) We adopt the TDMA-like MAC mechanism for the platoon beaconing to improve transmission reliability, while utilize CSMA-based MAC protocols for the safety message to maximize the channel utilization. 2) Beacon dissemination is coordinated by the platoon leader and the members’ beaconing time slots are adaptively allocated according to the current channel quality as well as the leader’s dynamics, which is based on the theoretical results in Section IV. 3) Self-configuring slot allocation for inter-platoon is proposed to avoid beacon collision among adjacent platoons. 4) For individual vehicles, we dynamically regulate the safety message sending time to avoid the collision with the platoon beacons.

#### A. Frame Structure

Based on the main ideas, a CCHI is divided into TDMA-based period (TS) for beacon dissemination and contention-based period (TC) for safety message dissemination, as shown in Fig. 2(a). TS contains one beacon slot for the leader $V_L$ and $k_m$ beacon slots for members $V_M$. The beacon from $V_L$ contains its kinetic information as well as beacon scheduling information, including the start slot of TS period $S_t$, the number of members in platoon $N_m$, the relative position of newly coming or leaving vehicles $P_n$ and the re-assign flag $R_a$ which is used to notify the members whether there are any changes with the platoon. For a given $N_m$ and $k_m$, the beaconing frequency $F$ of $V_M$ can be calculated as $F = 10k_m/N_m$. For instance, a beaconing frequency of 10/3Hz means $V_M$ send only one beacon every three CCHI, as illustrated in Fig. 2(b). $k_m$ can be dynamically adjusted by leader according to the current channel quality and the platoon dynamics. The TC period employs the CSMA protocol, mainly used for event-based safety message dissemination and newly coming nodes to reserve a slot for beaconing.
and future, the leader is able to derive a metric of the overall channel quality \( \epsilon \), which is a linear combination of \( N_b, N_c \) and \( S \), ranging from 0 to 1 (lower values describing a better channel quality) and can be calculated as follows [31], [34].

\[
\epsilon = \frac{N_b + w_c S + \frac{N}{2}}{1 + w_c}.
\]

We adopt a state machine to determine \( F \) for members, as depicted in Fig. 3. Three states are defined for beacon corresponding to different frequency in \( \{ F_{\text{min}}, F_{\text{def}}, F_{\text{max}} \} \), which shall be transferred by the transitions conditions including \( \alpha \) and \( \epsilon \). According to Eq. (9) as well as Theorem 1, the bigger \( \alpha \) is, the higher \( F \) is demanded. On the other hand, excessive number of beacons may lead to serious packet collision and channel overload, thus lower the packet transmission ratio. As a result, there is a tradeoff to decide \( F \), probably remaining a fixed value or even being reduced. The state transitions of the \( F \) shall be event driven according the following three rules:

**Rule 1:** In state \( F_{\text{min}} \), the state shall be switched to \( F_{\text{def}} \) if \( \alpha_L < \alpha <= \alpha_H \) and \( \epsilon <= \epsilon_H \), to \( F_{\text{max}} \) if \( \alpha > \alpha_H \) and \( \epsilon <= \epsilon_H \).

**In state \( F_{\text{min}} \),** \( F \) should be increased to maintain the platoon stability when \( \alpha \) becomes higher. However, the packet loss ratio will increase significantly when the channel quality is poor. So the \( F \) should remain fixed or even be reduced in this situation even though a high \( \alpha \) is detected.

**Rule 2:** In state \( F_{\text{def}} \), the state shall be switched to \( F_{\text{min}} \) if \( \alpha <= \alpha_L \) and \( \epsilon > \epsilon_L \), to \( F_{\text{max}} \) if \( \alpha > \alpha_H \) and \( \epsilon <= \epsilon_H \).

**In state \( F_{\text{def}} \),** when the platoon travels steadily, the excessive number of beacons will be meaningless for vehicle control. In this circumstance, the beaconing frequency could be reduced to alleviate the channel load so that a higher packet transmission ratio can be achieved.

**Rule 3:** In state \( F_{\text{max}} \), the state shall be switched to \( F_{\text{min}} \) if \( \epsilon > \epsilon_H \), to \( F_{\text{def}} \) if \( \alpha <= \alpha_H \) and \( \epsilon_L < \epsilon <= \epsilon_H \).

**In state \( F_{\text{max}} \),** a poor channel quality will dramatically increase the packet collision, thus beaconing frequency of members in platoon should be reduced to relieve the total channel load to guarantee a marginal packet loss. Since the setting of \( \alpha_L, \alpha_H, \epsilon_L, \) and \( \epsilon_H \) determine the trade-off between the channel condition and the platoon stability, we can set them according to the requirement of specific application. For example, if a better channel quality is required to improve the safety message transmission ratio, we can set smaller \( \alpha_L \) and \( \alpha_H \). On the other hand, we can set smaller \( \epsilon_L \) and \( \epsilon_H \) to improve the platoon stability. In addition, the general delay jitter among the members in platoon may impair the platooning performance. Consequently, we arrange the adjacent vehicles’ beaconing in different CCHI if \( F < 10 \), as illustrated in Fig. 2(c), to minimize the general delay jitter.

