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2-D DOA Estimation for L-shaped Array with Array Aperture and Snapshots Extension Techniques

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***Abstract*—A two dimensional (2-D) direction of arrival (DOA) estimation method for L-shaped array with automatic pairing is proposed. It exploits the conjugate symmetry property of the array manifold matrix to increase the effective array aperture and the number of virtual snapshots simultaneously, and then applies the principle of MUSIC to construct an angle cost function and transforms the conventional 2-D search into 1-D via a Rayleigh quotient, which can greatly reduce the computation complexity. Finally, the azimuth and elevation angles are estimated without pair matching. Simulation results show that the proposed method has a better performance and can resolve more sources than some existing computationally efficient methods.**

***Index Terms*—two dimensional, direction of arrival estimation, L-shaped array, Rayleigh quotient, pair matching.**

I. Introduction

Two dimensional (2-D) direction of arrival (DOA) estimation is a basic problem in array signal processing and has wide applications in wireless communications, radar, sonar, etc [1]. For 2-D DOA estimation, many geometrical structures have been developed, such as L-shaped array, circular array, parallel linear arrays, rectangular array, etc [1]–[11]. In particular, due to its simplicity and effectiveness, the L-shaped array has attracted a lot of attention in the past, based on which many computationally efficient algorithms have been proposed [12]–[25]. These algorithms can be divided into two classes. One is to estimate the angles corresponding to each uniform linear subarray via applying 1-D DOA estimation algorithms to the received data or reconstructed data of each subarray [12], [13], [18]–[22], [25]. However, additional angle pairing is needed, which may not work for some special cases and affect the overall performance [14]. To avoid this problem, the second class of algorithms can pair the angles automatically, such as the joint SVD [14], parallel factor analysis [15], and the effective array aperture extension method [23].

However, neither of them works when the number of sources is larger than the number of elements of each uniform linear subarray: to estimate angles of K sources, the total number of array elements of the L-shaped array M_{total} should satisfy $M_{total} > 2K$. Although the maximum identifiable source number of the maximum likelihood (ML) method [11] and the 2D-MUSIC method [26] can overcome this limit, they require multidimensional spectrum peak search, which are too high in computational cost for many real-time applications.

In this work, we aim to increase the maximum number of resolvable sources with automatic pairing for L-shaped arrays, while avoiding the multi-dimensional search. First, by utilizing the conjugate symmetry property of the uniform linear array (ULA) manifold matrix, a large array-received-data-like (LARDL) matrix is constructed, which is similar to the way followed in [23]; however, different from [23], we also increase the number of virtual snapshots of the LARDL matrix, which is crucial to increase the maximum number of resolvable sources; then, we apply the 2-D MUSIC principle and obtain an unconstrained 2-D optimization problem. To solve the problem without 2-D search, we transform it to a Rayleigh quotient form by adding a constraint. Finally, the azimuth angles are estimated with 1-D search and the elevation angles are estimated via the special structure of the resultant eigenvectors. Simulation results show that the proposed method yields better results than the classic JSVD [14], PARAFAC [15], CODE [18], CESA [20], EAET [23], and AAEA [25]. Furthermore, the maximum number of identifiable sources by the proposed method is $M_{total} - 2$.

Notations: Matrices and vectors are denoted by boldfaced capital letters and lower-case letters, respectively. $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ stand for conjugate, transpose, and conjugate transpose, respectively. $E\{\cdot\}$, $\otimes$, $\mathbf{I}_D$, $\mathbf{J}_D$, $\mathbf{0}_{m\times n}$, $\mathrm{diag}\{\cdot\}$, and $\mathrm{angle}(\cdot)$ denote the statistical expectation, Kronecker product, a $D\times D$ identity matrix, a $D\times D$ exchange matrix with ones on its antidiagonal and zeros elsewhere, an $m\times n$ zero matrix, diagonalization and phase angle operator for complex number, respectively. $\mathbf{D}(:,p:q)$ represents a submatrix consisting of the pth to the qth columns of matrix $\mathbf{D}$.

