Robust Beamforming Techniques for Non-Orthogonal Multiple Access Systems with Bounded Channel Uncertainties

Faezeh Alavi, Kanapathippillai Cumanan, Zhiguo Ding, Alister G. Burr

***Abstract*—In this letter, we propose a robust beamforming design for non-orthogonal multiple access (NOMA) based multiple-input single-output (MISO) downlink systems. In particular, the robust power minimization problem is studied with imperfect channel state information (CSI), where the beamformers are designed by incorporating norm-bounded channel uncertainties to provide the required quality of service at each user. This robust scheme is developed based on the worst-case performance optimization framework. In terms of beamforming vectors, the original robust design is not convex and therefore, the robust beamformers cannot be obtained directly. To circumvent this non-convex issue, the original intractable problem is reformulated into a convex problem, where the non-convex constraint is converted into a linear matrix inequality (LMI) by exploiting S-Procedure. Finally, simulation results are provided to demonstrate the effectiveness of the proposed robust design.**

***Index Terms*—Non-orthogonal multiple access (NOMA), multiple-input single-output (MISO), robust beamforming, worst-case performance optimization**

I. Introduction

Non-orthogonal multiple access (NOMA) is a promising multiple access technique for 5G networks which has the potential to address the issues associated with the exponential growth of data traffic such as spectrum scarcity and massive connectivity [1]–[5]. In contrast to conventional multiple access schemes, NOMA allows different users to efficiently share the same resources (i.e., time, frequency and code) at different power levels so that the user with lower channel gain is served with a higher power and vice versa. In this technique, a successive interference cancellation (SIC) approach is employed at receivers to separate multi-user signals, which significantly enhances the overall spectral efficiency. In other words, NOMA has the capability to control the interference by sharing resources while increasing system throughput with a reasonable additional complexity [5].

Recently, a significant amount of research has focused in studying several practical issues in NOMA scheme. In particular, beamforming designs for multiple antenna NOMA networks have received a great deal of interest in the research community due to their additional degrees of freedom and diversity gains [6]–[8]. A general framework for a multiple-input multiple-output (MIMO) NOMA system has been developed for both the downlink and the uplink in [6] whereas the throughput maximization problem was studied for a two-user MIMO NOMA system in [7]. The sum rate maximization problem for a multiple-input single-output (MISO) NOMA has been investigated in [8] through the minorization maximization algorithm. In most of the existing work, beamforming designs have been proposed for NOMA schemes with the assumption of perfect channel state information (CSI) at the transmitter [6]–[8]. However, this assumption might not be always valid for practical scenarios due to channel estimation and quantization errors [9]–[13]. On the other hand, channel uncertainties significantly influence the performance of the SIC based receivers as the decoding order of the received multi-user signals is determined with respect to the users' effective channel gains. Therefore, it is important to take into account the channel uncertainties especially in the beamforming design for NOMA networks. Motivated by this practical constraint, we focus on robust beamforming design based on the worst-case performance optimization framework to tackle the norm-bounded channel uncertainties [14]–[17]. In [14], the robust beamforming design has been developed for providing secure communication in wireless networks with imperfect CSI. By incorporating the bounded channel uncertainties, the robust sum power minimization problem is investigated in [15] for a downlink multicell network with the worst-case signal-to-interference-plus-noise-ratio (SINR) constraints whereas the robust weighted sum-rate maximization was studied for multicell downlink MISO systems in [16]. In [17], a robust minimum mean square error based beamforming technique is proposed for multi-antenna relay channels with imperfect CSI between the relay and the users. In the literature, there are two types of NOMA schemes considered: I) clustering NOMA [18]–[20], II) non-clustering NOMA [14]–[17]. In the clustering NOMA scheme, all the users in a cell are grouped into N clusters with two users in each cluster, for which a transmit beamforming vector is designed to support those two users through conventional multiuser beamforming designs. The users in each cluster are supported by a NOMA beamforming scheme. However, in the non-clustering NOMA scheme, there is no clustering and each user is supported by its own NOMA based beamforming vector. In [20], the authors studied a robust NOMA scheme for the MISO channel to maximize the worst-case achievable sum rate with a total transmit power constraint.