C. Self-configuring slot Allocation for Inter-platoon

In this part, we consider the scenario of multi-platoons drive on a road. Since platoons initially choose the slots in the front period of CCHI for beacon dissemination, according to subsection V-B, beacon collision will happen when two platoons get too close, as shown in Fig. 4(a). In this scenario,
the allocated TDMA slots for platoon A and B are overlapping (see Fig. 4(b)), which can lead to a Carrier Sense Collision (CS-Collision) among the two platoons within the transmission range. Another potential issue is the Hidden Collision (HN-collision) problem when only part of members in platoon A, except the leader A0, can be detected by B0, as shown in Fig. 4(c). In this case, packet loss will occur on the rear vehicles in platoon A if A0 and B0 transmit simultaneously.

To mitigate the problems of the CS-collision and HN-collision, we propose a self-configuring slot allocation for inter-platoon. A leader should first identify if there are any overlapping slot allocations among adjacent platoons. In our scheme, we design a simple sensing procedure for the leader of a platoon. First, in every CCHI, the leader broadcasts a schedule that allocates the beaconing slots for platoon members. If members receive the schedule correctly, they shall follow the schedule and send messages to the leader. In this manner, the leader will receive beacon messages from all the members under normal circumstance. On the other hand, if a leader cannot receive the beacons from its members in several successive CCHIs (at least 2 CCHIs), it will infer that the beacon losses are caused by the slot overlapping among nearby platoons. In existence of overlapping slot allocations, the leader will adaptively rearrange the TDMA-based period and temporarily choose the slots next to the overlapping slots to avoid the collisions. In case of no overlapping slot allocations, the leader will reset its original slots occupation.

To implement such self-configuring slot allocation, two practical situations are supposed to be taken into account. In case of two platoons A and B moving close on the same direction, then, the front platoon A maintains the TC period unchanged and the following platoon B is required to delay its TS period to avoid CS-collision and HN-collision, as shown in Fig. 4(d). This is because the front vehicle’s information is more important to a stable platooning control. Another reason is that compared to the front leader A0, the following leader B0 can sense the overlapping slots (by sensing the beacon loss of B2 and B3 in Fig. 4(c)) much earlier and adopt timing adjustments rapidly. In case of two platoons A and B moving on the opposite directions and the distance between two leaders being less than the transmission range, the leader’s beacons disseminated at the first time slot of TS will collide with each other. According to the leader’s beaconing scheme in V-B, the leader have another chance to disseminate the beacon randomly during TC period. Therefore, the leader (e.g., B0) which first receives the beacon will delay its TC period, and the other one (A0) keeps its TS period unchanged. In addition, to further improve the beacon reception ratio in case of congested channel load, the leader can disseminate an additional beacon at the end of TS period.

The workflow of the platooning system is briefly presented as follows. At the beginning of each CCHI, the platoon leader broadcasts its kinetic status as well as the time slots allocation for members based on current channel condition and platoon dynamics, the members then broadcast their information at the scheduled slots. At the end of each CCHI, each member will implement the consensus control algorithms of equation (4) based on the latest received neighboring information.
D. Safety Message Dissemination for Individual Vehicles

In general, safety message dissemination of individual vehicles is event-driven, which can maximize the channel utilization compared to the beaconing. Due to the coexistence of beacons and safety messages, the envisioned safety message dissemination scheme for \( V_f \) is to not only guarantee the safety message transmission performance, but also avoid impairing the beaconing process of the platoon.

As stated previously, safety messages are supposed to be disseminated within the TC period. To do that, \( V_f \)s need to estimate the duration of \( T_i \), which can be done as follows: \( V_f \) overhears the packets from neighbors and obtains the packet type (This can be identified based on the different packet length of beacons and safety messages), analyzing the corresponding received packet temporal distribution \([9]\). Since beacons are uniformly disseminated within TS period, its boundary, i.e. the duration of \( T_i \), can be approximately estimated by the unique distribution profile. Accordingly, those messages generated during \( T_S \) period will be delayed to TC period for dissemination.

VI. Analytical Model for Performance Evaluation

In this section, we theoretically analyze the performance of ABSD under different traffic conditions, where we assume that individual vehicles \( D_n \) on the road are spatially Poisson distributed with the mean value of \( \lambda_d \) and safety messages generated from \( V_f \)s subject to a temporally Poisson distribution with average \( \lambda_s \).

A. Beacon/Safety message Transmission Ratio

Beacon/safety message transmission ratio (PTR) for \( V_L \), \( V_M \) and \( V_f \), denoted by \( P_l \), \( P_m \) and \( P_f \) respectively, can be calculated by the probability that no other vehicles within transmission range send packets at the same time slot. \( V_f \)s adopt the high-priority access class 3 specified in 802.11p, the corresponding contention window size is \( W_s \).