II. Signal Model

As shown in Fig.1, an L-shaped array consists of two orthogonal M-element uniform linear arrays with inter-sensor spacing d along x and z axes, respectively. K narrowband far-field uncorrelated signals $\{s_k(n)\}_{k=1}^K$ ($n=1,\cdots,N$, N is the number of snapshots) of wavelength λ impinge from distinct directions with azimuth and elevation angles $\{(\theta_k,\phi_k)\}_{k=1}^K$ (note that here the azimuth angle definition is different from the traditional one). Therefore, we can express the array

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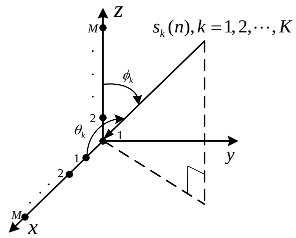

Fig. 1. L-shaped array configuration for 2-D DOA estimation.

manifold matrices of x and z subarrays as

$$\mathbf{A}_x(\boldsymbol{\theta})= [\mathbf{a}_x(\theta_1), \mathbf{a}_x(\theta_2), \cdots, \mathbf{a}_x(\theta_K)], \tag{1}$$

$$\mathbf{A}_z(\boldsymbol{\phi})= [\mathbf{a}_z(\phi_1), \mathbf{a}_z(\phi_2), \cdots, \mathbf{a}_z(\phi_K)], \tag{2}$$

where

$$\begin{aligned}\mathbf{a}_x(\theta_k)&=[a_{x,1}(\theta_k), a_{x,2}(\theta_k), \cdots, a_{x,M}(\theta_k)]^T\\ &=[e^{j2\pi\cos\theta_k d/\lambda}, \cdots, e^{j2\pi M\cos\theta_k d/\lambda}]^T,\\ \mathbf{a}_z(\phi_k)&=[a_{z,1}(\phi_k), a_{z,2}(\phi_k), \cdots, a_{z,M}(\phi_k)]^T\\ &=[1, e^{j2\pi\cos\phi_k d/\lambda}, \cdots, e^{j2\pi(M-1)\cos\phi_k d/\lambda}]^T.\end{aligned}$$

Hence, the received signal of x and z subarrays at the nth snapshot $\mathbf{x}(n)$ and $\mathbf{z}(n)$ can be represented by

$$\mathbf{x}(n)= \mathbf{A}_x(\boldsymbol{\theta})\mathbf{s}(n) + \mathbf{w}_x(n), \tag{3}$$

$$\mathbf{z}(n)= \mathbf{A}_z(\boldsymbol{\phi})\mathbf{s}(n) + \mathbf{w}_z(n), \tag{4}$$

where $\mathbf{x}(n) = [x_1(n), \cdots, x_M(n)]^T$, $\mathbf{z}(n) = [z_1(n), \cdots, z_M(n)]^T$, $\mathbf{s}(n) = [s_1(n), \cdots, s_K(n)]^T$ represents the source signal vector, and $\mathbf{w}_x(n) = [w_{x,1}(n), \cdots, w_{x,M}(n)]^T$ and $\mathbf{w}_z(n) = [w_{z,1}(n), \cdots, w_{z,M}(n)]^T$ denote the additive noise vectors corresponding to the x and z subarrays, respectively. Similar to [23], it is assumed that the additive noises are temporally and spatially white with zero-mean and variance σ_w^2, and are uncorrelated with the incident signals.

III. Proposed Method

A. Array Aperture and Snapshots Extension

According to [23], [27], [28], the manifold matrix for an M-element ULA is a Vandermonde matrix and possesses the conjugate symmetry property, i.e.,

$$\mathbf{J}_M(\mathbf{A}_x(\boldsymbol{\theta}))^* = \mathbf{A}_x(\boldsymbol{\theta})\tilde{\boldsymbol{\Phi}}_{xr}(\boldsymbol{\theta}), \tag{5}$$

$$\mathbf{J}_M(\mathbf{A}_z(\boldsymbol{\phi}))^* = \mathbf{A}_z(\boldsymbol{\phi})\tilde{\boldsymbol{\Phi}}_{zr}(\boldsymbol{\phi}), \tag{6}$$

where

$$\begin{aligned}\tilde{\boldsymbol{\Phi}}_{xr}(\boldsymbol{\theta})&=\mathrm{diag}\{e^{-j2\pi(M-1)\cos\theta_1 d/\lambda},\\ &\qquad \cdots, e^{-j2\pi(M-1)\cos\theta_K d/\lambda}\},\\ \tilde{\boldsymbol{\Phi}}_{zr}(\boldsymbol{\phi})&=\mathrm{diag}\{e^{-j2\pi(M-1)\cos\phi_1 d/\lambda},\\ &\qquad \cdots, e^{-j2\pi(M-1)\cos\phi_K d/\lambda}\}.\end{aligned}$$