In this letter, we follow the second class of research where NOMA scheme applied between all users and there is the spectrum sharing between all users in cell. Then, we propose a robust beamforming design for NOMA-based MISO downlink systems. In particular, the robust power minimization problem is solved based on worst-case optimization framework to provide the required quality of service at each user regardless of the associated channel uncertainties. By exploiting

S-Procedure, the original non-convex problem is converted into a convex one by recasting the non-convex constraints into linear matrix inequality (LMI) forms. Simulation results are provided to validate the effectiveness of the robust design by comparing the performance of the robust scheme with that of the non-robust approach. The work in [20] also studied the worst-case based robust scheme for MISO NOMA system, however, there are main differences between our proposed scheme and the work in [20]. A clustering NOMA scheme is developed in [20] by grouping users in each cluster. In this scheme, a single beamformer is designed to transmit the signals for all users in the same cluster whereas, in this letter, the signal for each user is transmitted with a dedicated beamformer. In addition, both beamforming designs are completely different as the work in [20] proposes robust sum-rate maximization based design whereas this letter solves robust power minimization problem with rate constraint on each user. In terms of solutions, the work in [20] exploits the relationship between MSE and achievable rate and derives an equivalent non-convex problem, which is decoupled into four sub-problems and those problems are iteratively solved to realize the solution of the original problem. In this letter, the robust power minimization problem is formulated by deriving the worst-case achievable rate. The original problem formulation turns out to be non-convex and we exploit S-Procedure and semidefinite relaxation to convert it to a convex one. Hence, the work in [20] and the proposed work in this letter are different including problem formulation and the solution approaches.

II. System Model and Problem Formulation

We consider NOMA-based downlink transmission where a base station (BS) sends information to K users $U_1, U_2, \ldots, U_K$. It is assumed that the BS is equipped with M antennas whereas each user consists of a single antenna. The channel coefficient vector between the BS and the k^{th} user U_k is denoted by $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ $(k = 1, \ldots, K)$ and $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ represents the corresponding beamforming vector of the k^{th} user U_k. The received signal at U_k is given by

$$y_k = \mathbf{h}_k^H \mathbf{w}_k s_k + \sum_{m\neq k} \mathbf{h}_k^H \mathbf{w}_m s_m + n_k, \qquad \forall k, \qquad (1)$$

where s_k denotes the symbol intended for U_k and $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ represents a zero-mean additive white Gaussian noise with variance σ_k^2. The power of the symbol s_k is assumed to be unity, i.e., $\mathbb{E}(|s_k|^2) = 1$. In practical scenarios, it is difficult to provide perfect CSI at the transmitter due to channel estimation and quantization errors. Therefore, we consider a robust beamforming design to overcome these channel uncertainties. In particular, we incorporate norm-bounded channel uncertainties [14]–[17] in the design as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \Delta\hat{\mathbf{h}}_k, \quad \|\Delta\hat{\mathbf{h}}_k\|_2 = \|\mathbf{h}_k - \hat{\mathbf{h}}_k\|_2 \leq \epsilon, \qquad (2)$$

where $\hat{\mathbf{h}}_k$, $\Delta\hat{\mathbf{h}}_k$ and $\epsilon \geq 0$ denote the estimate of $\mathbf{h}_k$, the norm-bounded channel estimation error and the channel estimation error bound, respectively.

In the NOMA scheme, user multiplexing is performed in the power domain and the SIC approach is employed at receivers to separate signals between different users. In this scheme, users are sorted based on the norm of their channels, i.e., $\|\mathbf{h}_1\|_2 \leq \|\mathbf{h}_2\|_2 \leq \ldots \leq \|\mathbf{h}_K\|_2$. For example, the k^{th} user decodes the signals intended for the users from U_1 to U_{k-1} using the SIC approach whereas the signals intended for the rest of the users (i.e., $U_{k+1}, \ldots, U_K$) are treated as interference at the k^{th} user. Based on this SIC approach, the l^{th} user can detect and remove the k^{th} user's signals for $1 \leq k < l$ [8]. Hence, the signal at the l^{th} user after removing the first $k-1$ users' signals to detect the k^{th} user is represented as

$$y_l^k = \mathbf{h}_l^H \mathbf{w}_k s_k + \sum_{m=1}^{k-1} \Delta\hat{\mathbf{h}}_l \mathbf{w}_m s_m + \sum_{m=k+1}^{K} \mathbf{h}_l^H \mathbf{w}_m s_m + n_l, \quad \forall k,\ l \in \{k, k+1, \ldots, K\}, \qquad (3)$$