In terms of traffic flow transition, two phases are considered: initial phase and steady phase. The initial phase for a \( V_f \) is the time interval from when it first meets a platoon until the time when beaconing duration \( T_i \) is estimated. In initial phase, \( V_f \) transmits the safety message with a probability \( P_{uns} \), which can be be calculated as:

\[
P_{uns} = \frac{2(1-p)^2}{2pW_s - 3p} (1 - e^{-\lambda_s T_{ss}}), \tag{14}
\]

where the first term indicates the probability that the packet is transmit when the counter reaches zero, and the second term means the vehicle has a safety message ready to send, \( p \) is the probability when a busy channel is sensed. \( T_{ss} \) is the average service time needed to transmit the safety message since it is arrived \([35]\). Let \( T_d \) denote the duration for a beacon transmission, \( P_l \) in this situation can be derived as:

\[
P_l = \frac{T_d}{T_{cch}} + \frac{T_{cch}-T_d}{T_{cch}} (1 - P_{uns})^{2R\lambda_d}. \tag{15}
\]

The initial phase for the platoon is the transition period for vehicles forming into a platoon. \( P_l \), \( P_m \) are given by the probability that \( V_f \)s within its transmission range do not sent message at the same time. and can be calculated by \( P_l = P_m = (1 - P_{uns})^{2R\lambda_d}. \)

Fig. 5. Markov chain for the channel contention.

The steady phase for a \( V_f \) is the period during which it moves within the transmission range of the platoon and estimated duration of \( T_i \). For an arbitrary individual vehicle, the contention process can be characterized by a two-dimensional Markov chain as illustrated in Fig. 5, in which each state variable is represented by \( \{s(t), b(t)\} \), where \( s(t) \in \{0,1\} \) represents that the vehicle has a safety message ready for transmission during non-TC or TC period, and \( b(t) \in \{0,1,2,...,W_s-1\} \) represents the backoff time counter. The transition probability of the Markov chain can be derived as follow:

\[
\begin{align*}
P\{0,k|0,k+1\} &= 1-p, k \in [0,W_s-2] \\
P\{0,k|0,k\} &= p \\
P\{0,k|0,0\} &= p(1-G_s)/W_s \\
P\{1,k|0,0\} &= pG_s/W_s \\
P\{0,k|1,k\} &= G_s \\
P\{1,k|1,k\} &= 1-G_s 
\end{align*}
\tag{16}
\]

where, except the first line, \( k \in \{0,1,2,...,W_s-1\} \). In this model, \( G_s \) and \( G_s \) are supposed to be constant and independent values. \( G_i \) is the probability that a safety message is generated in non-TC period, while \( G_s \) is the probability that the safety message is ready to send. Since the safety messages are generated uniformly over time, \( G_i = \frac{T_i}{T_{CCH} + T_{SCH}} \), and \( G_s = \frac{T_{SCH}}{T_{CCH} + T_{SCH}} \). Let \( b_{i,k} = \lim_{t \to \infty} P\{s(t) = i, b(t) = k\} \), and \( T_{ss} \) denotes the average service time. Thus the probability that an individual vehicle transmits in a randomly chosen slot time can be calculated as \( P_{uns}' = b(0,0)(1 - e^{-\lambda_s T_{ss}}) \), \( P_i \) in steady phase can be calculated as \( P_i = (1 - P_{uns}')^{2R\lambda_d} \). As for the platoon, PTR is related to the mean number of vehicles newly coming into their transmission range during a CCHI denoted by \( E(N_i) \). \( P_i \) and \( P_m \) can be calculated as:

\[
P_i = P_m = (1 - P_{uns}')^{E(N_i)}. \]

B. Beacon/safety message Reception Ratio

Due to potential simultaneous broadcasts (failure of random back-off) and the presence of hidden nodes, not every targeted receiver can receive the broadcast message successfully.
Beacon/safety message reception ratio (PRR) is defined as the ratio of the number of vehicles successfully received the Beacon/safety message to the number of target nodes. $P_{tr}$ for $V_L$ indicates the proportion of $V_{Ms}$ which receive the beacons from the leader. It is assumed that the leader $l_0$ locate at 0, and the position of given effective interference source vehicle $X$, $Y$, and $Z$ is within $(-l - R, -l], (-l, 0]$, and $[0, R]$, as illustrated in Fig. 6. $P_{tr}$ can be derived as:

$$P_{tr} = \int_{-l-R}^{-l} \int_{-l}^{0} \int_{0}^{R} (1 - \frac{\tilde{N}_{IR}}{N_m}) P(X = x) P(Y = y) P(Z = z) \, dx \, dy \, dz \tag{17}$$

where $\tilde{N}_{IR}$ is the number of vehicles within the interfered region (IR). $P(X = x)$ is the probability that an effective interference source located at $-x$ which can be expressed as: $P(X = x) = \tilde{r}_x \lambda_d e^{x \lambda_d \tilde{r}_x}$, in which $\tilde{r}_x$ is the average transmission rate within $(-x, 0)$. $P(Y=y)$ and $P(Z=z)$ can be calculated in the same way [36]. Similarly, we can also obtain $P_{mr}$ and $P_{tr}$.