We can see that the effective array aperture of a ULA can be increased via the conjugate operation. This is an effective technique and can improve the estimation performance significantly [23].

However, although the EAET method in [23] only increases the array aperture, the number of virtual snapshots in the LARDL matrix (viz. Eq.(16) in [23]) remains $M-1$, which limits the maximum number of resolvable sources K_{max} to $K_{max} \leq (M-1)$. To increase the array aperture and the virtual snapshots simultaneously, we first construct the cross-correlation matrix $\mathbf{R}_{xz}$ as follows,

$$\mathbf{R}_{xz} = E\{\mathbf{x}(n)\mathbf{z}^H(n)\} = \mathbf{A}_x(\boldsymbol{\theta})\mathbf{R}_{ss}\mathbf{A}_z^H(\boldsymbol{\phi}), \tag{7}$$

where $\mathbf{R}_{ss} = \mathrm{diag}\{p_1, \cdots, p_K\}$ and $\{p_k\}_{k=1}^K$ represent the signal power set. With the assumptions made in Section II, this procedure can also reduce the effect of noise.

Similar to [14] and [23], $\mathbf{R}_{xz}$ can be divided into two $M\times(M-1)$ matrices as follows,

$$\mathbf{Y}_1{=}\mathbf{R}_{xz}(:, 1:M-1) = \mathbf{A}_x(\boldsymbol{\theta})\mathbf{R}_{ss}\mathbf{A}_{z1}^H(\boldsymbol{\phi}), \tag{8}$$

$$\mathbf{Y}_2{=}\mathbf{R}_{xz}(:, 2:M) = \mathbf{A}_x(\boldsymbol{\theta})\mathbf{R}_{ss}\mathbf{A}_{z2}^H(\boldsymbol{\phi}), \tag{9}$$

where $\mathbf{A}_{z1}(\boldsymbol{\phi})$ and $\mathbf{A}_{z2}(\boldsymbol{\phi})$ stand for the first and the last $(M-1)$ rows of $\mathbf{A}_z(\boldsymbol{\phi})$, $\mathbf{A}_{z2}(\boldsymbol{\phi}) = \mathbf{A}_{z1}(\boldsymbol{\phi})\boldsymbol{\Phi}_z(\boldsymbol{\phi})$ and $\boldsymbol{\Phi}_z(\boldsymbol{\phi}) = \mathrm{diag}\{e^{j2\pi\cos\phi_1 d/\lambda}, \cdots, e^{j2\pi\cos\phi_K d/\lambda}\}$.

Considering (5) and using (8)-(9), we can construct a new LARDL matrix as follows,

$$\mathbf{Y} = \begin{bmatrix}\mathbf{Y}_1, \mathbf{J}_M\mathbf{Y}_2^*\\ \mathbf{Y}_2, \mathbf{J}_M\mathbf{Y}_1^*\end{bmatrix} = \mathbf{A}_g(\boldsymbol{\theta},\boldsymbol{\phi})\mathbf{S}_g(\boldsymbol{\theta},\boldsymbol{\phi}), \tag{10}$$

where

$$\mathbf{A}_g(\boldsymbol{\theta},\boldsymbol{\phi}) = \left[\mathbf{A}_x^T(\boldsymbol{\theta}), (\mathbf{A}_x(\boldsymbol{\theta})\boldsymbol{\Phi}_z^*(\boldsymbol{\phi}))^T\right]^T,$$

$$\mathbf{S}_g(\boldsymbol{\theta},\boldsymbol{\phi}) = \left[\mathbf{R}_{ss}\mathbf{A}_{z1}^H(\boldsymbol{\phi}), \boldsymbol{\Phi}_z(\boldsymbol{\phi})\tilde{\boldsymbol{\Phi}}_{xr}(\boldsymbol{\theta})\mathbf{R}_{ss}\mathbf{A}_{z1}^T(\boldsymbol{\phi})\right].$$

In this way, both the array aperture and the number of virtual snapshots have been increased.