where the first term is the desired signal to detect s_k and the second term is due to imperfect CSI at the receivers during the SIC process. Due to the channel uncertainties, the signals intended for the users $U_1, \ldots, U_{k-1}$ cannot be completely removed by the l^{th} user. The third term is the interference introduced by the signals intended to the users $U_{k+1}, \ldots, U_K$. According to the SIC based NOMA scheme, the l^{th} user should be able to detect all k^{th} $(k < l)$ user signals. Thus, the achievable rate of U_k can be defined as follows:

$$R_k = \log_2\Big(1 + \min_{l\in\{k,k+1,\ldots,K\}} \mathrm{SINR}_l^k\Big), \qquad (4)$$

where SINR_l^k denotes the SINR of the k^{th} user's signal at the l^{th} user which can be written as

$$\mathrm{SINR}_l^k = \frac{\mathbf{h}_l^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{h}_l}{\sum_{m=1}^{k-1} \Delta\hat{\mathbf{h}}_l^H \mathbf{w}_m \mathbf{w}_m^H \Delta\hat{\mathbf{h}}_l + \sum_{m=k+1}^{K} \mathbf{h}_l^H \mathbf{w}_m \mathbf{w}_m^H \mathbf{h}_l + \sigma_l^2}. \qquad (5)$$

For this network setup, we study robust power minimization by incorporating channel uncertainties to satisfy the required SINR at each user. This robust beamforming design is developed by considering the worst-case SINR of each user, which can be formulated as

$$\min_{\mathbf{w}_k\in\mathbb{C}^{M\times 1}} \sum_{k=1}^{K} \|\mathbf{w}_k\|_2^2, \qquad (6a)$$

$$\mathbf{s.t.}\ \min_{\|\Delta\hat{\mathbf{h}}_l\|_2\leq\epsilon} \Big(\min_{l\in\{k,k+1,\ldots,K\}} \mathrm{SINR}_l^k\Big) \geq \gamma_k^{min},\ \forall k, \qquad (6b)$$

where $\gamma_k^{min} = (2^{R_k^{min}} - 1)$ is the minimum required SINR to achieve a target rate R_k^{min} at U_k.

III. Robust Beamforming Design

The problem formulation in (6) is not convex and the optimal robust beamformers cannot be obtained directly. To tackle this issue, we introduce a new matrix variable $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ and reformulate the original robust problem in (6) into the following optimization framework without loss of generality:

$$\min_{\mathbf{W}_k\in\mathbb{C}^{M\times M}} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{W}_k), \qquad (7a)$$

$$\mathbf{s.t.}\ \varpi_{kl}, \quad \forall k,\ l = k, \ldots, K, \qquad (7b)$$

$$\mathbf{W}_k \succeq \mathbf{0},\ \mathrm{rank}(\mathbf{W}_k) = 1, \qquad \forall k, \qquad (7c)$$

where ϖ_{kl} is defined in Appendix A.

However, the reformulated problem in (7) is still not convex for two reasons; the rank-one constraint and unknown channel uncertainties, i.e., $\Delta\hat{\mathbf{h}}_k$, which lead to an intractable

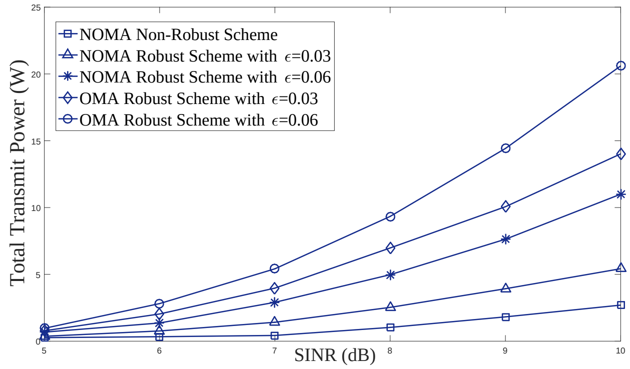

Fig. 1. Total transmit power versus different SINR thresholds for the robust and non-robust schemes with different channel estimation error bounds, ϵ.

