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Proceedings Paper:

https://doi.org/10.1109/ICPR.2016.7899915
Absorptive Scattering Model for Rough Laminar Surfaces

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Abstract—This paper introduces a new light scattering model for surfaces with rough boundaries and absorption. This is an extension to Ragheb-Hancock model. The new model adds an absorption term proportional of the squared cosine of the light incidence angle, and satisfies conservation of energy. To test the accuracy of the model, we have used the CURET database. The model was compared with alternatives such as the Jensen model, the Oren-Nayar model, and the original Ragheb-Hancock model. The results show that the new model produces the best fits to the data. Interestingly the model is capable of predicting absorption in dominant colored samples, a feature not possible with the original models studied. The absorption parameter of the new model provides is also informative of surface structure and composition, especially for layered dielectric materials.

I. INTRODUCTION

Light scattering models are of strong current interest in surface analysis for computer vision and graphics. The difficulty in predicting how light scatters from surfaces has stimulated the investigation of different modelling approaches for specific types of surface. For light scattering from rough surfaces, there are multiple parameters that contribute to the scattering behavior, and which must be considered. An effective model must consider all the relevant physical surface parameters. Unfortunately some of these can prove difficult to control or impractical to measure. These parameters are often ignored in surface modelling, and this, in turn, limits the effectiveness of the underlying model.

Light absorption measurements are especially useful for modeling and analyzing the chromatic properties of materials. They also have significant application in the biomedical imaging domain. In this paper we therefore aim to improve on existing light scattering models, and develop a modified version of the Ragheb-Hancock light scattering model for layered rough surface [1], by adding a wavelength dependant absorption term.

II. OVERVIEW

A. Prior Work

The Bidirectional Reflectance Distribution Function (BDRF) [2] [3] is a general tool for characterizing light reflectance distributions from different surfaces. The function describes the angular distribution of reflected radiance in terms of the corresponding distribution of incident radiance. Most existing models are developments or refinements of the classical Phong model, Torrance-Sparrow model, or the Oren-Nayar model, including terms for specular and diffuse reflectance [3].

Torrance and Sparrow first introduced a model for specular reflection from rough surfaces [4] [3]. Here roughness is modelled using microscopic concavities which have a V-form and are of equal length, referred to as microfacets. The microfacets have random orientations whose distribution is controlled by a number of model parameters. The model allows surfaces of varying degrees of roughness to be simulated. The Torrance-Sparrow model is considered as precursor to more recent scattering models. For instance, Oren and Nayar [5] developed a diffuse reflectance model, based on [4]. It is an improved version of the classic Lambertian interpretation of light scattering from diffuse materials, where each microfacet follow Lamberts law and which can be derived using geometrical optics.

In nature, many dielectric surfaces have a laminar structure, and are composed of translucent and opaque layers, each exhibiting their own roughness. Other models that aim to account for the scattering effects in layered surfaces are: a) the Stam model [6] which critically analyzes the problem of scattering in rough layered surfaces; b) the Matusik et al [7] model which makes empirical BRDF estimates for both metals and dielectrics; and c) the Ragheb and Hancock [1] model which details light scattering for layered rough dielectric surfaces. However, none of these models have taken light absorption into account.

The parameter of the absorption model is important for accurately reproducing the chromatic properties of materials and also for analyzing material absorption characteristics. It is also important for modelling and analyzing biological materials such as human skin, which not only improves the synthesis of realistic surface appearance but can also be used for the analysis of such surfaces [8]. Donner et al [9] introduce a layered, heterogeneous spectral reflectance model for human skin which accounts for absorption by introducing infinitesimally thin absorbing layers between the scattering layers. Jensen et al. [10] use the absorption coefficient in their subsurface scattering synthesize model. Both of these models uses an absorption term that is designed according to the domain specific aims of the study in hand. However, the chosen parameters can be intractable to measure directly or to estimate. Based on this observation in this paper we propose a new unit-less absorption model whose parameters are more
easily estimated and which is hence to easier to control.