Note that the PM-ESPRIT method in [23] cannot be used for (10), as it requires two $(M-1)$ array data sets to construct the matrix consisting of the azimuth rotation invariance factor. However, both the 2-D MUSIC and ML method can be applied to (10) to obtain 2-D DOA estimations with a multi-dimensional search. Next, we develop a novel estimation method to avoid the multidimensional search.

Remark 1: The dimensions of $\mathbf{A}_g(\boldsymbol{\theta},\boldsymbol{\phi})$ and $\mathbf{S}_g(\boldsymbol{\theta},\boldsymbol{\phi})$ are $2M\times K$ and $K\times 2(M-1)$, respectively, which results in the case that the number of array elements is larger than the number of snapshots. According to the subspace theory, the maximum number of identifiable sources cannot exceed $\max\{2M-1, 2(M-1)\}$, i.e., $2(M-1)$.

B. 2-D DOA Estimation Based on Rayleigh Quotient

Applying eigenvalue decomposition (EVD) to $\mathbf{R}_{yy} = \mathbf{Y}\mathbf{Y}^H$, we have

$$\mathbf{R}_{yy} = \mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_w\boldsymbol{\Lambda}_w\mathbf{U}_w^H, \tag{11}$$

where $\mathbf{U}_s$ and $\mathbf{U}_w$ represent the signal subspace and the noise subspace, respectively.[1] We can minimize the following cost function with 2-D search and obtain the angle estimation results, i.e.,

$$\{(\hat{\theta}_k, \hat{\phi}_k)\}_{k=1}^K = \arg\min_{\theta,\phi} f(\theta,\phi), \tag{12}$$

[1]For the non-ideal case with finite number of snapshots, the noise effect cannot be mitigated fully via the cross-correlation in (7), which will result in additional residual noise terms in (8)-(10), and the following (11).

where $f(\theta,\phi) = \mathbf{a}_g^H(\theta,\phi)\mathbf{U}_w\mathbf{U}_w^H\mathbf{a}_g(\theta,\phi)$, $\mathbf{a}_g(\theta,\phi) = [\mathbf{a}_x^T(\theta), e^{-j2\pi d cos\phi/\lambda}\mathbf{a}_x^T(\theta)]^T$. It is noticed that

$$\begin{aligned}\mathbf{a}_g(\theta,\phi) &= [1, e^{-j2\pi d cos\phi/\lambda}]^T \otimes \mathbf{a}_x(\theta) = \mathbf{q}(\phi)\otimes\mathbf{a}_x(\theta)\\ &= (\mathbf{I}_2\cdot\mathbf{q}(\phi))\otimes(\mathbf{a}_x(\theta)\cdot 1) = (\mathbf{I}_2\otimes\mathbf{a}_x(\theta))\,\mathbf{q}(\phi).\end{aligned} \tag{13}$$

Hence, we can rewrite the cost function in (12) as

$$f(\theta,\phi) = \mathbf{q}^H(\phi)\mathbf{F}(\theta)\mathbf{q}(\phi), \tag{14}$$

where $\mathbf{F}(\theta) = (\mathbf{I}_2\otimes\mathbf{a}_x(\theta))^H\mathbf{U}_w\mathbf{U}_w^H(\mathbf{I}_2\otimes\mathbf{a}_x(\theta))$. As a result, θ and ϕ can be separated, and hence the unconstrained 2-D optimization problem in (12) can be transformed into a 1-D optimization problem by adding a specific constraint $\mathbf{q}^H(\phi)\mathbf{q}(\phi) = 2$ and then solving the following problem,

$$\{(\hat{\theta}_k,\hat{\phi}_k)\}_{k=1}^K = \arg\min_{\theta,\phi}\frac{\mathbf{q}^H(\phi)\mathbf{F}(\theta)\mathbf{q}(\phi)}{\mathbf{q}^H(\phi)\mathbf{q}(\phi)}. \tag{15}$$