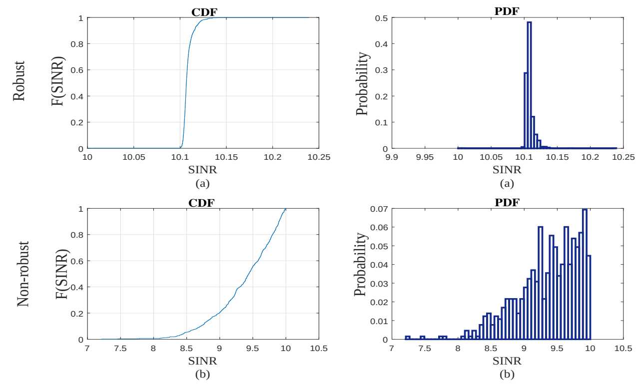

Fig. 2. Comparison CDF and PDF of minimum achieved SINR for (a) the robust scheme and (b) the non-robust scheme with $\epsilon = 0.06$, $\gamma_k^{min} = 10$ dB.

problem. The rank-one constraint in (7c) can be relaxed by exploiting semi-definite relaxation (SDR). To remove the unknown channel uncertainties and solve the original problem with available knowledge of imperfect CSI (error bound), we employ S-procedure to recast the non-convex constraints into LMIs.

Lemma 1: By relaxing the rank-one constraints on $\mathbf{W}_k$, the original problem in (7) can be recast into the following convex problem:

$$\min_{\substack{\mathbf{W}_k \in \mathbb{C}^{M\times M}, \\ \lambda_{kl} \geq 0}} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{W}_k), \tag{8a}$$

$$\text{s.t. } \mathbf{W}_k \succeq \mathbf{0},\ \mathbf{C}_{kl} \succeq \mathbf{0}, \quad \forall k,\ l = k, \ldots, K \tag{8b}$$

where $\mathbf{C}_{kl}$ is defined in Appendix B.

Proof: Please refer to Appendix B. ■

The problem in (8) is a standard semidefinite programming (SDP) and can be efficiently solved using interior-point methods. The optimal solution for the original problem in (6) can be obtained through extracting the eigenvector corresponding to the maximum eigenvalue of the rank-one solution of (8). Thus, the following lemma holds to show that the optimal solution to (8) is rank one.

Lemma 2: Provided the problem in (8) is feasible, there always exists a rank-one optimal solution $\{\mathbf{W}_k^*\}$.

Proof: Please refer to Appendix C. ■

IV. Simulation Results

To assess the performance of the proposed robust beamforming approach, we consider a single cell downlink transmission, where a multi-antenna BS serves single-antenna users which are uniformly distributed over the circle with a radius of 1000 meters around the BS, but no closer than $d_0 = 100$ meters. The small-scale fading of the channels is Rayleigh which represents an isotropic scattering environment. We model the large-scale fading effects as the product of path loss and shadowing fading. The log-normal shadowing is considered with standard deviation $\sigma_0 = 8$ dB, scaled by $(\frac{d_k}{d_0})^{-\beta}$ to incorporate the path-loss effects where d_k is the distance between U_k and the BS, measured in meters and $\beta = 3.8$ is the path-loss exponent. Throughout the simulations, it is assumed that the BS is equipped with eight antennas ($M = 8$) and it serves three users ($K = 3$). The noise variance at each user is assumed to be 0.01 (i.e., $\sigma_k^2 = 0.01$) and the target rates for all users are the same. The term "Non-robust scheme" refers to the scheme where the BS has imperfect CSI without any information on the channel uncertainties and the beamforming vectors are designed based on imperfect CSI without incorporating channel uncertainty information.

First, we study the impact of channel uncertainties on the required total transmit power. Fig. 1 depicts the required total transmit power against different SINR thresholds for the robust and the non-robust NOMA schemes as well as OMA scheme with different error bounds. As seen in Fig. 1, the robust scheme requires more transmit power than that of the non-robust scheme. This is because the robust scheme satisfies the required SINR all the time, at the price of more transmit power at the BS whereas the non-robust scheme does not. The difference between the required transmit power for the robust and the non-robust schemes increases with error bounds. This is because incorporating all possible sets of errors in the beamforming design to satisfy high SINR thresholds requires more transmit power in the robust scheme. Moreover, as seen in Fig. 1, the conventional framework, orthogonal multiple access (OMA), requires more transmit power to achieve the same rate in comparison with NOMA scheme. This demonstrates that the NOMA scheme yields a better performance in terms of spectral and energy efficiencies.