B. Contribution

The new model presented in this paper is a modification of the Ragheb and Hancock light scattering model for layered dielectrics with rough surface boundaries [1]. Using the wave scattering theory, the model assumes that the diffuse radiance is scattered from bi-layered rough surfaces, consisting of an opaque sub-surfaceayer below a transparent one. The model is detailed and produces remarkably good agreement with the experimental data studied. However, unlike our improved model, their model does not account for absorption. Hence, the new model introduced here is an extension of the Ragheb and Hancock model with the inclusion of an absorption term which is derived using the conservation energy for light transmission, reflectance, and absorption. This simplifies the analysis of reflectance without overcomplicating the model. Moreover, the absorption parameter is unit-less, and provides an alternative representation of light absorption in a dielectric.

III. METHODOLOGY

A. Ragheb and Hancock’s Light Scattering Model for Rough Layered Dielectric

The surface scattering geometry of the Ragheb’s model [1] was based on Kirchoff theory, as shown in Fig. 1 (a). The vector S points in the direction of the light source, which means that incident light with radiance $L_i$ propagates in the $-S$ direction. The scattered radiance $L_o$ is in the direction $V$, which is the position of the viewer. The light beam is incident on the surface with zenith angle $\theta_i$ and azimuth angle $\phi_i$. Additionally, Beckmann’s geometry applies so $\phi_i = \pi$ [1]. The light beam is then scattered at zenith angle $\theta_s$ and azimuth angle $\phi_s$.

In the layered surface geometry under study, in Fig. 1 (b) i) light first enters the surface at angle $\theta_i$, ii) is then refracted to angle $\theta_s$; iii) then undergoes single scattering on the lower surface layer (lower boundary), at angle $\theta_{s'}$, and iv) finally exits the surface layer (upper boundary) with zenith and azimuth angles $\theta_s$ and $\phi_s$. Both of the outgoing radiance components (surface and subsurface) are identical. The total outgoing radiance is the linear combination of both components with $\beta$ as its relative balance control. The notation used is summarized in Table I.

In [1], two different surface roughness model variants are studied, referred to as i) the Gaussian and ii) the Exponential, which refer to the nature of the correlation function for the surface and subsurface roughness. The scattered surface radiance $L^{sf}_{G}$ ($\theta_s$, $\phi_s$, $\sigma/T$) (which we refer to as $L^{sf}$) when the surface correlation function is Gaussian and is given by:

\[
L^{sf}_{G} = K_G \left[ \frac{\cos(\theta_s)}{v_r^2(\theta_s)} \right] \times \exp \left[ \frac{-T^2 v_{2y}^2(\theta_i, \theta_s, \phi_s)}{4 \sigma^2 v_r^2(\theta_i, \theta_s)} \right]
\]

and when surface correlation function is Exponential:

\[
L^{sf}_{E} = K_E \left[ \frac{\cos(\theta_s)}{v_r^2(\theta_s)} \right] \times \left[ 1 + \left( \frac{T^2 v_{2y}^2(\theta_i, \theta_s, \phi_s)}{\sigma^2 v_r^2(\theta_i, \theta_s)} \right) \right]^{-\frac{1}{2}}
\]

where $v_{2y}^2(\theta_i, \theta_s, \phi_s) = [k(\sin(\theta_i) - \sin(\theta_s) \cos(\phi_s))]^2 + [k(\sin(\theta_s) \sin(\phi_s))]^2$, $v_r(\theta_i, \theta_s) = -k(\cos(\theta_i) - \cos(\theta_s))$; and $k = 2\pi/\lambda$. The coefficients $K_G$ and $K_E$ are both proportional to $(\sigma/T)^2$ and can be normalized. Meanwhile, the subsurface scattered radiance $L^{sb}_{G}$ ($\theta_s$, $\phi_s$, $\sigma'/T'$, $n'$) (which we refer to as $L^{sb}$) when the correlation function is Gaussian is given by:

\[
L^{sb}_{G} = L^{sf}_{G} (\theta_i', \theta_s', \phi_s, \sigma'/T') \times [1 - f(\theta_i, n)] [1 - f(\theta_s', 1/n)] d\omega'
\]

and when the subsurface correlation function is Exponential:

\[
L^{sb}_{E} = L^{sf}_{E} (\theta_i', \theta_s', \phi_s, \sigma'/T') \times [1 - f(\theta_i, n)] [1 - f(\theta_s', 1/n)] d\omega'
\]

where, the solid angle is:

\[
d\omega' = \frac{\cos(\theta_i')}{n'^2 \cos(\theta'_i)} d\omega
\]

and when surface correlation function is Exponential:

\[
L^{sf}_{E} = K_E \left[ \frac{\cos(\theta_i)}{v_r^2(\theta_i)} \right] \times \left[ 1 + \left( \frac{T^2 v_{2y}^2(\theta_i, \theta_s, \phi_s)}{\sigma^2 v_r^2(\theta_i, \theta_s)} \right) \right]^{-\frac{1}{2}}
\]

where, the Fresnel coefficient, which models the refraction effects of the layers is given

\[
f(\alpha_i, r) = \left[ \frac{\sin(\alpha_i - \alpha_t)}{2 \sin^2(\alpha_i + \alpha_t)} \right] \times \left[ 1 + \frac{\cos(\alpha_i + \alpha_t)}{\cos^2(\alpha_i - \alpha_t)} \right]
\]

\[
r = \frac{\sin(\alpha_i)}{\sin(\alpha_s)} \text{ and } \alpha_t = \sin^{-1} \left[ \frac{\sin(\alpha_i)}{r} \right]
\]

In equation (7), where light is transmitted from air to dielectric, then $r = n$ and $\alpha_i = \theta_i$. If, on the other hand, light is transmitted from dielectric to air, then $r = 1/n$ and $\alpha_i = \sin^{-1}[\sin(\theta_i)/n]$. The overall outgoing scattered radiance $L_o$ is then given by:

\[
L_o = \beta L^{sb} + (1 - \beta) L^{sf}
\]

B. Absorption In the Subsurface Layer

To convey the degree of light absorption in a material, different measurements can be used. One example is the complex refractive index, which was used in Mie Theory to describe the absorption of electromagnetic radiation by spherical particles [11]. However, deriving a light scattering or reflectance model using the parameter can be difficult, resulting from problems either in measuring its value or solving for its imaginary component.

Instead of using a predefined function as the absorption term (e.g. complex refractive index), our new model derives the absorption term from first principles using the principle of conservation of energy during light transfer. In Ragheb and Hancock’s model, the reflectance is governed by the Fresnel co-efficient and the conservation energy was assumed to be
satisfied provided the normalisation $1 = R + T$ held. However, for the new model, the conservation energy is expressed via the different normalisation:

$$1 = R + T + A$$ (9)

In this equation, the amount of absorbed light $A$ is assumed to be proportional to the cosine squared of the incident angle $\theta_i$. As a result, the absorption is greatest (100%) when the incident light is normal to the surface and smallest (0%) when the incident light is perpendicular to the surface normal. The absorption term is defined as:

$$A_1 = A(a, \theta_i) = a(\cos^2(\theta_i))[1 - f(\theta_i, n)]$$ (10)

$$A_2 = A(a, \theta_{s2}) = a(\cos^2(\theta_{s2}))[1 - f(\theta', 1/n)]$$ (11)

where $a$ is the fractional absorption parameter, used to control how strongly light is absorbed. Equation 10 is used when incident light is transmitted from air to the material. On the other hand, Equation 11 is used when the incident light is transmitted from the material to air. Substituting (10) and (11) into (3) and (4), the subsurface scattering component now becomes:

$$L_E^{sb}(\theta_i, \phi_s, \sigma'/T', n) = L_E^{sf}(\theta_i', \phi_s', \sigma'/T')\times[1 - f(\theta_i, n) - A(a, \theta_i)][1 - f(\theta_i', 1/n) - A(a, \theta_{s2})]d\omega'$$ (12)

$$L_E^{sb}(\theta_i, \phi_s, \sigma'/T', n) = L_E^{sf}(\theta_i', \phi_s', \sigma'/T')\times[1 - f(\theta_i, n) - A(a, \theta_i)][1 - f(\theta_i', 1/n) - A(a, \theta_{s2})]d\omega'$$ (13)

where

$$\theta_{s2} = \sin^{-1}\left[\frac{\sin(\theta_i')}{1/n}\right]$$ (14)

By conservation of energy, a change in the absorption will cause a change in the transmission. Fig. 2 shows how the different values of $a$ affect the behavior of both the transmission and the absorption as the incident angle varies (for a medium with $n = 1.7$).