$\mathbf{F}(\theta)$ is a positive semi-definite Hermitian matrix, and (15) is a Rayleigh quotient problem. The solution to (15) is [2]

$$\{\hat{\theta}_k\}_{k=1}^K = \arg\min_{\theta,\phi}\frac{\mathbf{q}^H(\phi)\mathbf{F}(\theta)\mathbf{q}(\phi)}{\mathbf{q}^H(\phi)\mathbf{q}(\phi)} = \arg\min_{\theta}\lambda_{\min}(\mathbf{F}(\theta)), \tag{16}$$

where $\lambda_{\min}(\mathbf{F}(\theta))$ denotes the minimal eigenvalue of $\mathbf{F}(\theta)$. Then, we can estimate $\{\hat{\theta}_k\}_{k=1}^K$ via minimizing the minimal eigenvalue of $\mathbf{F}(\theta)$, and the elevation angle $\{\hat{\phi}_k\}_{k=1}^K$ can be estimated via the eigenvector corresponding to the minimum eigenvalue of $\mathbf{F}(\hat{\theta}_k)$,

$$\hat{\phi}_k = \arccos\left\{\text{angle}\left[-\frac{\mathbf{e}_{\min}^1(\mathbf{F}(\hat{\theta}_k))}{\mathbf{e}_{\min}^2(\mathbf{F}(\hat{\theta}_k))}\right]\frac{\lambda}{2\pi d}\right\}, \tag{17}$$

where $\mathbf{e}_{\min}^1(\mathbf{F}(\hat{\theta}_k))$ and $\mathbf{e}_{\min}^2(\mathbf{F}(\hat{\theta}_k))$ represent the first and second elements of the eigenvector corresponding to $\lambda_{\min}(\mathbf{F}(\hat{\theta}_k))$. Therefore, the elevation angles are obtained and paired with the azimuth angles automatically.

$Remark$ 2: To solve (16), a large number of computationally expensive EVD operations are required. When the search angle is equal to the true azimuth, the minimal eigenvalue of $\mathbf{F}(\theta)$ is close to 0. By the relationship between eigenvalue and determinant, (16) can be rewritten as

$$\hat{\theta}_k = \arg\max_{\theta} 1/\det(\mathbf{F}(\theta)). \tag{18}$$

$Remark$ 3: We can see that the azimuth and elevation estimations of the proposed method are different. Since the azimuth estimation benefits from $2M$-element data, while the elevation estimation only benefits from M-element data, the azimuth estimation performance is better than that of the elevation. However, the estimation performance can be reversed when $\mathbf{R}_{zx} = E\{\mathbf{z}(n)\mathbf{x}^H(n)\}$ is constructed.

[2]When the search angle θ is equal to any of K azimuth angles, $\lambda_{\min}(\mathbf{F}(\theta)) = 0$. That is, the single minimization problem (16) has K different solutions. For numerical computations, with the effect of noise and a finite number of snapshots, we choose K different $\mathbf{F}(\theta)$ whose minimum eigenvalues are close to zero and their corresponding search angles are then the K estimates of the true azimuth angles. For details, please refer to the online supplementary material of this letter.

C. Algorithm Analysis

According to *Remark 1* in Sec. III-A and the fact that the proposed Rayleigh quotient based 2-D DOA estimation method is just a dimension-reduction version for the 2-D MUSIC method, the maximum number of identifiable sources is $2(M-1)$, and it can estimate the azimuth and elevation angles simultaneously without pair matching.