Next, we evaluate the performance of the proposed robust and non-robust schemes in terms of the minimum achieved SINR between users. Fig. 2 provides cumulative distribution function (CDF) and probability density function (PDF) obtained from 1000 random sets of channels with error bounds of 0.06 ($\epsilon = 0.06$) where the SINR threshold has been set to 10 dB at each user. As evidenced by the results, the robust scheme outperforms the non-robust scheme in terms of minimum achieved SINRs. In addition, the robust scheme satisfies the SINR thresholds all the time regardless of the channel uncertainties whereas the non-robust design fails to satisfy the minimum SINR requirements.

V. Conclusion

In this letter, we propose a robust beamforming design for the downlink of a NOMA based MISO network by taking into account the norm-bounded channel uncertainties. However, the original robust problem formulation is not convex due to the imperfect CSI. To cope with this challenge, we exploited S-procedure to reformulate the original non-convex problem into a convex optimization framework by recasting the original non-convex constraints into an LMI form. Simulation results demonstrate that the proposed robust scheme offers a better performance than the non-robust approach by satisfying the SINR requirement at each user all the time regardless of associated channel uncertainties.

APPENDIX A
DERIVATION OF ϖ_{kl}

The equivalent transformations of (6b) can be obtained as ϖ_{kl} as follows:

$$\begin{cases} \min\limits_{\|\Delta\hat{\mathbf{h}}_k\|_2\le\epsilon}\Big(\dfrac{\mathbf{h}_k^H\mathbf{W}_k\mathbf{h}_k}{\sum\limits_{m=1}^{k-1}\Delta\hat{\mathbf{h}}_k^H\mathbf{W}_m\Delta\hat{\mathbf{h}}_k+\sum\limits_{m=k+1}^{K}h_k^H\mathbf{W}_m\mathbf{h}_k+\sigma_k^2}\Big)\le\gamma_k^{min}, \\ \min\limits_{\|\Delta\hat{\mathbf{h}}_{k+1}\|_2\le\epsilon}\Big(\dfrac{\mathbf{h}_{k+1}^H\mathbf{W}_k\mathbf{h}_{k+1}}{\sum\limits_{m=1}^{k-1}\Delta\hat{\mathbf{h}}_{k+1}\mathbf{W}_m\Delta\hat{\mathbf{h}}_{k+1}+\sum\limits_{m=k+1}^{K}\mathbf{h}_{k+1}^H\mathbf{W}_m\mathbf{h}_{k+1}+\sigma_{k+1}^2}\Big) \\ \qquad\le\gamma_k^{min}, \\ \vdots \\ \min\limits_{\|\Delta\hat{\mathbf{h}}_K\|_2\le\epsilon}\Big(\dfrac{\mathbf{h}_K^H\mathbf{W}_k\mathbf{h}_K}{\sum\limits_{m=1}^{k-1}\Delta\hat{\mathbf{h}}_K\mathbf{W}_m\Delta\hat{\mathbf{h}}_K+\sum\limits_{m=k+1}^{K}\mathbf{h}_K^H\mathbf{W}_m\mathbf{h}_K+\sigma_K^2}\Big)\le\gamma_k^{min}, \end{cases}$$

$$\Leftrightarrow \min_{\|\Delta\hat{\mathbf{h}}_l\|_2\le\epsilon}\Big(\frac{\mathbf{h}_l^H\mathbf{W}_k\mathbf{h}_l}{\sum\limits_{m=1}^{k-1}\Delta\hat{\mathbf{h}}_l^H\mathbf{W}_m\Delta\hat{\mathbf{h}}_l+\sum\limits_{m=k+1}^{K}\mathbf{h}_l^H\mathbf{W}_m\mathbf{h}_l+\sigma_l^2}\Big)\le\gamma_k^{min}$$
$$\triangleq \varpi_{kl},\ \ \forall k,\ l=k,\dots,K. \quad \text{(A.1)}$$