C. Beacon/safety message Sending Delay

The safety message sending delay is defined as the time interval from a safety message arriving at the sending queue until it is successfully sent. Since information contained in beacons is outdated for the next transmission, there are no queuing and back-off process involved in beacon transmission and the beacon will be dropped if collision happens. Consequently, the sending delay of beacons equals to its transmission delay. Accordingly, the sending delay of safety message denoted by $T_{sd}$ includes the queuing delay $T_{sq}$, time delay due to back-off process $T_{sf}$ and transmission delay $T_{st} = L/R_d + T_{DIFFS} + \delta$, where $\delta$ is the channel propagation delay. Thus, we have $T_{sd} = T_{sq} + T_{sf} + T_{st}$. According to the Markov chain in Fig. 5, the average time delay $T_{sf}$ can be derived as:

$$E[T_{sf}] = \sum_{i=0}^{W_s} \frac{(1 - G_i)p}{W_s} \sum_{i=0}^{W_s} \left( pT_i \right) + \sum_{i=0}^{W_s} \frac{G_ip}{W_s} \sum_{i=0}^{W_s} \left( 1 - G_i \right) \bar{T}_i \tag{18}$$

its second moment can be derived as:

$$E[T_{sf}^2] = \sum_{i=0}^{W_s} \left( \frac{1 - G_i}{W_s} \right) \left( \sum_{i=0}^{W_s} \left( pT_i \right) \right)^2 + \sum_{i=0}^{W_s} \left( \frac{G_ip}{W_s} \right) \left( \sum_{i=0}^{W_s} \left( 1 - G_i \right) \bar{T}_i \right)^2$$

$$= \frac{\bar{T}_i^2 (W_s - 1)(2W_s - 1)(p^2 - G_ip^2 + G_i - G_iG_i^2)}{6} \tag{19}$$

For a stable system, the crucial need is to ensure that the total load $\lambda_s T_{serv} < 1$, where the average safety message’s service time can be calculated as $T_{serv} = E[T_{sf}] + T_{st}$, and its second moment as $E[T_{serv}^2] = E[(T_{sf} + T_{st})^2]$. Hence we can derive the queuing delay $T_{sq}$ as in Pollaczek-Khintchine formula [37]:

$$T_{sq} = \frac{\lambda_s E[T_{sf}^2]}{2(1 - \lambda_s T_{serv})} \tag{20}$$

VII. NUMERICAL RESULTS

In this section, we first explain the experiment settings, then evaluate the performance for the proposed protocol, and finally validate the control mechanism.

A. Simulation Settings

In our experiments, we choose the Veins simulator [38], which combines OMNeT++ for event-driven network simulation and SUMO for the generation of traffic environment and vehicle movement. For the traffic scenario, unless specified otherwise, we consider a 10-kilometer highway segment with 4 lanes (one for platoon), on which the traffic flow is composed of several platoons and $V_T$s, and all vehicles are moving on the same direction. The $V_T$s are moving with speeds from 12m/s to 41m/s and their positions are subject to Poisson distribution. A screenshot of a simulation scenario is shown in Fig. 7. The system parameters for both communication model and platoon control are specified in Table II and Table III, respectively. It shall be noted that Free-Space path loss model ($\alpha = 2.0$) and Nakagami-m fading model [39] are employed here. The appropriate transmitting power is set to meet the requirement of the communication range with $R=300m$ for each vehicle. It shall be noticed that the channel quality $\epsilon$ ranges from 0 to 1, $\epsilon_L$ and $\epsilon_H$ are set to 0.3 and 0.7, representing typical non-congested and congested channel conditions, respectively. Similarly, the acceleration $\alpha$ ranges from 0 to 2.5, and $\alpha_L$ and $\alpha_H$ are set to 1.0 and 2.0, representing typical steady and drastic traffic scenarios, respectively. In addition, to model a