IV. EXPERIMENTAL SETUP AND RESULTS

To test our model, we use the CURET database [12]. Here we excluded the BRDF measurements that occur in the specular direction, and which total 198 non-specular measurements. In total 13 different material samples were selected for the experiment. The test was performed on the different normalisation: $\sigma/T$ and $\sigma'/T'$, coefficients for the surface equations of Gaussian and Exponential respectively.

The datasets were converted into normalized outgoing radiance $L_o(\theta_i, \phi_s, \sigma_s, \phi_s) = v(\theta_i, \phi_s, \phi_s) \cos(\theta_i)d\omega$. We experimented with fitting four different models to the CURET data, namely a) the proposed model with an Exponential correlation function, b) the proposed model with Gaussian correlation function, c) the Oren-Nayar model. The different models are used to explore how the absorption parameter affects the overall quality of fit.
A. Model Fitting

The normalized predicted radiance of the models is fitted to the normalized measured radiance data from the CURet database. This is done by varying the model parameters to find their smallest value of the root-mean-square error $\Delta_{RMS}$. The RMS fitting error is given by:

$$\Delta_{RMS} = 100 \times \frac{1}{K} \left( \sum_{k=1}^{K} \left[ L^I_O \left( \phi^k_i, \phi^k_s, \sigma^k_i, \beta^k_i \right) - L^P_O \left( \phi^k_i, \phi^k_s, \sigma^k_i, \beta^k_i \right) \right] \right)^{\frac{1}{2}}$$

where $L^I_O$ is the normalized BRDF from the CURet database, $L^P_O$ is the normalized radiance from the model prediction and $k$ runs over the index number of the BRDF measurements used ($K$).

There are four parameters in the proposed model (modified Exponential and modified Gaussian) that are varied in the exhaustive search for a best-fit. The values of $\sigma/T$ and $\sigma'/T'$ are made equal in this search. The ranges for the parameters used in the experiment are: $\sigma/T = \sigma'/T'$ which ranges from [0.12, 4.1] with 50 equal intervals, $\beta$ which ranges from [0.01, 1] with 100 equal intervals, the index of refraction, with a range of [1.3, 1.5] with 10 equal intervals, and $\phi$ with a range of [0, 1] with 101 equal intervals. The range for $\phi_a$ and $\phi_s$ for the Jensen model [10] are varied between [0.01, 1] with 100 intervals. Meanwhile, for the Oren-Nayar model [12], the parameter values were chosen based on the tabulated data given, but using only the diffuse component. The results are shown as plots of normalized measured data versus the normalized radiance predicted by the different models. The fitting and parameter estimation results are shown in Table II-IV.

B. Chi-Square per Degree of Freedom Test

To measure the discrepancy existing between the best-fit error (observation data) and its expected error, the chi-square per degree of freedom test was used to check whether the Ragheb-Hancock model and the absorption model differ significantly. The Chi-squared statistic is given by:

$$\chi^2 = \sum_{n=1}^{198} \frac{(Model_n - Data_n)^2}{Data_n}$$

After obtaining the chi-squared statistic, it is then divided by the number of degrees of freedom to give $\chi^2/v$ the chi-squared per degree of freedom, where $v = d - p$. Here, $d = 198$ which is the number of data samples and $p$ is the number of model parameters. For the Ragheb-Hancock model, $p = 3$ while the proposed model has $p = 4$. A comparison of the Ragheb-Hancock model and the proposed absorption model is given in Table V.

V. DISCUSSION

From the best-fit models and their associated parameters, there are several conclusions that can be drawn

1) When the absorption fraction $a$ in the modified absorption model is zero, the model is equivalent the Ragheb-Hancock model.