APPENDIX B
PROOF OF LEMMA 1

To incorporate the channel uncertainties in the robust optimization framework, we exploit S-procedure to convert the non-convex constraint into LMI form. By applying S-procedure [21], the constraint (7b) is derived as

$$\Delta\hat{\mathbf{h}}_l^H\mathbf{I}\Delta\hat{\mathbf{h}}_l-\epsilon^2\le 0\Rightarrow\Delta\hat{\mathbf{h}}_l^H(\sum_{m\ne k}\mathbf{W}_m-\mathbf{W}_k/\gamma_k^{min})\Delta\hat{\mathbf{h}}_l$$
$$+2\mathrm{Re}\{\hat{\mathbf{h}}_l^H(\sum_{m=k+1}^{K}\mathbf{W}_m-\mathbf{W}_k/\gamma_k^{min})\Delta\hat{\mathbf{h}}_l\}+\hat{\mathbf{h}}_l^H(\sum_{m=k+1}^{K}\mathbf{W}_m$$
$$-\mathbf{W}_k/\gamma_k^{min})\hat{\mathbf{h}}_l+\sigma_l^2\le 0, \quad \text{(B.1)}$$

Then, the constraint (7b) can be reformulated with $\lambda_{kl}\ge 0$ as the following semidefinite constraint

$$\mathbf{C}_{kl}=\begin{bmatrix}\lambda_{kl}\mathbf{I}+\phi_k+\nu_k & \phi_k\hat{\mathbf{h}}_l\\ \hat{\mathbf{h}}_l^H\phi_k & \hat{\mathbf{h}}_l^H\phi_k\hat{\mathbf{h}}_l-\sigma_k^2-\lambda_{kl}\epsilon^2\end{bmatrix}\succeq\mathbf{0}, \quad \text{(B.2)}$$

where $\phi_k=\frac{\mathbf{W}_k}{\gamma_k^{min}}-\sum_{m=k+1}^{K}\mathbf{W}_m$ and $\nu_k=-\sum_{m=1}^{k-1}\mathbf{W}_m$.

This completes the proof of Lemma 1. ■

APPENDIX C
PROOF OF LEMMA 2

To prove Lemma 2, we examine the Karush-Kuhn-Tucker (KKT) conditions of (8). First, let $\mathbf{Y}_k\in\mathbb{C}^{M\times M}$, $\mathbf{T}_{kl}\in\mathbb{C}^{(M+1)\times(M+1)}$ and $\mu_{kl}\in\mathbb{R}_+$ denote the dual variable of the constraints in (8b), respectively. Then, the Lagrangian dual function of (8) can be written as

$$\mathcal{L}(\mathbf{W}_k,\lambda_{kl},\mathbf{T}_{kl},\mu_{kl},\mathbf{Y}_k)=\sum_k\mathrm{Tr}(\mathbf{W}_k)-\sum_k\mathrm{Tr}(\mathbf{Y}_k\mathbf{W}_k)-$$
$$\sum_{k,l}\mathrm{Tr}(\mathbf{T}_{kl}\mathbf{A_1})-\sum_{k,l}\mathrm{Tr}[\mathbf{T}_{kl}\mathbf{H}_l^H\phi_k\mathbf{H}_l]-\sum_{k,l}\mathrm{Tr}(\mathbf{T}_{kl}\mathbf{A_2}), \quad \text{(C.1)}$$

where $\mathbf{H}_l=[\mathbf{I}\quad\mathbf{h}_l]$ and

$$\mathbf{A_1}=\begin{pmatrix}\lambda_{kl}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\sigma_k^2-\lambda_{kl}\epsilon^2\end{pmatrix},\quad \mathbf{A_2}=\begin{pmatrix}\nu_k & \mathbf{0}\\ \mathbf{0} & 0\end{pmatrix}.$$

The following KKT conditions hold for (8)

$$\frac{\partial\mathcal{L}}{\partial\mathbf{W}_k}=\mathbf{0}\Rightarrow\mathbf{Y}_k+\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H/\gamma_k^{min}=\mathbf{I}$$
$$+\sum_{j=1}^{k-1}\mathbf{H}_l\mathbf{T}_{jl}\mathbf{H}_l^H+\sum_{j=k+1}^{K}\mathbf{T}_{jl}, \quad \text{(C.2)}$$
$$\mathbf{W}_k\mathbf{Y}_k=\mathbf{0}, \quad \text{(C.3)}$$
$$(\mathbf{A_1}+\mathbf{H}_l^H\phi_k\mathbf{H}_l+\mathbf{A_2})\mathbf{T}_{kl}=\mathbf{0}. \quad \text{(C.4)}$$