In the following, we provide the computational complexity of the proposed method in comparison with existing ones in terms of the number of complex-valued multiplications as follows,

$$\begin{aligned}C_{\text{JSVD}} =& O\{M^2N + 2M(M-1)^2 + 2MK^2 + K^3\\ &+ N_sM^2K\},\end{aligned} \tag{19}$$

$$\begin{aligned}C_{\text{PARAFAC}} =& O\{4(M-1)^2N + N_{it}[(M-1)^2K^2 + 8(M\\ &-1)K + 13(M-1)^2K + 8(M-1)K^2]\},\end{aligned} \tag{20}$$

$$\begin{aligned}C_{\text{CODE}} =& O\{4M^2N + 2[2M^3 + 2MK^2 + K^3] + 2M^{\eta}\\ &+ 4MK^2 + 12M^3 + 4M^2K^2\},\end{aligned} \tag{21}$$

$$\begin{aligned}C_{\text{CESA}} =& O\{M^2N + 2(2M-K)K^2 + 3(2M-K)^2K\\ &+ 4N_s(2M-K)^2 + (2M-K)^2K^2\},\end{aligned} \tag{22}$$

$$\begin{aligned}C_{\text{EAET}} =& O\{M^2N + 8MK^2 + 4M(4M-K)K\\ &+ 8(M-1)K^2\},\end{aligned} \tag{23}$$

$$\begin{aligned}C_{\text{AAEA}} =& O\{M^2N + 9M^3 + 6M^2K + 2MK^3 + 18MK^2\\ &+ 3K^3\},\end{aligned} \tag{24}$$

$$\begin{aligned}C_{\text{Propose}} =& O\{\underbrace{M^2N}_{(7)} + \underbrace{16M^3}_{(11)} +\\ &\underbrace{N_s[16M^2 + 2 + 4M^2(2M-K)]}_{(18)}\},\end{aligned} \tag{25}$$

where N_s and N_{it} represent the total number of searches and the total number of iterations for parallel factor analysis. Since $N_s \gg M$, $N_s \gg K$, and $M > K$, the computation complexity of the proposed method is higher than that of JSVD, CODE, CESA and AAEA. Similar to [23], for the PARAFAC method, N_{it} largely depends on the received data and varies from 10 to 100 or even larger, and it is difficult to compare the proposed method with the PARAFAC method directly.

$Remark$ 4: For CODE, $\eta \gg 2$ and it is a relatively large integer for known polynomial rooting algorithms [29]. We will see later that the CODE method may not be computationally efficient for very large M's. However, its complexity is still lower than the ML and 2-D MUSIC methods for reasonable and large M's.

IV. Simulation Results

In this section, we compare the performance of the proposed method with JSVD [14], PARAFAC [15], CODE [18], CESA [20], EAET [23], AAEA [25], and the Cramer-Rao bound (CRB) [30], [31]. All the algorithms are implemented in MATLAB R2013b using a PC with Intel(R) Core(TM) i5-2320 CPU @3.00GHz and 4G RAM. It is assumed that $d = \lambda/2$, and all sources have the same power σ_s^2. We set the search ranges for azimuth and elevation as $[0°, 180°]$ with an interval of $0.1°$.

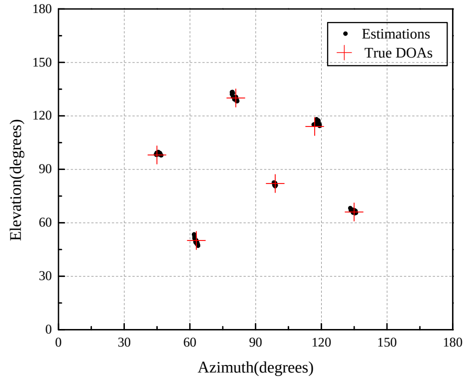

Fig. 2. Estimation results of 6 signals, M = 4, SNR = 10 dB, N = 500.

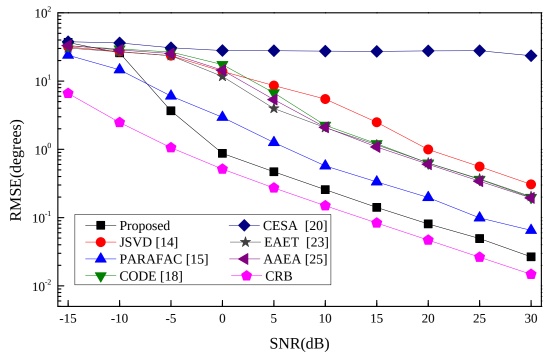

Fig. 3. RMSE versus SNR, K = 3, M = 4, N = 500.