2) The modified absorption model gave the best-fit overall. The Jensen model overestimated the radiance data while the Oren-Nayar model underestimated it.

3) A total of 7 samples gave the best fit when the absorption was zero. However, a total of 6 chromatic samples gave better results using the proposed absorption model; these samples were Rug-B (red), velvet (red), Quarry tile (pale red), Brown bread, Orange peel and Moss (green). This shows that the proposed model accounts well for chromatics effects in colored samples.

4) For samples that are dominated by one colour, e.g. Rug-B - red, the parameters $\sigma/T$ and $\beta$ are larger and the parameter $a$ is smaller in the dominant color channel.

5) The velvet sample gives the poorest fit of all the samples for both the modified Exponent and Gaussian models. This is probably due to measurement noise. Nevertheless, the proposed model still gave the best fit compared to the alternative models.

6) The absorption model variant with an exponential correlation function gives the best overall fit for all 13 samples on all color channels, followed by the absorption model variant with a Gaussian correlation function.

7) There is no significant difference between the chi-square test for the Ragheb-Hancock model and the proposed absorption model.

In comparison to the Ragheb-Hancock model, the new absorption model provides improvements in the quality of fit while allowing us to estimate the absorption fraction $a$, thus providing information concerning the absorption characteristics of the incident light.
In this paper, we have introduced a new light scattering model for layered rough surfaces with absorption. For the CURET database, we demonstrate that the model offers improvements over a number of alternative light scattering models including the Ragheb-Hancock model, which is an absorption-free version of the new method. The new method extends the Ragheb-Hancock model not only for the purposes of analyzing subsurface roughness, but also for analyzing the absorption characteristics of surfaces. This is a significant advantage when studying biological materials such as the skin and plant leaf. In the future, further experiments will be conducted on both highly chromatic and dielectrics with rough surface boundaries.

VI. CONCLUSION

The model parameters estimated for the modified Gaussian model, corresponding to the RMS fit errors for the 13 CURET samples are:

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<th>Sample (No.)</th>
<th>σ/T</th>
<th>β</th>
<th>n</th>
<th>a</th>
<th>σ/T</th>
<th>β</th>
<th>n</th>
<th>a</th>
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REFERENCES


Received March 28, 2016.
### TABLE V

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<td>0.0189</td>
<td>0.0120</td>
<td>0.0512</td>
<td>0.0438</td>
<td>0.0220</td>
<td>0.0208</td>
</tr>
<tr>
<td>Corn Husk (51)</td>
<td>0.0298</td>
<td>0.0294</td>
<td>0.0240</td>
<td>0.0572</td>
<td>0.0288</td>
<td>0.0219</td>
<td>0.0297</td>
</tr>
<tr>
<td>White Bread (52)</td>
<td>0.0438</td>
<td>0.0381</td>
<td>0.0731</td>
<td>0.0746</td>
<td>0.0623</td>
<td>0.1845</td>
<td>0.0436</td>
</tr>
<tr>
<td>Soleirolia Plant (53)</td>
<td>0.0231</td>
<td>0.0183</td>
<td>0.0182</td>
<td>0.0588</td>
<td>0.0247</td>
<td>0.0285</td>
<td>0.0230</td>
</tr>
<tr>
<td>Orange Peel (55)</td>
<td>0.0912</td>
<td>0.0218</td>
<td>0.0255</td>
<td>0.0962</td>
<td>0.0296</td>
<td>0.0424</td>
<td>0.0907</td>
</tr>
<tr>
<td>Moss (61)</td>
<td>0.0122</td>
<td>0.0112</td>
<td>0.0356</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0287</td>
<td>0.0231</td>
</tr>
</tbody>
</table>

**Fig. 3.** CUReT samples: normalized data against the normalized radiance predicted by the modified Exponential model. Samples from top to bottom are: Sponge (21), Brown Bread (48), Quarry Tile (25), Rug-B (19).

**Fig. 4.** CUReT samples: normalized data against the normalized radiance predicted by the modified Gaussian model. Samples from top to bottom are: Sponge (21), Brown Bread (48), Quarry Tile (25), Rug-B (19).