We premultiply (C.2) by $\mathbf{W}_k$, i.e.,

$$\mathbf{W}_k\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H/\gamma_k^{min}=\mathbf{W}_k\big(\mathbf{I}+\sum_{j=1}^{k-1}\mathbf{H}_l\mathbf{T}_{jl}\mathbf{H}_l^H+\sum_{j=k+1}^{K}\mathbf{T}_{jl}\big), \quad \text{(C.5)}$$

Then, we can write the following rank relation

$$\mathrm{rank}(\mathbf{W}_k)=\mathrm{rank}\Big[\mathbf{W}_k\big(\mathbf{I}+\sum_{j=1}^{k-1}\mathbf{H}_l\mathbf{T}_{jl}\mathbf{H}_l^H+\sum_{j=k+1}^{K}\mathbf{T}_{jl}\big)\Big]$$
$$=\mathrm{rank}(\mathbf{W}_k\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H)$$
$$\le\min\{\mathrm{rank}(\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H),\mathrm{rank}(\mathbf{W}_k)\}. \quad \text{(C.6)}$$

Based on (C.6), it is required to show $\mathrm{rank}(\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H)\le 1$ if we claim $\mathrm{rank}(\mathbf{W}_k)\le 1$.

First, we consider the following equations and Lemma 3:

$$[\mathbf{I}\quad\mathbf{0}]\mathbf{H}_l^H=\mathbf{I},\quad[\mathbf{I}\quad\mathbf{0}]\mathbf{A_1}=\lambda_{kl}(\mathbf{H}_l^H-[\mathbf{0}_M\quad\mathbf{h}_l]),$$
$$[\mathbf{I}\quad\mathbf{0}]\mathbf{A_2}=\nu_k(\mathbf{H}_l^H-[\mathbf{0}_M\quad\mathbf{h}_l]), \quad \text{(C.7)}$$

Lemma 3: If a block Hermitian matrix $\mathbf{B}=\begin{bmatrix}\mathbf{B_1} & \mathbf{B_2}\\ \mathbf{B_3} & \mathbf{B_4}\end{bmatrix}\succeq 0$ then the main diagonal matrices $\mathbf{B_1}$ and $\mathbf{B_4}$ must be positive definite (PSD) matrices [21].

We pre-multiply $[\mathbf{I}\quad\mathbf{0}]$ and post-multiply $\mathbf{H}_l^H$ by (C.4), respectively, and applying the equalities in (C.7):

$$\lambda_{kl}(\mathbf{H}_l^H-[\mathbf{0}_M\ \mathbf{h}_l])\mathbf{T}_{kl}\mathbf{H}_l^H+\nu_k(\mathbf{H}_l^H-[\mathbf{0}_M\ \mathbf{h}_l])\mathbf{T}_{kl}\mathbf{H}_l^H$$
$$+\phi_k\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H=\mathbf{0}\Rightarrow$$
$$(\lambda_{kl}\mathbf{I}+\phi_k+\nu_k)\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H=(\lambda_{kl}\mathbf{I}+\nu_k)[\mathbf{0}_M\ \mathbf{h}_l]\mathbf{T}_{kl}\mathbf{H}_l^H \quad \text{(C.8)}$$

By applying Lemma 3 to (B.2), we can claim $(\lambda_{kl}\mathbf{I}+\phi_k+\nu_k)\succeq\mathbf{0}$ and is nonsingular; thus, multiplying by a nonsingular matrix will not change the matrix rank. Thus, the following rank relation holds:

$$\mathrm{rank}(\mathbf{H}_l\mathbf{T}_{kl}\mathbf{H}_l^H)=\mathrm{rank}((\lambda_{kl}\mathbf{I}+\nu_k)[\mathbf{0}\quad\mathbf{h}_l]\mathbf{T}_{kl}\mathbf{H}_l^H)$$
$$\le\mathrm{rank}([\mathbf{0}\quad\mathbf{h}_l])\le 1. \quad \text{(C.9)}$$

This completes the proof of Lemma 2. ■

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