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical/Mac protocol</td>
<td>IEEE 802.11p</td>
</tr>
<tr>
<td>Path loss model</td>
<td>Free-space ($\alpha=2$)</td>
</tr>
<tr>
<td>Fading Model</td>
<td>Nakagami-m ($m=3$)</td>
</tr>
<tr>
<td>Transmission power</td>
<td>20 dBm</td>
</tr>
<tr>
<td>Beacon frequency for $V_L$</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Transmission range $R$</td>
<td>300m</td>
</tr>
<tr>
<td>Safety message rate $\lambda_s$</td>
<td>5 packets/sec</td>
</tr>
<tr>
<td>Beacon slot time $\varphi$</td>
<td>0.5 ms</td>
</tr>
<tr>
<td>back-off slot $\varnothing$</td>
<td>16</td>
</tr>
<tr>
<td>Min.CW for safety message</td>
<td>3</td>
</tr>
<tr>
<td>CW for beacon</td>
<td>15$\mu$s</td>
</tr>
<tr>
<td>Data rate</td>
<td>6 Mb/s</td>
</tr>
<tr>
<td>Beacon size</td>
<td>200 bytes</td>
</tr>
<tr>
<td>Safety message size</td>
<td>512 bytes</td>
</tr>
<tr>
<td>Weight factor $c$</td>
<td>2</td>
</tr>
<tr>
<td>$\epsilon_L$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>1 m/s$^2$</td>
</tr>
<tr>
<td>$\alpha_H$</td>
<td>2 m/s$^2$</td>
</tr>
</tbody>
</table>
Fig. 7. The screenshot of a simulation scenario, in which individual vehicles are represented by white arrows while vehicles within a platoon are represented by yellow (leader) or red (member) arrows.

TABLE III
TRAFFIC RELATED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle length</td>
<td>5 m</td>
<td>Max. acceleration α</td>
<td>2.5 m/s</td>
</tr>
<tr>
<td>Platoon size N</td>
<td>9</td>
<td>Max. deceleration</td>
<td>6 m/s</td>
</tr>
<tr>
<td>Intra-platoon spacing d</td>
<td>10 m</td>
<td>Road length l</td>
<td>10 km</td>
</tr>
<tr>
<td>Max. λd</td>
<td>0.52 vehicles/m</td>
<td>Average speed</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Control gains</td>
<td>γ1 = 1, γ2 = 2</td>
<td>Max. speed</td>
<td>41 m/s</td>
</tr>
<tr>
<td>Actuator lag</td>
<td>0.25 s</td>
<td>Control gain</td>
<td>β = 10</td>
</tr>
</tbody>
</table>

We also compare the performance of V_f’s safety message dissemination with and without the ABSD scheme. The results show that ABSD has little impact on the performance of V_f’s safety message dissemination.

We consider a scenario with time-varying traffic conditions. The simulation results are slightly larger than [39], ABSD has the similar performance of transmission delay. The reason is that, although the V_f’s have low probability to collide with the platooning beacons. We also compare the performance of V_f’s safety message dissemination with and without the ABSD scheme. The results show that ABSD can lead to much higher PRRs of beacon messages only during the TS period, the collision probability becomes longer and it takes several CCHIs for V_f’s to maintain at a high level during the whole process, and the PRR of V_M drops slightly when F jumps to a higher value. This is because, in this case, T_i becomes longer and it takes several CCHIs for V_f to estimate the new T_i, so the collision rate will be relatively high during the transition period. Moreover, by comparing the PRR of V_f with and without ABSD in Fig. 9(b), we can see that ABSD can lead to much higher PRRs of V_f and V_M.

Fig. 10 shows that the safety message transmission delay increases with the growth of λd, which is due to the high probability of channel contention and collisions in dense traffic condition. The simulation results are slightly larger than the analytical results for ABSD. Moreover, compared to the adaptive and mobility based algorithm (AMBA) proposed in [39], ABSD has the similar performance of transmission delay.

B. Performance of Communication

We first evaluate the communication performance of the proposed ABSD scheme in a scenario with only one platoon and several V_f’s. Fig. 8(a) and Fig. 8(b) show the PTR and PRR versus λd respectively. In Fig. 8(a), we assume that all vehicles move steadily and F=5. We can see that the simulation results match well with the analytical results. Moreover, PRRs of V_L and V_M are rather high (more than 90%) if λd is within the range of [0, 0.32]. We also compare the performance of V_f’s safety message dissemination with and without the ABSD scheme. The results show that ABSD has little impact on the performance of V_f’s safety message dissemination.

Fig. 8(b) shows the PRR of V_L versus λd under different beaconing frequencies of V_M, which indicates a stable and reliable link between V_L and V_M. It is obvious that, if λd increases, the PRR for V_L will decrease. Moreover, a smaller F leads to a higher PRR for V_L. This is because beacons have a higher probability to collide with safety messages from V_M with a larger F. In Fig. 8(b), we also compare ABSD with ATB proposed in [31]. The simulation results show that ABSD substantially outperforms ATB in terms of PRR under different traffic conditions.

To understand the dynamics of PTR and PRR of vehicles, we consider a scenario with time-varying F in Fig. 9. Specifically, we assume that, from 0 to 25 CCHI, λd increases linearly, and then from 25 to 50 CCHI, it decreases linearly to initial value while the acceleration α begins to increase linearly from 0 to 2.5. According to our protocol designed in Section V, F is 10 initially, then decreases to 5 and 2.5 at 10 and 20 CCHI respectively, and increases to 5 and 10 at the 30 and 40 CCHI respectively. We can observe from Fig. 9(a) that the PRRs of both V_L and V_M can maintain at a high level during the whole process, and the PTR of V_M drops slightly when F jumps to a higher value. This is because, in this case, T_i becomes longer and it takes several CCHIs for V_f to estimate the new T_i, so the collision rate will be relatively high during the transition period. Moreover, by comparing the PRR of V_L with and without ABSD in Fig. 9(b), we can see that ABSD can lead to much higher PRRs of V_L and V_M.