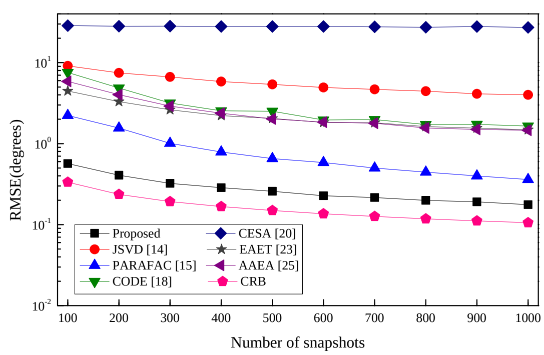

Fig. 4. RMSE versus number of snapshots, K = 3, M = 4, SNR = 10 dB.

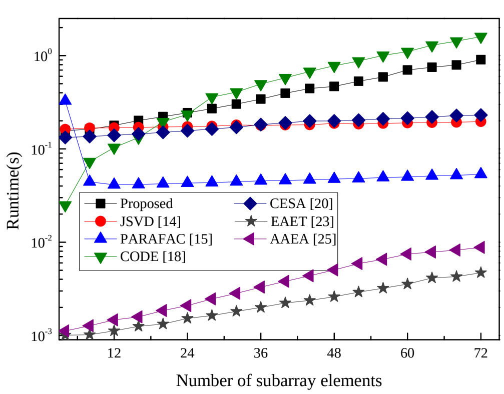

Fig. 5. Runtime versus number of subarray elements, K = 3, N = 500, SNR = 10 dB.

Example 1: The number of subarray elements M, the number of snapshots N, and signal to noise ratio (SNR) are fixed at 4, 500 and 10 dB, respectively. There are $2(M-1)=6$ uncorrelated signals. The results are obtained via 500 Monte Carlo trials, shown in Fig. 2. We can see that as expected, the proposed method can handle the 6 sources effectively. This distinct ability is a significant advantage over the other six methods and of great value for many practical applications.

Example 2: In this example, the performance of the proposed method with respect to SNR is investigated. Azimuth and elevation angles of 3 sources are set to $(85°, 76°)$, $(105°, 85°)$, and $(130°, 67°)$. With $M = 4$ and $N = 500$, the input SNR varies from -15 dB to 30 dB with an interval of 5 dB. For each fixed SNR, 500 Monte Carlo trials are conducted. The RMSE results are shown in Fig. 3. As shown, for SNR ≥ -5 dB, the root-mean-square-error (RMSE) using the proposed method decreases as SNR increases, and the RMSE curve of the proposed method is the lowest among all considered methods, achieving the best performance. However, for SNR ≤ -10 dB, none of the methods works due to a very large estimation error.

Example 3: In this example, we examine the performance of the proposed method against the number of snapshots. The simulation conditions are the same as above except that SNR = 10 dB and N ranges from 100 to 1000 with an interval 100. The results are shown in Fig. 4, where we can see that the increase of the number of snapshots has improved the estimation result of the proposed method, which is again the best among all the examined methods.

Example 4: Now, the running time with respect to the number of subarray elements is presented. The conditions are similar to those of Example 3 except that $N = 500$ and M varies from 4 to 72. The CPU running time is shown in Fig. 5. We can see that the proposed method has the second highest running time for large M's. However, the running time of the proposed method and that of the CODE method are very similar for $M \geq 24$. Another observation is that the running time of the PARAFAC method is very large for small M's, which results from a large number of required iterations. Overall, we can say that the proposed method is computationally comparable with the existing efficient methods.

V. Conclusions

A 2-D DOA estimation method for L-shaped arrays with automatic pairing has been introduced. The proposed method utilizes the conjugate symmetry property of the ULA manifold matrix to increase the effective array aperture and virtual snapshots number simultaneously, capable of handling $2(M-1)$ sources, with M being the number of elements of each subarray. To reduce computational complexity, a 1-D search method was then derived through a Rayleigh quotient formulation, which pairs the azimuth and elevation angles automatically. As demonstrated by simulation results, the proposed method has achieved a better performance than many existing algorithms, with a comparable computational complexity. However, the proposed method cannot deal with the angle ambiguity problem described in [32], and further research is needed.

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