Fig. 10 shows that the safety message transmission delay increases with the growth of λd, which is due to the high probability of channel contention and collisions in dense traffic condition. The simulation results are slightly larger than the analytical results for ABSD. Moreover, compared to the adaptive and mobility based algorithm (AMBA) proposed in [39], ABSD has the similar performance of transmission delay. The reason is that, although the V_f’s can transmit the safety messages only during the TS period, the collision probability is lower because all the platoon beacons are disseminated in the TC period. The simulation results in Fig. 8-10 indicate that ABSD can significantly enhance the PTR and PRR of beacon dissemination, meanwhile it does not significantly compromise.

more realistic vehicle dynamics, the actuator lag (i.e., the delay between the acceleration command and its actual realization in the vehicle due to inertial and mechanical limits) is considered and implemented in the simulation.
safety message transmission and delay performance.

Next, we set a normal individual vehicle density to 0.12 vehicles/m and investigate the communication performance when two platoons are approaching. Fig. 11(a) displays the PRR of $V_L$ in traffic scenario that platoon $B$ is approaching platoon $A$ on the same direction, and the speed difference between $B0$ and $A0$ is 10m/s. We can see that the PRRs of both leaders keep a steady and high level in most of the time, but drop about 10% during a short transition period (about 4 CCHIs). This is because, during the transition period, some packets from leaders are colliding with safety messages from $V_M$; and meanwhile, platoon leader $B0$ can detect the beacons from the rear members of platoon $A$ so it can delay its TS period to avoid more collisions. Fig. 11(b) displays the performance in the scenario that platoon $B$ is moving closer to platoon $A$ on the opposite directions. It can be seen that the PRRs of the two leaders simultaneously decrease about 20% when they first sense each other (about 300m of distance.
between $A_0$ and $B_0$), and then return to the previous level after a short transition phase (about 4 CCHIs).

We then consider a more general traffic scenario wherein platoon $B$ follows $A$ on the eastward direction, and platoon $D$ follows $C$ on the westward direction. In addition, all vehicles move with the constant speed of $30m/s$, and the distance between $A_0$ and $B_0$ (or $C_0$ and $D_0$) is $330m$. We can see in Fig. 12 that the PRRs of platoon leaders are about $95\%$ most of the time. The first anomaly happens at about CCHI=4 when the PRRs of $A_0$ and $C_0$ decrease to about $75\%$. At this moment, they can sense each other with about $300m$ distance between them. Shortly after that, when CCHI is about 6, the PRRs of platoon leaders $B_0$ and $D_0$ decrease slightly. This is because, after leaders $A_0$ and $C_0$ notice each other, they attempt to mitigate beacon collisions by randomly send another beacon in the TC period. According to our protocol, the leader who receives the beacon first will delay its TS period in the next CCHI, which accordingly affects the following platoon leader’s TS period. In Fig. 12, the next notable PRR decrease of all $V_L$s happens at CCHI=60, when platoon leaders $B_0$ and $C_0$ (or $A_0$ and $D_0$) enter into each others’ transmission range, i.e., the distance between $B_0$ and $C_0$ (or $A_0$ and $D_0$) is about $300m$. From the results we can also find that, after all PRRs can be recovered quickly, in about 4 CCHI.

As shown in the Fig. 9, Fig. 11 and Fig. 12, the density of individual vehicle have impact on the decreasing level of PRR for both $V_L$ and $V_M$ during the transition period when $F$ becomes higher or platoons get closer. Thus, In Fig. 13, we explore the minimum PRR for both $V_M$ and $V_L$ versus $\lambda_d$ in the transition period. We can see that the PRR for $V_L$ can keep in a high value when $F$ jump from 5Hz to 10Hz. While, the PRR for $V_M$ decrease obviously when the density of individual vehicles becomes high. This is because the leader keeps using the first slot in a CCHI to broadcast the beacons, while some members use the new slots when $F$ becomes higher. Thus the probability of packet collision between $V_M$ and $V_L$ becomes higher during the transition period. We also investigate the minimum decrease of PRR for both $V_M$ and $V_L$ versus $\lambda_d$ in the transition period when platoon $B$ is moving closer to platoon $A$ on the opposite directions. We can see in Fig. 13 that the PRR of both $V_L$ and $V_M$ decrease with a higher individual vehicles density. This is because one of the platoons will delay its whole TS period, which leads to the decreases of the PRR of both $V_L$ and $V_M$.

To summarize, although there are several transient drops of PRR during the transition period, we can still observe from Fig. 9, Fig. 11 and Fig. 12 that the PRR of a leader is about $95\%$ in the steady phase with ABSD, which significantly outperforms the existed beacon dissemination scheme. We can see that the decrease of PRR happens mainly due to the collision with the safety messages from individual vehicles. Moreover, according to Fig. 13, the decrease will become more obvious with a higher density of individual vehicles. In our future work, we will try to address the transient decline of PRR during the transition period. We can also find that PRRs can be recovered quickly, in less than 4 CCHIs. This is because it takes about 4 CCHIs for all the nearby individual vehicles to identify the new TS period. The simulation results in the figures verify the efficiency of ABSD on solving the problem of overlapping slots occupation among platoons.

### C. Performance of Platooning System

In this subsection, we evaluate the impact of beacon dissemination on the platoon-based cooperative driving. We assume one platoon with 1 leader and 8 members, and adopt the parameter settings of consensus control algorithm in Table III. We evaluate the system performance under a perturbation scenario, where the leader’s velocity is experiencing a sinusoidal disturbance.

$$\delta(t) = A \sin(0.2\pi t), A = 5m/s.$$ 

(21)
We first evaluate the system performance under a general beaconing scheme where all vehicles randomly transmit the beacons in each CCHI with the same probability. Fig. 14 plots the state errors of the 4-th member, which is the vehicle at the center of the platoon, with respect to the leader under different beacon reception ratios: $P_{br} = P_{mr} = 90\%$, 80\%, and 70\%. We can see that, with more packet loss in beaconing, the magnitude of state error (both velocity and position) increases accordingly, which is consistent with our conclusion in Theorem 1. The reason is that the leader’s beacon loss introduces the estimation error of leader’s state in the proposed consensus algorithms. A larger packet loss ratio can lead to a larger estimation error on leader’s state, which increases the state error between the members and the leader.

Finally, we evaluate the platooning performance under the proposed ABSD scheme. Fig. 15 shows the simulation results. We can observe that, when a leader’s beacon dissemination has high reception ratio ($P_{br} = 0.95$), members’ beacon loss has little impact on the system performance, e.g., the state errors are approximately equal in both $P_{mr} = 70\%$ and $P_{mr} = 50\%$ cases. These results can justify our protocol design that gives priority to leader’s beacon dissemination. Moreover, for the same beacon reception ratio, the magnitude of the state error with the ABSD scheme is smaller than that with the general beaconing scheme, as shown in Fig. 15, when we compare the case of $P_{br} = P_{mr} = 0.8$ and the case of $P_{br} = 0.95, P_{mr} = 0.7$. Such simulation results verify the efficiency of the proposed beaconing scheme.

VIII. CONCLUSION

In this paper, we have systematically investigated how to facilitate a reliable vehicle platoon in realistic traffic conditions with both platoons and individual vehicles. Specifically, we first proposed a consensus-based control mechanism and theoretically analyzed how the stability of the platoon can be affected by various parameters, including message loss due to imperfect inter-vehicle communication. Based on the understanding, we designed and analyzed an adaptive communication protocol that takes into account different periodical control messages generated by the platoon control mechanism, as well as event-triggered safety messages from individual cars. We conducted extensive experiments that confirm the effectiveness of the proposed control mechanism and communication protocol.

APPENDIX

Proof of Theorem 1

Before the proof of Theorem 1, we first introduce Lyapunov-Razumikhin Theorem: Let $C([-r, 0], \mathbb{R}^n)$ be a Banach space of continuous functions defined on an interval $[-r,0]$ and taking values in $\mathbb{R}^n$ with a norm $||\phi||_c = \max_{\theta \in [-r,0]} ||\phi(\theta)||$. Consider the following time-delay system:

$$
\dot{x} = f(t, x_t), t > 0,
$$

$$
x(\theta) = \phi(\theta), \theta \in [-r, 0]
$$

where $x_t(\theta) = x(t + \theta), \forall \theta \in [-r, 0]$, $f : \mathbb{R} \times C([-r, 0], \mathbb{R}^n) \to \mathbb{R}$ is a continuous function and $f(t, 0) = 0$, $\forall t \in \mathbb{R}$. Then we hold:

Lemma 2 (Lyapunov-Razumikhin Theorem [40]): Let $\phi_1$, $\phi_2$ and $\phi_3$ be continuous, nonnegative, nondecreasing functions with $\phi_1(s) > 0$, $\phi_2(s) > 0$ and $\phi_3(s) > 0$ for $s > 0$ and $\phi_1(0) = \phi_2(0) = 0$. If there is a continuous function $V(t, x)$ such that

$$
\phi_1(||x||) \leq V(t, x) \leq \phi_2(||x||), t \in \mathbb{R}, x \in \mathbb{R}^n,
$$

In addition, there exists a continuous nondecreasing function $\phi(s)$ with $\phi(s) > s$, $s > 0$ such that the derivative of $V$ along the solution $x(t)$ of Eq. (22) satisfies

$$
\dot{V}(t, x(t)) \leq -\phi_3(||x||)
$$

if $V(t + \theta, x(t + \theta)) < \phi(V(t, x(t))), \theta \in [-r, 0]$

then the solution $x = 0$ is uniformly asymptotically stable.

Proof:

Based on Lemma 1, $F_n$ is Hurwitz stable. Therefore, there exists a positive definite matrix $\Phi \in \mathbb{R}^{2N \times 2N}$ such that

$$
\Phi F_n + F_n^T \Phi = -I_{2N \times 2N}
$$

Consider Lyapunov-Razumikhin candidate function $V(\check{x}_k) = \check{x}_k^T \Phi \check{x}_k$, then

$$
\dot{V}(\check{x}) = \check{x}^T (\Phi F_n + F_n^T \Phi) \check{x}
$$

$$
- 2 \sum_{j=1}^N \sum_{i=0}^N \check{x}^T \Phi A_{j,\sigma} A_{i,\sigma} \int_{-\tau_j}^{0} \check{x}(t + s - \tau_j) ds
$$

$$
- 2 \sum_{j=1}^N \check{x}^T \Phi A_{j,\sigma} \int_{-\tau_j}^{0} \Delta(t + s) ds + 2\check{x}^T \Phi \Delta
$$
It is well known that for any $a, b \in \mathbb{R}^n$ and any positive-definite matrix $\Omega \in \mathbb{R}^{n \times n}$, $2a^T b \leq a^T \Omega^{-1} a + b^T \Omega b$. Thus
\[
\dot{V}(\chi) \leq \chi^T (\Phi F \chi + F^T \Phi) \chi + \tau_j \sum_{j=1}^N \sum_{i=0}^N \chi^T (\Phi A_{j_i}^T A_{i_j}^T \Phi^{-1} A_{i_j} A_{j_i} \Phi) \chi \\
+ \sum_{j=1}^N \sum_{i=0}^N \int_{-\tau_j}^0 \dot{\chi}^T (t + s - \tau_j) \Phi \dot{\chi} (t + s - \tau_j) ds \\
+ \tau_j \sum_{j=1}^N \chi^T \Phi A_{j_i} \Phi^{-1} A_{j_i}^T \Phi \chi \\
+ 2 \chi^T \Phi \Delta
\]
Choose $\phi_i = \zeta s$ with the constant $\zeta > 1$, in case of $V(\chi(t + s - \tau_j)) = \chi_k^T (t + s - \tau_j) \Phi \chi_k (t + s - \tau_j) \leq \zeta V(\chi), \tau_j \leq \tau/2$, we then have
\[
\dot{V}(\chi) \leq -\chi^T \left\{ I - \frac{\tau}{2} \left( \sum_{j=1}^N \sum_{i=0}^N (\Phi A_{j_i}^T A_{i_j}^T \Phi^{-1} A_{i_j} A_{j_i} \Phi) \\
+ \zeta \Phi + \sum_{j=1}^N \Phi A_{j_i} \Phi^{-1} A_{j_i}^T \Phi \right) \right\} \chi \\
+ 2 \chi^T \Phi \Delta
\]
We denote $\psi := \max \sum_{j=1}^N \sum_{i=0}^N (\| \Phi A_{j_i}^T A_{i_j}^T \Phi^{-1} A_{i_j} A_{j_i} \Phi \|_2)$
\[+ \sum_{j=1}^N (\| \Phi A_{j_i} \Phi^{-1} A_{j_i}^T \Phi \|_2).\]
Obviously, for $V(\chi) = \chi^T \Phi \chi$, we have
\[
\Delta \| \chi \|^2 \leq V(\chi) \leq \bar{\lambda} \| \chi \|^2
\]
where $\lambda$ and $\bar{\lambda}$ are the minimum and maximum of the eigenvalues of $\Psi$, which means
\[
\| \chi \| \leq \sqrt{\frac{\Delta}{\bar{\lambda}}} \sqrt{V(\chi)}
\]
On the other hand,
\[
\min \frac{\chi^T \chi}{\chi^T (\Phi) \chi} \geq \frac{1}{\bar{\lambda}} = 2 \kappa
\]
where $\kappa = \frac{1}{2 \bar{\lambda}} > 0$. Then we have
\[
\dot{V}(\chi) \leq -2 \kappa V(\chi) + \frac{\tau}{2} \psi \| \chi \|^2 + 2 \sqrt{\frac{\| \delta \|^2 \chi^2}{\kappa \Delta}} \kappa V(\chi)
\leq - (\kappa \Delta - \frac{\tau \psi}{2}) \| \chi \|^2 + \frac{\chi^2}{\kappa \Delta} \| \delta \|^2
\]
Therefore, if $\tau < \frac{2 \kappa \Delta}{\psi}$, then $\dot{V}(\chi) < -\eta \chi^T \chi + C_0$ for some $\eta > 0$, where $C_0 = \frac{\chi^2}{\kappa \Delta} \| \delta \|^2$. Based on Lemma 2, $\chi$ is uniformly ultimately bounded by $C_0$.

REFERENCES


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